

# Authenticated Encryption

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## Purpose of AE

Provide both secrecy guarantees of **secure ciphers** and authenticity guarantees of **MACs**.

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Several possible combinations, **not** all of them secure?

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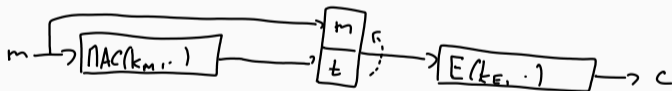
- **Encrypt-then-MAC**

✓



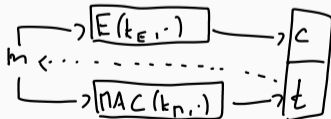
- **MAC-then-encrypt**

ok



- **Encrypt-and-MAC**

✗



Instead of requiring the user to construct the correct combination manually, AE aims to provide a single primitive with both secrecy/authenticity guarantees.

## Definition 1: Cipher for AE

An AE-enabled cipher over  $(\mathcal{K}, \mathcal{M}, \mathcal{C})$  is a tuple  $\mathcal{E} = (E, D)$ , where

- $E: \mathcal{K} \times \mathcal{M} \rightarrow \mathcal{D}(\mathcal{C})$  is a (possibly probabilistic) encryption algorithm
- $D: \mathcal{K} \times \mathcal{C} \rightarrow \mathcal{M} \cup \{\perp\}$  is a decryption algorithm s.t. for all  $k \in \mathcal{K}, m \in \mathcal{M}$  it holds

↓  
"reject"

$$D(k, E(k, m)) = m.$$

## Definition 2: AE Security

Let  $\mathcal{E}$  be an AE-enabled cipher. We say that  $\mathcal{E}$  is  $(\epsilon, \delta)$ -AE secure if:

- $\mathcal{E}$  is  $\epsilon$ -CPA secure; and
- $\mathcal{E}$  has an  $\delta$ -ciphertext integrity (see next slide)

AE-secure ciphers provide security against both chosen plaintext and **chosen ciphertext attacks (CCAs)**.

## Ciphertext integrity attack game

Let  $\mathcal{E} = (E, D)$  be an AE-enabled cipher over  $(\mathcal{K}, \mathcal{M}, \mathcal{C})$ . The **ciphertext integrity attack game** between the challenger and the adversary  $\mathcal{A}$  proceeds as follows:

- The challenger samples a key  $k \in \mathcal{K}$  uniformly at random.
- $\mathcal{A}$  chooses a number of rounds  $N$  of the game. In each round  $1 \leq i \leq N$ :
  - $\mathcal{A}$  computes a message  $m_i \in \mathcal{M}$  and sends it to the challenger;
  - the challenger computes  $c_i = E(k, m_i)$  and sends  $c_i$  to the adversary.
- After the final round,  $\mathcal{A}$  computes a ciphertext  $c \in \mathcal{C}$  such that  $c \notin \{c_1, c_2, \dots, c_N\}$ .

$\mathcal{A}$  **wins** the game if  $D(k, c) \neq \perp$ . We denote by  $\mathbb{P}$  the probability measure corresponding to the game.

The CI-advantage of  $\mathcal{A}$  against  $\mathcal{E}$  is the quantity

$$ADV_{CI}(\mathcal{E}, \mathcal{A}) = \mathbb{P}(\mathcal{A} \text{ wins the AE attack game}).$$

We say that  $\mathcal{E}$  has  $\delta$ -ciphertext integrity if  $ADV_{CI}(\mathcal{E}, \mathcal{A}) \leq \delta$  for all efficient adversaries  $\mathcal{A}$ .

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Let  $(E, D)$  be a cipher over  $(\mathcal{K}, \mathcal{M}, \mathcal{C})$ . The CCA attack game against  $(E, D)$  is played as follows:

## Stage 1:

- The challenger samples a key  $k \in \mathcal{K}$  and a bit  $i \in \{0, 1\}$ , both uniformly at random. Neither is revealed to the adversary.
- The adversary announces a number of rounds  $q$  for which the game will be played.



# CCA attack game

Let  $(E, D)$  be a cipher over  $(\mathcal{K}, \mathcal{M}, \mathcal{C})$ . The **CCA attack game against  $(E, D)$**  is played as follows:

## Stage 2:

- In each round  $1 \leq j \leq q$ , the adversary chooses to perform either a **plaintext query** or a **ciphertext query**:
  - Plaintext query: the **adversary** computes **two messages**,  $m_0^j$  and  $m_1^j$  **of the same length** and sends them to the challenger, who responds with  $c^j = E(k, m_i^j)$ .
  - Ciphertext query: the **adversary** computes  $c_i' \in \mathcal{C}$  s.t.

$$c_i' \notin \{c_j \mid \text{plaintext query was done in round } j < i\},$$

and sends  $c_i'$  to the challenger, who responds with  $m_i' = D(k, c_i')$ .

## Stage 3:

- Finally, **adversary** outputs a **guess**  $g \in \{0, 1\}$ .

The **adversary wins** the game if  $g = i$ , otherwise it loses.

# CCA attack game: picture

Challenger

$$b \xleftarrow{R} \{0,1\}^n$$

$$k \xleftarrow{R} \mathcal{K}$$

$$c_i = E(k, m_{i,b})$$

$$D(k, c_i)$$

Adversary

select no. of rounds  $N$

in each round  $i$ :

PLAINTEXT QUERY

create  $m_{i,0}$  and  $m_{i,1}$

CIPHERTEXT QUERY

create  $c_i$  s.t.  $c_i \notin \{c_1, \dots, c_{i-1}\}$

guess  $g \in \{0,1\}$

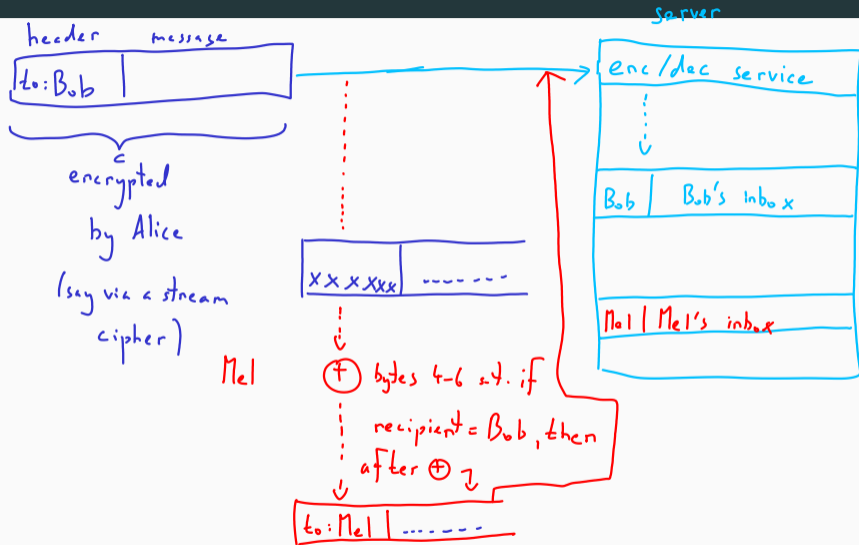


We define the **CCA advantage** of  $\mathcal{A}$  against  $\mathcal{E}$  as the quantity

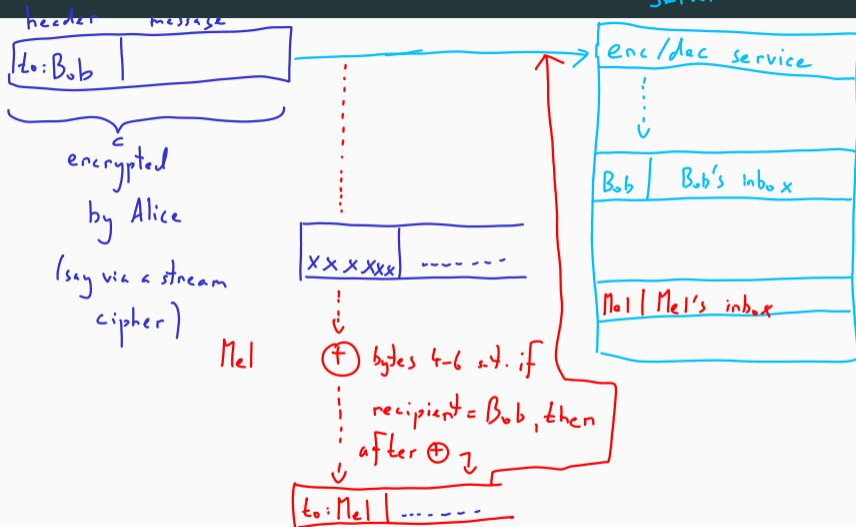
$$ADV_{CCA}(\mathcal{E}, \mathcal{A}) = \left| \mathbb{P}(\mathcal{A} \text{ wins the game against } \mathcal{E}) - \frac{1}{2} \right|.$$

We say that  $\mathcal{E}$  is an  **$\varepsilon$ -CCA secure cipher** (where  $\varepsilon > 0$ ) if **for every efficient adversary** it holds  $ADV_{CCA}(\mathcal{E}, \mathcal{A}) \leq \varepsilon$ .

# CC attack: simplified example



# CC attack: simplified example



For a practical example, see, e.g. the [padding oracle](#) CC attack POODLE (2014) completely breaking the security of SSL 3.0 (CBC-based MAC-then-encrypt).

# AE security provides CCA security

## Theorem 1

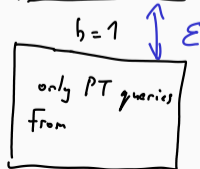
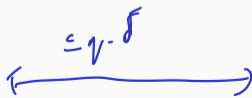
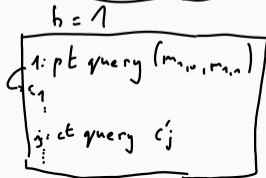
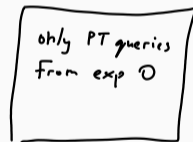
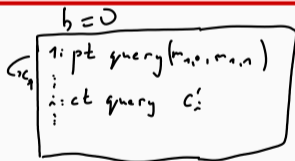
Let  $\mathcal{E} = (E, D)$  be an  $(\epsilon, \delta)$ -AE secure cipher. Then, for any efficient adversary  $\mathcal{A}$  who makes at most  $q$  queries it holds

$$\text{ADV}_{\text{CCA}}(\mathcal{E}, \mathcal{A}) \leq \epsilon + 2q\delta.$$

*ciphertext queries removed*

$b=0$

Proof sketch:



## Definition 3: MAC-then-Encrypt (MtE) system

Let  $\underline{MAC} = (S, V)$  be a MAC over  $(\mathcal{M}, \mathcal{H}, \mathcal{K}_M)$  and let  $\underline{\mathcal{E}} = (E, D)$  be a (classical) cipher over  $(\underline{\mathcal{M} \times \mathcal{H}}, \mathcal{C}, \mathcal{K}_E)$ . An  $\text{MtE}$  system built from  $\text{MAC}$  and  $\mathcal{E}$  is an AE-enabled cipher  $\mathcal{E}_{\text{MtE}} = (E_{\text{MtE}}, D_{\text{MtE}})$  over  $(\mathcal{M}, \mathcal{C}, \mathcal{K}_M \times \mathcal{K}_E)$  defined as follows:

- for every  $m \in \mathcal{M}$ ,  $(k_M, k_E) \in \mathcal{K}_M \times \mathcal{K}_E$  we put

$$E_{\text{MtE}}((k_M, k_E), m) = E(k_E, (m, S(k_M, m)))$$

- for every  $c \in \mathcal{C}$ ,  $(k_M, k_E) \in \mathcal{K}_M \times \mathcal{K}_E$ , the decryption process  $D_{\text{MtE}}((k_M, k_E), c)$ :
  - first computes  $(m, t) = D(k_E, c)$  (and outputs  $\perp$  if  $(m, t) \notin \mathcal{M} \times \mathcal{H}$ )
  - checks that  $V(k_M, m, t) = \text{true}$ ; if yes, the procedure outputs  $m$ , otherwise it outputs  $\perp$ .

MtE systems generally do not provide AE security (see the POODLE attack against SSL 3.0). However, security guarantees can be recovered for certain underlying ciphers and additional assumptions.

## Theorem 2

Let  $\mathcal{E}$  be an  $\varepsilon$ -CPA secure cipher which is a randomized counter mode of some block cipher over the block space  $\mathcal{X}$ . Moreover, let  $MAC$  be an  $\alpha$ -one-time secure MAC. Then, when restricting to adversaries that make at most  $q$  queries, the MtE system built from  $MAC$  and  $\mathcal{E}$  is  $(\varepsilon, \delta)$ -AE-secure for

$$\delta = \frac{q^2}{2|\mathcal{X}|} + (q + 1) \cdot \alpha$$

For randomized CBC mode + secure MAC, similar theorem can be obtained assuming that  $m || t$  is never padded (all messages consist of full blocks, tag one full block).



Nevertheless, due to the difficulty of implementing an MtE system correctly, its usage (in particular the design of new primitives based on this approach) is discouraged.

# Encrypt-then-MAC

## Definition 4: Encrypt-then-MAC (EtM) system

Let  $\mathcal{E} = (E, D)$  be a (classical) cipher over  $(\mathcal{M}, \mathcal{C}, \mathcal{K}_E)$  and let  $MAC = (S, V)$  be a MAC over  $(\mathcal{C}, \mathcal{H}, \mathcal{K}_M)$ . An **EtM** system built from  $\mathcal{E}$  and  $MAC$  is an AE-enabled cipher  $\mathcal{E}_{EtM} = (E_{EtM}, D_{EtM})$  over  $(\mathcal{M}, \mathcal{C} \times \mathcal{H}, \mathcal{K}_E \times \mathcal{K}_M)$  defined as follows:

- for every  $m \in \mathcal{M}$ ,  $(k_E, k_M) \in \mathcal{K}_E \times \mathcal{K}_M$  we put

$$E_{EtM}((k_E, k_M), m) = (E(k_E, m), S(k_M, E(k_E, m)))$$

- for every  $(c, t) \in \mathcal{C} \times \mathcal{H}$ ,  $(k_E, k_M) \in \mathcal{K}_E \times \mathcal{K}_M$ , the decryption process  $D_{EtM}((k_E, k_M), (c, t))$ :
  - first computes  $m = D(k_E, c)$  and then checks whether  $V(k_M, m, t) = true$ ; if yes, the procedure outputs  $m$ , otherwise it outputs  $\perp$ .

## Theorem 3

Let  $\mathcal{E}$  be an  $\varepsilon$ -CPA secure cipher, and let  $MAC$  be a  $\delta$ -secure MAC. Then, when restricting to adversaries the EtM system built from  $MAC$  and  $\mathcal{E}$  is  $(\varepsilon, \delta)$ -AE-secure.

# Practical AE-enabled ciphers

Typically dedicated modes of block ciphers:

- **EAX**: basically nonce-based CTR then CMAC.
- **CCM**: CBC-MAC then nonce-based CTR encryption (e.g. WiFi traffic)
- **GCM**: **Galois counter mode**, nonce-based CTR then a Carter-Wegman-style MAC based using the GHASH one-time MAC (like the polynomial MAC from previous lecture but working over an appropriate Galois field rather than over  $\mathbb{Z}_p$ )
  - fastest, in particular using the acceleration via the PCLMULQDQ instruction

$$\hat{r} \xrightarrow{\text{random}} (r, E(k_1, r) \oplus S_{\text{ot}}(k_2, m))$$

OCB

# Authenticated encryption with associated data

1. All the state-of-the-art AE modes do support **authenticated encryption with associated data (AEAD)**.
  - message - encrypted and authenticated
  - associated data - only authenticated

