IA159 Formal Methods for Software Analysis

Bounded Model Checking, *k*-Induction

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Focus and sources

focus

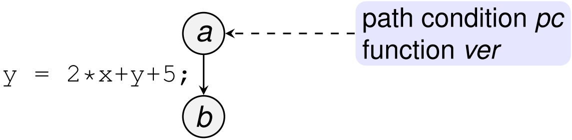
- memoryless version of symbolic execution
- bounded model checking (BMC)
- *k*-induction

source

- A. F. Donaldson, L. Haller, D. Kroening, and P. Rümmer: Software Verification Using *k*-Induction, SAS 2011.

Memoryless version of symbolic execution

- does not use symbolic memory, the assignments are stored to path condition
- to do that, we need to consider another instance of each variable after each assignment to it and remeber its current instance
- let *ver* : $Vars \to \mathbb{N}$ be the function keeping the current instances
- initially, $ver(\mathtt{x}) = 1$ for each $x \in Vars$



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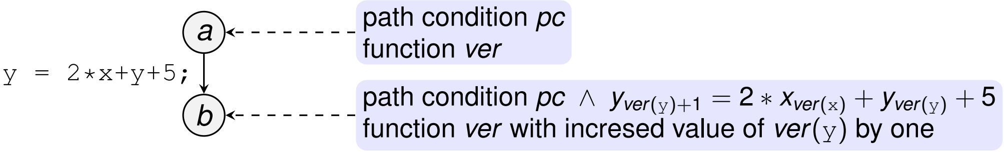

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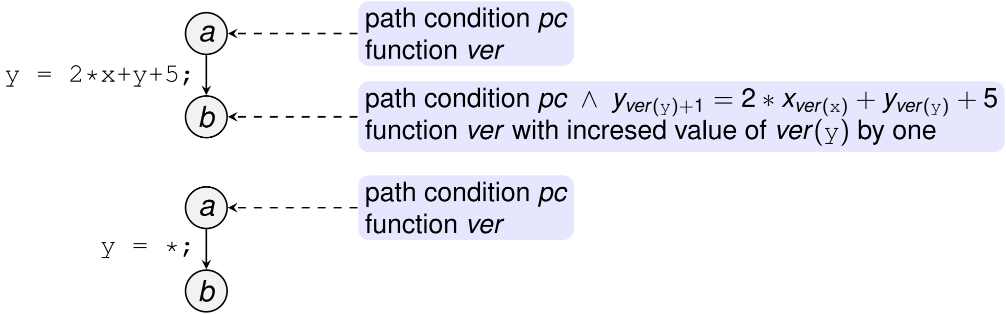

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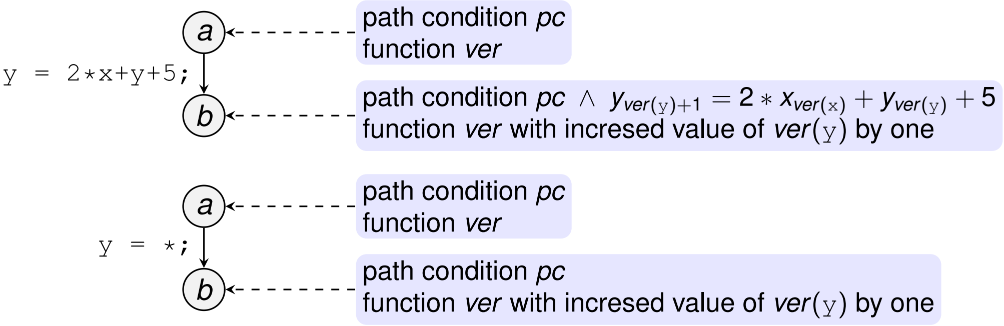

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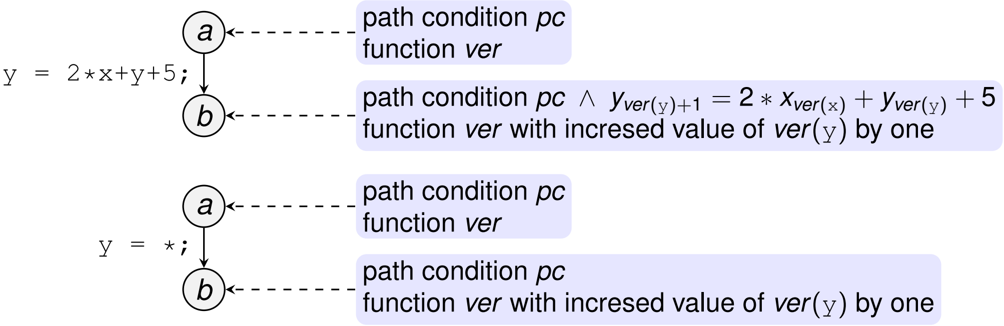

- symbolic execution of branching statements is modified similarly

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- workflow

```
unwind all loops and recursion k-times for a given bound k
compute the error reaching formula φ from the unwound program
if φ is satisfiable then
    return bug found
else
    compute the bound reaching formula ψ from the unwound program
    if the ψ is unsatisfiable then
        return the program is correct
    else
        return unknown (increase bound and start again)
```

Example

original program

```
unsigned char n = input();
if (n == 0) {return 0};
unsigned char v = 0;
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unwound program for $k = 3$

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error reaching formula φ

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satisfiable

variable types are considered

bitvector arithmetic is used

$n_1 = 2$

$v_1 = 0$	$s_1 = 0$	$i_1 = 0$
$v_2 = 224$	$s_2 = 224$	$i_2 = 1$
$v_3 = 63$	$s_3 = 31$	$i_3 = 2$

bug found!

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satisfiable $\implies$ unknown (bound reachable)

Notes on BMC

- very efficient in finding bugs
- uses a sort of SSA when constructing the formula
- constant propagation can simplify the program and the formula and it can reveal that the bound is unreachable
- implemented for example in CBMC
  - tool for bounded model checking of C and C++ programs
  - supports C89, C99, most of C11 and most extensions of gcc and Visual Studio
  - the winner of SV-COMP 2014
  - `https://www.cprover.org/cbmc/`
  - a version for Java programs called JBMC

k-induction

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- extension of BMC that can prove correctness more often
- very successful on symbolic transition systems
- to prove program correctness, we show
  1. base case:
     all feasible paths starting in an initial state of length at most k are correct
  2. induction step:
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- the idea can be applied to programs in different ways
  - k-induction on single-loop programs
- k-induction does semantically the same as backward symbolic execution
  - M. Chalupa and J. Strejček: Backward Symbolic Execution with Loop Folding, SAS 2021.

k-induction on single-loop programs

```
n,x,i = *,0,0;
a,b,c = 1,2,3;
while (i < n) {
  assert(a != b);
  a,b,c = b,c,a;
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assert(x = 0);
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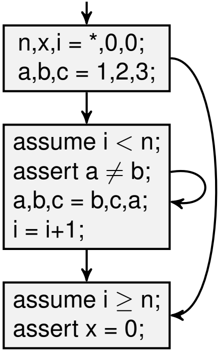

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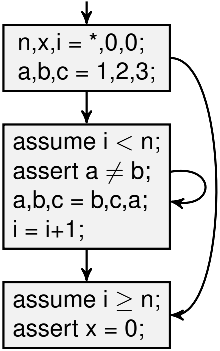

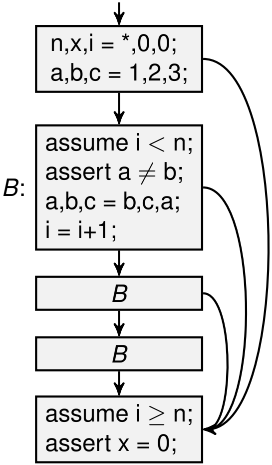

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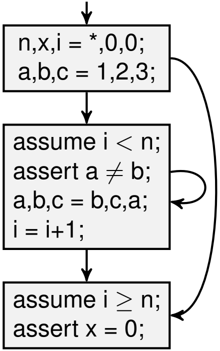

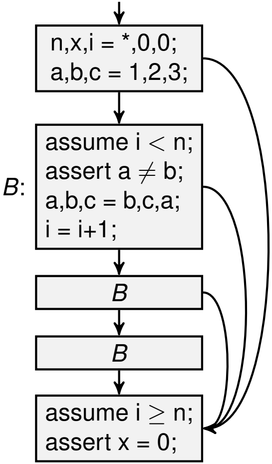

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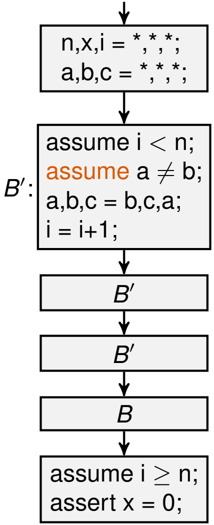

induction step for $k = 3$

The End