IA168 — Problem set 1

Throughout this problem set, we consider only two-player strategic-form games. Except for Problem 4, we consider only **pure** strategies.

Definitions. A strategy profile $s \in S$ *Pareto-dominates* a strategy profile $s' \in S$ iff $u_i(s) \geq u_i(s')$ for all players $i \in N$ and $u_i(s) > u_i(s')$ for some player $i \in N$.
A strategy profile $s \in S$ is *Pareto-optimal* iff it is not Pareto-dominated by any other strategy profile.

Problem 1 [2 points]

Consider a game where Player 1 selects a positive even integer less than 7 and, at the same time, Player 2 selects a positive odd integer less than 7. The payoff of the player with the higher selected number is the absolute difference of the selected numbers and the payoff of the other player is the sum of the selected numbers. Give a formal description of this game as a two-player strategic-form game (i.e., according to the definition from the lectures).

Problem 2 [4 points]

Consider a **zero-sum** game, where each player has exactly four strategies, called A_1, B_1, C_1, D_1, and A_2, B_2, C_2, D_2, respectively. Define the utility function of this game so that for both $i \in \{1, 2\}$, all of the following conditions are satisfied:

- the strategy A_i of player i is strictly dominated;
- the strategy B_i of player i is never-best-response, but not strictly dominated;
- the strategy C_i of player i is not never-best-response;
- (D_1, D_2) is the only Nash equilibrium of the game.

Problem 3 [3 points]

For each $n \in \{0, 1, \ldots, 6\}$, give an example of a game where Player 1 has exactly 2 strategies, Player 2 has exactly 3 strategies and there are exactly n Nash equilibria.

Problem 4 [6 points]

Consider a game with **mixed** strategies, where each player has exactly two pure strategies, called A_1, B_1, and A_2, B_2, respectively. The utility functions are defined by the following table:

	A_2	B_2
A_1	$(a, 4)$	$(-a, 2)$
B_1	$(3, 1)$	$(1, 3)$

In dependence on the parameter $a \in \mathbb{R}$, find all Nash equilibria of this game, and for each of them, decide whether it is Pareto-optimal. Justify your reasoning.

Problem 5 [5 points]

Prove or disprove:

a) IESDS creates no new Nash equilibria in any game with finitely many strategies.

b) IESDS creates no new Nash equilibria in any game.