IA159 Formal Methods for Software Analysis

Verification via Automata, Symbolic Execution, and Interpolation

Jan Strejček

Faculty of Informatics
Masaryk University

Focus and sources

focus

- basic principle
- interpolation
- example

ULTIMATE

sources

- M. Heizmann, J. Hoenicke, A. Podelski: Software Model Checking for People Who Love Automata, CAV 2013.
- M. Heizmann, J. Hoenicke, A. Podelski: Termination Analysis by Learning Terminating Programs, CAV 2014.

Special thanks to M. Heizmann for providing me with his slides.

Programs as languages

New view on programs

A program defines a language over program statements.

- alphabet = the set of program statements
- finite automaton = control flow graph
- accepting states = error locations

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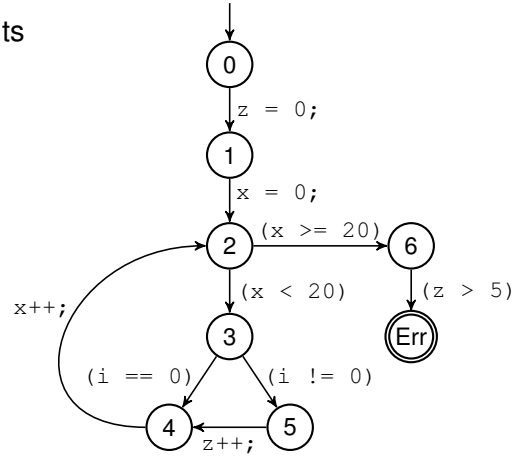


The goal

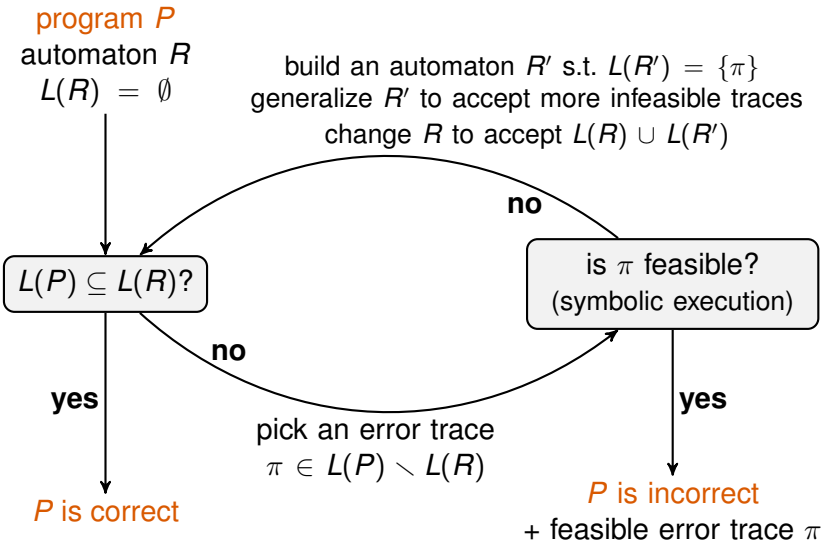
- such an automaton accepts error traces
- not all error traces are feasible

The goal

To decide whether there exists a feasible error trace accepted by the automaton.

The program is correct iff all error traces accepted by the automaton are infeasible.

The algorithm



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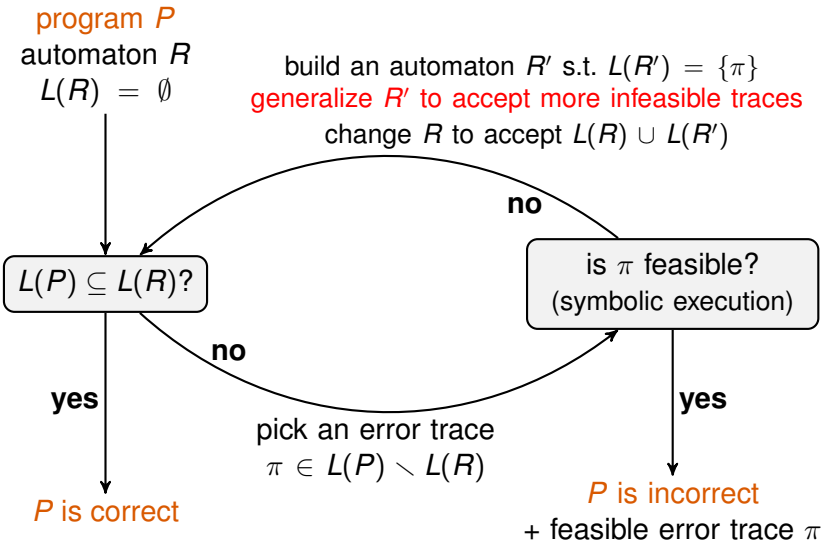

Craig interpolation

Theorem (William Craig, 1957)

Let φ, ψ be two first-order formulae such that $\varphi \implies \psi$. Then there exists a first order-formula θ called interpolant such that

- *all non-logical symbols (i.e., variables and uninterpreted function and predicate symbols) in θ occur in both φ and ψ,*
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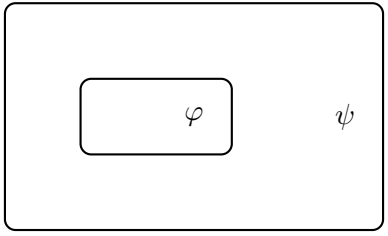

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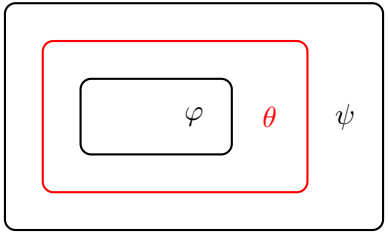

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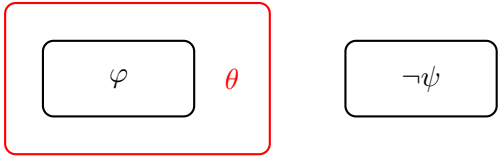

Example: program

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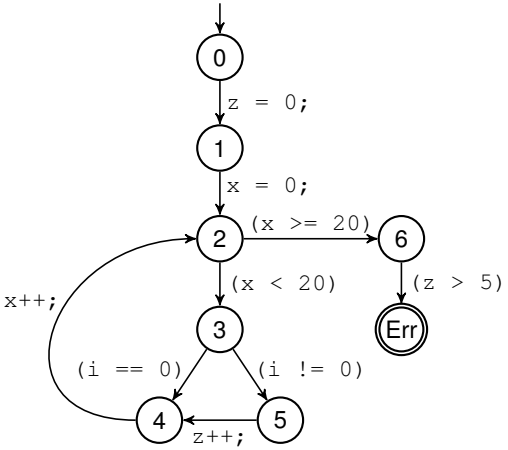

Example: error trace

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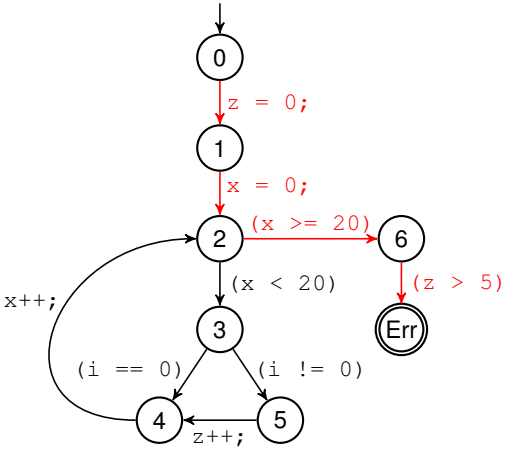
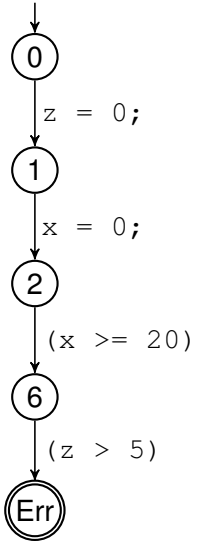
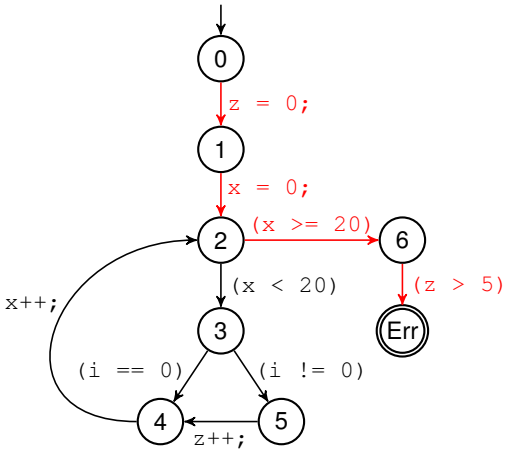
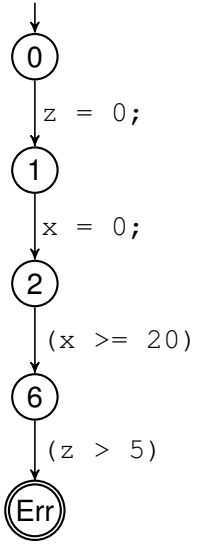


Example: error path





Example: feasability analysis

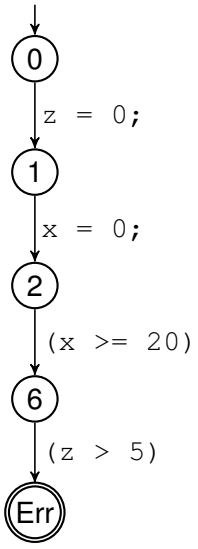


symbolic execution produces

$z = 0 \wedge x = 0 \wedge x \geq 20 \wedge z > 5 \equiv \mathit{false}$

$\Longrightarrow$ the error trace is infeasible

Example: generalization of infeasibility arguments



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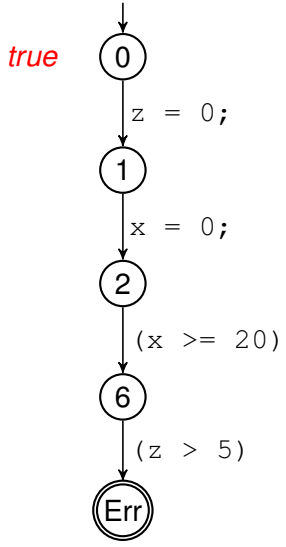

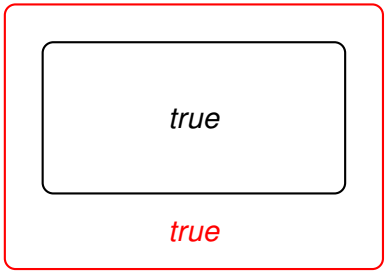

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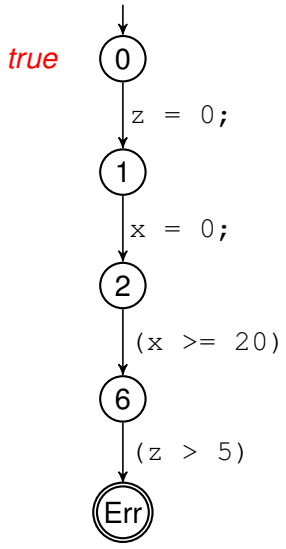

$true \wedge z = 0$

$x = 0 \wedge$
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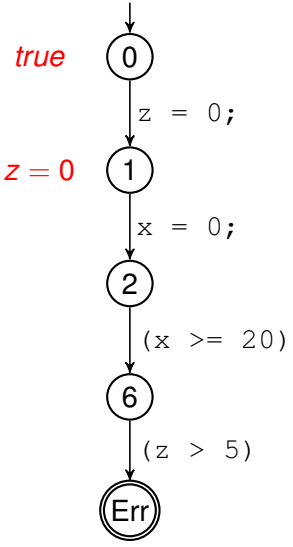

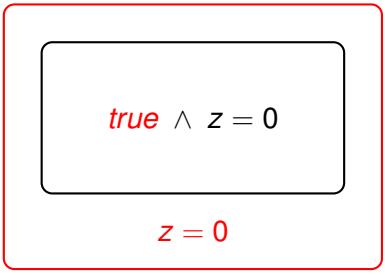

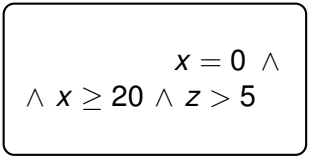

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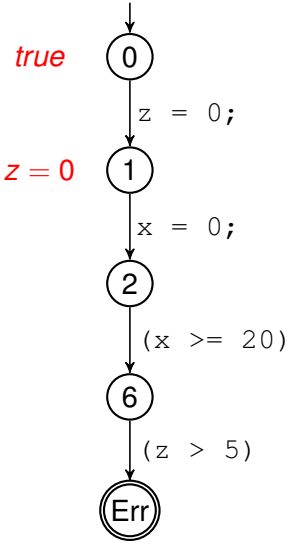

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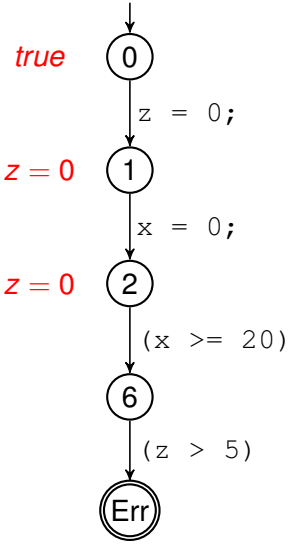

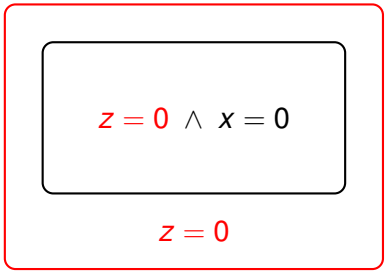

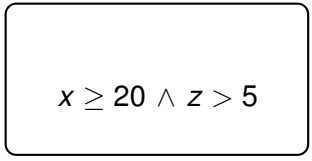

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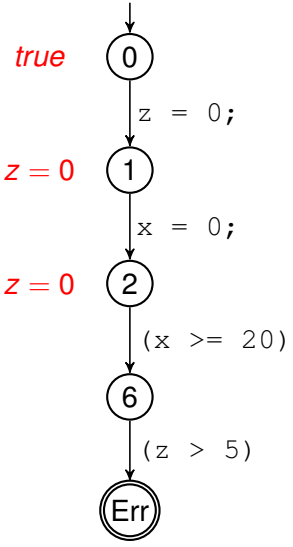

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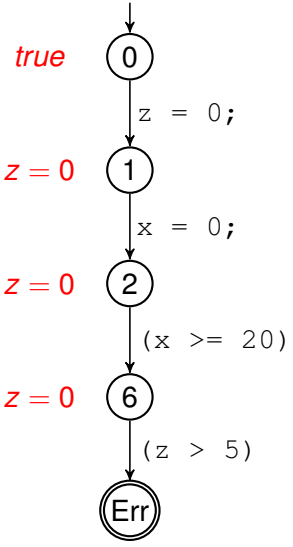

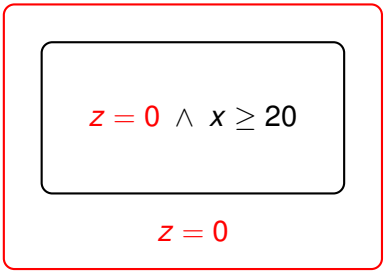

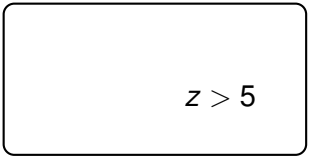

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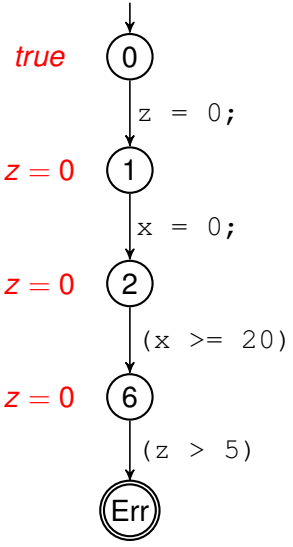

$z = 0 \wedge z > 5$

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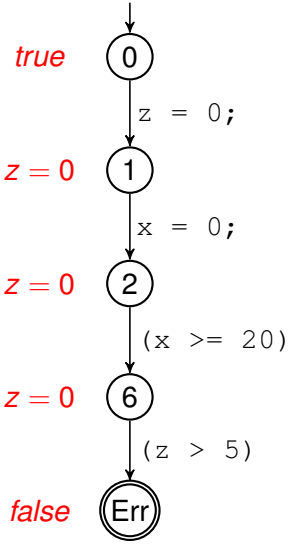

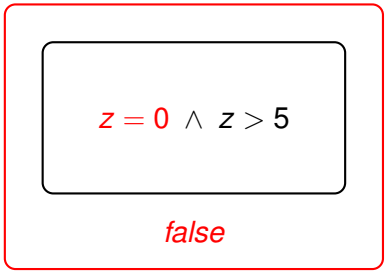

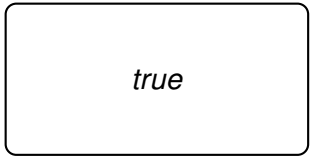

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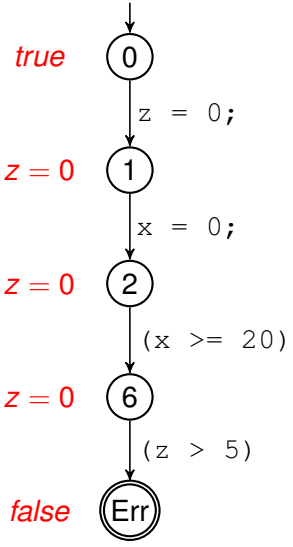

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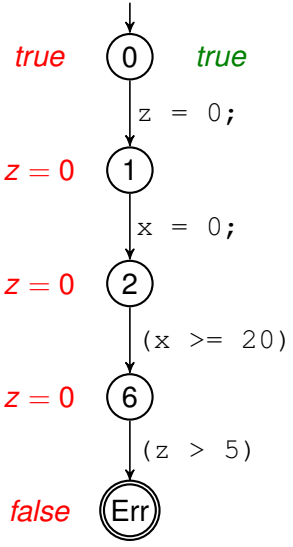

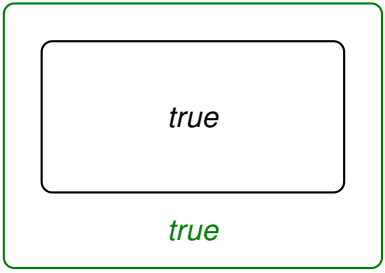

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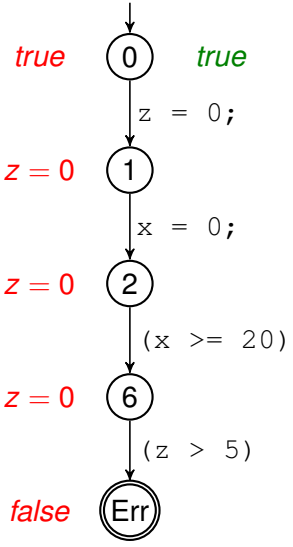

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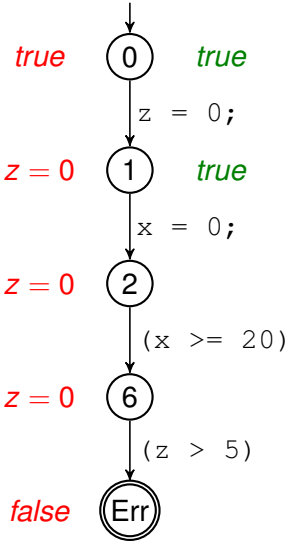

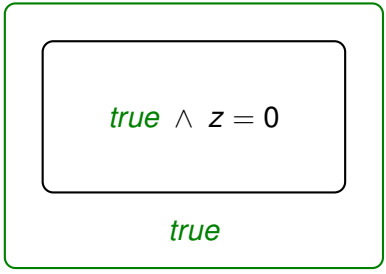

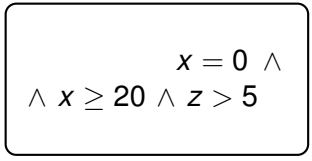

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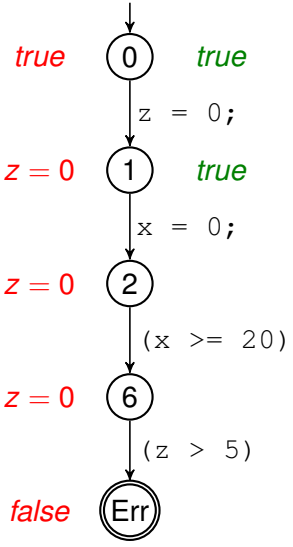

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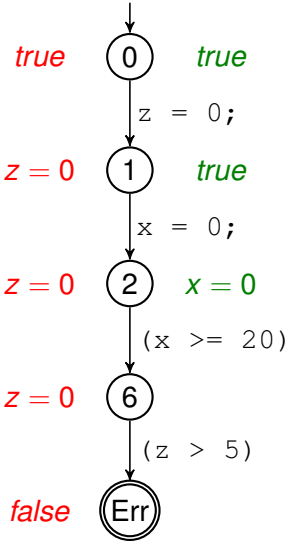

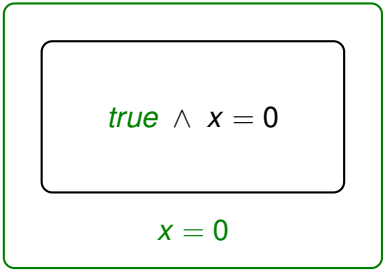

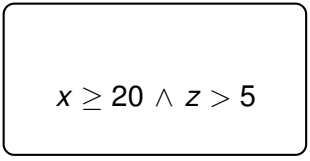

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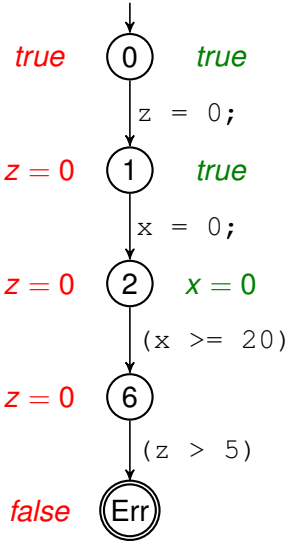

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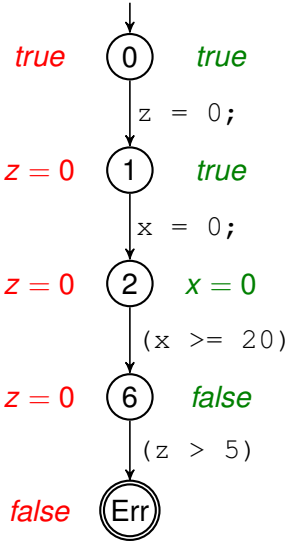

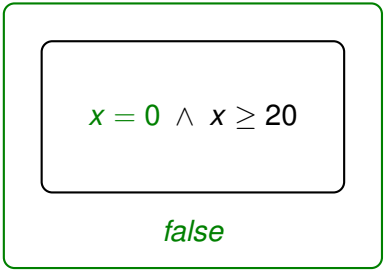

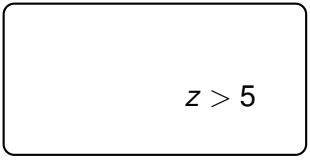

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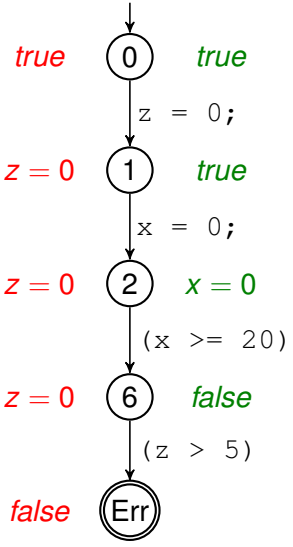

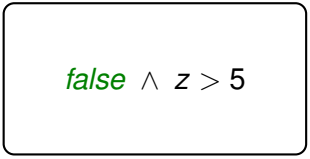

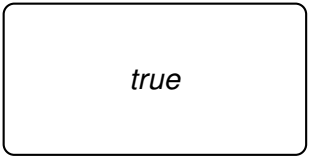

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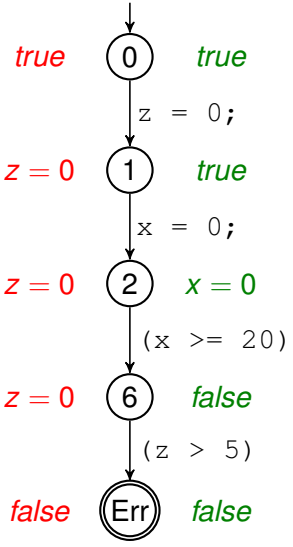
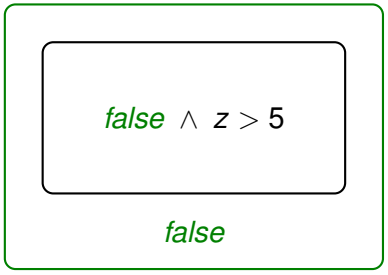
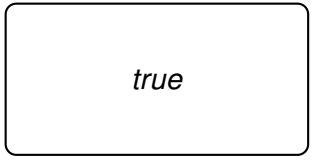
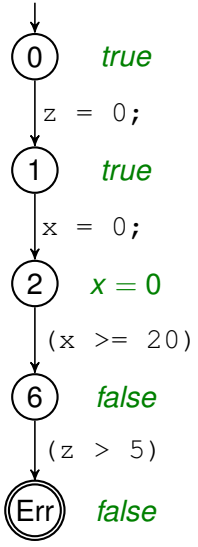
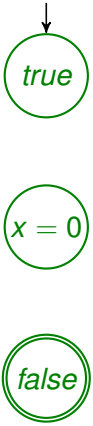






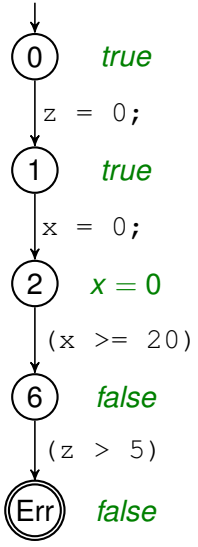
Example: generalization of infeasibility arguments

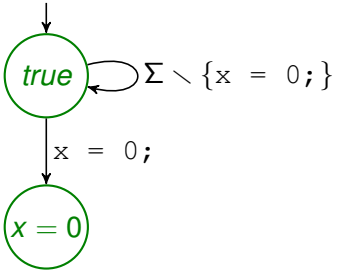




$\Sigma = \{$`z = 0;`, `x = 0;`, `(x >= 20)`,
`(x < 20)`, `(i == 0)`, `(i != 0)`,
`z++;`, `x++;`, `(z > 5)`$\}$

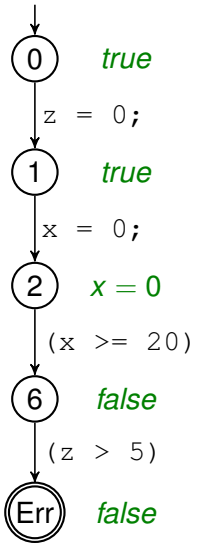
Example: generalization of infeasibility arguments

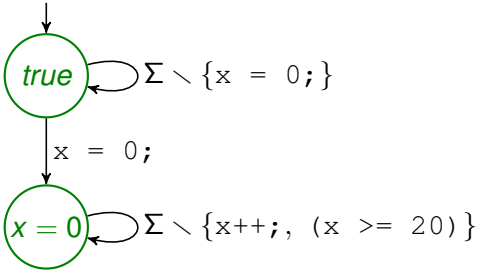




$$\Sigma = \{\texttt{z = 0;}, \texttt{x = 0;}, \texttt{(x >= 20)}, \texttt{(x < 20)}, \texttt{(i == 0)}, \texttt{(i != 0)}, \texttt{z++;}, \texttt{x++;}, \texttt{(z > 5)}\}$$

Example: generalization of infeasibility arguments





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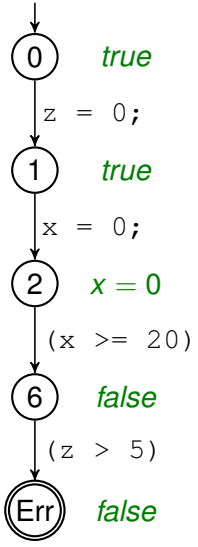
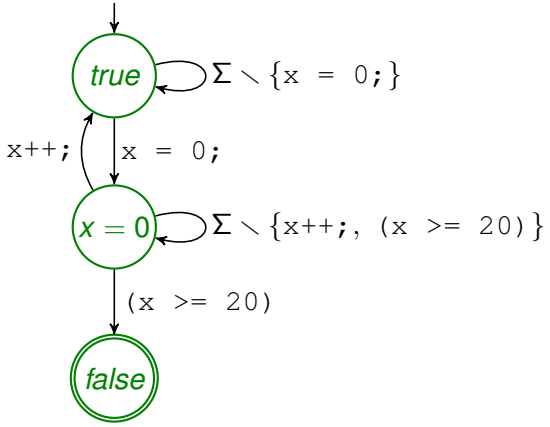
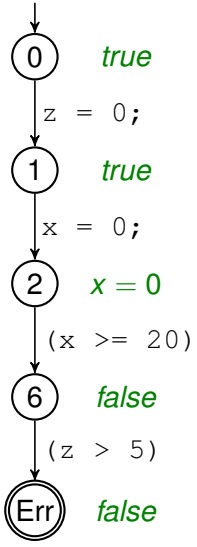
Example: generalization of infeasibility arguments

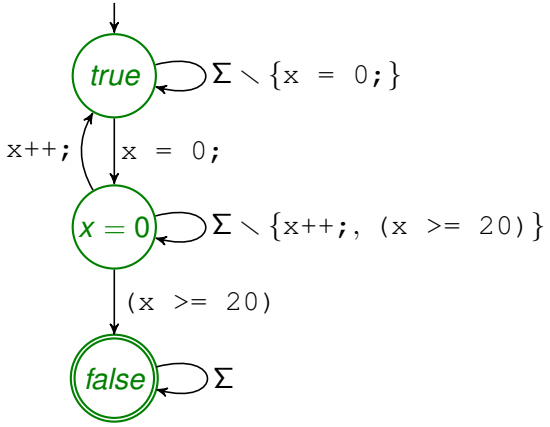




$$\Sigma = \{\texttt{z = 0;}, \texttt{x = 0;}, \texttt{(x >= 20)},\ \texttt{(x < 20)}, \texttt{(i == 0)}, \texttt{(i != 0)},\ \texttt{z++;}, \texttt{x++;}, \texttt{(z > 5)}\}$$

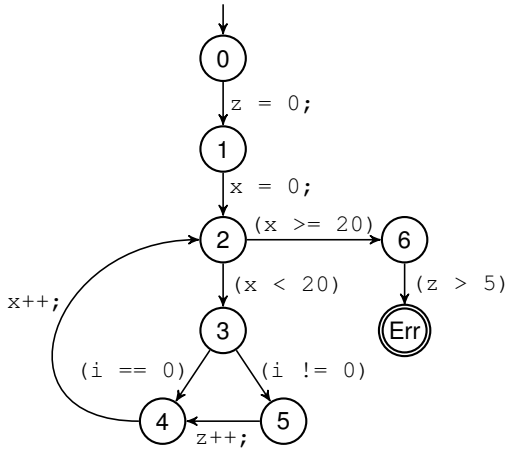
Example: generalization of infeasibility arguments

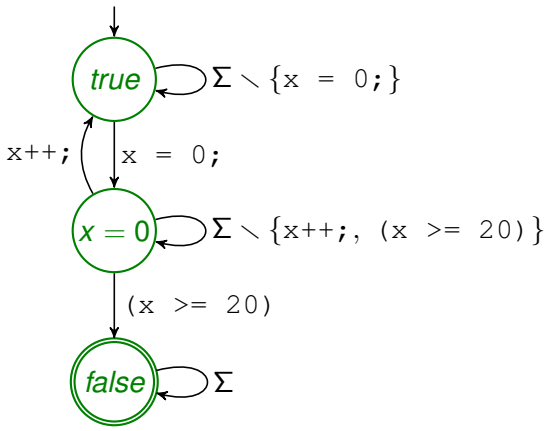




$$\Sigma = \{\texttt{z = 0;}, \texttt{x = 0;}, \texttt{(x >= 20)}, \texttt{(x < 20)}, \texttt{(i == 0)}, \texttt{(i != 0)}, \texttt{z++;}, \texttt{x++;}, \texttt{(z > 5)}\}$$

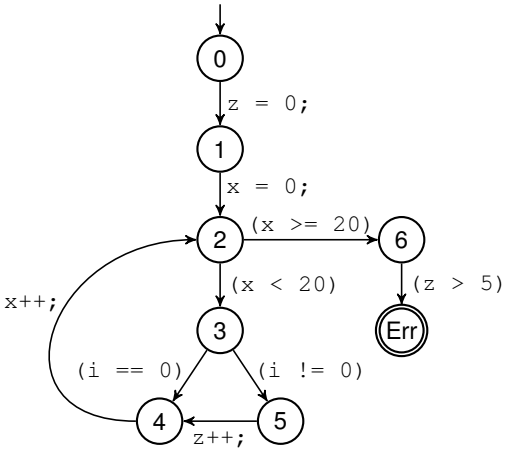
Example: $L(P) \subseteq L(R)$?

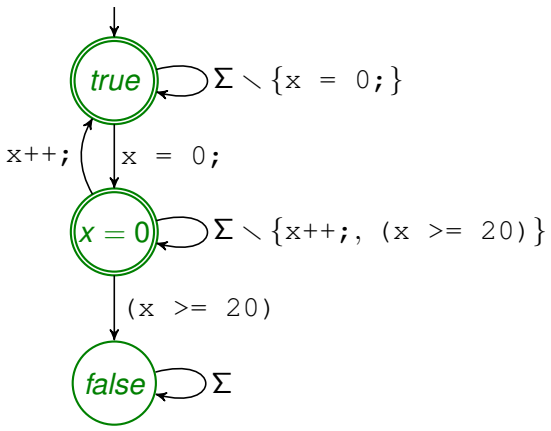




$L(P) \subseteq L(R)$ iff the language of the synchronous product of P and complemented R is empty.

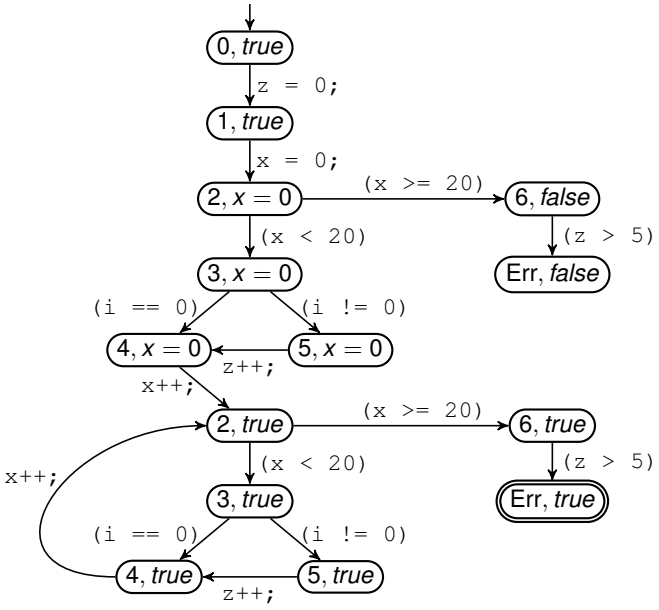
Example: $L(P) \subseteq L(R)$?



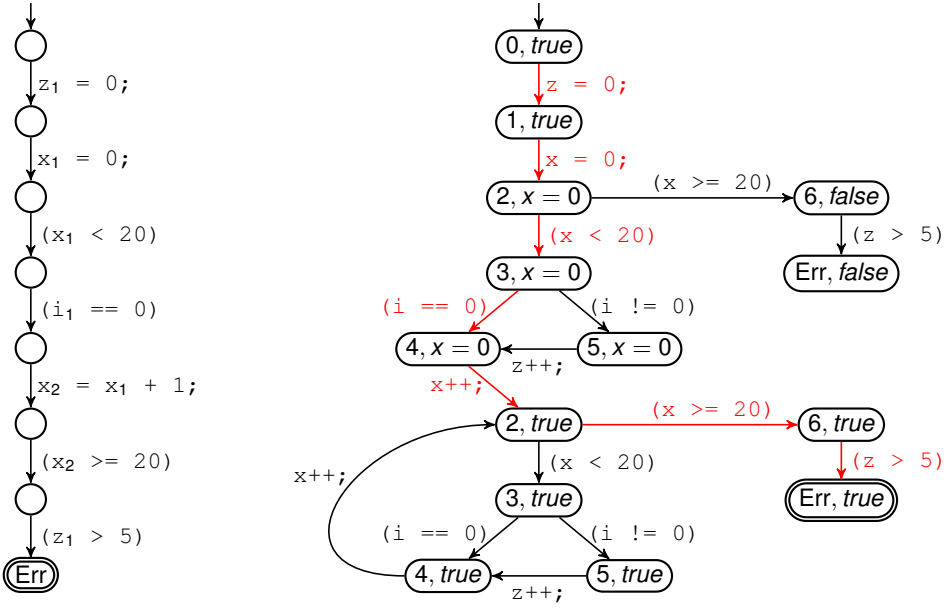


$L(P) \subseteq L(R)$ iff the language of the synchronous product of P and complemented R is empty.

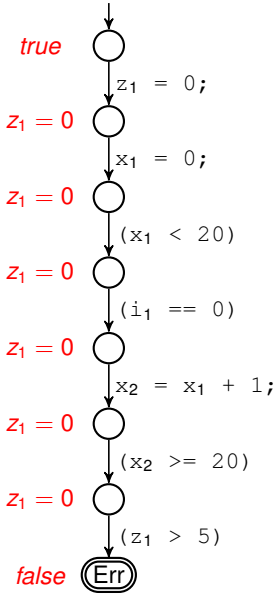
Example: the product

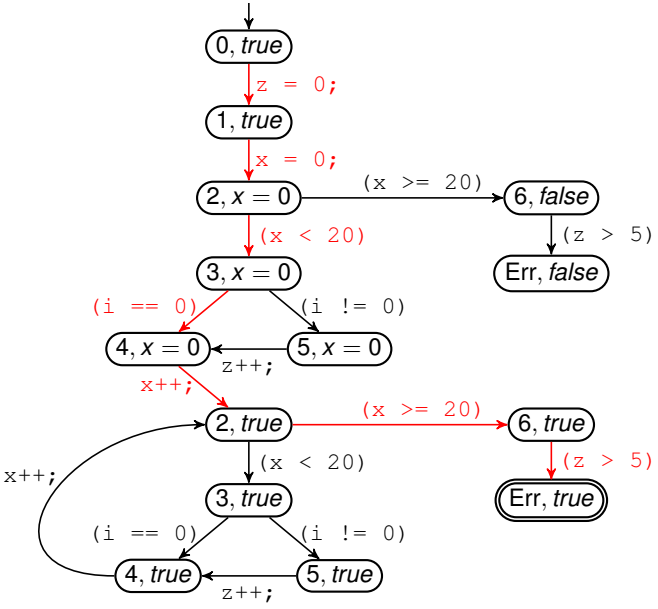


Example: error trace

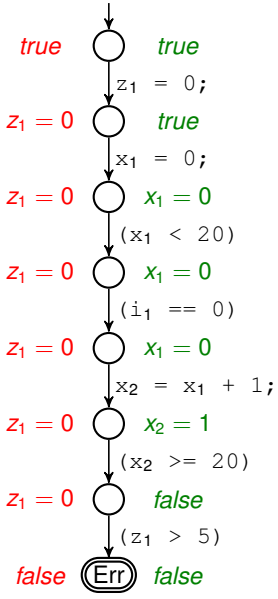


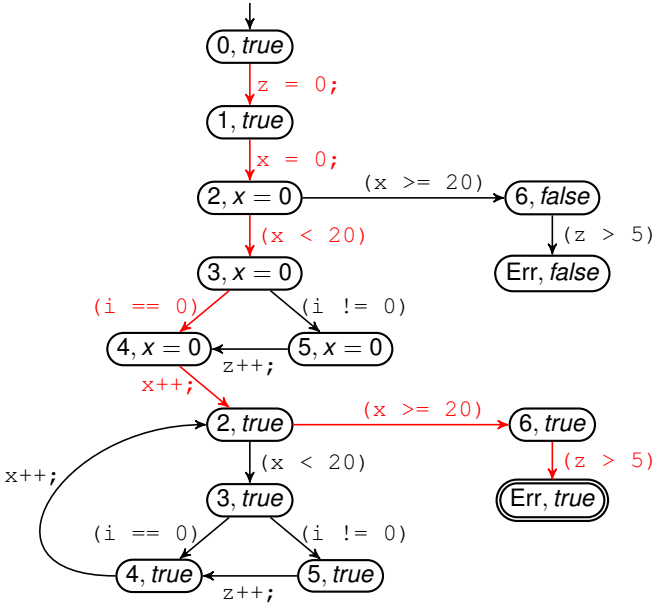
Example: generalization of infeasibility arguments





Example: generalization of infeasibility arguments





Notes

- interpolation algorithms exist only for some logics/theories
- to handle function calls and unbounded recursion, nested word automata and nested interpolants are needed
- implemented in Ultimate Automizer with numerous optimizations, e.g. on-the-fly complementation and emptiness check of the product
- extensions
  - for termination analysis: Ultimate Büchi Automizer
  - for LTL properties: Ultimate LTL Automizer
  - ...

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`https://www.ultimate-pa.org/`