$\frac{1-x^6}{1-x} \cdot \frac{1}{1-x} \cdot \frac{1}{1-x}$ koef. u x^7

$\binom{9}{7} - 3$

↑ výběry bez omezení

$(a+b)^n = \sum \binom{n}{k} a^{n-k} b^k$

$a \to 1$, $b \to x$, $a^{n-k} \to 1$, $b^k \to x^k$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$(a+b)^5 = a^0b^5 + 5a^1b^4 + 10a^2b^3 + 10a^3b^2 + 5a^4b^1 + a^5b^0$

$$\sum \binom{n}{k} x^k = (1+x)^n$$

$$\sum \binom{n}{k} k\, x^{k-1} = n(1+x)^{n-1} \qquad \Big| x=1$$

$$\begin{tikzcd}
(1,1,1,\text{---}) \arrow[rr, leftrightarrow] \arrow[dr, leftrightarrow] & & \sum\limits_{k=0}^{\infty} x^k \arrow[dl, leftrightarrow] \\
& \frac{1}{1-x} &
\end{tikzcd}$$

$$\begin{tikzcd}
(F_0, F_1, F_2, \ldots) \arrow[rr, leftrightarrow] \arrow[dr, leftrightarrow] & & \sum\limits_{k=0}^{\infty} F_k x^k \arrow[dl, leftrightarrow] \\
& f(x) &
\end{tikzcd}$$

$$F_k = F_{k-1} + F_{k-2}, \quad F_0 = 0, \quad F_1 = 1$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k \qquad \Big/ \int_0^x dx$$

$$-\ln(1-x) = \sum_{k=0}^{\infty} \frac{x^{k+1}}{k+1} = \frac{x^1}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \dots$$

$k=0 \quad k=1 \quad k=2$

$$\ln\frac{1}{1-x} = \sum_{k=1}^{\infty} \frac{x^k}{k}$$

$k=1 \quad k=2 \quad k=3$

$$r \in \mathbb{R} \Rightarrow \binom{r}{k} = \frac{r\cdot(r-1)\dots(r-k+1)}{k\cdot(k-1)\dots 1}$$

$$\binom{-4}{3} = \frac{(-4)\cdot(-5)\cdot(-6)}{3\cdot 2\cdot 1}$$

$$= -\frac{6\cdot 5\cdot 4}{3\cdot 2\cdot 1} = -\binom{6}{3}$$

$$f(x) = (1+x)^r = \sum a_k \cdot x^k \qquad f'(x) = r\cdot(1+x)^{r-1}$$

$$f''(x) = r\cdot(r-1)\cdot(1+x)^{r-2}$$

$$a_k = \frac{f^{(k)}(0)}{k!} = \frac{r\cdot(r-1)\dots(r-k+1)}{k\cdot(k-1)\dots 1}$$

$$(1+x)^r = \sum \binom{r}{k} x^k$$

$x \leftsquigarrow -x$

$r \leftsquigarrow -n$

$$(1-x)^{-n} = \sum \binom{-n}{k} (-x)^k$$

$$\frac{1}{(1-x)^n} = \sum \binom{k+n-1}{n-1} \underset{\substack{\uparrow \\ (-1)^k \cdot (-x)^k}}{x^k}$$

$$\frac{1-x^{31}}{1-x} \cdot \frac{1-x^{41}}{1-x} \cdot \frac{1-x^{51}}{1-x}$$

$$= (1-x)^{-3} (1-x^{31})(1-x^{41})(1-x^{51})$$

$$=$$

			2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

3.1. zrušit rezervaci

$$(a_0+b_0)+(a_1+b_1)x+(a_2+b_2)x^2+\cdots$$

$$=(a_0+a_1x+a_2x^2+\cdots)$$
$$+(b_0+b_1x+b_2x^2+\cdots)$$

$$\alpha\cdot a_0+(\alpha\cdot a_1)x+(\alpha\cdot a_2)x^2+\cdots$$

$$=\alpha(a_0+a_1x+a_2x^2+\cdots)$$

$$x^n(a_0+a_1x+a_2x^2+\cdots)$$

$$=0+0x+0x^2+\cdots+0x^{n-1}$$
$$+a_0x^n+a_1x^{n+1}+\cdots$$

$$(a_0+a_1x+a_2x^2+a_3x^3+\cdots)'$$

$$=a_1+2a_2x+3a_3x^2+\cdots$$

$$(a_0+a_1x+a_2x^2+\cdots)(b_0+b_1x+b_2x^2+\cdots)$$

$$= \cdots + (a_0 b_k + a_1 b_{k-1} + \cdots + a_k b_0)x^k + \cdots$$

$$\uparrow$$

$$a_0 \cdot b_k x^k \qquad a_1 x \cdot b_{k-1} x^{k-1} \qquad a_k x^k \cdot b_0$$

$$x^n \cdot \sum a_k x^k = \sum a_k x^{n+k}$$

$$= \sum a_{k-n} x^k$$

$$(1, 1, 1, \text{—}) \qquad (a_0, a_1, a_2, \text{—}) \qquad (0, \tfrac{1}{1}, \tfrac{1}{2}, \ldots)$$

$$\updownarrow \qquad \updownarrow \qquad \updownarrow$$

$$\frac{1}{1-x} \qquad a(x) \qquad \ln\frac{1}{1-x}$$

$$(-, a_0 + a_1 + \cdots + a_k, \sim)$$

$$\updownarrow$$

$$\frac{1}{1-x} \cdot a(x)$$

$$H_k = 0 + \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{k}$$

$$\frac{1}{1-x} \qquad (1, 1, 1, - \quad -) \qquad a_k = 1$$

$$\frac{1}{(1-x)^2} \qquad (1, 2, 3, \text{———}) \qquad a_k = k+1$$

$$\frac{1}{(1-x)^3} \qquad (1, 3, 6, \text{———}) \qquad a_k = (k+1)\frac{k+2}{2}$$

$$= \binom{k+2}{2}$$

$(1, 2, 4, 8, \ldots)$

$$\sum 2^i \cdot x^i = \sum (2x)^i = \frac{1}{1-2x}$$

$(1, 1+2, 1+2+4, 1+2+4+8, \ldots)$

$$\frac{1}{1-x} \cdot \frac{1}{1-2x}$$

↑ částečné součty

$k \geq 2$

$$a_k = 5a_{k-1} - 6a_{k-2} \qquad a_0 = 0,\ a_1 = 1$$

$k=1$: $\quad a_1 = 5a_0 - 6a_{-1} + [1=1]$

$$1 = 0 - 0 + 1$$

$k=0$: $\quad a_0 = 5a_{-1} - 6a_{-2}$

$$0 = 0 - 0 \quad \checkmark$$

+1 pokud $k=1$

$$a_k = 5a_{k-1} - 6a_{k-2} + [k=1]$$

$$\downarrow \sum (\,)\cdot x^k$$

$$\sum a_k x^k = \sum \left(5a_{k-1} - 6a_{k-2} + [k=1]\right)\cdot x^k$$

$$\sum a_k x^k = 5\cdot\sum a_{k-1}x^k - 6\sum a_{k-2}x^k + \sum [k=1]x^k$$

|| def

$$a(x) = 5x\,a(x) - 6x^2 a(x) + x$$

$(0 + 1x + 0x^2 + 0x^3 + \dots)$

$$a(x) = 5x\,a(x) - 6x^2 a(x) + x$$

$$(1 - 5x + 6x^2)\,a(x) = x$$

$$a(x) = \frac{x}{1-5x+6x^2}$$

$$= \frac{1}{1-3x} - \frac{1}{1-2x}$$

$$= \sum \overbrace{\binom{k+0}{0}}^{1} \cdot 3^k x^k - \sum \overbrace{\binom{k+0}{0}}^{1} \cdot 2^k x^k$$

$$= \sum (3^k - 2^k)\, x^k$$

$$a_k = 3^k - 2^k$$