IA168 — Problem set 3

Problem 1 [5 points]

Consider the following two-player strategic-form game G:

	X	Y
A	$(5,5)$	$(-1,6)$
B	$(6,-1)$	$(1,1)$

a) In G^{avg}_{irep}, find a subgame-perfect equilibrium whose outcome is $(4.5, 4.2)$.

b) Calculate $\inf_{s \in SPE(G^{avg}_{irep})} u_1(s)$.

c) Calculate $\sup_{s \in SPE(G^{avg}_{irep})} u_1(s)$.

Justify your reasoning.

Problem 2 [4 points]

Consider the following two-player strategic-form game G, with real-valued parameters x, y:

	A	B
A	$(2,1)$	$(7,-1)$
B	$(-2,6)$	(x,y)

The players will play an infinite number of rounds, with a discount factor δ. Both will play the following strategy: If only B's have been played so far (i.e., the current history lies in $(B,B)^*$), then the player plays B; otherwise he plays A. Let s denote the corresponding strategy profile.

Find all pairs $(x,y) \in \mathbb{R} \times \mathbb{R}$ for which $\inf\{\delta \in \mathbb{R}\colon\ 0 < \delta < 1 \ \wedge\ s \text{ is a SPE in } G^{\delta}_{irep}\} = 3/5$.

Justify your reasoning.

Problem 3 [4 points]

Consider the incomplete-information game $G = (\{1,2\}, (\{A,B,C\},\{D,E,F\}), (\{P,Q\},\{R,S\}), (u_1,u_2)\})$, where u_1, u_2 are given by the following matrices:

$u_1(-,-,P)$	D	E	F
A	6	5	4
B	1	2	5
C	1	2	3

$u_1(-,-,Q)$	D	E	F
A	6	5	4
B	1	2	3
C	1	5	3

$u_2(-,-,R)$	D	E	F
A	6	1	1
B	5	1	1
C	4	1	2

$u_2(-,-,S)$	D	E	F
A	1	5	1
B	2	4	2
C	3	3	3

For each $X \in \{A,B,C,D,E,F\}$, find all strictly, weakly, and very weakly dominant strategies in game G_{-X}, where G_{-X} is created from G by deleting action X.

Problem 4 [7 points]

Consider the following Bayesian game: There are two players, they have two actions A, B, and they have two types S, R. Type S means the player wants to play the same action as the other player, R means he wants to play the other action. Specifically, the gain is $+3$ if this goal is achieved, plus there is bonus $+1$ for playing action A.

Formally: $G_P = (\{1,2\}, (\{A,B\},\{A,B\}), (\{S,R\},\{S,R\}), (u_1,u_2), P)$, where u_1, u_2 are given by the following matrices:

$u_1(-,-,S)$	A	B
A	4	1
B	0	3

$u_1(-,-,R)$	A	B
A	1	4
B	3	0

$u_2(-,-,S)$	A	B
A	4	0
B	1	3

$u_2(-,-,R)$	A	B
A	1	3
B	4	0

Let $BNE(G_P)$ denote the set of Bayesian Nash equilibria in game G_P. Moreover, let $UV|XY$ denote the strategy profile $(\{(S,U),(R,V)\},\{(S,X),(R,Y)\})$ (i.e., player 1 plays U if he is S and he plays V if he is R; similarly for player 2). Find a distribution P such that:

a) $BNE(G_P) = \emptyset$;

b) $BNE(G_P) = \{AA|AB, AB|AA\}$;

c) $BNE(G_P) = \{AB|AB\}$;

d) $BNE(G_P) = \{AB|AB, BA|BA\}$;

e) $BNE(G_P) = \{AA|AB\}$;

f) $|BNE(G_P)| = 5$.

Furthermore, P is required to satisfy that for every player $i \in \{1,2\}$ and every type $t \in \{S,R\}$, the probability that i is of type t is positive.