

Exercise Sheet 2: Extensive-Form Games

Due Date: December 16, 2024

Instructions

- Submit your exercises as a PDF file through the file vault (odevzdávárna) in the IS.
- Solutions will be graded based on their correctness and the *clarity* of the arguments presented.
- You do not have to provide proofs and justifications if you are not explicitly instructed to do so.
- Collaboration: While you may discuss the exercises with classmates, you must write down the solutions on your own.
- The bonus exercise is optional and will not be graded (although feedback will be provided if you submit a solution). Solving the exercise serves mainly your benefit.
- To pass the exercise sheet, you need to obtain **at least 45** points out of 85 possible.
- If you do not meet the threshold for the minimum points, you may resubmit your solution after the first trial is marked.

Exercises

Exercise 1: (max. 10 points)

Formalize rock-paper-scissors as a two-player zero-sum imperfect-information extensive-form game.

Exercise 2: (max. 25 points)

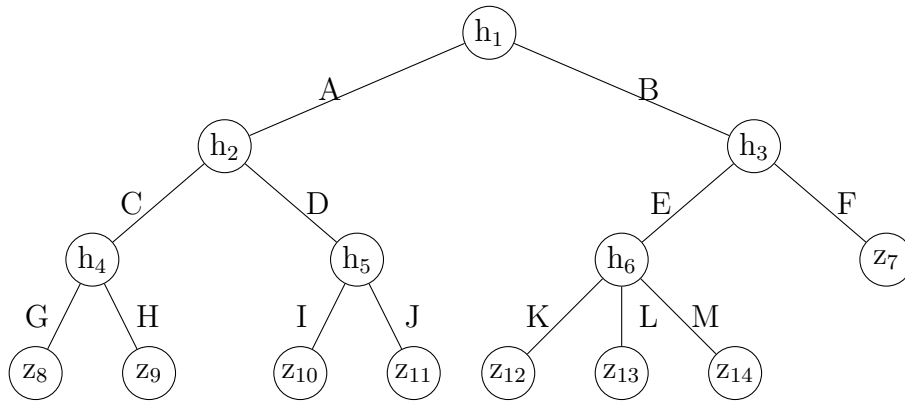
Find a two-player perfect-information extensive-form game where all of the following conditions are satisfied:

- there is a strategy profile whose outcome is for both players better than that of any Nash equilibrium;
- there is a Nash equilibrium whose outcome is for both players better than that of any subgame-perfect equilibrium;
- there are exactly two subgame-perfect equilibria s, s' , and the outcome of s is for both players better than that of s' .

Should you fail to find such a game, try your best (for partial points) to find a game which matches the requirements as closely as you can.

Exercise 3: (max. 25 points)

- a) [10 points] Consider the one-player perfect-information extensive-form game depicted below. In this game, consider a mixed strategy σ given as follows:



$$\begin{aligned} \sigma(ACEHJK) &= 1/15 \\ \sigma(ACFHJL) &= 1/15 \\ \sigma(ADFGIL) &= 3/15 \\ \sigma(BCEGIL) &= 1/4 \\ \sigma(BCFHJL) &= 1/18 \\ \sigma(BDEGIM) &= 1/4 \\ \sigma(BDFGIL) &= 2/18 \end{aligned}$$

Find an equivalent behavioral strategy β . Is it unique? Justify your answer.

- b) [15 points] Prove or disprove: In every zero-sum two-player perfect-information extensive-form game G , all subgame-perfect equilibria have the same outcome for player 1.

Exercise 4: (max. 25 points)

Consider the following two-player strategic-form game G :

	X	Y
A	5, 6	-1, 7
B	7, -1	2, 2

- a) [10 points] Find a **subgame-perfect** equilibrium in G_{irep}^{avg} whose value is $(3, 10/3)$.
 b) [8 points] Determine $\inf_{c \in \text{SPE}(G_{irep}^{avg})} u_1(c)$.
 c) [7 points] Determine $\sup_{c \in \text{SPE}(G_{irep}^{avg})} u_1(c)$.

Justify your answers.