IA159 Formal Methods for Software Analysis

Program Slicing and Points-to Analysis

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Focus and sources

focus

- slicing via dependence graphs
- points-to analysis
- static single assignment (SSA)
- data dependencies
- control dependencies

sources

- M. Chalupa: Program Slicing and Symbolic Execution for Verification, PhD thesis, 2021.
- B.Alpern, M. N. Wegman, and F. K. Zadeck: Detecting equality of variables in programs, POPL 1988.
-

Program slicing

Program slicing reduces a given program by removing statements that are irrelevant for a given slicing criterion.

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- introduced in *M. D. Weiser: Program Slicing, ICSE 1981*
- the approach based on dependence graphs presented in *K. J. Ottenstein and L. M. Ottenstein: The Program Dependence Graph in a Software Development Environment, SDE 1984*

Applications of program slicing

- program debugging
- code comprehension
- code optimization including automatic parallelization
- software verification
- …

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a typical application in software verification (implemented in Symbiotic)

1. find potentially erroneous statements by a cheap analysis
2. slice the program to preserve all executions of these statements
3. verify the sliced program

Simple example

Which statements are irrelevant for the `assert`?

```
z = z + 3;
if (z > 0) {
  x = z + 1;
  z = 3 * x;
} else {
  y = z + 5;
  x = x * x - z;
}
if (x > y)
  z = x - 1;
assert(x > 0);
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Basic slicing algorithm

1. build a dependence graph for the given program
    - nodes are statements
    - edges correspond to data and control dependencies
2. sliced program corresponds to the nodes that are backward reachable from the slicing criterion(s)

intuitive meanings

- a statement *r* is data dependent on a statement *w* if there exists a program execution where *r* reads a value from a memory that has been written by *w*
- a statement *n* is control dependent on a statement *b* if *b* is the closest point where a program execution may go some way that misses *n*
- in practice, we compute overapproximations

Simple dependency graph

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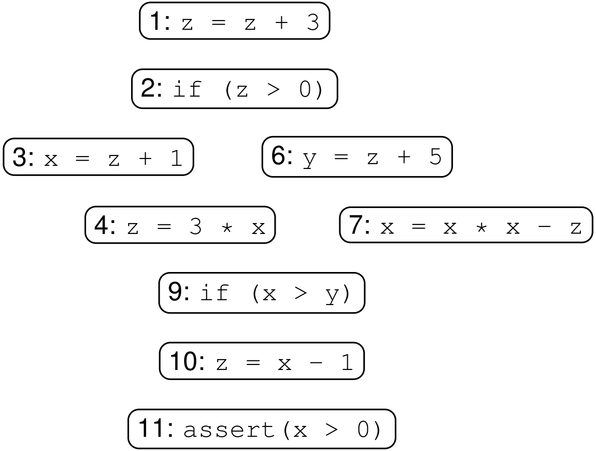

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9: `if (x > y)`

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$w \longrightarrow r$

r is data dependent on *w* if there exists a program execution where *r* reads a value from a memory that has been written by *w*

$b \dashrightarrow n$

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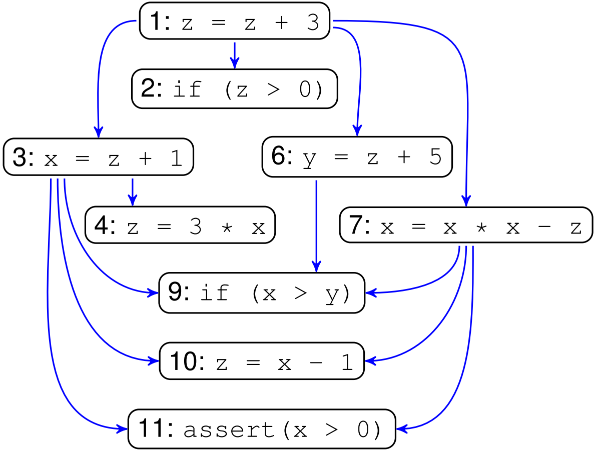

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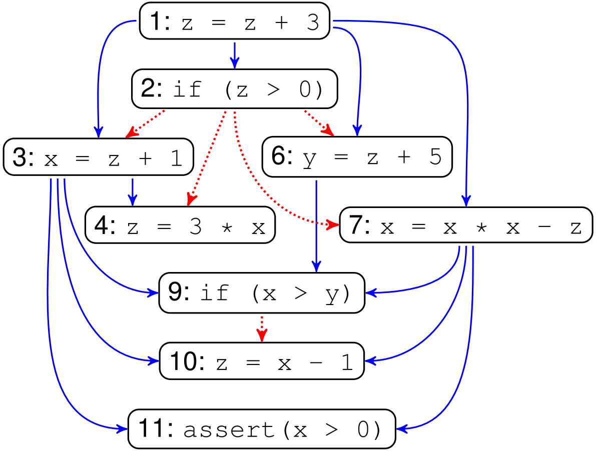

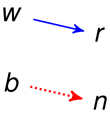

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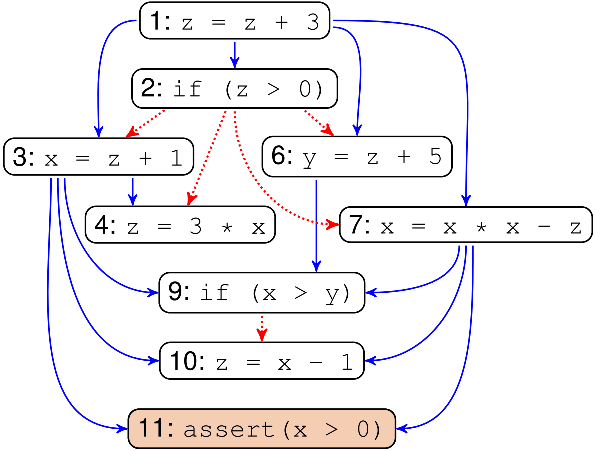

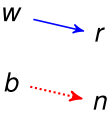

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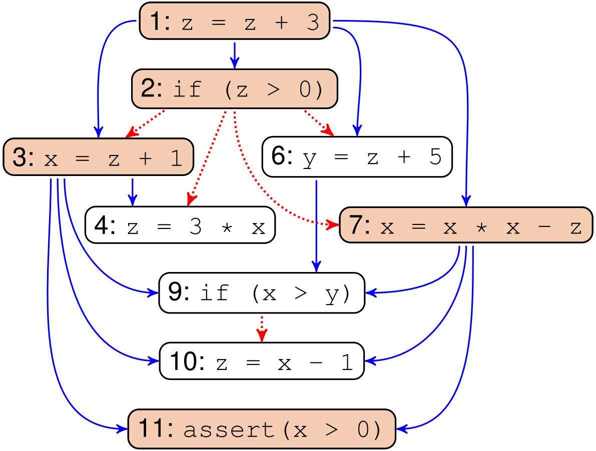

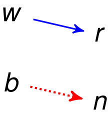

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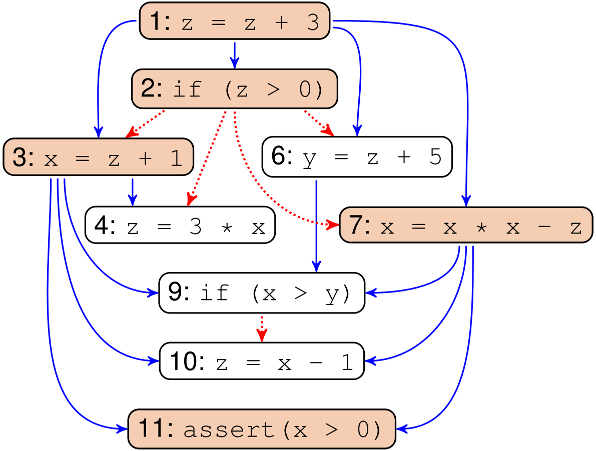

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Points-to analysis aka pointer analysis

Motivation

How data dependencies look like?

```
int x;
int *p;
int *q;
x = 5;
p = &x;
q = p;
*q = 7;
assert(x > 6);
```

Motivation

How data dependencies look like?

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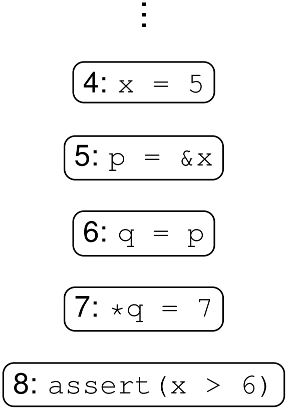

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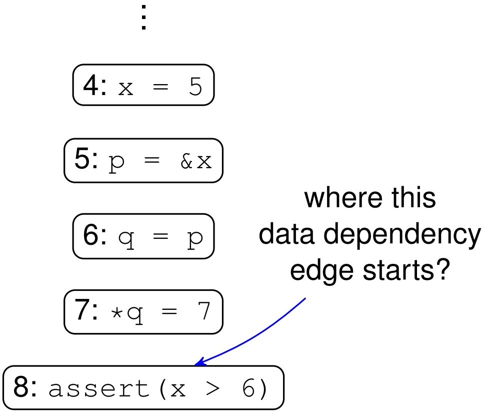

where this
data dependency
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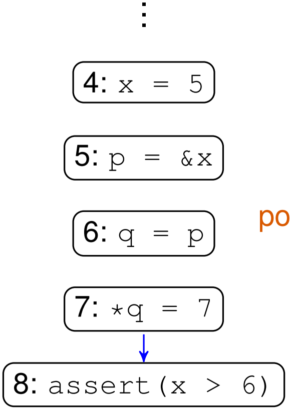

points-to analysis
needed

Points-to analysis

- assigns to each pointer *p* the points-to set that contains all memory locations *p* may point to
- memory locations are abstractions of concrete objects located in memory during program execution
  - often identified with allocation statements like 1: `int x` or 35: `malloc(128)`
  - can represent more concrete objects, e.g., for `malloc` in cycle

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  - can represent more concrete objects, e.g., for `malloc` in cycle
- we use two additional memory locations
  - `null` representing a pointer value NULL
  - `unknown` saying that the pointer can point anywhere
- additionally, it tracks which memory locations represent one concrete memory object and which are abstract
- can be computed by an abstract interpretation

Points-to analysis

- can be flow sensitive or insensitive
  - flow sensitive analysis assigns a points-to set to a pointer and a program location (more precise but more expensive)
  - flow insensitive analysis used mainly for programs in static single assignment (SSA) form

```
int y;
int *data = malloc(40);
...
int *p = &y;
if (y > 2) {
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} else {
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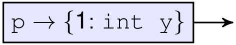

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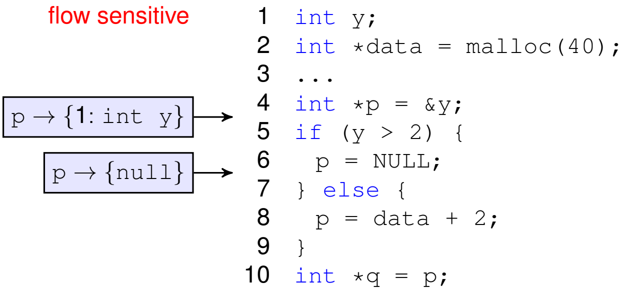

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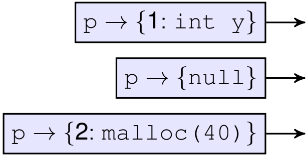

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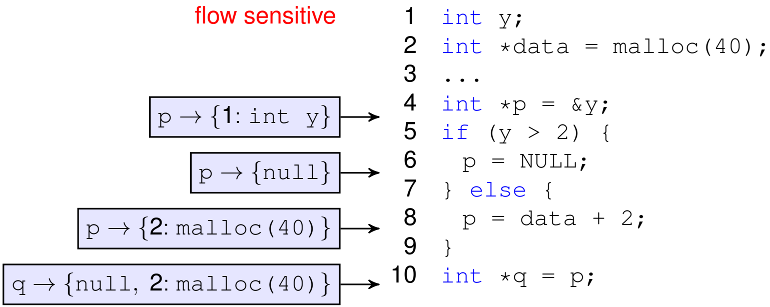

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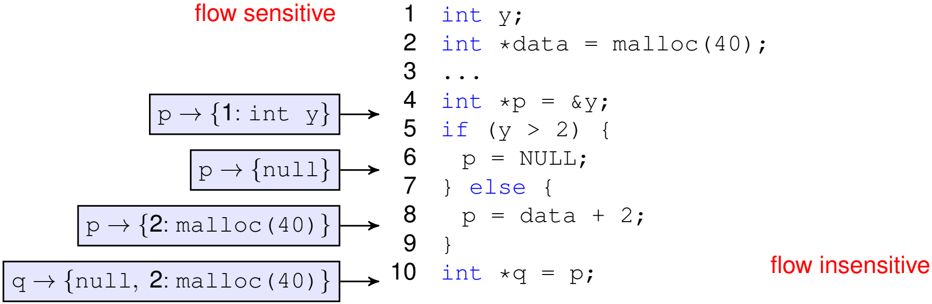

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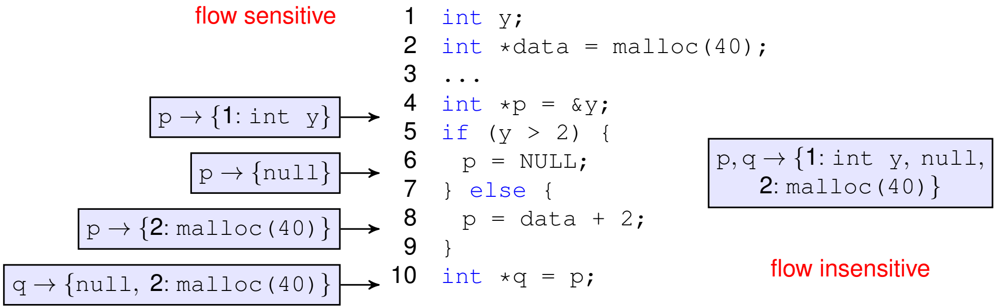

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p → {1: int y}

p → {null}

p → {2: malloc(40)}

q → {null, 2: malloc(40)}

p, q → {1: int y, null, 2: malloc(40)}

flow insensitive

Points-to analysis

- can be flow sensitive or insensitive
  - flow sensitive analysis assigns a points-to set to a pointer and a program location (more precise but more expensive)
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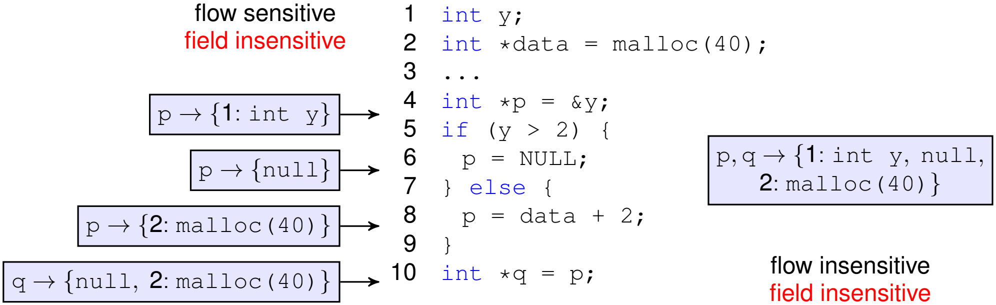
- can be field insensitive or sensitive
  - field sensitive analysis tracks also offsets
  - field sensitive analysis is more precise but more expensive



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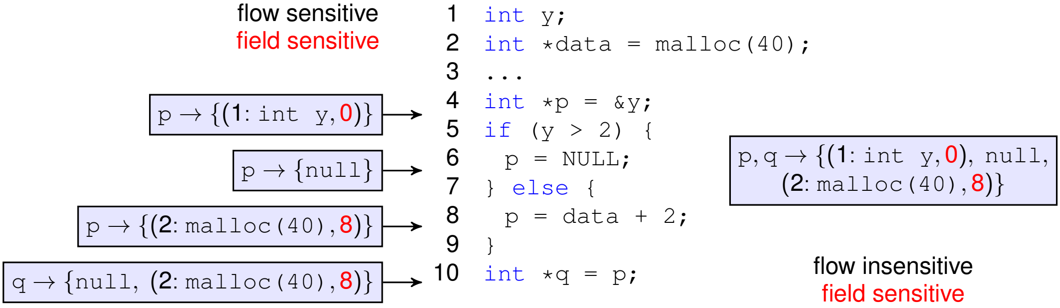

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p → {(1: `int y`, 0)} → line 4

p → {`null`} → line 6

p → {(2: `malloc(40)`, 8)} → line 8

q → {`null`, (2: `malloc(40)`, 8)} → line 10

p, q → {(1: `int y`, 0), `null`, (2: `malloc(40)`, 8)}

flow insensitive
field sensitive

Notes

- a popular algorithm for points-to analysis presented in *L. O. Andersen: Program Analysis and Specialization for the C Programming Language, PhD thesis, 1994*
- applications of points-to analysis
  - can prove that a program is memory safe, i.e., it contains no invalid pointer dereference and no invalid memory deallocation
  - can be used for computation of data dependencies
  - can help to identify functions called via a function pointer
  - …

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x = input();
z = x + 3;
if (z > 0) {
  x = z + 1;
  z = 3 * x;
} else {
  z = z + 5;
}


z = z + x;
```

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- special instructions called ϕ-nodes added

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z = z + x;
```

```
x_1 = input();
z_1 = x_1 + 3;
if (z_1 > 0) {
  x_2 = z_1 + 1;
  z_2 = 3 * x_2;
} else {
  z_3 = z_1 + 5;
}
x_3 = φ(x_2,x_1);
z_4 = φ(z_2,z_3);
z_5 = z_4 + x_3;
```

Applications of SSA

- simplifies static analysis
  - without SSA, x may have different values in different locations
  - with SSA, x_i has the same value everywhere
  - flow-insensitive analyses provide better results for programs in SSA
- used in many verification tools and also in compilers
- LLVM IR also uses SSA (sort of)

Data dependence

Data dependence

Consider a fixed control flow graph (CFG) with nodes V. We assume that for each node $n \in V$, we have sets:

- $sdef(n)$ of memory locations that must be written by n
- $wdef(n)$ of memory locations that may be written by n
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- $\texttt{null}, \texttt{unknown} \notin sdef(n)$ and $\texttt{null} \notin wdef(n) \cup ref(n)$
- $sdef(n) \subseteq wdef(n)$
- $sdef(n)$ contains only memory locations that represent one concrete object each time n is executed
- the sets can be computed by a field-sensitive points-to analysis

Data dependence

Definition (data dependence)

Let V be the set of nodes of a CFG. A node $n_r \in V$ is data dependent on a node $n_w \in V$ if there is a path $n_w = n_1, n_2, \ldots, n_k = n_r$ in the CFG such that

- $\texttt{unknown} \notin \mathit{wdef}(n_w) \cup \mathit{ref}(n_r)$ and $\mathit{wdef}(n_w) \cap \mathit{ref}(n_r) \not\subseteq \bigcup_{1<i<k} \mathit{sdef}(n_i)$ or
- $\texttt{unknown} \in \mathit{wdef}(n_w)$ and $\mathit{ref}(n_r) \not\subseteq \bigcup_{1<i<k} \mathit{sdef}(n_i)$ or
- $\texttt{unknown} \in \mathit{ref}(n_r)$ and $\mathit{wdef}(n_w) \not\subseteq \bigcup_{1<i<k} \mathit{sdef}(n_i)$.

Data dependence

Definition (reaching definition)

Consider a node n_w and $e \in \mathit{wdef}(n_w)$. We say that the definition of e at n_w reaches a node n if there is a path $n_w = n_1, n_2, \ldots, n_k = n$ and $e \notin \bigcup_{1<i<k} \mathit{sdef}(n_i)$.

Data dependence

Definition (reaching definition)

Consider a node n_w and $e \in \mathit{wdef}(n_w)$. We say that the definition of e at n_w reaches a node n if there is a path $n_w = n_1, n_2, \ldots, n_k = n$ and $e \notin \bigcup_{1<i<k} \mathit{sdef}(n_i)$.

- reaching definitions can be computed by an abstract interpretation
- `unknown` $\in \mathit{wdef}(n_w)$ reaches all nodes reachable from n_w

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- reaching definitions can be computed by an abstract interpretation
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Theorem

If n_r is data dependent on n_w, then

- *the definition of some $e \in \mathit{wdef}(n_w)$ at n_w reaches n_r and $e \in \mathit{ref}(n_r)$, or*
- $\texttt{unknown} \in \mathit{wdef}(n_w) \cup \mathit{ref}(n_r)$ *and* $\mathit{wdef}(n_w) \neq \emptyset$ *and* $\mathit{ref}(n_r) \neq \emptyset$.

Data dependence

- the previous theorem allows to compute an overapproximation of data dependencies with use reaching definitions
- computation is relatively slow because it computes more information than needed
- there are faster algorithms, e.g., byte-memory SSA algorithm presented in *M. Chalupa: Program Slicing and Symbolic Execution for Verification, PhD thesis, 2021*

Control dependence

Motivation

Which statements are irrelevant for the `assert`?

```
unsigned int i,n;
n = input();
i = 0;
while (i < n) {
  i++;
}
assert(false);
```

```
unsigned int i,n;
n = input();
i = 0;
while (i >= n) {
  i++;
}
assert(false);
```

Motivation

Which statements are irrelevant for the `assert`?

```






assert(false);
```

```
unsigned int i,n;
n = input();
i = 0;
while (i >= n) {
  i++;
}
assert(false);
```

Motivation

Which statements are irrelevant for the `assert`?

```
1
2
3
4
5
6
7  assert(false);
```

```
1  unsigned int i,n;
2  n = input();
3  i = 0;
4  while (i >= n) {
5    i++;
6  }
7  assert(false);
```

- removing a potentially non-terminating cycle can transform an unrechable code into a reachable
- line 7 on the right is unreachable if `input()` always returns `0`

Two notions of control dependence

Intuitively, a statement *n* is control dependent on a statement *b* if *b* is the closest point where the program may go some way that misses *n*.

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- assumes that every execution is finite
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weak control dependence

- assumes that every execution is finite
- an instance: standard control dependence

strong control dependence

- sensitive to program non-termination: there can be a dependence between two statements if one can infinitely delay the execution of the other
- an instance: non-termination sensitive control dependence

Standard control dependence

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Given an exit-CFG, its node b post-dominates a node a if b is on every path from a to exit. If $a \neq b$, we say that b strictly post-dominates a.

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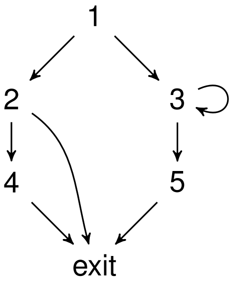
Definition (standard control dependence)

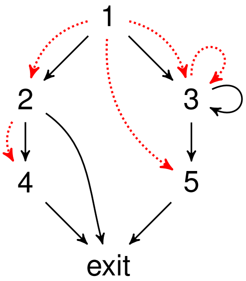
Given an exit-CFG, we say that node n is standard control dependent (SCD) on node b if

1. there exists a non-trivial path π from b to n with any node on π (excluding b) post-dominated by n and
2. b is not strictly post-dominated by n.

Standard control dependence



Standard control dependence



- the SCD relation for an exit-CFG (V, E) can be computed in time $\mathcal{O}(|E|)$ using the algorithm of *J. Ferrante et al.: The Program Dependence Graph and Its Use in Optimization, TOPLAS 1987*
- each CFG can be transformed into an exit-CFG

Non-termination sensitive control dependence

- predicate nodes in CFG are nodes corresponding to branching statements
- maximal path is a path that cannot be further prolonged, i.e., it is infinite or it ends in a node without any successor

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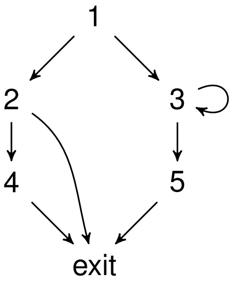
Definition (non-termination sensitive control dependence)

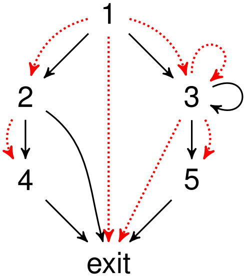
Given a CFG, a node n is non-termination sensitive control dependent (NTSCD) on a predicate node p if p has two successors s_1 and s_2 such that

1. all maximal paths from s_1 contain n and
2. there exists a maximal path from s_2 that does not contain n.

Non-termination sensitive control dependence



Non-termination sensitive control dependence



- the NTSCD relation for a CFG (V, E) can be computed in time $\mathcal{O}(|V|^2)$ using the algorithm of *M. Chalupa et al.: Fast Computation of Strong Control Dependencies, CAV 2021*
- NTSCD treats every program cycle as potentialy non-terminating

Notes

- we used program dependence graphs for programs without procedure calls
- there are also system dependence graphs for programs with procedure calls
- both NTSCD and SCD have applications in program slicing for software verification: SCD leads to smaller sliced programs and can only lead to produce false alarms, but not to false negatives
- there are other notions of control dependence, e.g., decisive order dependence (DOD)
- points-to analysis and slicer for LLVM implemented in DG
  `https://github.com/mchalupa/dg`