

IA168 — Problem set 3

Problem 1 [5 points]

Consider the following two-player strategic-form game G :

	X	Y
A	$(5, 5)$	$(-1, 6)$
B	$(6, -1)$	$(1, 1)$

- a) In G_{irep}^{avg} , find a subgame-perfect equilibrium whose outcome is $(4.5, 4.2)$.
- b) Calculate $\inf_{s \in SPE(G_{irep}^{avg})} u_1(s)$.
- c) Calculate $\sup_{s \in SPE(G_{irep}^{avg})} u_1(s)$.

Justify your reasoning.

Problem 2 [4 points]

Consider the following two-player strategic-form game G , with real-valued parameters x, y :

	A	B
A	$(2, 1)$	$(7, -1)$
B	$(-2, 6)$	(x, y)

The players will play an infinite number of rounds, with a discount factor δ . Both will play the following strategy: If only B 's have been played so far (i.e., the current history lies in $(B, B)^*$), then the player plays B ; otherwise he plays A . Let s denote the corresponding strategy profile.

Find all pairs $(x, y) \in \mathbb{R} \times \mathbb{R}$ for which $\inf\{\delta \in \mathbb{R} : 0 < \delta < 1 \wedge s \text{ is a SPE in } G_{irep}^\delta\} = 3/5$.
Justify your reasoning.

Problem 3 [4 points]

Consider the incomplete-information game $G = (\{1, 2\}, (\{A, B, C\}, \{D, E, F\}), (\{P, Q\}, \{R, S\}), (u_1, u_2))$, where u_1, u_2 are given by the following matrices:

$u_1(-, -, P)$		D	E	F	$u_1(-, -, Q)$		D	E	F
	A	6	5	4		A	6	5	4
	B	1	2	5		B	1	2	3
	C	1	2	3		C	1	5	3
$u_2(-, -, R)$		D	E	F	$u_2(-, -, S)$		D	E	F
	A	6	1	1		A	1	5	1
	B	5	1	1		B	2	4	2
	C	4	1	2		C	3	3	3

For each $X \in \{A, B, C, D, E, F\}$, find all strictly, weakly, and very weakly dominant strategies in game G_{-X} , where G_{-X} is created from G by deleting action X .

Problem 4 [7 points]

Consider the following Bayesian game: There are two players, they have two actions A, B , and they have two types S, R . Type S means the player wants to play the same action as the other player, R means he wants to play the other action. Specifically, the gain is $+3$ if this goal is achieved, plus there is bonus $+1$ for playing action A .

Formally: $G_P = (\{1, 2\}, (\{A, B\}, \{A, B\}), (\{S, R\}, \{S, R\}), (u_1, u_2), P)$, where u_1, u_2 are given by the following matrices:

$$\begin{array}{c|cc} u_1(-, -, S) & A & B \\ \hline A & 4 & 1 \\ B & 0 & 3 \end{array} \qquad \begin{array}{c|cc} u_1(-, -, R) & A & B \\ \hline A & 1 & 4 \\ B & 3 & 0 \end{array}$$

$$\begin{array}{c|cc} u_2(-, -, S) & A & B \\ \hline A & 4 & 0 \\ B & 1 & 3 \end{array} \qquad \begin{array}{c|cc} u_2(-, -, R) & A & B \\ \hline A & 1 & 3 \\ B & 4 & 0 \end{array}$$

Let $BNE(G_P)$ denote the set of Bayesian Nash equilibria in game G_P . Moreover, let $UV|XY$ denote the strategy profile $(\{(S, U), (R, V)\}, \{(S, X), (R, Y)\})$ (i.e., player 1 plays U if he is S and he plays V if he is R ; similarly for player 2). Find a distribution P such that:

- $BNE(G_P) = \emptyset$;
- $BNE(G_P) = \{AA|AB, AB|AA\}$;
- $BNE(G_P) = \{AB|AB\}$;
- $BNE(G_P) = \{AB|AB, BA|BA\}$;
- $BNE(G_P) = \{AA|AB\}$;
- $|BNE(G_P)| = 5$.

Furthermore, P is required to satisfy that for every player $i \in \{1, 2\}$ and every type $t \in \{S, R\}$, the probability that i is of type t is positive.