$$b_0 + b_1 x + \cdots + b_{k-1} x^{k-1}$$

$x^{n-k} \cdot$

$$\underbrace{c_0 + \cdots + c_{n-k-1} x^{n-k-1}}_{\text{kontrolní}} + \underbrace{b_0 x^{n-k} + b_1 x^{n-k+1} + \cdots + b_{k-1} x^{n-1}}_{\text{informační}}$$

$$v(x) = q(x) \cdot (x+1) + q(-1)$$

$r(x) = q(-1) = q(1) =$ počet 1 v zápise q (mod 2)

---

nerozpoznání chyby $\iff$ $\underset{\text{vždy}}{v(x)}, u(x)$ kódové slovo

$\iff$ $e(x)$ kódové slovo

tj. $p(x) \mid e(x)$

jednoduchá chyba $\quad e(x) = x^i$

rozpoznatelná pro $p(x) \neq x^m$

dvojitá chyba $\quad e(x) = x^i + x^{i+l} = x^i(1 + x^l)$

$x^{2^m-1} = 1$ v $G(2^m)$

g "prim. kořen" $\Rightarrow$ g je kořen $\underset{\substack{\| \\ p_1(x) \cdots p_r(x)}}{x^{2^m-1} - 1}$

$g : (\mathbb{Z}_2)^k \longrightarrow (\mathbb{Z}_2)^n$ $\leftarrow$ n-tice zbytk. tříd mod 2
$(0/1, \ldots, 0/1)$

$u \longmapsto G \cdot u$

$\uparrow$ matice kódu

$(0,1) + (1,1) = (1,0)$

$$\begin{array}{r} 01 \\ 11 \\ \hline 100 \end{array} \quad \times$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow$ kódová slova = součty sloupců G

$$u(x) \longrightarrow r(x) + x^{n-k} \cdot u(x)$$

$$u'(x) \longrightarrow r'(x) + x^{n-k} \cdot u'(x)$$

$$u(x) + u'(x) \overset{?}{\longrightarrow} r(x) + r'(x) + x^{n-k} \cdot (u(x) + u'(x))$$

$$\left(\begin{matrix} \left.\begin{matrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{matrix}\right\} P \\ \hline \left.\begin{matrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{matrix}\right\} I \end{matrix}\right) = G$$

control

$$G \cdot u = \begin{pmatrix} P \\ \hline I \end{pmatrix} u = \begin{pmatrix} P \cdot u \\ \hline I \cdot u \end{pmatrix} = \begin{pmatrix} P \cdot u \\ \hline u \end{pmatrix}$$

info

Kdy v je tvaru $G \cdot u$ pro nějaké u?

$$v = \begin{pmatrix} w \\ \hline u \end{pmatrix}, \qquad G \cdot u = \begin{pmatrix} P \cdot u \\ \hline u \end{pmatrix}$$

$$\Leftrightarrow\ w = P \cdot u \ \Leftrightarrow\ I \cdot w + P \cdot u = 0 \ \Leftrightarrow\ (I \ P) \begin{pmatrix} w \\ u \end{pmatrix} = 0$$

$$
\begin{pmatrix}
1 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 \\
\hdashline
1 & & & \\
 & 1 & & \\
 & & 1 & \\
 & & & 1
\end{pmatrix} \leftarrow G
\cdot
\begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}
=
\begin{pmatrix} 1 \\ 0 \\ 0 \\ \hdashline 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}
\quad \Bigg\| \quad
\begin{pmatrix} 0 \\ 0 \\ 1 \\ \hdashline 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}
+
\begin{pmatrix} 0 \\ 0 \\ 0 \\ \hdashline 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}
$$

$$
\begin{pmatrix} 0 \\ 0 \\ 1 \\ \hdashline 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}
=
\begin{pmatrix} 1 \\ 0 \\ 0 \\ \hdashline 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}
+
\begin{pmatrix} 1 \\ 0 \\ 1 \\ \hdashline 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}
=
\left(
\begin{pmatrix} 1 \\ 0 \\ 0 \\ \hdashline 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}
+
\begin{pmatrix} 1 \\ 0 \\ 1 \\ \hdashline 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}
\right)
+
\left(
\begin{pmatrix} 1 \\ 0 \\ 1 \\ \hdashline 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}
+
\begin{pmatrix} 1 \\ 0 \\ 1 \\ \hdashline 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}
\right)
$$

$$C_n = n-1 + \frac{1}{n}(C_0 + C_{n-1}) + \cdots + \frac{1}{n}(C_{n-1} + C_0)$$

$$= n-1 + \frac{2}{n}(C_0 + \cdots + C_{n-1})$$

$$nC_n = n(n-1) + 2(C_0 + \cdots + C_{n-1})$$

$$(n-1)C_{n-1} = (n-1)(n-2) + 2(C_0 + \cdots + C_{n-2})$$

$$nC_n - (n-1)C_{n-1} = 2(n-1) + 2C_{n-1}$$

$$nC_n = (n+1)C_{n-1} + 2(n-1)$$

$$\frac{C_n}{n+1} = \frac{C_{n-1}}{n} + 2\frac{n-1}{n(n+1)}$$

$$= \left(\frac{C_{n-2}}{n-1} + 2\frac{n-2}{(n-1)n}\right) + 2\frac{n-1}{n(n+1)}$$

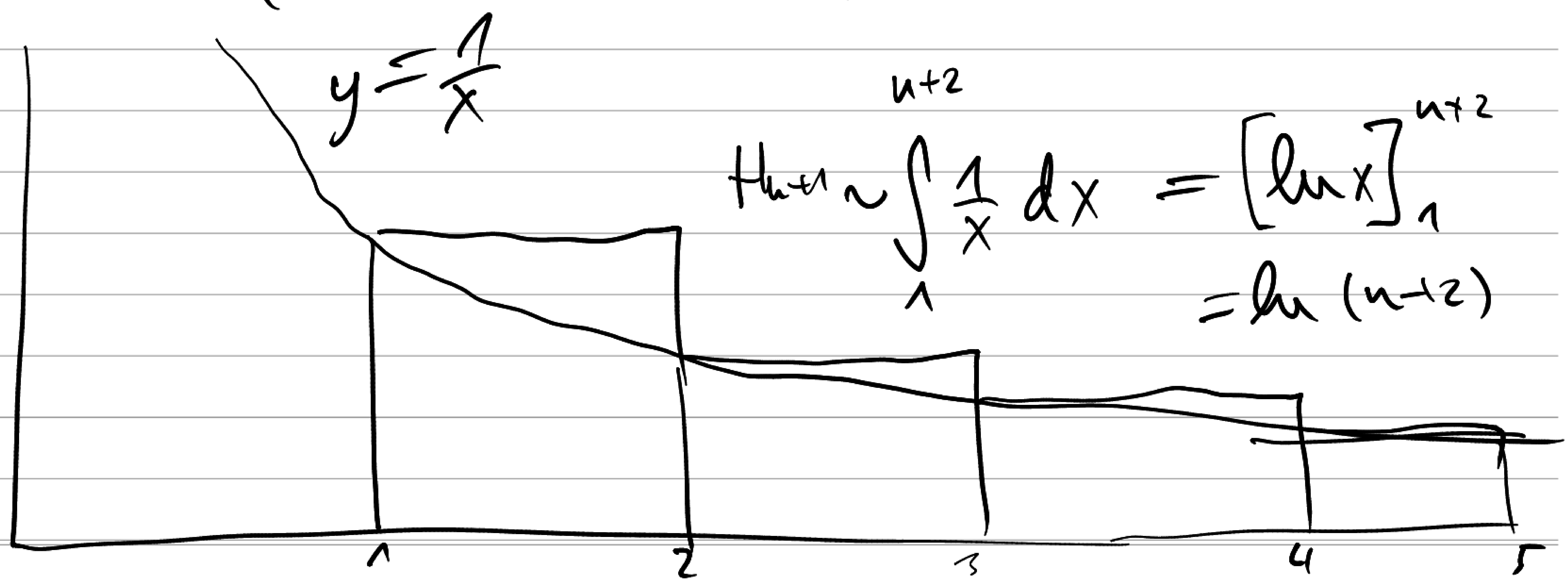
$$= \left(\frac{C_{n-3}}{n-2} + 2\frac{n-3}{(n-2)(n-1)}\right) + 2\frac{n-2}{(n-1)n} + 2\frac{n-1}{n(n+1)}$$

$$\frac{c_n}{n+1} = 2\left(\frac{n-1}{n(n+1)} + \frac{n-2}{(n-1)n} + \cdots + \frac{1}{2\cdot 3}\right) + \frac{c_1}{2}$$

$$\frac{n-1}{n(n+1)} = \frac{2}{n+1} - \frac{1}{n}, \qquad \frac{n-2}{(n-1)n} = \frac{2}{n} - \frac{1}{n-1}, \qquad \ldots, \qquad \frac{1}{2\cdot 3} = \frac{2}{3} - \frac{1}{2}$$

$$= 2\left(\frac{2}{n+1} + \frac{1}{n} + \frac{1}{n-1} + \cdots + \frac{1}{3} - \frac{1}{2}\right) + \frac{c_1}{2}, \qquad \frac{c_1}{2} = 0$$

$$= 2\left(\frac{4}{n+1} + H_{n+1} - 2\right)$$



$$H_{n+1} \sim \int_1^{n+2} \frac{1}{x}\,dx = [\ln x]_1^{n+2} = \ln(n+2)$$

$$\binom{n}{k} = \{T \subseteq S \mid |T| = k\}$$

$$|S| = n$$

$$|A \sqcup B| = |A| + |B|$$

$$|A \times B| = |A| \cdot |B|$$

$$\Big|\bigsqcup_{i \in A} B_i\Big| = |A| \cdot |B_i|$$

pokud nezávisí na i

$$|S_n| = n!$$

$$V(n,k) = \frac{n!}{(n-k)!} = C(n,k) \cdot k!$$

$$C(n,k) = \binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

$$\binom{10}{3} = \frac{10!}{7! \cdot 3!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}$$

$k = 2$ $\quad p_1 = \underset{1}{m}, \quad p_2 = \underset{0}{n-m}$

$$S \quad \begin{matrix} 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 & s_8 & s_9 \end{matrix} \longleftrightarrow \{s_1, s_3, s_4, s_7\} \quad \begin{matrix} 1 & m\times \\ 0 & (n-m)\times \end{matrix}$$

$$\rightsquigarrow \binom{n}{m} = \frac{n!}{m!\,(n-m)!}$$

$$\begin{array}{ccccccc} 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ s_1 & s_2 & s_3 & s_4 & s_5 & s_6 & s_7 \\ 2 & 1 & 0 & 1 & 0 & 0 & 3 \end{array}$$

$11 \mid 1 \mid \mid 1 \mid \mid \mid 111 \leftarrow$ k = počet jedniček

$n-1$ = počet $|$

$$\# = \binom{n+k-1}{k}$$