ještě k derivacím

$$(f_1(x)\cdot f_2(x)\cdot \ldots \cdot f_n(x))' = f_1'(x)\cdot f_2(x)\cdot \ldots \cdot f_n(x) + f_1(x)\cdot f_2'(x)\cdot f_3(x)\cdot \ldots \cdot f_n(x) + \ldots + f_1(x)\cdot \ldots \cdot f_{n-1}(x)\cdot f_n'(x)$$

$$\left(\frac{f}{g}\right)' = \left(f\cdot\frac{1}{g}\right)' = \left(f\cdot (g)^{-1}\right)' = f'\cdot (g)^{-1} + f\cdot(-1)\cdot(g)^{-2}\cdot g' = \frac{f'\cdot g - f\cdot g'}{g^2}$$

$\cdot\ f = \operatorname{arctg}(x^2)$ $\quad f'(1) = ?$ $\quad f''(x) = ?$ $\quad f^{(6)}$ $\quad f'''(x),\ f^{IV},\ f^{V}$

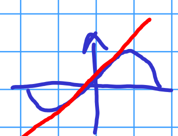
$$f'(x) = \frac{1}{(x^2)^2+1}\cdot 2x = \boxed{\frac{2x}{x^4+1}} \qquad f'(1) = \frac{2\cdot 1}{1^4+1} = 2$$

$$f''(x) = (f'(x))' = \frac{2(x^4+1) - 2x\cdot(4x^3+0)}{(x^4+1)^2} = \frac{2(x^4+1) - 8x^4}{(x^4+1)^2} = \frac{1-6x^4}{(x^4+1)^2}$$

L'Hospitalovo pravidlo

$\lim\limits_{x\to x_0} \frac{f(x)}{g(x)}$ $\frac{\infty}{\infty}$ $(\pm\infty)$ pokud $\lim\limits_{x\to x_0} |f(x)| = \lim\limits_{x\to x_0} |g(x)| = 0$ nebo ∞ $\quad \lim\limits_{x\to 0} \frac{\operatorname{tg}\left(x+\frac{\pi}{2}\right)}{\frac{1}{x}}$ $\frac{\pm\infty}{\pm\infty}$

a $\lim\limits_{x\to x_0} \frac{f'(x)}{g'(x)}$ existuje, pak $\lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$



$\frac{(\sin x)'}{(x)'} = \frac{\cos x}{1} = 1$

Jde tedy o to limity typu $0\cdot\infty,\ 1^\infty,\ 0^0,\ \frac{1}{0}-\frac{1}{0'},\ \infty-\infty'$ dostat do tvaru $\frac{0}{0}$ nebo $\frac{\pm\infty}{\pm\infty}$.

- typ $\overset{f}{0}\cdot\overset{g}{\infty} \to \frac{f\,0}{\frac{1}{g}\,\frac{1}{\infty}} = \boxed{\frac{0}{0}}$ $\quad$ přip. $\frac{\infty}{\frac{1}{0}} = \frac{\pm\infty}{\pm\infty}$ $\quad$ $0\cdot\ln 0 \to$ typ $0\cdot\infty$
- typ 1^∞ (0^0) $\to e^{\ln 1^\infty}$ (0^0) $= e^{\infty\cdot\ln 1} \to$ typ $\infty\cdot 0$
- typ $\frac{1}{0}-\frac{1}{0'} \to \frac{0'-0}{0\cdot 0'} = \boxed{\frac{0}{0}}$
- typ $\infty-\infty' \to \frac{1}{\frac{1}{\infty}} - \frac{1}{\frac{1}{\infty'}} \to \frac{\frac{1}{\infty'}-\frac{1}{\infty}}{\frac{1}{\infty\cdot\infty'}} \to$ typ $\frac{0}{0}$

různá chytrá úprava

(- typ $0^\infty = 0$)

$\bullet \lim_{x\to 0^+} \underbrace{\ln x}_{}\cdot \sin x = |\text{typ } (-\infty)\cdot 0| = \lim_{x\to 0} \frac{\ln x}{\frac{1}{\sin x}} = |\text{typ } \frac{-\infty}{\infty}|$

čitatel

$\stackrel{L'H}{=} \lim_{x\to 0} \frac{\frac{1}{x}}{\frac{-\cos x}{\sin^2 x}} = \lim_{x\to 0} \frac{\sin x \cdot \frac{\sin x}{x}}{-\cos x} = \frac{0\cdot 1}{-1} = 0$

nutné napsat

$\bullet \lim_{x\to 0} \frac{1}{\sin x} - \frac{1}{\ln(x+1)} = |\text{typ } \frac{1}{0} - \frac{1}{0}'| = \lim \frac{\ln(x+1) - \sin x}{\sin x \,\ln(x+1)}$ $|\frac{0}{0}|$

$\infty - \infty'$

zde se využije L'H $2x$

$\bullet \lim_{x\to\infty} \sqrt{x^2+x} - \sqrt{x^2+1} = |\text{typ } \infty - \infty'| =$

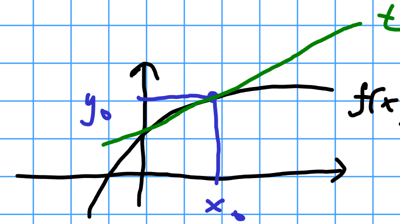
$= \lim_{x\to\infty} \frac{\frac{1}{\sqrt{x+1}} - \frac{1}{\sqrt{x^2+x}}}{\frac{1}{\sqrt{x^2+x}\,\sqrt{x+1}}} = |\text{typ } \frac{0}{0}| = \lim_{x\to\infty} \frac{\left(\frac{1}{\sqrt{x+1}} - \frac{1}{\sqrt{x^2+x}}\right)'}{\left(\frac{1}{\sqrt{x^2+x}\,\sqrt{x+1}}\right)'}$

(chytřeji :) $= \lim_{x\to\infty} x\left(\sqrt{1+\frac{1}{x}} - \sqrt{1+\frac{1}{x^2}}\right) = |\text{typ } \infty\cdot 0|$

(nebo jak minule rozšíříme $\frac{\sqrt{\cdots} + \sqrt{\cdots}}{-\,''\,-}$)

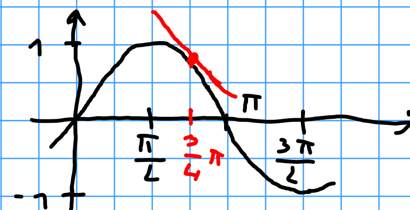
Rovnice tečny

tečna k $f(x)$ v bodě x_0



$t: \underbrace{y - y_0}_{f(x_0)} = \underbrace{k}_{f'(x_0)} \cdot (x - x_0)$ ✗

$\Rightarrow$ $\boxed{y = f(x_0) + f'(x_0) \cdot (x - x_0)}$

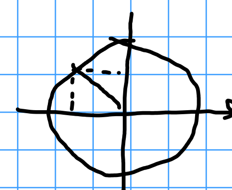


- tečna k $f(x) = \sin x$ v bodě $\frac{3}{4}\pi$

$f'(x)\ (\sin x)' = \cos x$

$f\left(\frac{3}{4}\pi\right) = \sin \frac{3}{4}\pi = \frac{-\sqrt{2}}{2}$

$f'\left(\frac{3}{4}\pi\right) = \cos \frac{3}{4}\pi = \frac{-\sqrt{2}}{2}$

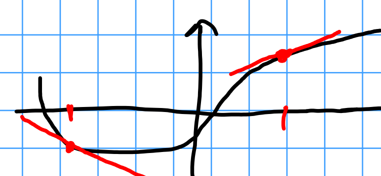


$t: y = \frac{-\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{3}{4}\pi\right)$

Průběh funkce

$f'(x_0) > 0$ ... rostoucí ↗

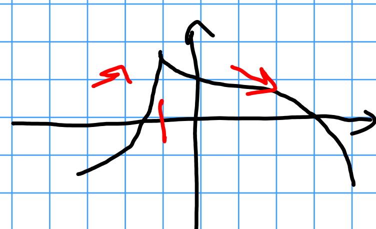
$f'(x_0) < 0$ ... klesající ↘

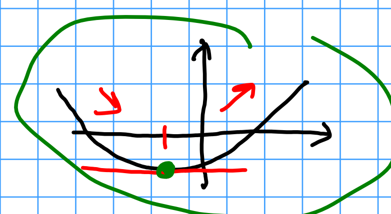


($\pm\infty$ se neuvažuje)

lok. max. / min

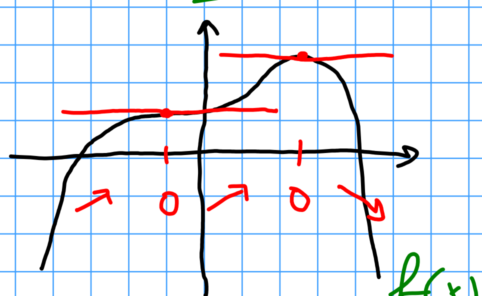
↗ | ↘ / ↘ | ↗





pokud v bodě x_0 lok. extrému existuje derivace,

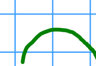
pak $f'(x_0) = 0$ ... tzv. stacionární bod → nejsme si extrémem jisti



$\Rightarrow$ musíme potvrdit a určit jaký

a) šipkami ↗ 0 ↘ (max)

b) $f''(x_0) = \begin{cases} > 0 & \text{min} \\ = 0 & \text{nevíme} \\ < 0 & \text{max} \end{cases}$



- $f(x) = \sqrt{(x^3-1)^2+1} = \sqrt{x^6-2x^3+2} = (x^6-2x^3+2)^{\frac{1}{2}}$ $\quad \boxed{D(f)=\mathbb{R}}$

$$f'(x) = \frac{1}{2}\,(x^6-2x^3+2)^{-1/2}\cdot(6x^5-6x^2) = \frac{6x^5-6x^2}{2\underbrace{\sqrt{x^6-2x^3+2}}_{\neq 0}}$$

rozhoduje o znaménku

$y = x^3$, $\sqrt{y^2-2y+2} \geq 0$, $\neq$

$D = 4-4\cdot 2 < 0$

$$6x^5-6x^2 = 6x^2(x^3-1)$$

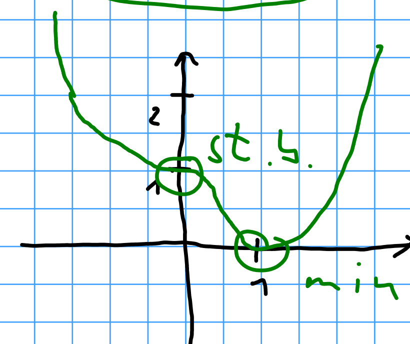
nulové body: 0, $\sqrt[3]{1}=1$

f $\oplus$

f': $\ominus$ $\;0\;$ $\ominus$ $\;0\;$ $\oplus$

-1 ↘ 0 ↘ 1 ↗

$$f'(-1) = 3\cdot\frac{(-1)^2((-1)^3-1)}{\sqrt{\dots}} < 0$$



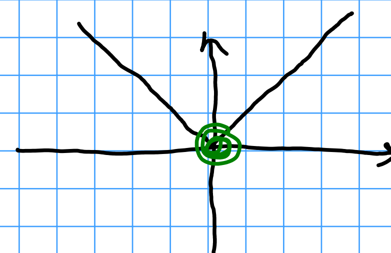
$f(0) =$

$f(1) =$

- $f(x) = |x|$ (absolutní hodnota)

$$= \begin{cases} -x & \text{pro } x<0 \\ 0 & x=0 \\ x & x>0 \end{cases}$$

$$f'(x) = \begin{cases} -1 & x<0 \\ \lim\limits_{h\to 0}\frac{|0+h|-0}{h} = \lim\limits_{h\to 0}\frac{|h|}{h} & \text{neex} \\ 1 & x>0 \end{cases}$$



$\boxed{f \geq 0}$

f': $-$ $\;0\;$ $+$

↘ 0 ↗

min

- $f(x) = \sqrt{\sin x - 1}$