

$$\frac{1-x^6}{1-x} \cdot \frac{1}{1-x} \cdot \frac{1}{1-x} \quad \text{wef. u } x^7$$

$$\binom{9}{7} - 3$$

↑ výběry bez omezení

$$(a+b)^n = \sum \binom{n}{k} a^{n-k} b^k$$

↑ ↑ ↑ ↑
1 x 1 x^k

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

$$(a+b)^5 = a^0 b^5 + 5 a^1 b^4 + 10 a^2 b^3 + 10 a^3 b^2 + 5 a^4 b^1 + a^5 b^0$$

$$\sum \binom{n}{k} x^k = (1+x)^n$$

$$\sum \binom{n}{k} k x^{k-1} = n(1+x)^{n-1} \quad |_{x=1}$$

$$(1, 1, 1, \dots) \longleftrightarrow \sum_{k=0}^{\infty} x^k$$

$$\frac{1}{1-x}$$

$$(F_0, F_1, F_2, \dots) \longleftrightarrow \sum_{k=0}^{\infty} F_k x^k$$

$$f(x)$$

$$F_k = F_{k-1} + F_{k-2}, \quad F_0 = 0, \quad F_1 = 1$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k \quad / \int_0^x dx$$

$$-\ln(1-x) = \sum_{k=0}^{\infty} \frac{x^{k+1}}{k+1} = \frac{x^1}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \dots$$

$$\ln \frac{1}{1-x} = \sum_{k=1}^{\infty} \frac{x^k}{k} \quad \begin{matrix} k=1 & k=2 & k=3 \end{matrix}$$

$$r \in \mathbb{R} \Rightarrow \binom{r}{k} = \frac{r \cdot (r-1) \cdot \dots \cdot (r-k+1)}{k \cdot (k-1) \cdot \dots \cdot 1}$$

$$\binom{-4}{3} = \frac{(-4) \cdot (-5) \cdot (-6)}{3 \cdot 2 \cdot 1}$$

$$= - \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = - \binom{6}{3}$$

$$f(x) = (1+x)^r = \sum a_k \cdot x^k$$

$$f'(x) = r \cdot (1+x)^{r-1}$$

$$f''(x) = r \cdot (r-1) \cdot (1+x)^{r-2}$$

$$a_k = \frac{f^{(k)}(0)}{k!} = \frac{r \cdot (r-1) \cdot \dots \cdot (r-k+1)}{k \cdot (k-1) \cdot \dots \cdot 1}$$

$$(1+x)^r = \sum \binom{r}{k} x^k$$

$$x \leftarrow -x$$

$$r \leftarrow -n$$

$$(1-x)^{-n} = \sum \binom{-n}{k} (-x)^k$$

$$\frac{1}{(1-x)^n} = \sum \binom{k+n-1}{n-1} x^k$$

↑
 $(-1)^k \cdot (-x)^k$



$$\frac{1-x^{3n}}{1-x} \cdot \frac{1-x^{4n}}{1-x} \cdot \frac{1-x^{5n}}{1-x}$$

$$= (1-x)^{-3} (1-x^{3n})(1-x^{4n})(1-x^{5n})$$

=

			2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

3.1. *zmsit* *rezervaci*

$$(a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2 + \dots$$

$$= (a_0 + a_1x + a_2x^2 + \dots) + (b_0 + b_1x + b_2x^2 + \dots)$$

$$\alpha \cdot a_0 + (\alpha \cdot a_1)x + (\alpha \cdot a_2)x^2 + \dots$$

$$= \alpha (a_0 + a_1x + a_2x^2 + \dots)$$

$$x^4 (a_0 + a_1x + a_2x^2 + \dots)$$

$$= 0 + 0x + 0x^2 + \dots + 0x^{4-1} + a_0x^4 + a_1x^{4+1} + \dots$$

$$(a_0 + a_1x + a_2x^2 + a_3x^3 + \dots)'$$

$$= a_1 + 2a_2x + 3a_3x^2 + \dots$$

$$(a_0 + a_1x + a_2x^2 + \dots)(b_0 + b_1x + b_2x^2 + \dots)$$

$$= \dots + (a_0b_k + a_1b_{k-1} + \dots + a_kb_0)x^k + \dots$$

$$\begin{array}{c} \uparrow \\ a_0 \cdot b_k x^k \quad a_1 x \cdot b_{k-1} x^{k-1} \quad a_k x^k \cdot b_0 \end{array}$$

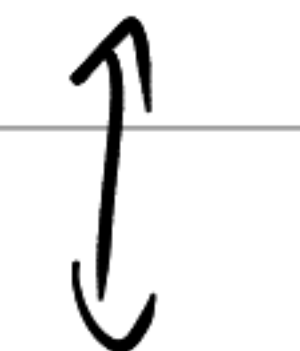
$$x^n \cdot \sum a_k x^k = \sum a_k x^{n+k}$$



$$= \sum a_{k-n} x^k \quad (0, 1, 1, 2, \dots)$$

$$(1, 1, 1, \dots)$$

$$(a_0, a_1, a_2, \dots)$$

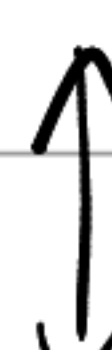


$$\frac{1}{1-x}$$

$$a(x)$$

$$\ln \frac{1}{1-x}$$

$$(-, a_0 + a_1 + \dots + a_2, \dots)$$



$$\frac{1}{1-x} \cdot a(x)$$

$$H_k = 0 + \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{k}$$

$$\frac{1}{1-x} \quad (1, 1, 1, - \dots) \quad a_k = 1$$

$$\frac{1}{(1-x)^2} \quad (1, 2, 3, \dots) \quad a_k = k+1$$

$$\frac{1}{(1-x)^3} \quad (1, 3, 6, \dots) \quad a_k = (k+1) \frac{k+2}{2}$$
$$= \binom{k+2}{2}$$

(1, 2, 4, 8, —)

$$\sum 2^k \cdot x^k = \sum (2x)^k = \frac{1}{1-2x}$$

(1, 1+2, 1+2+4, 1+2+4+8, —)

$$\frac{1}{1-x} \cdot \frac{1}{1-2x}$$

↑ cascade soucy

$$a_k = 5a_{k-1} - 6a_{k-2} \quad \leftarrow k \geq 2 \quad a_0 = 0, a_1 = 1$$

$$\{=1: \quad a_1 = 5a_0 - 6a_{-1} + [1=1]$$

$$1 = 0 - 0 + 1$$

$$\{=0: \quad a_0 = 5a_{-1} - 6a_{-2}$$

$$0 = 0 - 0$$



\swarrow +1 polind $\{=1$

$$a_k = 5a_{k-1} - 6a_{k-2} + [k=1]$$

$$\longrightarrow \sum (1) \cdot x^k$$

$$\sum a_k x^k = \sum (5a_{k-1} - 6a_{k-2} + [k=1]) \cdot x^k$$

$$\sum a_k x^k = 5 \cdot \sum a_{k-1} x^k - 6 \sum a_{k-2} x^k + \sum [k=1] x^k$$

|| def

$$a(x) = 5x a(x) - 6x^2 a(x) + x$$



$$(0 + 1x + 0x^2 + 0x^3 + \dots)$$

$$a(x) - 5x a(x) - 6x^2 a(x) + x$$

$$(1 - 5x + 6x^2) a(x) = x$$

$$a(x) = \frac{x}{1 - 5x + 6x^2}$$

$$= \frac{1}{1 - 3x} - \frac{1}{1 - 2x}$$

$$= \sum \binom{k+0}{0} \cdot 3^k x^k - \sum \binom{k+0}{0} \cdot 2^k x^k$$

$$= \sum (3^k - 2^k) x^k$$

$$a_k = 3^k - 2^k$$