Risk measurement

Overview

1. Probability and Statistics
    - Probability distribution
    - Mean
    - Standard deviation
2. Law of large numbers
    - Central limit theorem
3. Loss Forecasting
    - Probability analysis
    - Regression analysis
    - Forecasting based on loss distributions

- To determine expected losses, insurance actuaries apply probability and statistical analysis to given loss situations.
- The probability of an event is simply the long-run relative frequency of the event, given an infinite number of trials with no changes in the underlying conditions.
- The probability of some events can be determined without experimentation - e.g.: If a “fair”coin is flipped in the air, the probability the coin will come up ”heads”is 50%, and the probability it will come up ”tails”is also 50%.
- Other probabilities, such as the probability of dying during a specified year or the probability of being involved in an auto accident, can be estimated from past loss data.

- A convenient way of summarizing events and probabilities is through a probability distribution.
- A probability distribution lists events that could occur and the corresponding probability of each event’s occurrence.
- Probability distributions may be discrete, meaning that only distinct outcomes are possible, or continuous, meaning that any outcome over a range of outcomes could occur.
- e.g.: The number of runs scored in a baseball game is a discrete measure, as partial runs cannot be scored. Speed and temperature are continuous measures, as all values over the range of values can occur.

- Probability distributions are characterized by two important measures: central tendency and dispersion.
- Although there are several measures of central tendency, the measure most often employed is the mean (μ) or expected value (EV) of the distribution.
- (Other measures of central tendency are the median, which is the middle observation in a probability distribution, and the mode, which is the observation that occurs most often.)
- The mean or expected value is found by multiplying each outcome by the probability of occurrence, and then summing the resulting products:

$$\mu \text{ or } EV = \sum X_i P_i$$

e.g.: Assume that an actuary estimates the following probabilities of various losses for a certain risk:

Amount of Loss (Xi)		Probability of Loss (Pi)		X_iP_i
$0	X	0.30	=	$0
$360	X	0.50	=	$180
$600	X	0.20	=	$120
		$\sum X_iP_i$	=	$300

Thus, we could say that the mean or expected loss given the probability distribution is $300.

Although the mean value indicates central tendency, it does not tell us anything about the riskiness or dispersion of the distribution. Consider a 2nd probability-of-loss distribution:

Amount of Loss (Xi)		Probability of Loss (Pi)		X_iP_i
\$225	X	0.40	=	\$90
\$350	X	0.60	=	\$210
		$\sum X_iP_i$	=	\$300

This distribution also has a mean loss value of \$300. However, the 1st distribution is riskier because the range of possible outcomes is from \$0 to \$600. With the 2nd distribution, the range of possible outcomes is only \$125 (\$350 — \$225), so we are more certain about the outcome with the 2nd distribution.

- 2 standard measures of dispersion to characterize the variability or dispersion about the mean value: the variance (σ 2) and the standard deviation (σ)
- The variance of a probability distribution is the sum of the squared differences between the possible outcomes and the expected value, weighted by the probability of the outcomes:

$$\sigma^2 = \sum P_i(X_i - EV)^2$$

- is the average squared deviation between the possible outcomes and the mean

Because the variance is in ”squared units”, it is necessary to take the square root of the variance so that the central tendency and dispersion measures are in the same units. The square root of the variance is the standard deviation. The variance and standard deviation of the first distribution:

$$\sigma^2 = 0.30(0 - 300)2 + 0.50(360 - 300)2 + 0.20(600 - 300)2$$

$$= 27,000 + 1,800 + 18,000$$

$$= 46,800$$

$$\sigma = \sqrt{46,800} = 261.33$$

For the 2nd distribution, the variance and standard deviation:

$$\sigma^2 = 0.40(225 - 300)2 + 0.60(350 - 300)2$$

$$= 2,250 + 1,500$$

$$= 3,750$$

$$\sigma = \sqrt{3,750} = 61.24$$

Thus, while the means of the two distributions are the same, the standard deviations are significantly different.

- Higher standard deviations, relative to the mean, are associated with greater uncertainty of loss; therefore, risk is higher.
- Lower standard deviations, relative to the mean, are associated with less uncertainty of loss; therefore, risk is lower.

Probability distribution

- The two probability distributions used in the discussion of central tendency and dispersion are ”odd”in that only three and two possible outcomes, respectively, could occur.
- In addition, specific probabilities corresponding to the loss levels are assigned. In practice, estimating the frequency and severity of loss is difficult.
- Insurers can employ both actual loss data and theoretical loss distributions in estimating losses (e.g.: the binomial, Poisson and normal distribution).

Law of large numbers

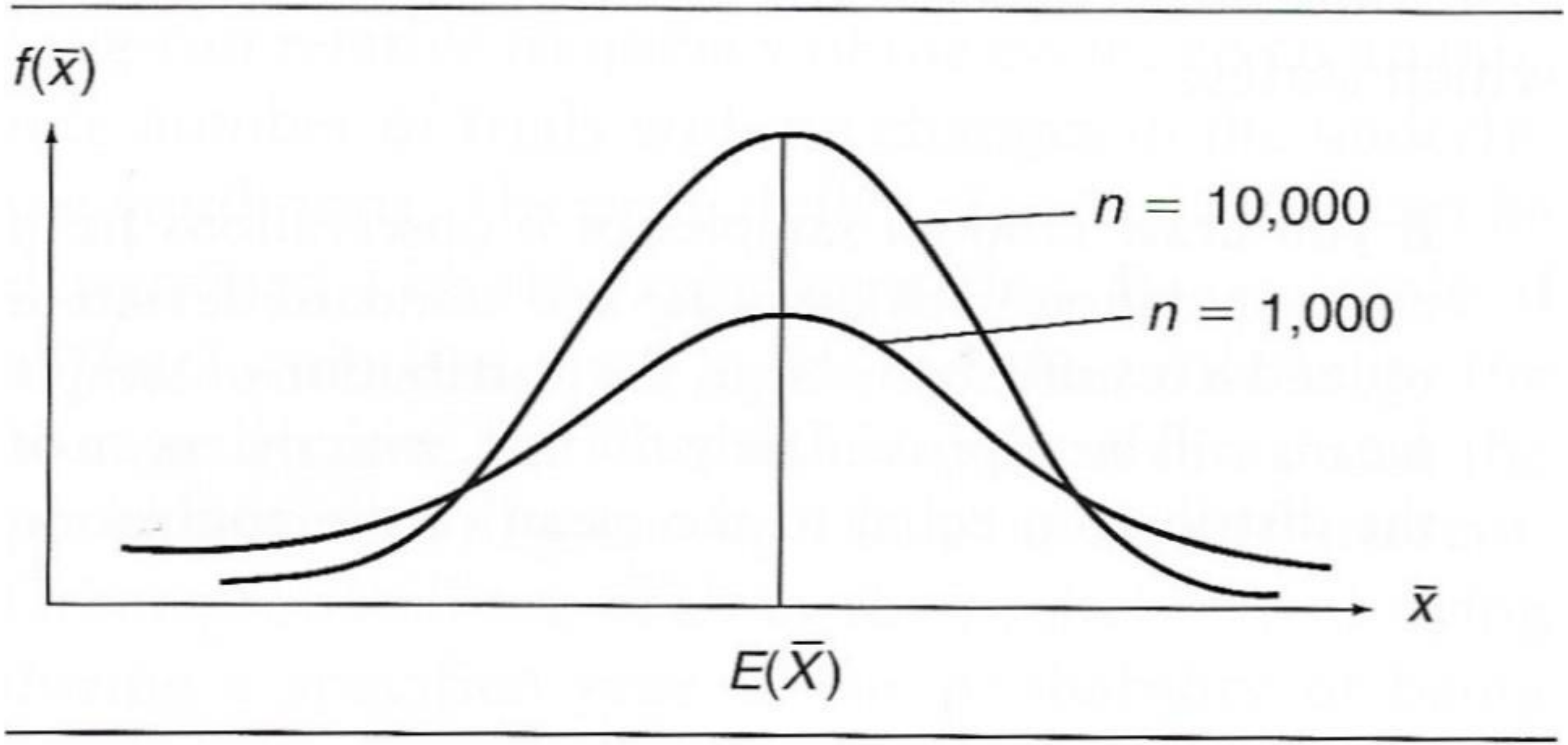
- Even if the characteristics of the population were known with certainty, insurers do not insure populations.
- Rather, they select a sample from the population and insure the sample.
- Obviously, the relationship between population parameters and the characteristics of the sample (mean and standard deviation) is important for insurers, since actual experience may vary significantly from the population parameters.
- The characteristics of the sampling distribution help to illustrate the law of large numbers, the mathematical foundation of insurance.

- It can be shown that the average losses for a random sample of n exposure units will follow a normal distribution because of the Central Limit Theorem, which states:
- If you draw random samples of *n* observations from any population with mean μ_x and standard deviation σ_x, and *n* is sufficiently large, the distribution of sample means will be approximately normal, with the mean of the distribution equal to the mean of the population $\mu_x = \mu_x$ and the standard error of the sample mean σ_x equal to the standard deviation of the population (σ_x) divided by the square root of $n(\sigma_x = \sigma_x / \sqrt{n})$.
- This approximation becomes increasingly accurate as the sample size, *n*, increases.

The central limit theorem - implications for insurers:

1. The sample distribution of means does not depend on the population distribution, provided *n* is sufficiently large. Regardless of the population distribution (bimodal, unimodal, symmetric, skewed right, skewed left, and so on), the distribution of sample means will approach the normal distribution as the sample size increases.

1. Sampling distribution versus sample size



- The normal distribution is a symmetric, bell-shaped curve.
- It is defined by the mean and standard deviation of the distribution.
- About 68% of the distribution lies within one standard deviation of the mean, and about 95% of the distribution lies within two standard deviations of the mean.
- The normal curve has many statistical applications (hypothesis testing, confidence intervals, and so on) and is easy to use.

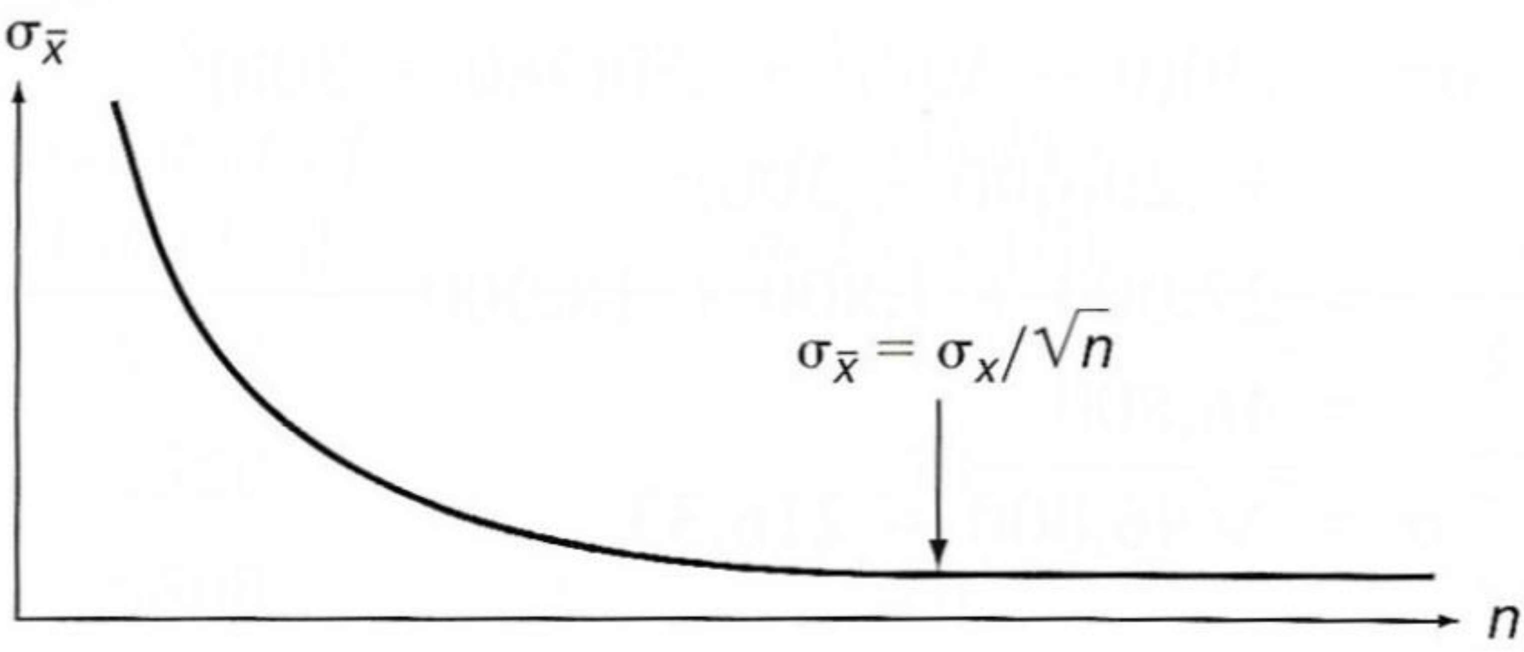
2. The standard error of the sample means distribution declines as the sample size increases.
   Recall that the standard error is defined as $\sigma_x = \sigma_x / \sqrt{n}$ the standard error of the sample mean loss distribution is equal to the standard deviation of the population divided by the square root of the sample size.
   Because the population standard deviation is independent of the sample size, the standard error of the sampling distribution, σ_x, can be reduced by simply increasing the sample size.

e.g.: Assume that an insurer would like to select a sample to insure from a population where the mean loss is $500 and the standard deviation is $350. As the insurer increases the number of units insured (*n*), the standard error of the sampling distribution σ_x will decline. The standard error for various sample sizes is summarized below:

n	σ_x
10	110.68
100	35.00
1,000	11.07
10,000	3.50
100,000	1.11

Thus, as the sample size increases, the difference between actual results and expected results decreases. Indeed, approaches zero as n gets very large.

2. Standard error of the sampling distribution versus sample size



2. Standard error of the sampling distribution versus sample size

- Obviously, when an insurer increases the size of the sample insured, underwriting risk (maximum insured losses) increases because more insured units could suffer a loss.
- The underwriting risk for an insurer is equal to the number of units insured multiplied by the standard error of the average loss distribution, σ_x.
- Recalling that σ_x is equal to $\sigma_x / \sqrt{n}$, the expression for underwriting risk: $n \times \sigma_x = n \times \sigma_x / \sqrt{n} = \sqrt{n} \times \sigma_x$
- Thus, while underwriting risk increases with an increase in the sample size, it does not increase proportionately.
- Insurance companies are in the loss business — they expect some losses will occur. It is the deviation between actual losses and expected losses that is the major concern. By insuring large samples, insurers reduce their objective risk. There truly is ”safety in numbers” for insurers.

- A risk manager must also identify the risks the organization faces, and then analyze the potential frequency and severity of these loss exposures.
- Although loss history provides valuable information, there is no guarantee that future losses will follow past loss trends.
- Risk managers can employ a number of techniques to assist in predicting loss levels:
    1. Probability analysis
    2. Regression analysis
    3. Forecasting based on loss distributions

- Chance of loss is the possibility that an adverse event will occur.
- The probability (P) of such an event is equal to the number of events likely to occur (X) divided by the number of exposure units (N).
- Thus, if a vehicle fleet has 500 vehicles and on average 100 vehicles suffer physical damage each year, the probability that a fleet vehicle will be damaged in any given year is:
  P(physical damage) = 100/500 = 0.20 or 20%

Types of events:

1. Independent events

   - the occurrence does not affect the occurrence of another event
   - e.g.: Assume that a business has production facilities in Prague and Brno, and that the probability of a fire at the Prague plant is 5% and that the probability of a fire at the Brno plant is 4%. Obviously, the occurrence of one of these events does not influence the occurrence of the other event. If events are independent, the probability that they will occur together is the product of the individual probabilities. Thus, the probability that both production facilities will be damaged by fire is:

     P(fire at Prague plant) × P(fire at Brno plant) = P(fire at both plants) = 0.04 × 0.05 = 0.002 or 0.2%

2. Dependent events

- the occurrence of one event affects the occurrence of the other
- If 2 buildings are located close together, and one building catches on fire, the probability that the other building will burn is increased.
- e.g.: Suppose that the individual probability of a fire loss at each building is 3%. The probability that the 2nd building will have a fire given that the 1st building has a fire, however, may be 40%. What is the probability of two fires? This probability is a conditional probability that is equal to the probability of the 1st event multiplied by the probability of the 2nd event given that the 1st event has occurred:

P(fire at one bldg) × P(fire at second plant given fire at first bldg) = P(both burn) = 0.03 × 0.40 = 0.012 or 1.20%

3. Mutually exclusive events

- if the occurrence of one event precludes the occurrence of the 2nd event
- e.g.: If a building is destroyed by fire, it cannot also be destroyed by flood.
- probabilities are additive – e.g.: If the probability that a building will be destroyed by fire is 2% and the probability that the building will be destroyed by flood is 1%, then the probability the building will be destroyed by either fire or flood is:

  P(fire destroys bldg) + P(flood destroys bldg) = P(fire or flood destroys bldg) = 0.02 + 0.01 = 0.03 or 3%

4. If the independent events are not mutually exclusive, then more than 1 event could occur.

   - Care must be taken not to ”double-count”when determining the probability that at least one event will occur.
   - e.g.: If the probability of minor fire damage is 4% and the probability of minor flood damage is 3%, then the probability of at least 1 of these events occurring is:

   P(minor fire) + P(minor flood) − P(minor fire and flood) =
   P(at least one event) $0.04 + 0.03 - (0.04 \times 0.03) = 0.0688$ or 6.88%

   Assigning probabilities to individual and joint events and analyzing the probabilities can assist the risk manager in formulating a risk treatment plan.

- characterizes the relationship between 2 or more variables and then uses this characterization to predict values of a variable
- 1 (the dependent) variable is hypothesized to be a function of 1 or more independent variables.
- It is not difficult to envision relationships that would be of interest to risk managers in which 1 variable is dependent on another variable.
- e.g.: Consider workers compensation claims. It is logical to hypothesize that the number of workers compensation claims should be positively related to some variable representing employment (such as the number of employees, payroll, or hours worked).
- e.g.: We would expect the number of physical damage claims for a fleet of vehicles to increase as the size of the fleet increases or as the number of miles driven each year by fleet vehicles increases.

- The number of claims is plotted against payroll.
- Regression analysis provides the coordinates of the line that best fits the points on the chart.
- This line will minimize the sum of the squared deviations of the points from the line. Our hypothesized relationship: Number of workers compensation claims = B_0 + (B_1 × P ayroll [in thousands]) where B_0 is a constant and B_1 is the coefficient of the independent variable.
  The coefficient of determination, R-square, ranges from 0 to 1 and measures the model fit. An R-square value close to 1 indicates that the model does a good company's annual payroll in thousands of dollars and job of predicting Y values. By substituting the estimated payroll for next year (in thousands), the risk manager estimates that 509 workers compensation claims will occur in the next year.

3. Forecasting based on loss distributions

- A loss distribution is a probability distribution of losses that could occur.
- Forecasting by using loss distributions works well if losses tend to follow a specified distribution and the sample size is large.
- Knowing the parameters that specify the loss distribution (for example, mean, standard deviation, and frequency of occurrence) enables the risk manager to estimate the number of events, severity, and confidence intervals.

- Assume that the number of weather-related property losses is normally distributed with a mean (m) equal to 16 and standard deviation (s) equal to 3. What is the probability that the number of weather-related property losses will be between 16 and 22?
- Assume the number of physical damage losses for a large fleet of vehicles is normally distributed with a mean of 400 and a standard deviation of 80. What is the probability that: a. More than 440 losses will occur? b. Between 320 and 480 losses will occur? c. Between 460 and 520 losses will occur?

Crisis matrix was designed by Klaus Winterling. The matrix is one of analytical techniques used in risk management.

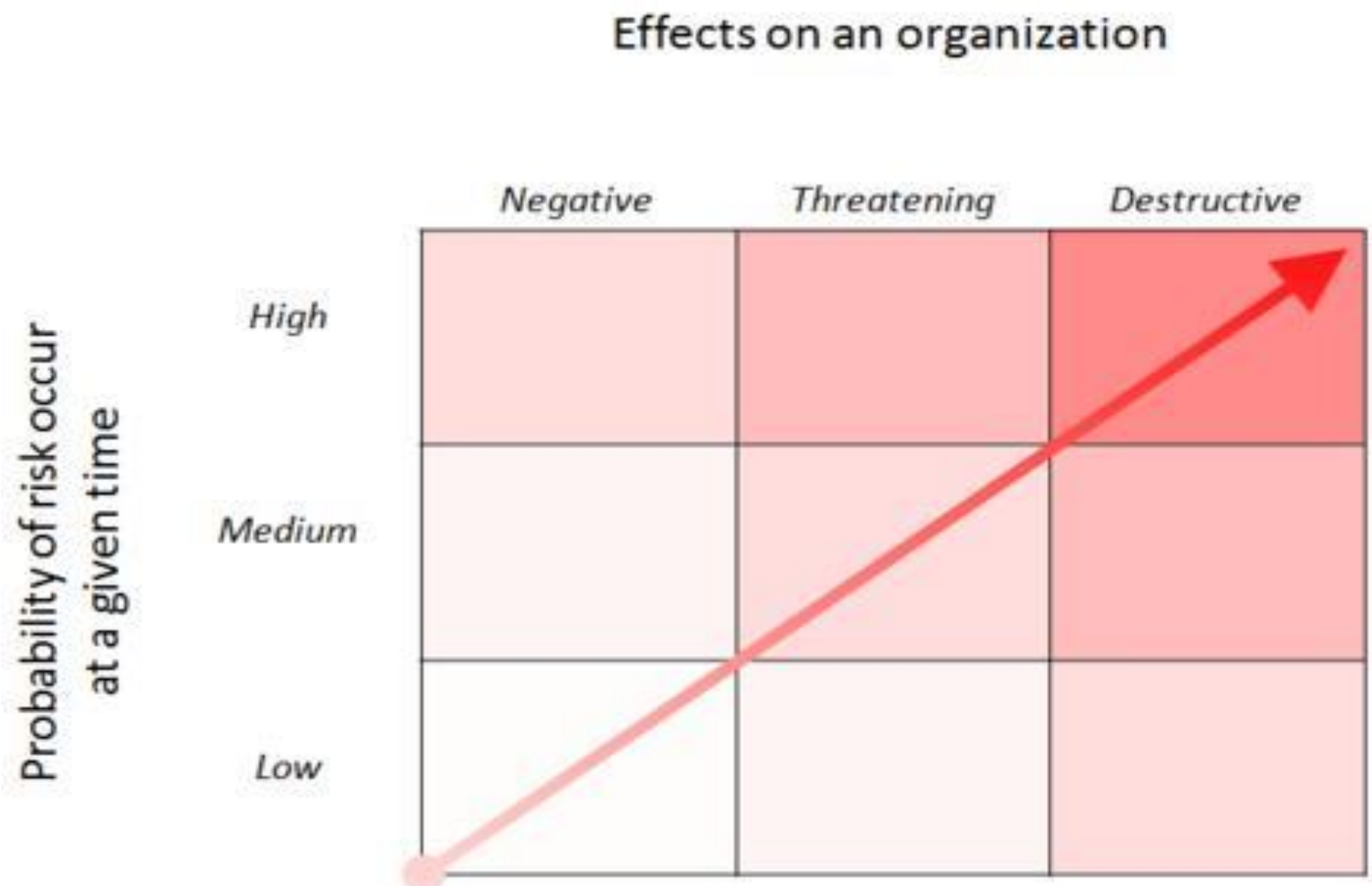
The matrix allows risks categorization by **two parameters**:

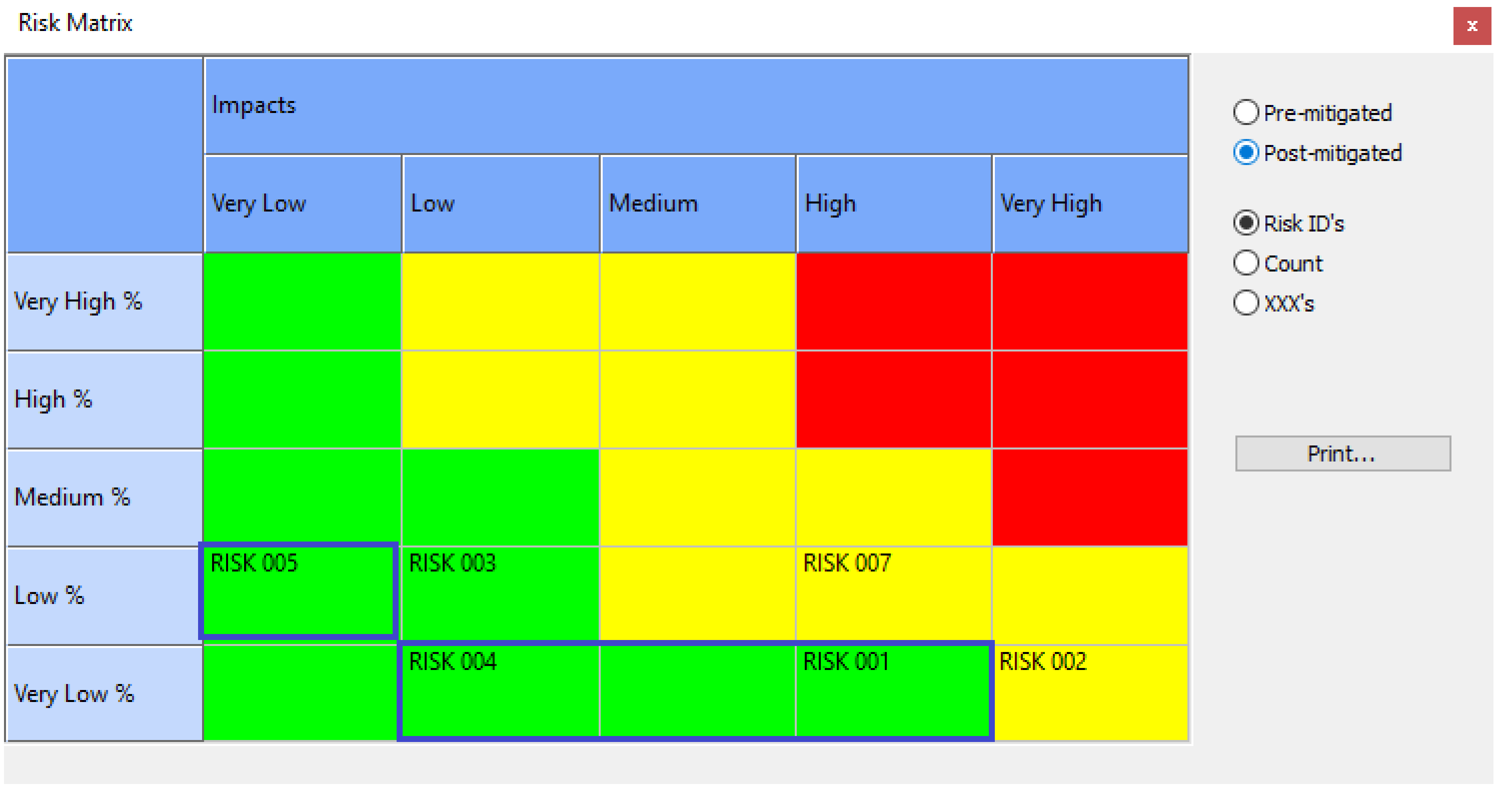
Probability of a risk occur at a given time - how real and probable is that the risk will actually occurs - matrix defines three levels of probability

- low
- medium
- high

Risk effects on an SBU - what what would be the impacts of the risk on an organization or department if the risk occurs - matrix defines three levels of effect

- negative,
- threatening
- destructive


Effects on an organization
Negative
Threatening
Destructive
High
Medium
Low
Probability of risk occur
at a given time


Risk Matrix
Impacts
Very Low
Low
Medium
High
Very High
Very High %
High %
Medium %
Low %
Very Low %
RISK 005
RISK 003
RISK 007
RISK 004
RISK 001
RISK 002
Pre-mitigated
Post-mitigated
Risk ID's
Count
XXX's
Print...

Risk Severity Matrix

Likelihood

Almost Certain (5)	Moderate	High	Extreme	Extreme	Extreme
Likely (4)	Moderate	Moderate	High	Extreme	Extreme
Possible (3)	Low	Moderate	Moderate	High	Extreme
Unlikely (2)	Low	Low	Moderate	High	High
Rare (1)	Low	Low	Low	Moderate	Moderate
	Insignificant (1)	Minor (2)	Moderate (3)	Major (4)	Critical (5)

Consequence

Thank you for your attention!