# IA169 System Verification and Assurance

## LTL Model Checking

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# Motivation

**Checking Quality**

- Testing is incomplete, gives no guarantees of correctness.
- Deductive verification is expensive and applicable to closed programs only.

**Typical reasons for system failure after deployment**

- Interaction with environment (unexpected input values).
- Interaction with other system components.
- Parallelism (difficult to test).

**Model Checking**

- Automated formal verification process for ...
- ... reactive systems.
- ... parallel/distributed systems.

# Verification of Reactive and Parallel Programs

# Parallel Programs

**Parallel Composition**

- Components concurrently contribute to the transformation of a computation state.
- The meaning comes from interleaving of actions (transformation steps) of individual components.

**Meaning Functions Do Not Compose**

- Meaning function of a composition cannot be obtain as composition of meaning functions of participating components.
- The result depends on particular interleaving.

# Example of Incomposability

**Parallel System**

- System: (y=x; y++; x=y) $\|$ (y=x; y++; x=y)
- Input-output variable $x$
- Meaning function of both processes is $\lambda$x->x+1.
- The composition is: $(\lambda$x->x+1$)\cdot(\lambda$x->x+1$)$.
- $(\lambda$x->x+1$)\cdot(\lambda$x->x+1$)\ 0 = 2$

**Two Different System Runs**

- State $= (x, y_1, y_2)$
- (0,-,-) $\xrightarrow{y_1=x}$ (0,0,-) $\xrightarrow{y_2=x}$ (0,0,0) $\xrightarrow{y_1++}$ $\xrightarrow{x=y_1}$ (1,1,0) $\xrightarrow{y_2++}$ $\xrightarrow{x=y_2}$ (**1**,1,1)
- (0,-,-) $\xrightarrow{y_1=x}$ (0,0,-) $\xrightarrow{y_1++}$ $\xrightarrow{x=y_1}$ (1,1,-) $\xrightarrow{y_2=x}$ (1,1,1) $\xrightarrow{y_2++}$ $\xrightarrow{x=y_2}$ (**2**,1,2)

# Consequence of Incomposability

**Consequence 1**

- In general, we cannot deduce correctness of the system from the correctness of its components.

**Consequence 2**

- In general, we cannot decompose the problem of checking correctness of the system into subproblems, i.e. to reduce it to the verification of correctness of all the participating system components.

# Specification for Reactive/Parallel Programs

**Problem of Specification**

- The number of inputs and the input values are unknown before a particular execution of a reactive system.
- **How to specify the system properties?**

**Examples of Specification**

- The system will not crash.
- Events A and B happens before event C.
- User is not allowed to enter a new value until the system processes the previous one.
- Procedure X cannot be executed simultaneously by processes P and Q (mutual exclusion).
- Every action A is immediately followed by a sequence of actions B,C and D.

# Model Checking Principles

## Model Checking

- Approach to formal verification of systems.
- Based on the system's state-space exploration.

## State-space Exploration

- Generate and explore all possible states of computation the system under verification may get to.
- Requires description of system behavior (code, or model).
- Requires description of possible input actions (environment).

## System Specification

- Amir Pnueli, 1977
- Specification may be given with logic formulae.
- Use of Modal and Temporal Logics

# Model Checking

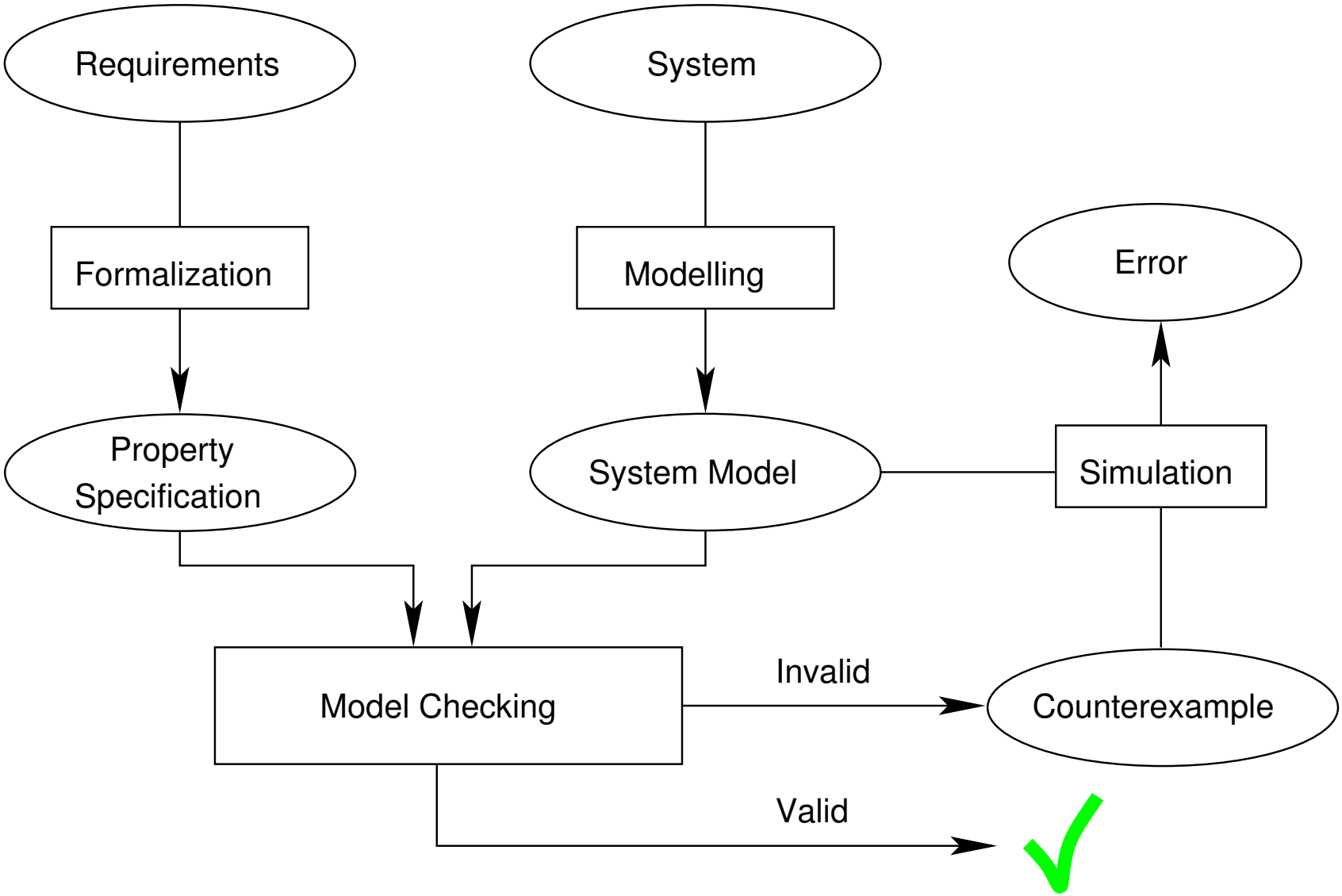
## Model Checking – Overview

- Build a formal model $\mathcal{M}$ of the system under verification.
- Express specification as a formula $\varphi$ of selected temporal logic.
- Decide, if $\mathcal{M} \models \varphi$. That is, if $\mathcal{M}$ is a model of formula $\varphi$. (Hence the name.)

## Optionally

- As a side effect of the decision a **counterexample** may be produced.
- The counterexample is a sequence of states witnessing violation (in the case the system is erroneous) of the formula.
- **Model checking (the decision process) can be fully automated for all finite (and some infinite) models of systems.**

# Model Checking – Schema



# Automated Tools for Model Checking

**Model Checkers**

- Software tools that can decide validity of a formula over a model of system under verification.
- SPIN, UppAal, SMV, Prism, DIVINE ...

**Modelling Languages**

- Processes described as extended finite state machines.
- Extension allows to use shared or local variables and guard execution of a transition with a Boolean expression.
- Optionally, some transitions may be synchronised with transitions of other finite state machines/processes.

# Modelling and Formalization of Verified Systems

# Atomic Proposition

**Reminder**

- System can be viewed as a set of states that are walked along by executing instructions of the program.
- State = valuation of modelled variables.

**Atomic Propositions**

- Basic statements describing qualities of individual states, for example: $max(x, y) \geq 3$.
- Validity of atomic proposition for a given state must be decidable with information merely encoded by the state.
- Amount of observable events and facts depends on amount of abstraction used during the system modelling.

# Kripke Structure

## Kripke Structure

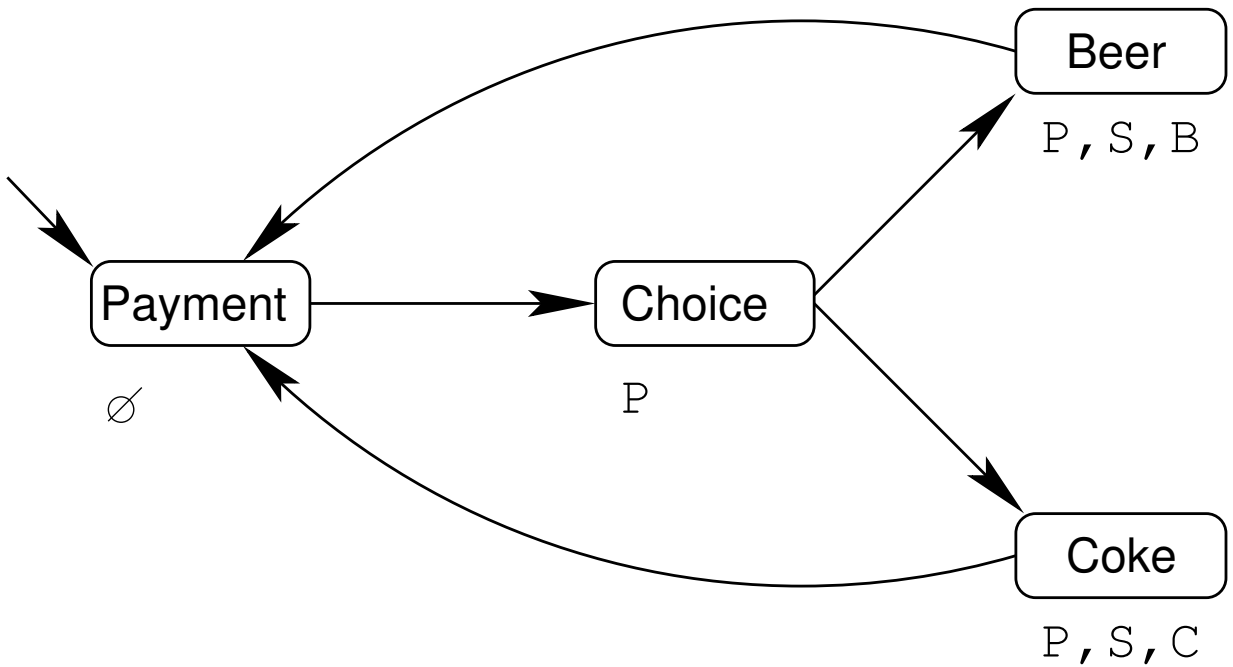
- Let $AP$ be a set of atomic propositions.
- Kripke structure is a quadruple $(S, T, I, s_0)$, where
  - $S$ is a (finite) set of states,
  - $T \subseteq S \times S$ is a transition relation,
  - $I : S \to 2^{AP}$ is an interpretation of AP.
  - $s_0 \in S$ is an initial state.

## Kripke Transition System

- Let $Act$ be a set of instructions executable by the program.
- Kripke structure can be extended with transition labelling to form a Kripke Transitions System.
- Kripke Transition System is a five-tuple $(S, T, I, s_0, \mathcal{L})$, where
  - $(S, T, I, s_0)$ is Kripke Structure,
  - $\mathcal{L} : T \to Act$ is labelling function.

# Kripke Structure – Example

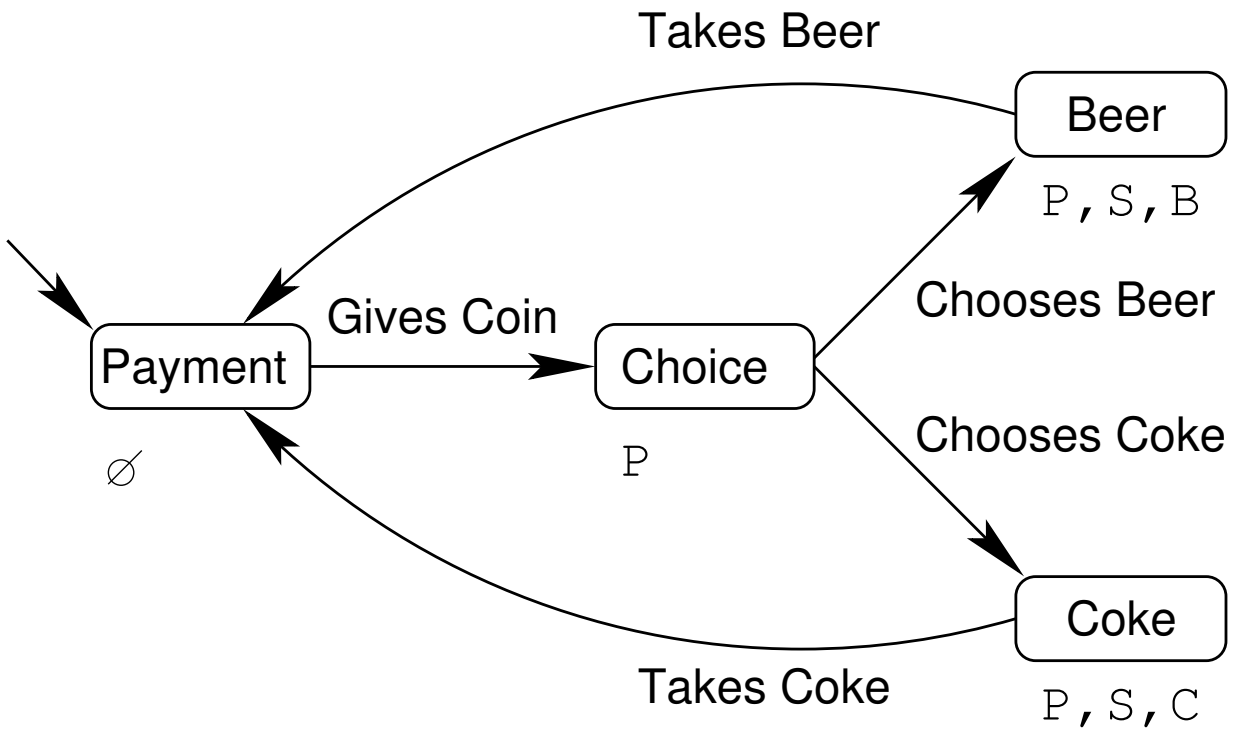
Kripke Structure




AP={P – Paid, S – Served, C – Coke, B – Beer}

# Kripke Structure – Example

Kripke Transition System




AP={P – Paid, S – Served, C – Coke, B – Beer}

# System Run

**Run**

- Maximal path (such that it cannot be extended) in the graph induced by Kripke Structure starting at the initial state.
- Let $M = (S, T, I, s_0)$ be a Kripke structure. Run is a sequence of states $\pi = s_0, s_1, s_2, \ldots$ such that $\forall i \in \mathbb{N}_0.(s_i, s_{i+1}) \in T$.

**Finite Paths and Runs**

- Some finite path $\pi = s_0, s_1, s_2, \ldots, s_k$ cannot be extended if $\nexists s_{k+1} \in S.(s_k, s_{k+1}) \in T$.
- Technically, we will turn maximal finite path into infinite by repeating the very last state.
- Maximal path $s_0, \ldots, s_k$ will be understood as infinite run $s_0, \ldots, s_k, s_k, s_k, \ldots$.

# Implicit and Explicit System Description

**Observation**

- Usually, Kripke structure that captures system behaviour is not given by full enumeration of states and transitions (explicitly), but it is given by the program source code (implicitly).
- Implicit description tends to be exponentially more succinct.

**State-Space Generation**

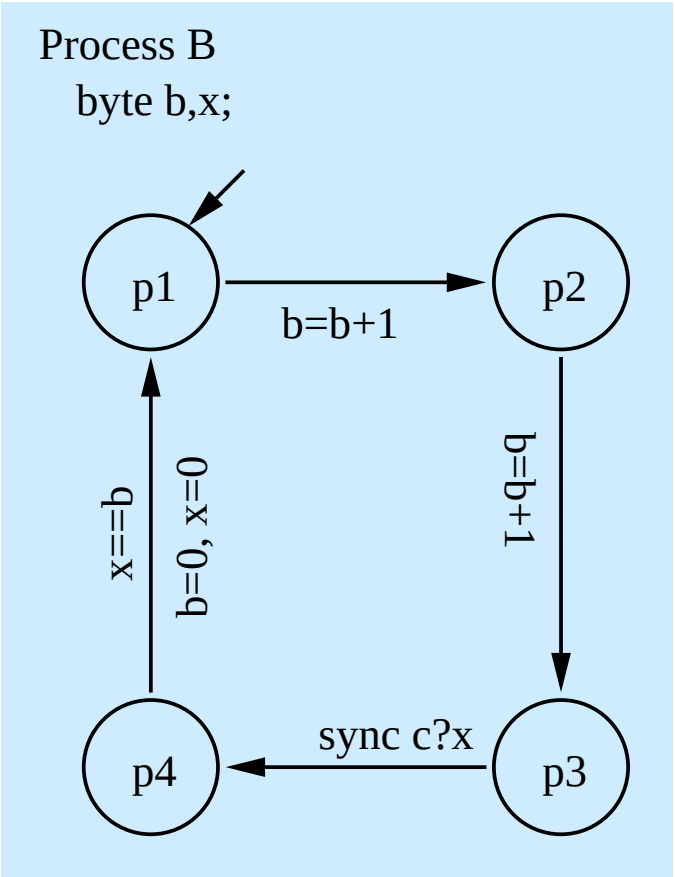
- Computation of explicit representation from the implicit one.
- Interpretation of implicit representation must be formally precise.

**Practice**

- Programming languages do not have precise formal semantics.
- Model checkers often build on top of modelling languages.

# An Example of Modelling Language – DVE

- Finite Automaton
  - States (Locations)
  - Initial state
  - Transitions
  - (Accepting states)
- Transitions Extended with
  - Guards
  - Synchronisation and Value Passing
  - Effect (Assignment)
- Local Variables
  - `integer`, `byte`
  - `channel`



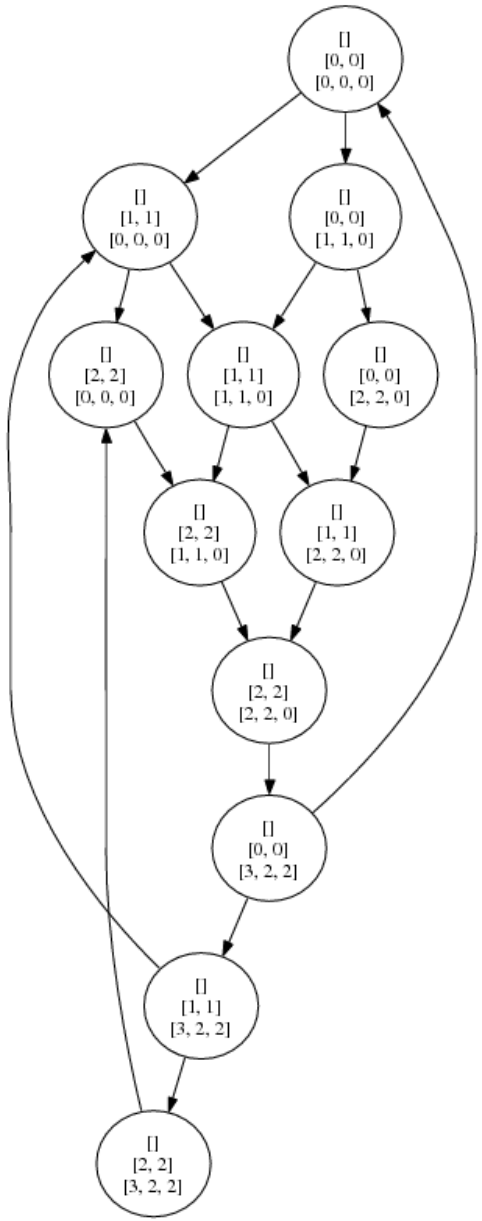
# Example of System Described in DVE Language

```
channel {byte} c[0];

process A {
byte a;
state q1,q2,q3;
init q1;
trans
q1→q2 { effect a=a+1; },
q2→q3 { effect a=a+1; },
q3→q1 { sync c!a; effect a=0; };
}

process B {
byte b,x;
state p1,p2,p3,p4;
init p1;
trans
p1→p2 { effect b=b+1; },
p2→p3 { effect b=b+1; },
p3→p4 { sync c?x; },
p4→p1 { guard x==b; effect b=0, x=0; };
}

system async;
```



# Semantics Shown By Interpretation

State: []; A:[q1, a:0]; B:[p1, b:0, x:0]
0 ⟨0.0⟩: q1 → q2 { effect a = a+1; }
1 ⟨1.0⟩: p1 → p2 { effect b = b+1; }
Command:1

---

State: []; A:[q1, a:0]; B:[p2, b:1, x:0]
0 ⟨0.0⟩: q1 → q2 { effect a = a+1; }
1 ⟨1.1⟩: p2 → p3 { effect b = b+1; }
Command:1

---

State: []; A:[q1, a:0]; B:[p3, b:2, x:0]
0 ⟨0.0⟩: q1 → q2 { effect a = a+1; }
Command:0

---

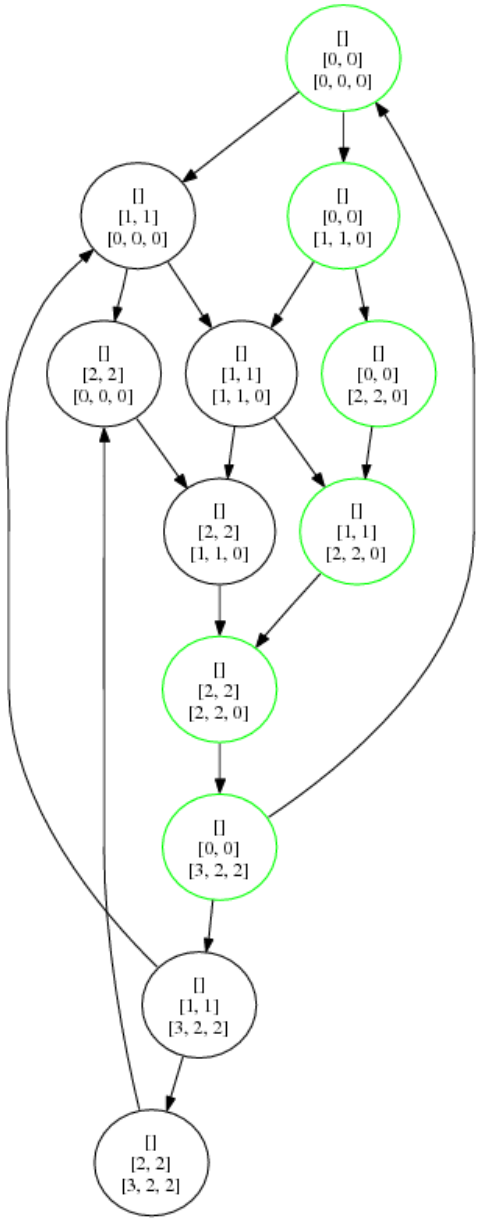
State: []; A:[q2, a:1]; B:[p3, b:2, x:0]
0 ⟨0.1⟩: q2 → q3 { effect a = a+1; }
Command:0

---

State: []; A:[q3, a:2]; B:[p3, b:2, x:0]
0 ⟨0.2&1.2⟩: q3 → q1 { sync c!a; effect a = 0; }
p3 → p4 { sync c?x; }
Command:0

---

State: []; A:[q1, a:0]; B:[p4, b:2, x:2]



# Formalizing System Properties

# Specification as Languages of Infinite Words

**Problem**

- How to formally describe properties of a single run?
- How to mechanically check for their satisfaction?

**Solution**

- Employ finite automaton as a mechanical observer of run.
- Runs are infinite.
- Finite automata for infinite words ($\omega$-regular languages).
- Büchi acceptance condition – automaton accepts a word if it passes through an accepting state infinitely many often.

# Automata over infinite words

## Büchi automata

- Büchi automaton is a tuple $A = (\Sigma, S, s, \delta, F)$, where
  - $\Sigma$ is a finite set of symbols,
  - $S$ is a finite set f states,
  - $s \in S$ is an initial state,
  - $\delta : S \times \Sigma \to 2^S$ is transition relation, and
  - $F \subseteq S$ is a set of accepting states.

## Language accepted by a Büchi automaton

- Run $\rho$ of automaton $A$ over infinite word $w = a_1 a_2 \ldots$ is a sequence of states $\rho = s_0, s_1, \ldots$ such that $s_0 \equiv s$ and $\forall i : s_i \in \delta(s_{i-1}, a_i)$.
- $inf(\rho)$ – Set of states that appear infinitely many time in $\rho$.
- Run $\rho$ is accepting if and only if $inf(\rho) \cap F \neq \emptyset$.
- Language accepted with an automaton $A$ is a set of all words for which an accepting run exists. Denoted as $L(A)$.

# Shortcuts in Transition Guards

**Observation**

- Let AP={X,Y,Z}.
- Transition labelled with $\{X\}$ denotes that $X$ must hold true upon execution of the transition, while $Y$ and $Z$ are false.
- If we want to express that $X$ is true, $Z$ is false, and for $Y$ we do not care, we have to create two transitions labelled with $\{X\}$ and $\{X, Y\}$.

**APs as Boolean Formulae**

- Transitions between the two same states may be combined and labelled with a Boolean formula over atomic propositions.

**Example**

- Transitions {X}, {Y}, {X,Y}, {X,Z}, {Y,Z} a {X,Y,Z} can be combined into a single one labelled with $X \vee Y$.
- If there are no restrictions upon execution of the transition, it may be labelled with $\mathit{true} \equiv X \vee \neg X$.

# Task: Express with a Büchi automaton

**System**

- Vending machine as seen before.
- $\Sigma = 2^{\{P,S,C,B\}}$,
- $\mathit{Paid} = \{A \in \Sigma \mid P \in A\}$, $\mathit{Served} = \{A \in \Sigma \mid S \in A\}$, ...

**Express the following properties**

- Vending machine serves at least one drink.
- Vending machine serves at least one coke.

- Vending machine serves infinitely many drinks.
- Vending machine serves infinitely many beers.

- Vending machine does not serve a drink without being paid.
- After being paid, vending machine always serve a drink.

# Linear Temporal Logic

# Linear Temporal Logic (LTL) Informally

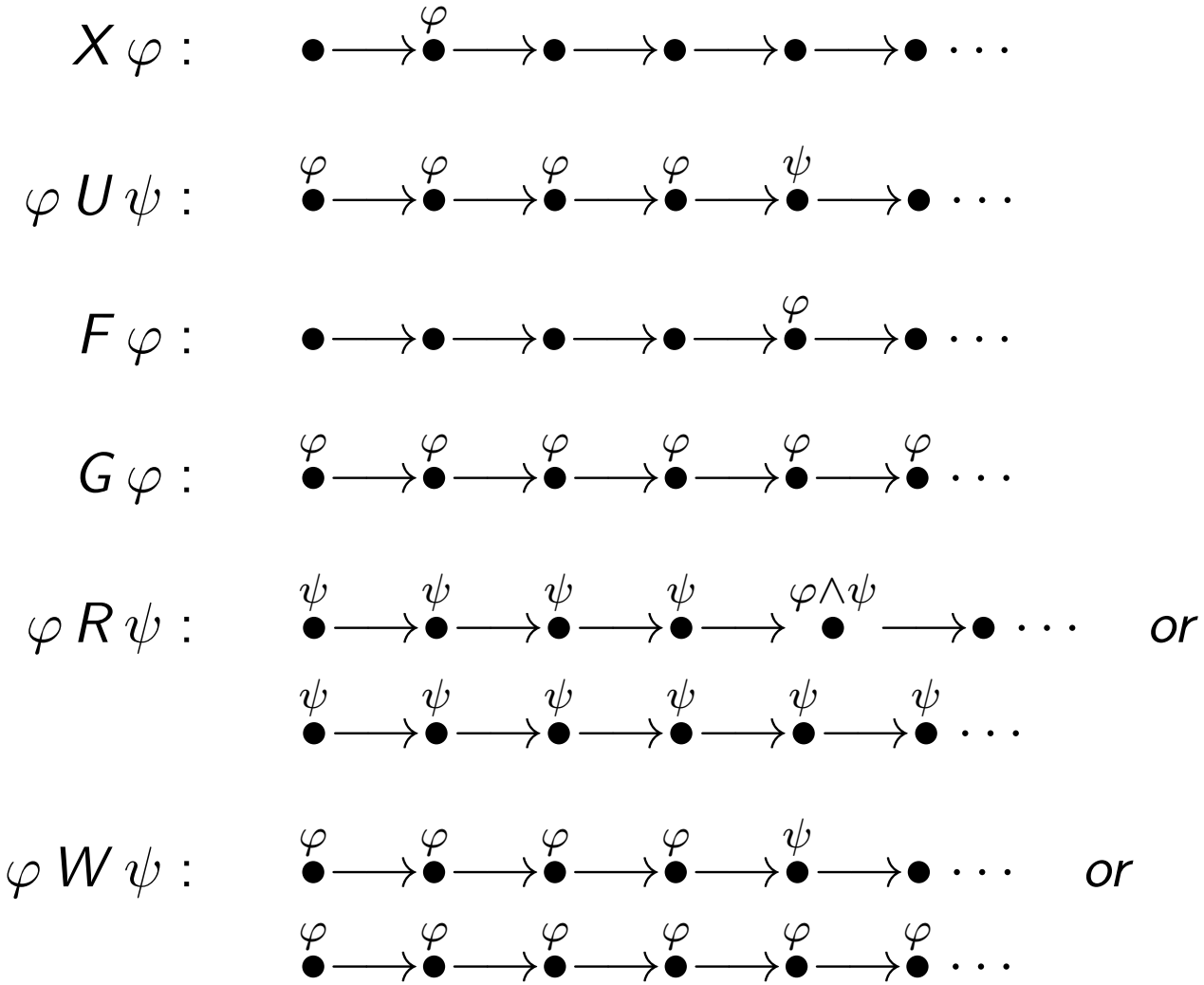
**Formula $\varphi$**

- Is evaluated on top of a single run of Kripke structure.
- Express validity of APs in the states along the given run.

**Temporal Operators of LTL**

- $F\,\varphi$ — $\varphi$ holds true eventually (Future).
- $G\,\varphi$ — $\varphi$ holds true all the time (Globally).
- $\varphi\,U\,\psi$ — $\varphi$ holds true until eventually $\psi$ holds true (Until).
- $X\,\varphi$ — $\varphi$ is valid after execution of one transition (Next).
- $\varphi\,R\,\psi$ — $\psi$ holds true until $\varphi \wedge \psi$ holds true (Release).
- $\varphi\,W\,\psi$ — until, but $\psi$ may never become true (Weak Until).

# Graphical Representation of LTL Temporal Operators



# Syntax of LTL

Let $AP$ be a set of atomic propositions.

- If $p \in AP$, then $p$ is an LTL formula.
- If $\varphi$ is an LTL formula, then $\neg\varphi$ is an LTL formula.
- If $\varphi$ and $\psi$ are LTL formulae, then $\varphi \vee \psi$ is an LTL formula.
- If $\varphi$ is an LTL formula, then $X\,\varphi$ is an LTL formula.
- If $\varphi$ and $\psi$ are LTL formulae, then $\varphi\,U\,\psi$ is an LTL formula.

Alternatively

$$\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid X\,\varphi \mid \varphi\,U\,\varphi$$

# Syntactic shortcuts

Propositional Logic

- $\varphi \wedge \psi \equiv \neg(\neg\varphi \vee \neg\psi)$
- $\varphi \Rightarrow \psi \equiv \neg\varphi \vee \psi$
- $\varphi \Leftrightarrow \psi \equiv (\varphi \Rightarrow \psi) \wedge (\psi \Rightarrow \varphi)$

Temporal operators

- $F\,\varphi \equiv \mathit{true}\,U\,\varphi$
- $G\,\varphi \equiv \neg F\,\neg\varphi$
- $\varphi\,R\,\psi \equiv \neg(\neg\varphi\,U\,\neg\psi)$
- $\varphi\,W\,\psi \equiv \varphi\,U\,\psi \vee G\,\varphi$

Alternative syntax

- $F\varphi \equiv \diamond\varphi$
- $G\varphi \equiv \Box\varphi$
- $X\varphi \equiv \circ\varphi$

# Models of LTL Formulae

**Model of an LTL formula**

- Let $AP$ be a set of atomic propositions.
- Model of an LTL formula is a run $\pi$ of Kripke structure.

**Notation**

- Let $\pi = s_0, s_1, s_2, \ldots$.
- Suffix of run $\pi$ starting at $s_k$ is denoted as $\pi^k = s_k, s_{k+1}, s_{k+2}, \ldots$.
- K-th state of the run, is referred to as $\pi(k) = s_k$.

# Semantics of LTL

**Assumptions**

- Let $AP$ be a set of atomic propositions.
- Let $\pi$ be a run of Kripke structure $M = (S, T, I, s_0)$.
- Let $\varphi$, $\psi$ be syntactically correct LTL formulae.
- Let $p \in AP$ denote atomic proposition.

**Semantics**

$$
\begin{aligned}
\pi \models p \quad &\text{iff} \quad p \in I(\pi(0)) \\
\pi \models \neg\varphi \quad &\text{iff} \quad \pi \not\models \varphi \\
\pi \models \varphi \vee \psi \quad &\text{iff} \quad \pi \models \varphi \text{ or } \pi \models \psi \\
\pi \models X\,\varphi \quad &\text{iff} \quad \pi^1 \models \varphi \\
\pi \models \varphi\, U\, \psi \quad &\text{iff} \quad \exists k.0 \leq k, \pi^k \models \psi \text{ and} \\
&\qquad\quad \forall i.0 \leq i < k, \pi^i \models \varphi
\end{aligned}
$$

# Semantics of Other Temporal Operators

$$\pi \models F\,\varphi \quad \text{iff} \quad \exists k.k \geq 0, \pi^k \models \varphi$$

$$\pi \models G\,\varphi \quad \text{iff} \quad \forall k.k \geq 0, \pi^k \models \varphi$$

$$\begin{aligned}\pi \models \varphi\,R\,\psi \quad \text{iff} \quad & (\exists k.0 \leq k, \pi^k \models \varphi \wedge \psi \text{ and} \\ & \forall i.0 \leq i < k, \pi^i \models \psi) \\ & \text{or } (\forall k.k \geq 0, \pi^k \models \psi)\end{aligned}$$

$$\begin{aligned}\pi \models \varphi\,W\,\psi \quad \text{iff} \quad & (\exists k.0 \leq k, \pi^k \models \psi \text{ and} \\ & \forall i.0 \leq i < k, \pi^i \models \varphi) \\ & \text{or } (\forall k.k \geq 0, \pi^k \models \varphi)\end{aligned}$$

# LTL Model Checking

**Verification Employing LTL**

- System is viewed as a set of runs.
- System is satisfies LTL formula if and only if all system runs satisfy the formula.
- In other words, any run violating the formula is a witness that the system does not satisfy the formula.

**Lemma**

- If a finite state system does not satisfy an LTL formula then this may be witnessed with a **lasso-shaped** run.
- Run $\pi$ is lasso-shaped if $\pi = \pi_1 \cdot (\pi_2)^\omega$, where

  $\pi_1 = s_0, s_1, \ldots, s_k$

  $\pi_2 = s_{k+1}, s_{k+2}, \ldots, s_{k+n}$, where $s_k \equiv s_{k+n}$.
- Note that $\pi^\omega$ denotes infinite repetition of $\pi$.

# Self-study

**Get acquainted with famous SPIN model checker**

- Model Peterson's mutual exclusion protocol in ProMeLa.
- State expected LTL properties of Peterson's protocol.
- Verify them using SPIN model checker.