PV181 Laboratory of security and applied cryptography

Asymmetric cryptography

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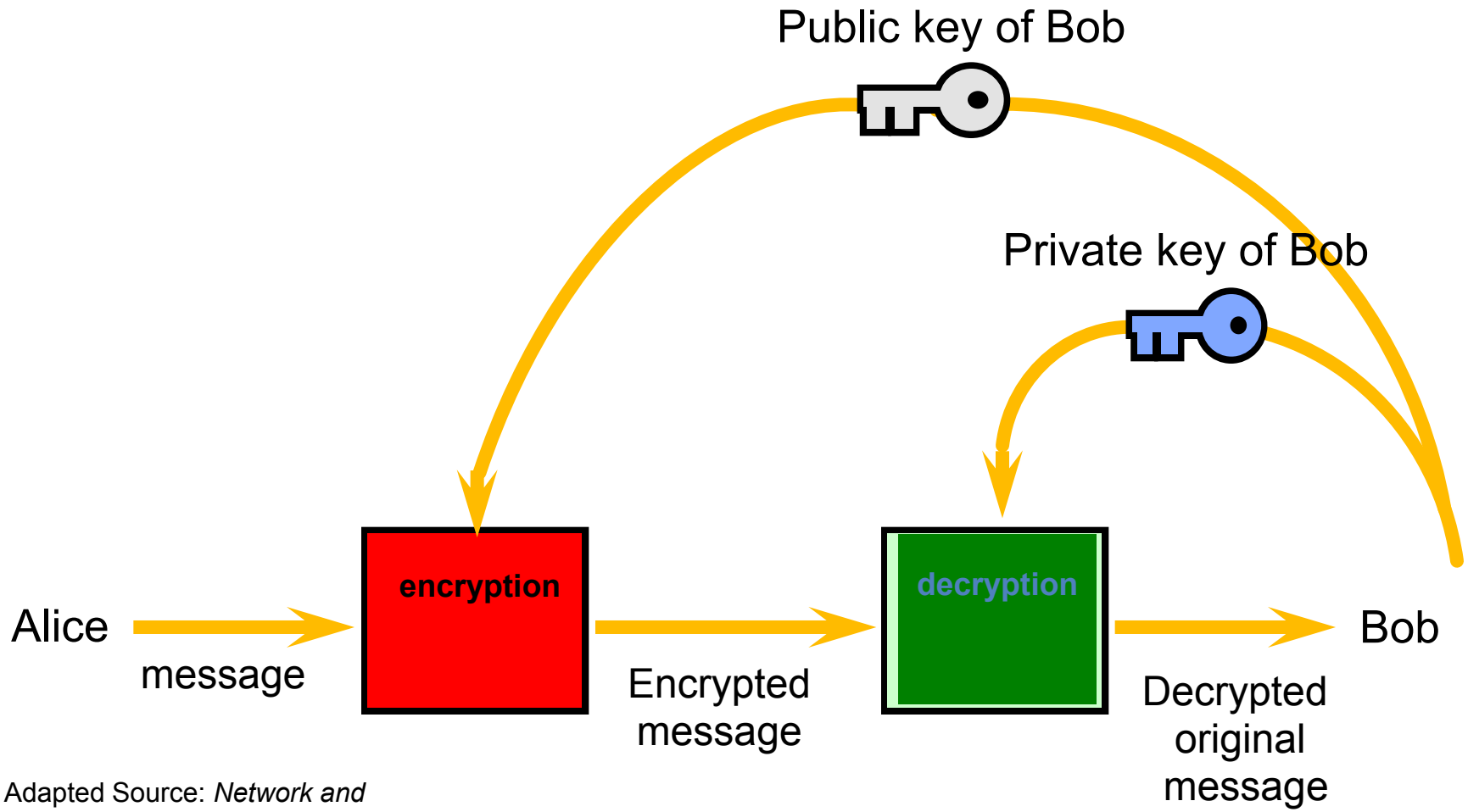
Public vs private key cryptography

- Private (symmetric)
  - both parties share secret (**private**)
  - Pros: fast encryption
  - Cons: key distribution requires **secure channel**
- Public (asymmetric)
  - one key is **public**
  - Pros - key distribution – **insecure** channel is OK
  - Cons - slow encryption
- Practice - private + public:
  - **public** used to establish key for **private** key system

Asymmetric cryptography

- Two related keys – created by **one** party
  - different inverse operations (encryption - decryption, signing – signature verification)
- Properties - hard to compute private from public key
  - based on hard mathematical problems
- Hard problems and cryptosystems:
  - Integer factorization – RSA, Rabin, …
  - Discrete logarithm problem (DLP): ElGamal, EC, DSA, …
  - Others (DH, decoding,…) – Diffie-Helman, McElliece,…

Asymmetric cryptosystem



Adapted Source: *Network and Internetwork Security* (Stallings)

Asymmetric cryptosystem

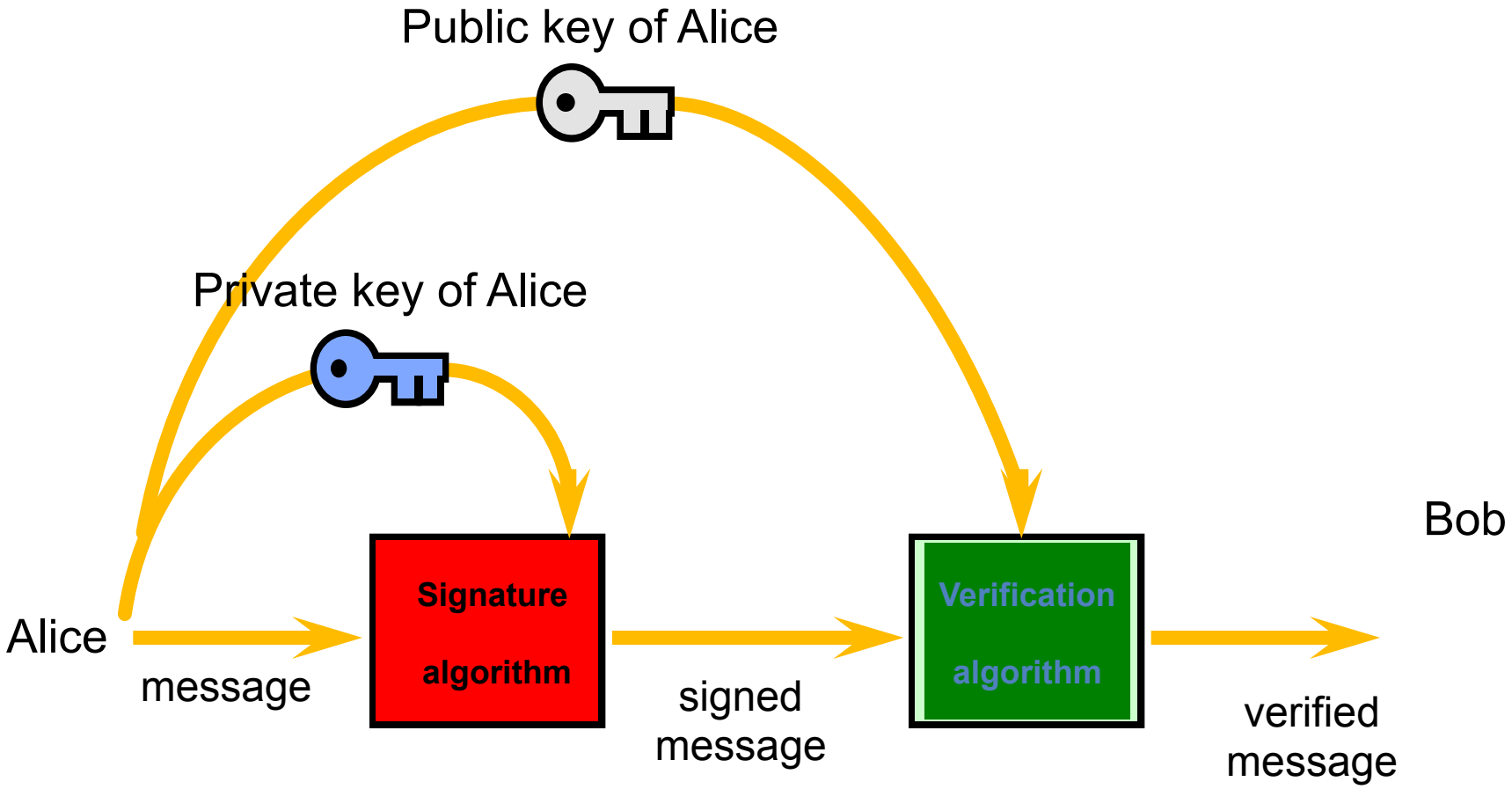
- Bob generates both keys:
  - Public is sent to Alice
  - Private is kept secret
- Alice encrypt message with her public key and send it to Bob
- Bob decrypt message using his private key

Digital signature

- Asymmetric cryptography
  - Private key – signature generation (usually only **hash** of data is signed **not** data itself)
  - Public key – verification procedure
- Data integrity + data origin + non-repudiation:
- Non-repudiation - correct signatures can be generated only by those having the private key

- The digital signature itself does not give any guarantees with respect to signing time.

Digital signature scheme



Source: *Network and Internetwork Security* (Stallings)

Digital signature

- Alice generates key pair
  - Public key is published (sent to Bob) for verification of signature
- Alice sign a document using her private key
- Bob use public key to verify the digital signature

RSA: matematics

1. Secret primes p, q: $n = p.q$
2. Public exponent e:
$$\gcd(e, (p-1)) = \gcd(e, (q-1)) = 1$$
3. Private exponent d : $de \equiv 1 \, mod \, \varphi(n)$

Encryption (public n, e): $E(m) = m^e \, mod \, n = c$

Decryption (public n, d): $D(c) = c^d \, mod \, n = m$

RSA-1024: means n has 1024 bits and $m < n$

RSA example

- Intentionally small numbers (**not** secure).
- We generate parameters: $p = 17$, $q = 7$, $n = p.q = 119$

- Public exponent is selected e = **3,5** $(\gcd(3,7-1) = 3)$
- Private exponent computed:
  ed = 5d = 1 (mod 96) to have **d = 77.**
- The public key: **(n = 119, e = 5),**
  The private key: **(n = 119, d = 77)**

- Encryption/decryption:
  - Message **m** = 'C' = 65
  - Encryption **m'** = 65^5 mod 119 = 46.
  - Decryption m = 46^{77} mod 119 = 65

RSA Padding example (PKCS#1 v1.5)

- Document
  - “00 01 02 03 04 05 06 07 07 06 05 04 03 02 01”
- Hash of the document (sha-1)
  - “b3 39 90 4c d2 a0 10 e6 19 37 eb e5 b5 83 37 8c 5d 10 51 95”
- Padded hash
  - “00 01 ff ff ff ff ff ff ff ff ff ff ff ff ff ff ff ff ff ff ff ff ff ff ff ff ff ff ff 00 30 21 30 09 06 05 2b 0e 03 02 1a 05 00 04 14 b3 39 90 4c d2 a0 10 e6 19 37 eb e5 b5 83 37 8c 5d 10 51 95”

RSA in practice: Padding

- $\mu(M)$ = 6b bb … bb ba || Hash(M) || 3x cc
  where x = 3 for SHA-1, 1 for RIPEMD-160
  – ANSI X9.31

- $\mu(M)$ = 00 01 ff … ff 00 || HashAlgID || Hash(M)
  – PKCS #1 v1.5

- $\mu(M)$ = 00 || H || $G(H) \oplus$ [$salt$ || 00 … 00]
  where H = Hash($salt$, M), $salt$ is random, and G is a mask generation function
  – Probabilistic Signature Scheme (PSS)

Hard problems

- Integer factorization
  - for **n** find divisor **p** of **n**
- Discrete logarithm problem:
  - in Z_p, Elliptic curves (EC)
    for $y = g * g * \cdots * g = g^x$ find x
  - $*$ represents operation (*, +) for given domain (integers, EC)
  - Domain parameters:
    - $g, n = ord(g)$ - n should be large
    - params defining algebraic structure: Z_p or **EC**

DLP for integers

For g, p, y find integer x such that

$$y \equiv g^x \bmod p$$

$g = 2, p = 31$

$g^x \bmod p$: $2,4,8,16,1 = 3^5 \bmod 31$ (order = 5)

$g = 3, p = 31$

$g^x \bmod p$: $3,9,27,19, \ldots, 3^{30} \bmod 31 = 1$

full order = 30, 3 is generator (all numbers)

Digital Signature Standard (DSS)

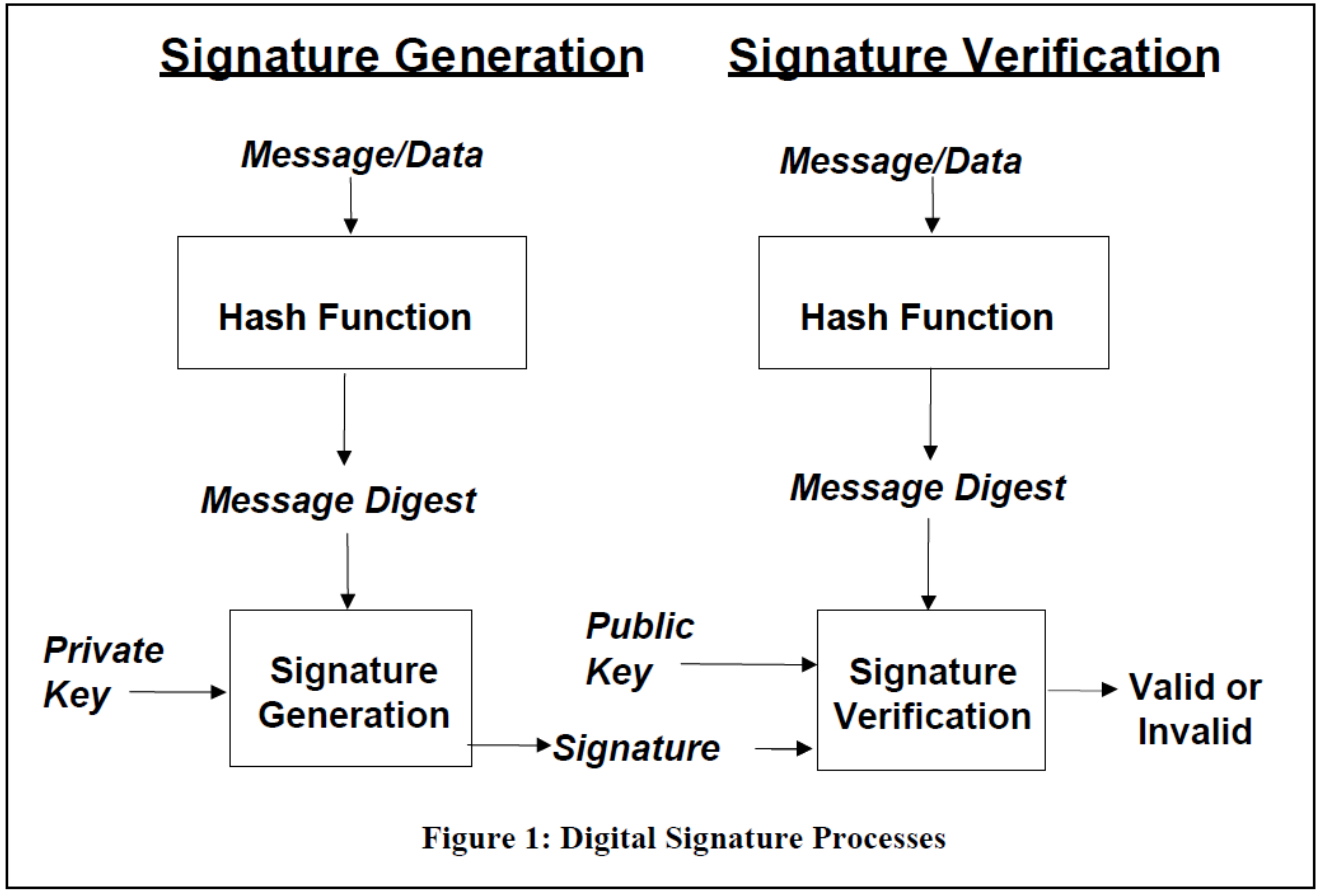


Figure 1: Digital Signature Processes

Digital Signature Standard (DSS)

- Signing:
  - Hash H(m) of message m is computed
  - H(m) is signed (E(H(m))) with private key
- Verification:
  - Public key is applied to E(H(m)) to get H(m) of original m
  - m' (obtained along with signature) is hashed to H(m')
  - if H(m) is equal to H(m') signature is verified

Digital Signature Algorithm (DSA)

- Proposed in 1991 by NIST
- In 1994 the selection procedure for Digital Signature Standard (DSS) was concluded – DSA (Digital Signature Algorithm) was selected.
- Modified version of ElGamal algorithm, based on discrete logarithm in Z_p.
- Became FIPS standard FIPS 186 in 1993.
- Slightly modified in 1996 as FIPS 186-1.
- Extended in 2000 as FIPS 186-2.
- Updated in 2009 as FIPS 186-3 (new key sizes).

- Now NIST FIPS 186-3 supports RSA & DSA & ECDSA.

DSS

- Selection of Parameter Sizes and Hash Functions

- Domain Parameter Generation
  - only for DSA, ECDSA
- Signature Generation

- Signature Verification and Validation

DSA: mathematics

- Key generation – **domain parameters**
  - Decide on a key length **L** and **N**, e.g. (1024,160).
    - N must be less than or equal to the hash output length
  - Choose an N-bit prime **q**. ["order of **g** w.r.t **p**"]
  - Choose an L-bit prime modulus **p** such that p–1 is a multiple of q.
  - Choose **g**, a number whose multiplicative order modulo p is q, e.g. $g = h^{(p-1)/q} \bmod p$ for some arbitrary h ($1 < h < p-1$). ["**generator**"]
  - Domain parameters **(p, q, g)** may be shared between different users of the DSA system.

DSA: mathematics II

- Key generation
  - Choose random **x**, such that $0 < x < q$.
  - Calculate **y** = $g^x \bmod p$.
- Private key: **x**.
- Public key: y & (p, q, g).

DSA: mathematics III

- Signature generation
  - Generate a random per-message value **k** such that $0 < k < q$.
  - Calculate **r** = $(g^k \bmod p) \bmod q$
  - Calculate **s** = $(k^{-1}(H(m) + x*r)) \bmod q$
  - The signature is (r, s).
- Signature verification
  - **w** = $(\mathbf{s})^{-1} \bmod q$
  - **u1** = $(\mathbf{H(m)}*w) \bmod q$
  - **u2** = $(\mathbf{r}*w) \bmod q$
  - **v** = $((g^{u1}*y^{u2}) \bmod p) \bmod q$
  - The signature is valid if **v = r**
- For DSA (1024,160) the signature size will be 2x160 bits.

DSA: Padding

- Decide on lengths **L** and **N**, e.g. (1024,160).
  - N must be less than or equal to the hash output length
    - E.g. for (1024,160) sha-1 is typically used, sha-256 would be ok as well and only first 160 bits would be used
  - $s = (k^{-1}(\mathbf{H(m)} + x*r)) \bmod q$
- "It is recommended that the security strength of the (L, N) pair and the security strength of the hash function used for the generation of digital signatures be the same unless an agreement has been made between participating entities to use a stronger hash function. When the length of the output of the hash function is greater than N (i.e., the bit length of q), then the leftmost N bits of the hash function output block shall be used in any calculation using the hash function output during the generation or verification of a digital signature. A hash function that provides a lower security strength than the (L, N) pair ordinarily should not be used, since this would reduce the security strength of the digital signature process to a level no greater than that provided by the hash function." [FIPS 186-3]

Elliptic curve DSA (ECDSA)

- Elliptic curves invented by Koblitz & Miller in 1985.
- ECDSA proposed in 1992 by Vanstone
- Became ISO standard (ISO 14888-3) in 1998
- Became ANSI standard (ANSI X9.62) in 1999

- ECDSA is a version of DSA based on elliptic curves.

ECDSA: Elliptic curve domain parameters

- **(field,a,b,G,n,h)**
  - Finite field
    - **p** for F_p
    - **m, bases (trinomial, pentanomial)** for $\mathbf{F_2^m}$
  - Coefficients **a, b**: $y^2 = x^3 + ax + b$
  - Group generator: **G**
  - Order of the G: **n**
  - Optional cofactor: **h**
    - (h = number of elements in field / order n)
  - The base point G generates a cyclic subgroup of order n in the field.

ECDSA: Keys

- Generating key pair
  - Select a random integer **d** from [1,n − 1]
  - Compute **P** = d*G;
- Private key: **d**
- Public key: **P**

- For 256-bit curve
  - the private key **d** will be approx. 256-bit long
  - the public key **P** is a point on the curve – will be approx 512-bit long

ECDSA: Signatures

- Generate signature
  - Select a random integer **k** from [1,n − 1]
  - $(\mathbf{x}_1,\mathbf{y}_1) = k*G$
  - Calculate $\mathbf{r} = x_1 \pmod n$
  - Calculate $\mathbf{s} = k^{-1}(M + r*d) \pmod n$
  - Signature is **(r,s)**.
- Signature verification
  - Calculate $\mathbf{w} = s^{-1} \pmod n$
  - Calculate $\mathbf{u_1} = z*w \pmod n$ & $\mathbf{u_2} = r*w \pmod n$
  - Calculate $(\mathbf{x}_1,\mathbf{y}_1) = u_1*G + u_2*P$
  - The signature is valid if $\mathbf{r = x_1 \pmod n}$.
- For 256-bit curve the signature length will be approx. 512 bits

ECDSA: Padding

- Rules are same as for DSA
- “It is recommended that the security strength associated with the bit length of n and the security strength of the hash function be the same unless an agreement has been made between participating entities to use a stronger hash function. When the length of the output of the hash function is greater than the bit length of n, then the leftmost n bits of the hash function output block shall be used in any calculation using the hash function output during the generation or verification of a digital signature. A hash function that provides a lower security strength than the security strength associated with the bit length of n ordinarily should not be used, since this would reduce the security strength of the digital signature process to a level no greater than that provided by the hash function.” [FIPS 186-3]

Digital certificate

Digital certificate

- used to prove ownership of the public key
- binds public key to identity (identity, email,…)
- **Public key certificate** is **signed** by trusted third party – Certification Authority (CA)
- two models: centralized and decentralized

Trust models

- Public key infrastructure (PKI)
  - centralized – hierarchy of CA's
  - cert signed by party
  - used in web browsers
  - standard X.509
- Web of trust
  - decentralized model
  - signed by many parties
  - used in PGP, GPG
  - standard OpenPGP

Public key Infrastructure (PKI)

- set of roles and procedures:
  - issue, maintain, administer, revoke, suspend, reinstate, and renew digital certificates
  - create and manage a public key repository

PKI:

- CA – stores, issues, signs certs
- RA – verifies identity
- Central directory– cert requests issued and revoked,
- Management system
- Cert policy

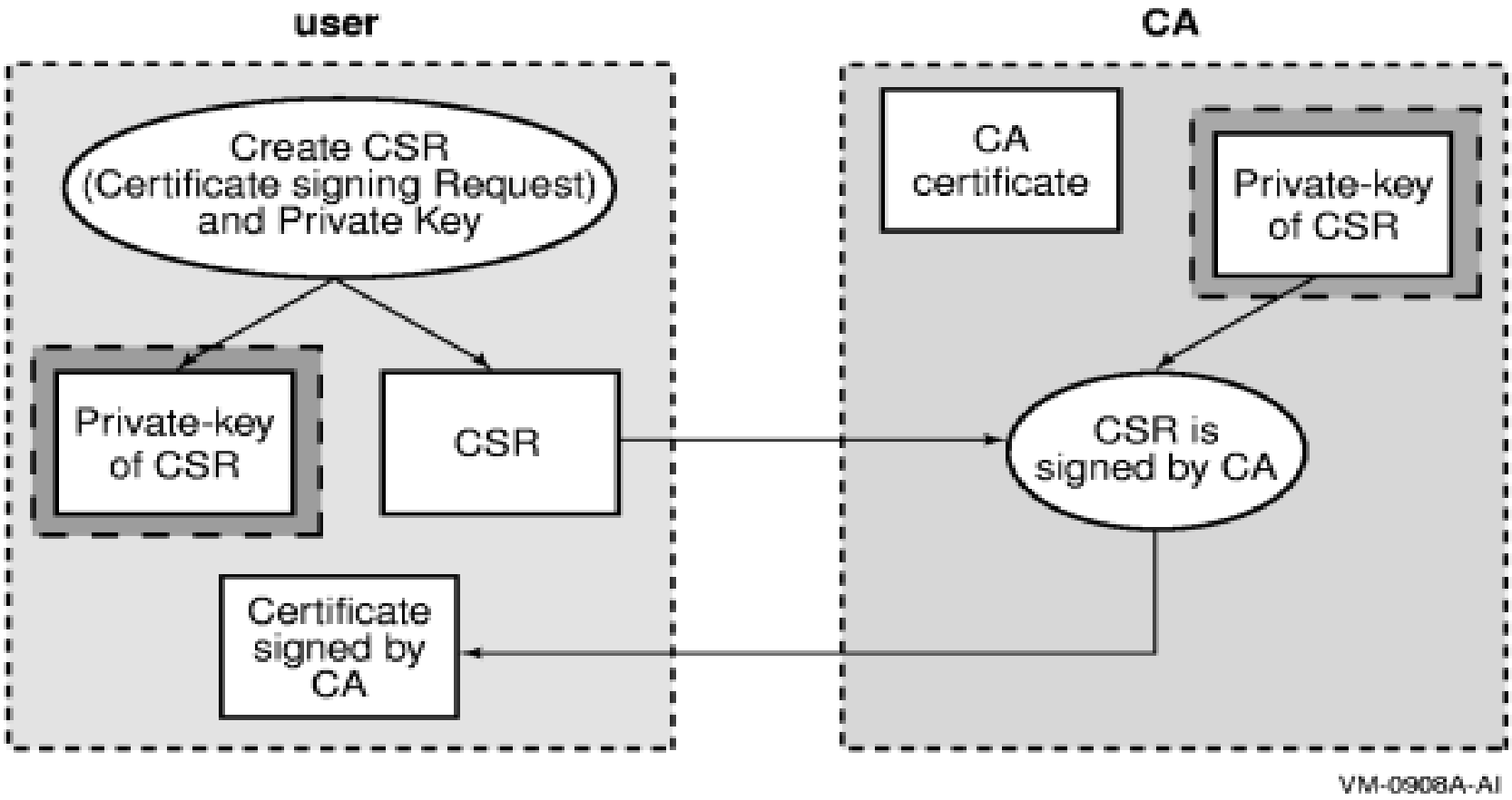
X.509 PKI certificate

- Certification Authority – trusted third party
- Certificate revocation lists (CRL) – certificates no longer be trusted (compromised key, CA,…)
- RFC5280 – defines format and semantics of certs and CRLs
- X.509 versions 1,2,3

X.509 PKI certificate content

Serial Number: unique ID of cert
Subject: ID of entity
Signature algorithm:
Signature:
Issuer: verifier of info and issued cert
Valid–From: date cert is first valid from
Valid–To: expiry date
Key-Usage: purpose of PK (signature, cert signing, …)
Public Key:
Thumbprint algorithm: to compute hash of PK cert
Thumbprint (fingerprint): hash of abbreviated PK cert

Certificate issuing



Certificate issuing

- User:
  - creates key pair
  - public + info is used to create - certification signing request
  - CSR(cert. s. request) is sent to CA
- CA
  - cert is created = CSR is signed by private key of CA
  - Cert is sent to user

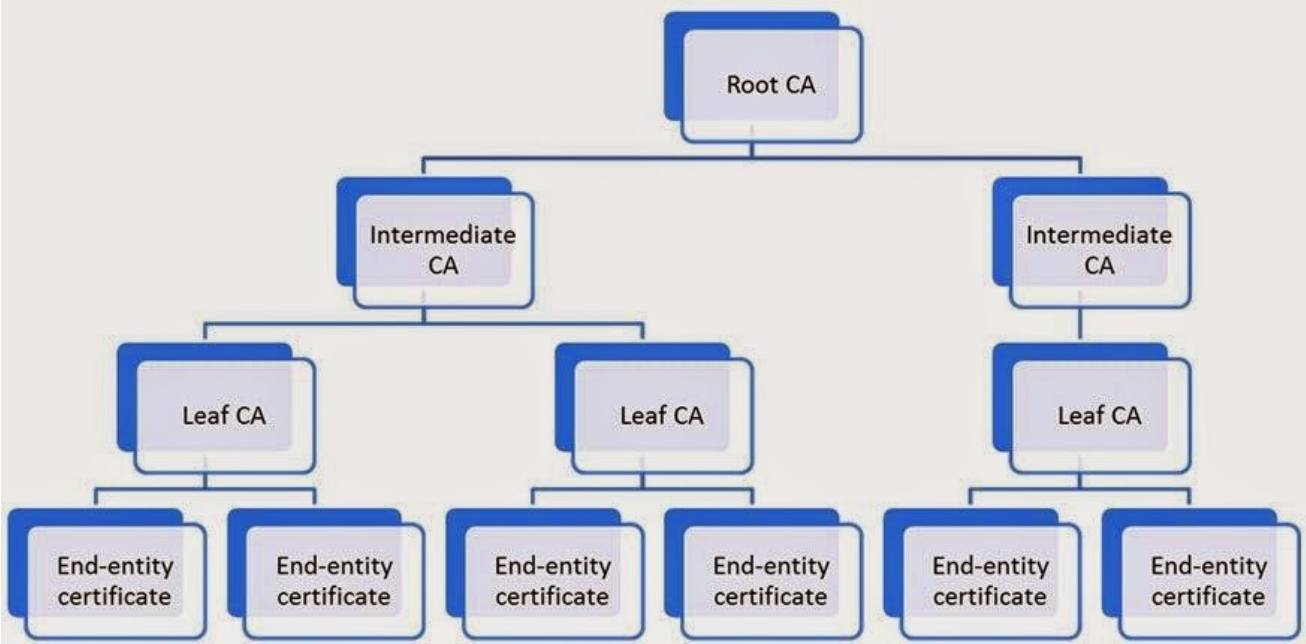
Certificate verification

Checking single cert:

- current **date** against validity period
- current validity of CA public key
- signature of CA on cert
- check whether certificate is revoked
  - CRL or OCSP
- policies

Certificates hierarchy

- **root CA (trust anchor)** - self-signed certificate
- Intermediate CA's
- **End entity** – user certificate



Certificates hierarchy

- **root CA (trust anchor)** - self-signed certificate
- Intermediate CA's
- **End entity** – user certificate

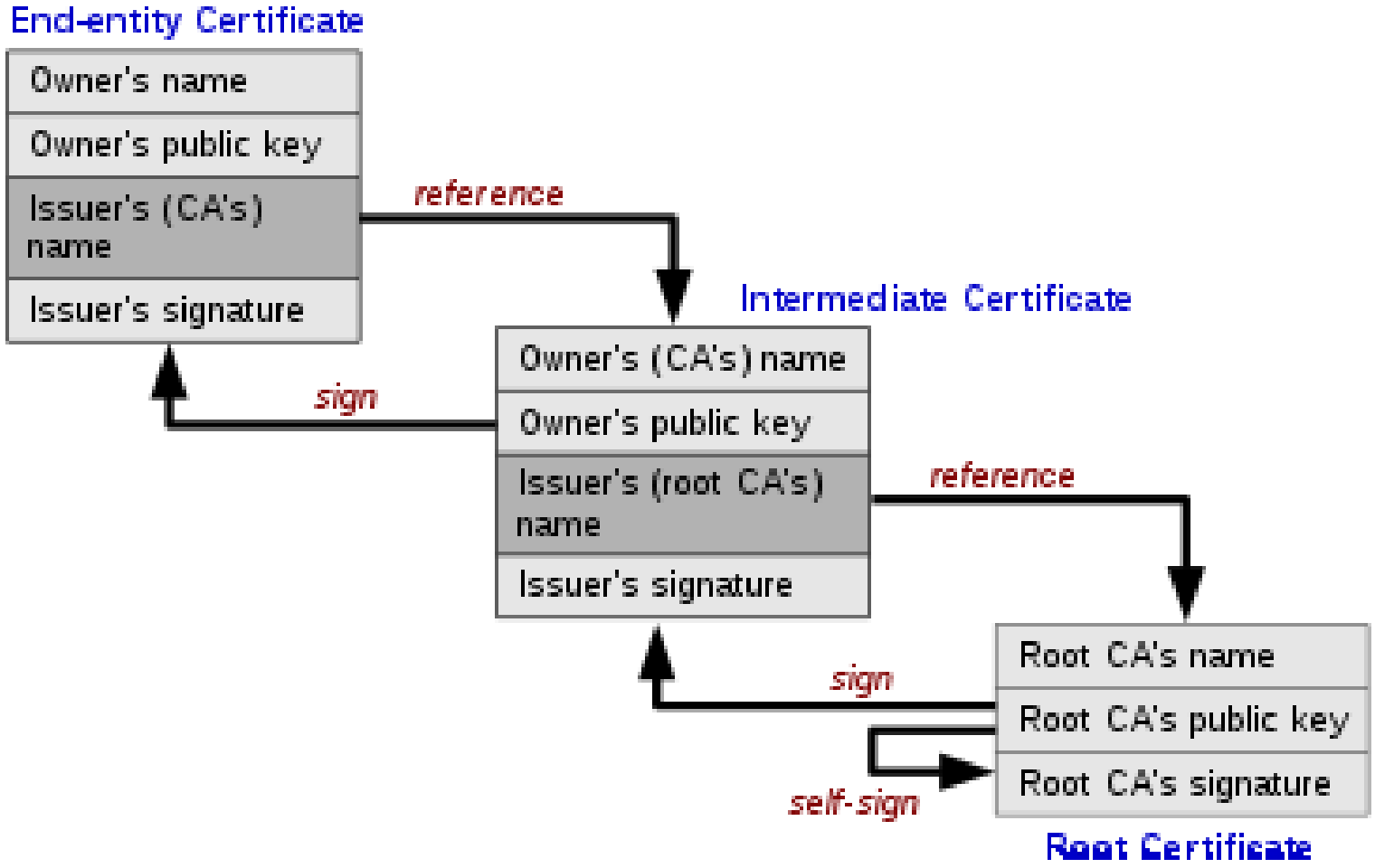
- Tree structure
  - root = root CA
  - nodes = intermediate CA's
  - leaves = end user certificates

Chain of trust

- Trust transfer – to lower CA's
- Root cert, intermediate certs, end-user cert.
- Chain:
  - end-user cert – signed by CA1
  - CA1 cert – signed by CA2…
  - root CA cert – signed by itself

Server – sends all certs up to root cert to browser

Chain of trust



Chain of trust - text

Cert = Owner's name, Owner PK, Issuer's name, issuer signature

1. End entity cert
   – issuer reference to 2.cert owner's name
2. Intermediate cert
   – issuer reference to 3.root cert owner's name
3. Root cert

Certificate path validation

Input: cert path, trust anchor

Path validation:

1. Check all certs if still valid
2. Check revocation status of certs
3. Check issuer = of previous cert subject
4. Check policy constraints
5. …

Revocation

- Reasons for revocation
  - **key compromise** (most common), CA compromise, affiliation change,…
- Two states:
  - revoked – irreversibly for compromised private key
  - hold – unsure user about key compromising, can be reinstalled
- Checked using:
  - CRL – list of revoked certs
  - Online Certificate Status Protocol – on demand

CRL

Issued by CA:

- Certificate Revocation List (CRL):
  - list of revoked certificates of end-users
- Authority Revocation list
  - List of revoked cert of CA's

1. Issuer name
2. Date list created
3. Date next CRL scheduled
4. Entries = serial number + revocation date of cert