$$|M \setminus (A \cup B)| = |M| - |A| - |B| + |A \cap B|$$

$$|M \setminus (A \cup B \cup C)| = |M| - |A| - |B| - |C| + |A \cap B| + |A \cap C| + |B \cap C| - |A \cap B \cap C|$$

$$\binom{r}{0} - \binom{r}{1} + \binom{r}{2} - \binom{r}{3} + \cdots = \begin{cases} 0 & \text{pro } r > 0 \\ 1 & \text{pro } r = 0 \end{cases}$$

$$\binom{15}{4} - \binom{13}{2} \qquad |M \setminus A| = |M| - |A|$$

všechny — obsahující oba dva prosl. posl.

$$a_0 + \cdots + a_n = (n+1) \cdot \frac{a_0 + a_n}{2}$$

arithm.

$$0 + \cdots + n = (n+1) \cdot \frac{n}{2}$$

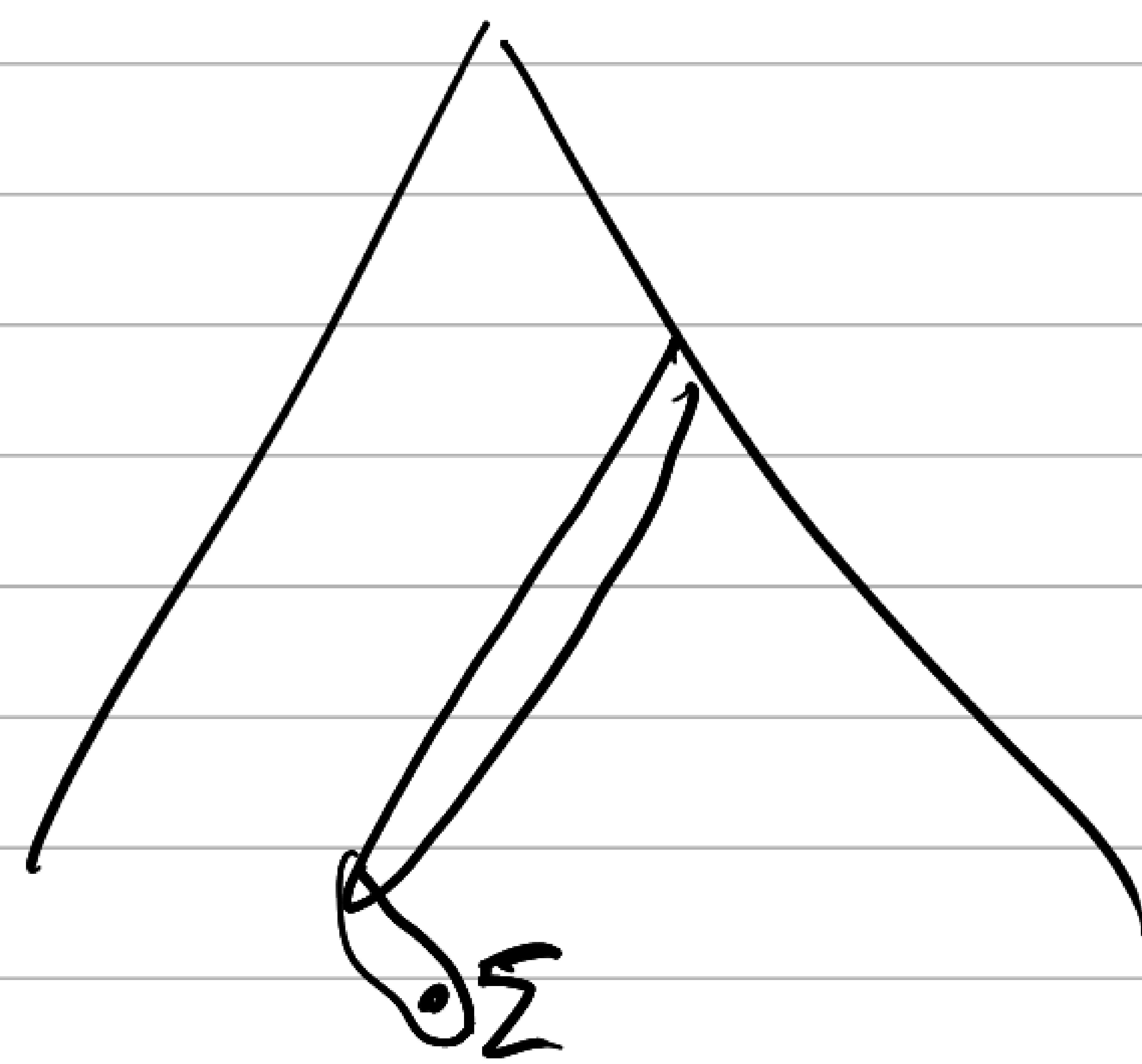
$$a_0 + \cdots + a_n = \frac{a_{n+1} - a_0}{q - 1}$$

geom s kvoc. q

$$1 + x + x^2 + \cdots + x^n = \frac{x^{n+1} - 1}{x - 1}$$

$(x+y) \cdots (x+y)$

$x^k y^{n-k}$ --- počet výběrů k závorek z n
↳ x, zbylé y



$x_1 < \dots < x_{m+1}$ z $\{1, \dots, n+1\}$

$x_{m+1} = n+1$: $x_1, \dots, x_m$ vybírané z $1, \dots, n$

$x_{m+1} = n$ $\quad 1, \dots, n-1$

$\vdots$

$x_{m+1} = m+1$ $\quad 1, \dots, m$

$1 \quad 2 \quad 3 \quad \dots \quad 100$

$\sigma(1) \quad \sigma(2) \quad \sigma(3) \quad \dots \quad \sigma(100)$

$$= 1-[\ln x]_{50}^{100} = 1-(\ln 100 - \ln 50) = 1-\ln\frac{100}{50} = 1-\ln 2$$

Chceme všechny cykly σ délky ≤ 50

obráceně: cyklus délky $k \geq 51$ ... přesně jeden

jejich počet $\binom{100}{k}\cdot\frac{k!}{k}\cdot(100-k)!$

↑ cyklické permutace $1,\dots,k$

$$\begin{tikzcd}
1 \arrow[r] & x_1 \arrow[r] & x_2 \arrow[r] & \dots \arrow[r] & x_{k-1} \arrow[llll, bend left]
\end{tikzcd}$$

prst.:

$$\frac{100! - \sum_{k=51}^{100}\binom{100}{k}\cdot\frac{k!}{k}(100-k)!}{100!}$$

$$= 1-\sum_{k=51}^{100}\frac{\frac{\cancel{100!}}{\cancel{k!}\,\cancel{(100-k)!}}\cdot\frac{\cancel{k!}}{k}\,\cancel{(100-k)!}}{\cancel{100!}} = 1-\sum_{k=51}^{100}\frac{1}{k} \approx 1-\int_{50}^{100}\frac{1}{x}\,dx$$

$$(1+x+x^2+\cdots+x^5)(1+x+\cdots)(1+x+\cdots)$$

$$= \frac{1-x^6}{1-x}\,\frac{1}{1-x}\,\frac{1}{1-x} = \frac{1-x^6}{(1-x)^3}$$

$$(y+x)^n = \sum \binom{n}{k} y^{n-k} x^k$$

$y=1$ $\qquad$ $y=1$

$$\begin{array}{c} \to 1 \\ \to 1\ 1 \\ \to 1\ 2\ 1 \\ 1\ 3\ 3\ 1 \\ 1\ 4\ 6\ 4\ 1 \end{array}$$

$$(1+x)^n = \sum \binom{n}{k} x^k \qquad / \ ()'$$

$$n(1+x)^{n-1} = \sum \binom{n}{k} k x^{k-1} \qquad / \ (1)$$

$$n\, 2^{n-1} = \sum \binom{n}{k} \cdot k$$

$$\binom{n}{k} = \frac{n \cdot (n-1) \cdots (n-k+1)}{k \cdot (k-1) \cdots 1}$$

$$= \frac{n}{k} \cdot \binom{n-1}{k-1}$$

$$(a_0, a_1, a_2, \dots) \rightleftarrows a_0 + a_1 x + a_2 x^2 + \dots$$

sečtení

Taylorův rozvoj

$$a(x) = \sum a_k x^k \qquad a(x) = \sum \frac{a^{(k)}(0)}{k!} x^k$$

Příklad: $1 + 2 + \dots + n = \frac{n(n+1)}{2}$

$$\frac{1}{1-x} = \sum_{k \geq 0} x^k \qquad \Big/ \int_0^x dx$$

$$-\ln(1-x) = \sum_{k \geq 0} \frac{x^{k+1}}{k+1} = \sum_{k \geq 1} \frac{x^k}{k}$$

subst. $k+1 \rightsquigarrow k$

$$\frac{x^1}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \dots$$

$$(1+x)^\alpha = f(x) = \sum_{k\geq 0} \frac{f^{(k)}(0)}{k!} x^k \qquad \alpha \in \mathbb{R}$$

$$= \sum_{k\geq 0} \frac{\alpha(\alpha-1)\cdots(\alpha-k+1)}{k!} x^k$$

$$\alpha(1+x)^{\alpha-1} = f'(x)$$

$$\alpha(\alpha-1)(1+x)^{\alpha-2} = f''(x)$$

$$\vdots$$

$$\alpha(\alpha-1)\cdots(\alpha-k+1)(1+x)^{\alpha-k} = f^{(k)}(x)$$

Značení: $\binom{\alpha}{k} = \frac{\alpha(\alpha-1)\cdots(\alpha-k+1)}{k(k-1)\cdots 1}$

$$(1-x)^{-n} = \sum \binom{-n}{k} (-x)^k$$

$$= \sum \frac{(-n)(-n-1)\cdots(-n-k+1)}{k(k-1)\cdots 1} (-1)^k x^k$$

$$= \sum \frac{\cancel{(-1)^k}\,(n+k-1)(n+k-2)\cdots(n+1)n}{k(k-1)\cdots 2\cdot 1} \cancel{(-1)^k} x^k$$

$$= \sum \binom{n+k-1}{k} x^k = \sum \binom{k+n-1}{n-1} x^k$$

$n=1$: $\frac{1}{(1-x)^1} = \sum \binom{k+0}{0} x^k$ tj. $\frac{1}{1-x} = \sum x^k$

$n=2$: $\frac{1}{(1-x)^2} = \sum \binom{k+1}{1} x^k$ tj. $\frac{1}{(1-x)^2} = \sum (k+1) x^k$

$n=3$: $\frac{1}{(1-x)^3} = \sum \binom{k+2}{2} x^k$ tj. $\frac{1}{(1-x)^3} = \sum \frac{(k+2)(k+1)}{2} x^k$

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pravidla: $(a_0 + a_1 x + a_2 x^2 + \dots) + (b_0 + b_1 x + b_2 x^2 + \dots)$

$= (a_0 + b_0) + (a_1 + b_1) x + (a_2 + b_2) x^2 + \dots$

$\alpha \cdot (a_0 + a_1 x + a_2 x^2 + \dots)$

$= (\alpha \cdot a_0) + (\alpha \cdot a_1) x + (\alpha \cdot a_2) x^2 + \dots$

$$a(x)\,b(x) = (a_0 + a_1x + a_2x^2 + \dots)(b_0 + b_1x + b_2x^2 + \dots)$$

$$= a_0b_0 + (a_1b_0 + a_0b_1)x + (a_2b_0 + a_1b_1 + a_0b_2)x^2 + \dots$$

$$a(x) = 1 + x + x^2 + \dots = \frac{1}{1-x}$$

$$\frac{1}{1-x}\,b(x) = b_0 + (b_0 + b_1)x + (b_0 + b_1 + b_2)x^2 + \dots$$

$$\frac{1}{1-x} \cdot \ln\frac{1}{1-x}$$

$$(1, 1, \dots) * (0, \tfrac{1}{1}, \tfrac{1}{2}, \dots) = (0, \tfrac{1}{1}, \tfrac{1}{1} + \tfrac{1}{2}, \dots)$$

$$\frac{1}{1-x} \cdot \frac{1}{1-x}$$

$a_k = k+1$

$$(1, 1, \dots) * (1, 1, \dots) = (1, 2, 3, \dots)$$

$$\frac{1}{1-x} \cdot \frac{1}{(1-x)^2}$$

$\binom{k+2}{2}$

$$(1, 1, \dots) * (1, 2, 3, \dots) = (1, 1+2, 1+2+3, \dots)$$