$\bullet \int \operatorname{tg} x \, dx = -\int \frac{-\sin x}{\cos x} dx = -\ln|\cos x| + C$

Pravidlo $\int f(ax+b)\, dx = \frac{1}{a} F(ax+b) + C$, kde $\int f(x)\, dx = F(x) + C$

$\bullet \int \frac{A}{(x-x_0)^n} dx = A \int (x-x_0)^{-n} dx = A \cdot \frac{(x-x_0)^{-n+1}}{-n+1} + C$

$\bullet \int \frac{1}{(x-x_0)^2 + a^2} dx = \int \frac{1}{a^2} \frac{1}{\left(\frac{x-x_0}{a}\right)^2 + 1} dx = \frac{1}{a^{\cancel{2}}} \cdot \cancel{a} \operatorname{arctg}\left(\frac{x-x_0}{a}\right)$

$\frac{1}{a}x - \frac{x_0}{a}$

$\bullet \int \frac{5x-3}{x^2+2x+3} dx \; {}_{D<0} = \int \frac{\frac{5}{2}\left(2x - \frac{2\cdot 3}{5}\right)}{x^2+2x+3} dx = \frac{5}{2}\left(\int \frac{2x+2}{x^2+2x+3} dx + \int \frac{-2-\frac{6}{5}}{x^2+2x+3} dx\right)$

$= \frac{5}{2} \ln|x^2+2x+3| + \int \frac{-5-3}{x^2+2x+3} dx = \left| \begin{array}{l} x^2+2x+3 = \\ = (x+1)^2 + 2 \end{array} \right|$

$= \frac{5}{2} \ln|x^2+2x+3| - 8 \int \frac{1}{(x+1)^2+2} dx$ (proto $D<0$)

$= \frac{5}{2} \ln|x^2+2x+3| - \frac{8}{2} \cdot \sqrt{2} \operatorname{arctg}\left(\frac{x+1}{\sqrt{2}}\right) + C$

Pravidlo per-partes

$(u(x) \cdot v(x))' = u'(x) \cdot v(x) + u(x) \cdot v'(x)$

$\int u(x) v'(x) \, dx = u(x) v(x) - \int u'(x) \cdot v(x) \, dx$

$\bullet \int x \ln x \, dx = \left| \begin{array}{ll} u = \ln x & u' = \frac{1}{x} \\ v' = x & v = \frac{x^2}{2} \end{array} \right| = \ln x \cdot \frac{x^2}{2} - \int \frac{1}{\cancel{x}} \cdot \frac{x^{\cancel{2}}}{2} dx$

$= \frac{x^2}{2} \ln x - \int \frac{x}{2} dx = \frac{x^2}{2} \ln x - \frac{x^2}{4} + c$

- $\int x e^x \, dx = \left| \begin{array}{ll} u = x & u' = 1 \\ v' = e^x & v = e^x \end{array} \right| = x e^x - \int e^x \, dx = x e^x - e^x + C$
$= e^x (x-1) + C$

- $\int x^2 \sin x \, dx = \left| \begin{array}{ll} u = x^2 & u' = 2x \\ v' = \sin x & v = -\cos x \end{array} \right| = -x^2 \cos x + \int 2x (\cancel{-} \cos x) \, dx$
$\overset{I-I}{=} -x^2 \cos x + 2 \left(x \sin x - \int \sin x \, dx \right)$
$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$

- $\int 4x \arctan x \, dx = \left| \begin{array}{ll} u = \arctan x & u' = \frac{1}{x^2+1} \\ v' = 4x & v = 2x^2 \end{array} \right| = 2x^2 \arctan x -$
$- \int \frac{\overset{2(x^2+1-1)}{2x^2}}{x^2+1} dx = 2x^2 \arctan x - 2 \int 1 - \frac{1}{x^2+1} dx = 2x^2 \arctan x - 2x + \arctan x + C$

- $\underbrace{\int e^x \sin x \, dx}_{=: I} = \left| \begin{array}{ll} u = e^x & u' = e^x \\ v' = \sin x & v = -\cos x \end{array} \right| = -e^x \cos x - \int (-e^x \cos x) \, dx$
$\overset{I-I}{=} -e^x \cos x + \left(e^x \sin x - \underbrace{\int e^x \sin x \, dx}_{= I} \right)$
$2I = e^x (\sin x - \cos x) \rightarrow I = \frac{e^x}{2} (\sin x - \cos x) + C$

- $\int 1 \cdot \arctan x \, dx = \left| \begin{array}{ll} u = \arctan x & u' = \frac{1}{x^2+1} \\ v' = 1 & v = x \end{array} \right| = x \arctan x - \frac{1}{2} \int \frac{2x}{x^2+1} dx$
$= x \arctan x - \frac{1}{2} \ln |x^2+1| + C$

Substituční metoda

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

$$\int f'(\varphi(x))\,\varphi'(x)\,dx = \left| \begin{array}{l} t = \varphi(x) \\ dt = \varphi'(x)\,dx \\ x = \varphi^{-1}(t) \\ dx = (\varphi^{-1}(t))'\,dt \end{array} \right| = \int f'(t)\,dt = F(t) + C = F(\varphi(x)) + C$$

Mnohdy se taky hodí udělat substituci naopak:

$$\left| \begin{array}{l} x = \psi(t) \\ dx = \psi'(t)\,dt \end{array} \right|$$

- $$\int \frac{\overbrace{\operatorname{arctg} x}^{t}}{x^2+1}\,\overbrace{dx}^{dt} = \left| \begin{array}{l} t = \operatorname{arctg} x \\ dt = \frac{1}{x^2+1}\,dx \end{array} \right| = \int \operatorname{arctg} x \cdot \frac{1}{x^2+1}\,dx$$

$$= \int t\,dt = \frac{t^2}{2} + C = \frac{1}{2}\operatorname{arctg}^2 x + C$$

- $$\int \frac{\sqrt{x} + \sqrt[3]{x^2}}{x + \sqrt{x}}\,dx = \left| \begin{array}{l} t = \sqrt[6]{x} \\ x = t^6 \\ dx = 6t^5\,dt \end{array} \right| = \int \frac{t^3 + t^4}{t^6 + t^3} \cdot 6t^5\,dt$$

$$= \int \frac{t^3(1+t)}{t^3(t^3+1)} \cdot 6t^5\,dt = 6\int \frac{t^5+t^6}{t^3+1}\,dt$$

$$\left| \begin{array}{l} A^3+B^3 = \\ (A+B)(A^2-AB+B^2) \end{array} \right| = 6\int \frac{t^5+t^6}{(t+1)\underbrace{(t^2-t+1)}_{D<0}}\,dt = (\text{parciální zlomky})$$

$\sin^a x \cdot \cos^b x$

- $$\int \sin^3 x\,dx = \left| \begin{array}{l} t = \cos x \\ dt = \sin x\,dx \end{array} \right| = \int 1 - t^2\,dt = t - \frac{t^3}{3} + C$$

$\sin^2 x \cdot \sin x$

$(1 - \underbrace{\cos^2 x}_{t^2}) \cdot \underbrace{\sin x\,dx}_{dt}$

$$= \cos x - \frac{\cos^3 x}{3} + C$$

• $\int \sin^2 x \cdot \cos^2 x \, dx = \left| \begin{array}{l} \cos^2 x = \frac{1+\cos 2x}{2} \\ \sin^2 x = \frac{1-\cos 2x}{2} \end{array} \right| = \int \frac{1}{4}(1+\cos 2x)(1-\cos 2x)\, dx$

$\left(\cos 2x = \underset{(1-\sin^2 x)}{\cos^2 x} - \underset{(1-\cos^2 x)}{\sin^2 x} = \begin{cases} 2\cos^2 x - 1 \\ 1 - 2\sin^2 x \end{cases} \right)$

$= \frac{1}{4} \int 1 - \cos^2(2x)\, dx = \frac{1}{4} x - \frac{1}{4}\int \cos^2(2x)\, dx =$

$= \frac{1}{4} x - \frac{1}{4}\int \left(\frac{1}{2} + \frac{\cos(4x)}{2}\right) dx = \frac{1}{4} x - \frac{1}{8} x - \frac{1}{8}\cdot\left(\frac{1}{4}(\sin 4x)\right)$

$= \frac{1}{8} x + \frac{1}{32} \sin(4x) + C$

• $\int \overset{t^3}{\sin^3 x} \cdot \overset{dt}{\cos x \, dx} = \left| \begin{array}{l} t = \sin x \\ dt = \cos x \, dx \end{array} \right| = \int t^3 \, dt = \frac{t^4}{4} + C$

$= \frac{\sin^4(x)}{4} + C$

Integrály typu $\int R(\sin x, \cos x)\, dx$

1) $R(-\sin x, \cos x) = -R(\sin x, \cos x) \rightarrow t = \cos x$

2) $R(\sin x, -\cos x) = -R(\sin x, \cos x) \rightarrow t = \sin x$

3) $R(-\sin x, -\cos x) = R(\sin x, \cos x) \rightarrow t = \operatorname{tg} x$

4) univerzální substituce

$t = \operatorname{tg} \frac{x}{2} \rightarrow x = 2 \operatorname{arctg} t, \quad dx = \frac{2}{1+t^2}\, dt$

$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2}$

• $\int \frac{\sin x \cdot \cos x}{\cos x + 1}\, dx \overset{1)}{=} \left| \begin{array}{l} t = \cos x \\ dt = -\sin x \, dx \end{array} \right| = -\int \frac{t+1-1}{t+1}\, dt = -\int 1 - \frac{1}{t+1}\, dt$

$= -t + \ln|t+1| + C = -\cos x + \ln|\cos x + 1| + C$

• $\int \frac{\sin x}{\sin x + \cos x}\, dx \overset{3)}{=} \left| \begin{array}{l} t = \operatorname{tg} x \\ dt = \frac{1}{\cos^2 x}\, dx \end{array} \right| = \int \frac{\sin x}{\cos x\left(\frac{\sin x}{\cos x} + 1\right)}\, dx$

$= \int \frac{\operatorname{tg} x}{\operatorname{tg} x + 1}\, dx = \left| \begin{array}{l} x = \operatorname{arctg} t \\ dx = \frac{1}{t^2+1}\, dt \end{array} \right| = \int \frac{t}{(t+1)(t^2+1)}\, dt = \text{parc. zl.} \ldots$

• $\int \frac{1}{\cos x}\,dx \overset{2)}{=} \left|\begin{matrix} t=\sin x\\ dt=\cos x\,dx\end{matrix}\right| = \int \frac{\cos x}{\underbrace{\cos^2 x}_{(1-\sin^2 x)}}\,dx = \int \frac{1}{1-t^2}\,dt$

$= \int \frac{1}{(1-t)(1+t)}\,dt = \int \frac{1/2}{1-t} + \frac{1/2}{1+t}\,dt = \frac{1}{2}\ln|1-t| + \frac{1}{2}\ln|1+t|$

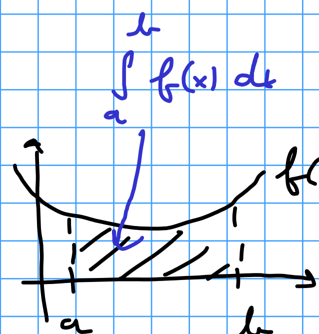
$= \frac{1}{2}\ln|1-\sin^2 x| + C$

• $\int \frac{\cos x - 1}{\sin x + 1}\,dx \overset{4)}{=} \left|\begin{matrix} \cos x = \frac{1-t^2}{1+t^2}\\ \sin x = \frac{2t}{1+t^2}\\ dx = \frac{2}{1+t^2}\,dt\end{matrix}\right| =$

$= \int \frac{\frac{1-t^2}{1+t^2}-1}{\frac{2t}{1+t^2}+1}\cdot\frac{2}{1+t^2}\,dt = \int \frac{\frac{1-t^2-(1+t^2)}{\cancel{1+t^2}}}{\frac{2t+1+t^2}{\cancel{1+t^2}}}\cdot\frac{2}{1+t^2}\,dt$

$= \int \frac{-4t^2}{(t+1)^2\cdot(1+t^2)}\,dt = \int \frac{A}{t+1} + \frac{B}{(t+1)^2} + \frac{Cx+D}{1+t^2}\,dt$

Určitý integrál



$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b) - F(a)$

• $\int_1^2 x^2 + \sqrt{x}\,dx = \left[\frac{x^3}{3} + \frac{x^{3/2}}{3/2}\right]_1^2 = \frac{8}{3} + \frac{2\sqrt{8}}{3} - \left(\frac{1}{3} + \frac{2}{3}\right)$

$= \frac{5+4\sqrt{2}}{3}$

• $\int_a^b u(x)\cdot v'(x)\,d = \left[u(x)\cdot v(x)\right]_a^b - \int_a^b u'(x)\cdot v(x)\,dx$