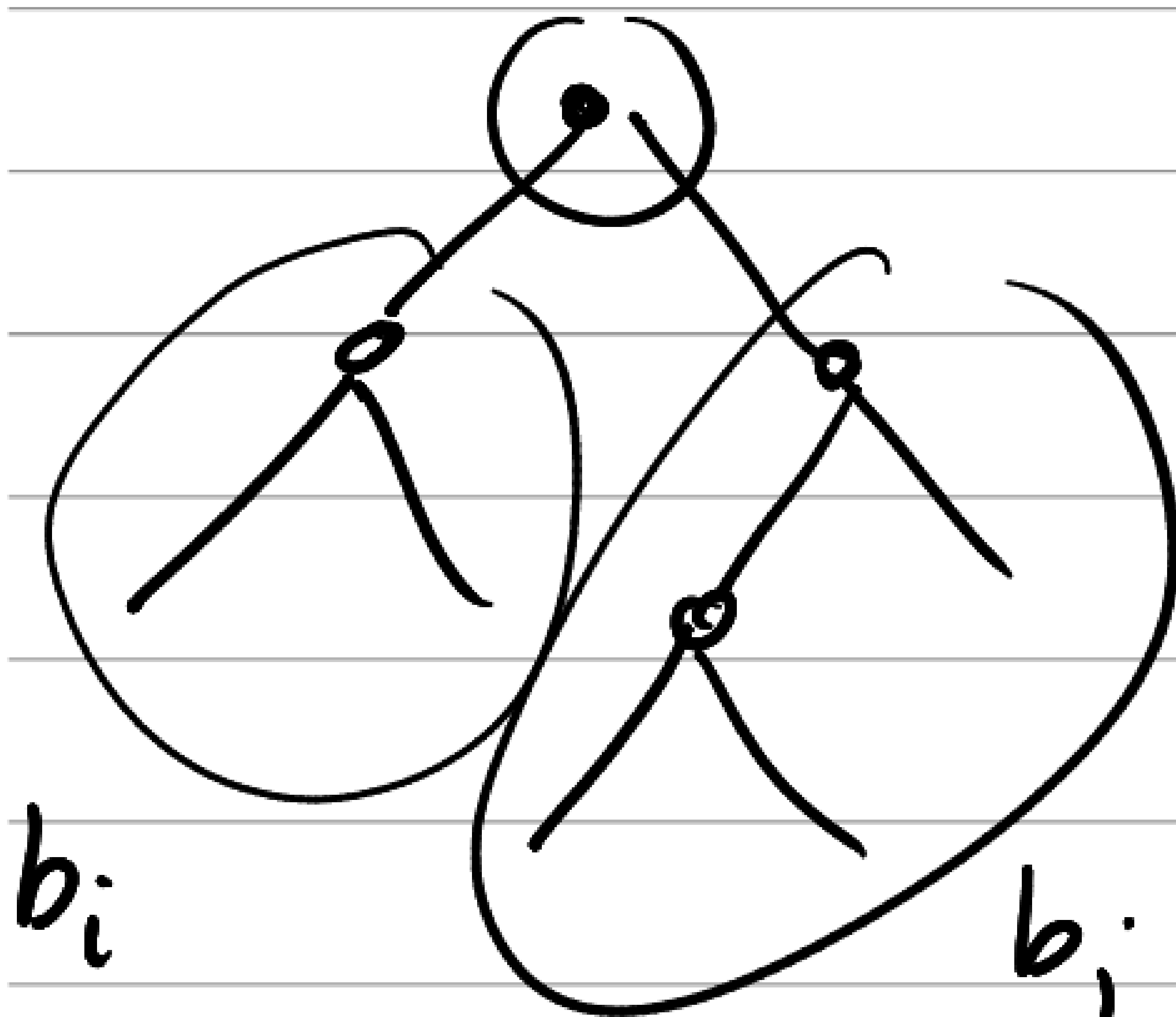
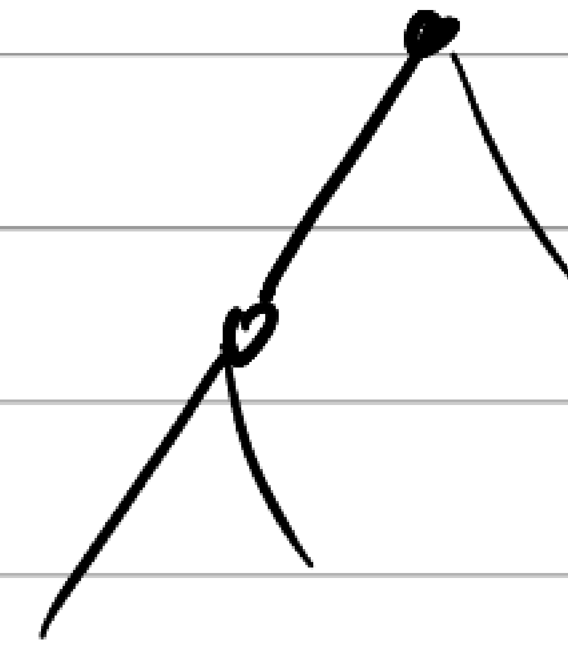
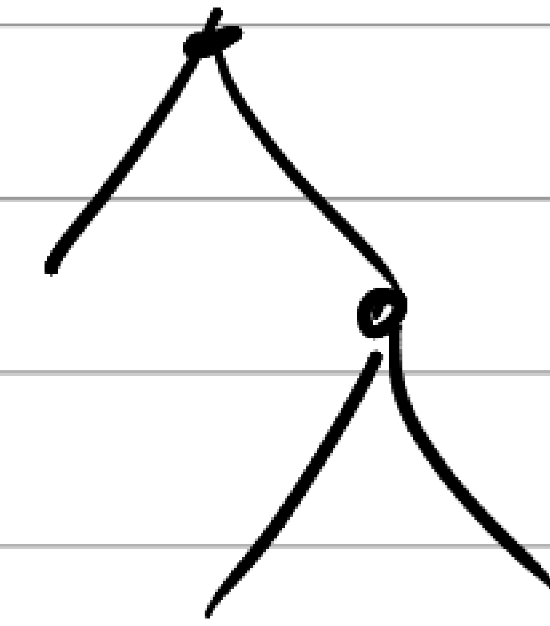






b_i b_j možností

$i+j = n-1$

$$(c_0 + c_1 x + c_2 x^2 + \dots)\cdot(d_0 + d_1 x + d_2 x^2 + \dots)$$

$$= \dots + (c_0 d_k + c_1 d_{k-1} + \dots + c_k d_0)\cdot x^k + \dots$$

$$b_n = \sum_{i+j=n-1} b_i b_j + [n=0] \quad / \cdot x^n \quad / \sum_n$$

$$B(x) = x \cdot B(x) B(x) + 1$$

$$1 - \frac{1}{2}(4x) + \dots$$

$$0 = x \cdot B(x)^2 - B(x) + 1 \Rightarrow B(x) = \frac{1 \pm \sqrt{1-4x}}{2x}$$

$$\frac{1}{\sqrt{1-4x}} = (1-4x)^{-1/2} = \sum_n \binom{-1/2}{n} (-4x)^n$$

$$(1+x)^r = \sum \binom{r}{n} \cdot x^n$$

$$= \sum_n \binom{2n}{n} \left(-\frac{1}{4}\right)^n (-4x)^n$$

$$= \sum_n \binom{2n}{n} x^n$$

$x^{n+1} \cdot 1$

$$B(x) = \frac{1 - \left(\sum_n \binom{2n}{n} x^n\right)(1-4x)}{2x}$$

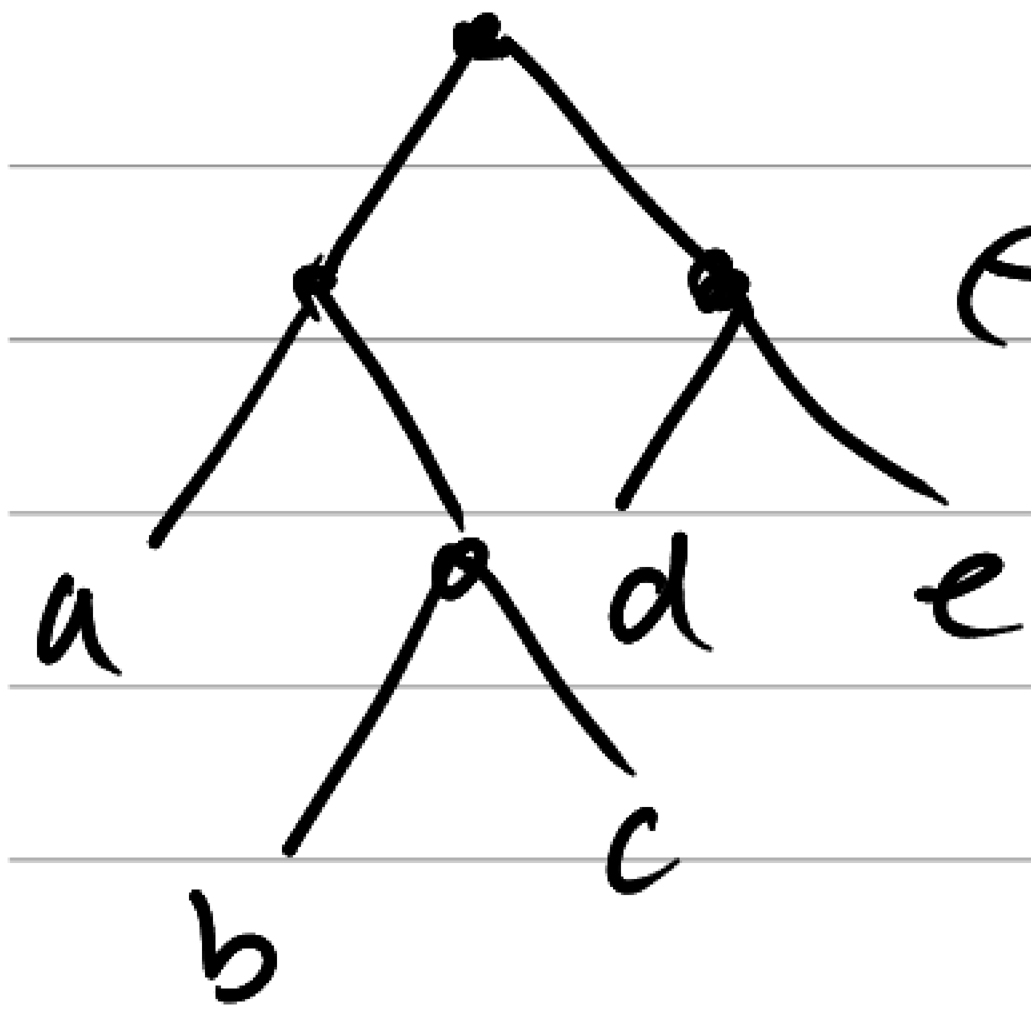
$$\text{Koef. } n \quad x^n : \quad -\frac{\binom{2n+2}{n+1}}{2} + \frac{\binom{2n}{n} \cdot 4}{2}$$

$$\binom{2n+2}{n+1} = \frac{(2n+2)(2n+1)(2n)!}{(n+1) \cdot n! \cdot (n+1) \cdot n!} = \frac{2(2n^2+3n+1)}{(n+1)^2} \binom{2n}{n}$$

$$\frac{2n^2+4n+2}{(n+1)^2}$$

$$= \dots = \frac{1}{n+1} \binom{2n}{n} =: C_n$$

$$\binom{2n}{n}\left(2 - \frac{2n^2+3n+1}{(n+1)^2}\right) = \binom{2n}{n} \cdot \frac{n+1}{(n+1)^2} = \binom{2n}{n} \cdot \frac{1}{n+1}$$

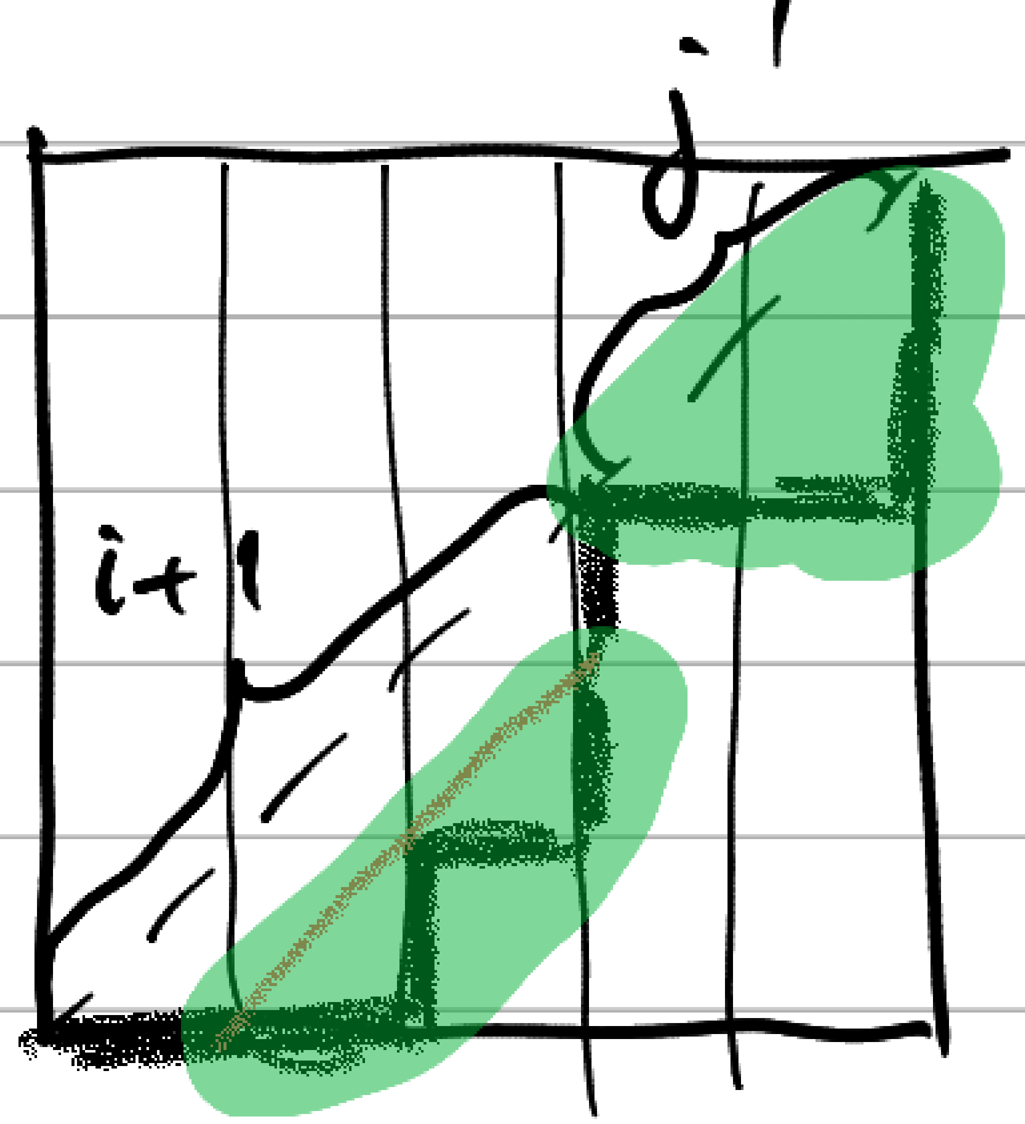


$\longleftrightarrow \underbrace{(a(bc))}_{i\times\,\cdot}\ \cdot\ \underbrace{(de)}_{j\times\,\cdot}$ 4 operace ·

$1\times\,\cdot$

4 vrcholy

$\Rightarrow C_n = \sum_{i+j=n-1} C_i C_j$

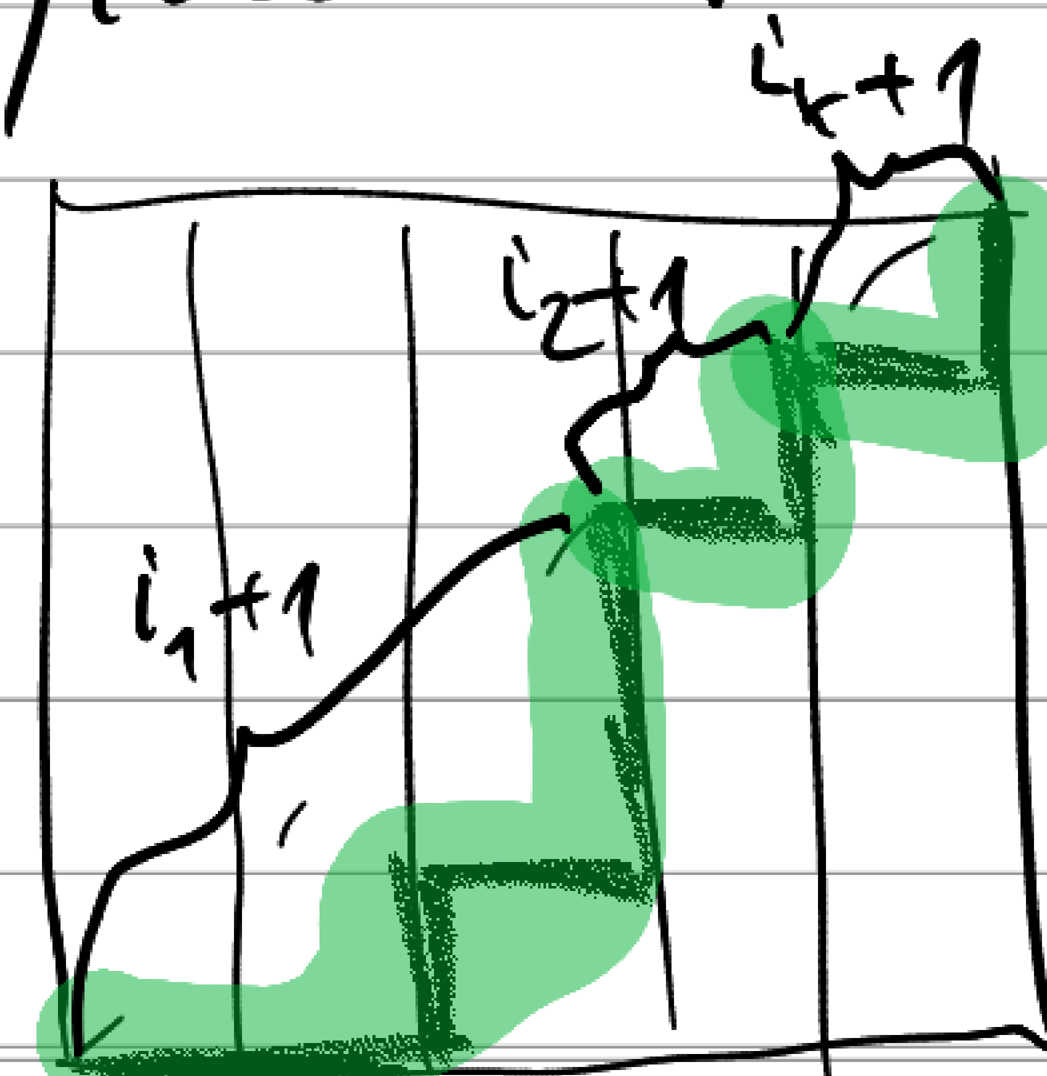


cesta délky j

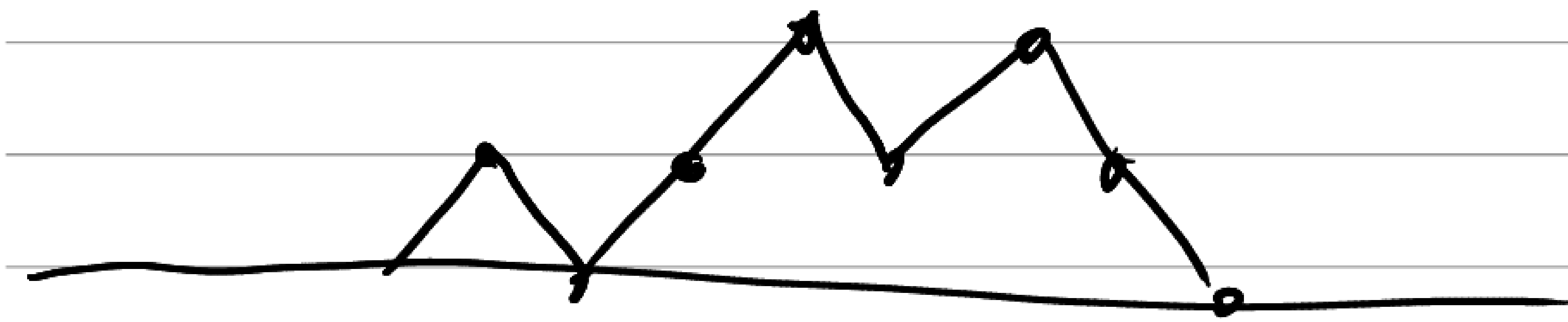
stejná rekurence

cesta délky i

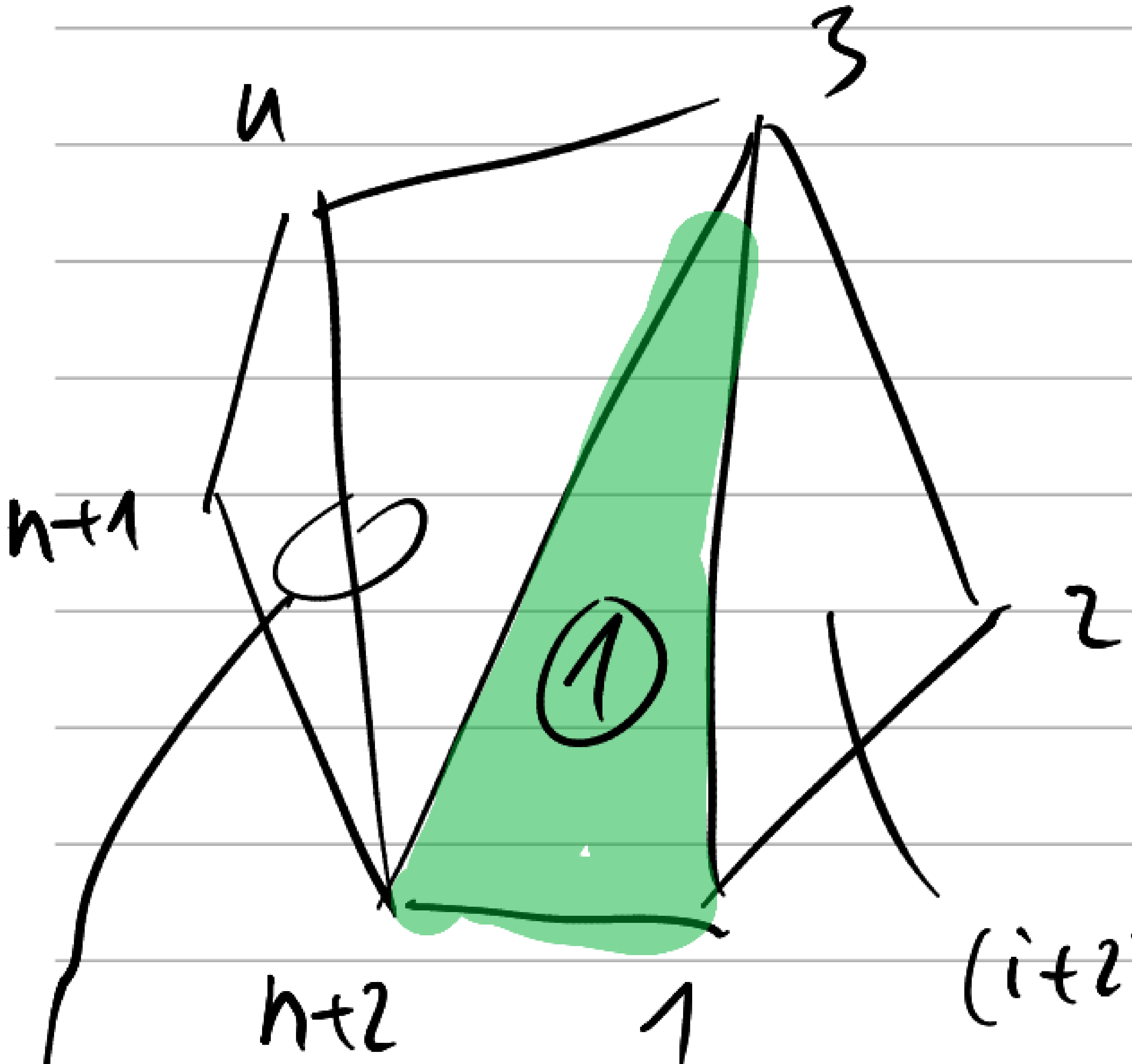
jiná rekurence :



$$C_n = \sum_{i_1+\dots+i_k = n-k} C_{i_1}\cdot \dots \cdot C_{i_k}$$

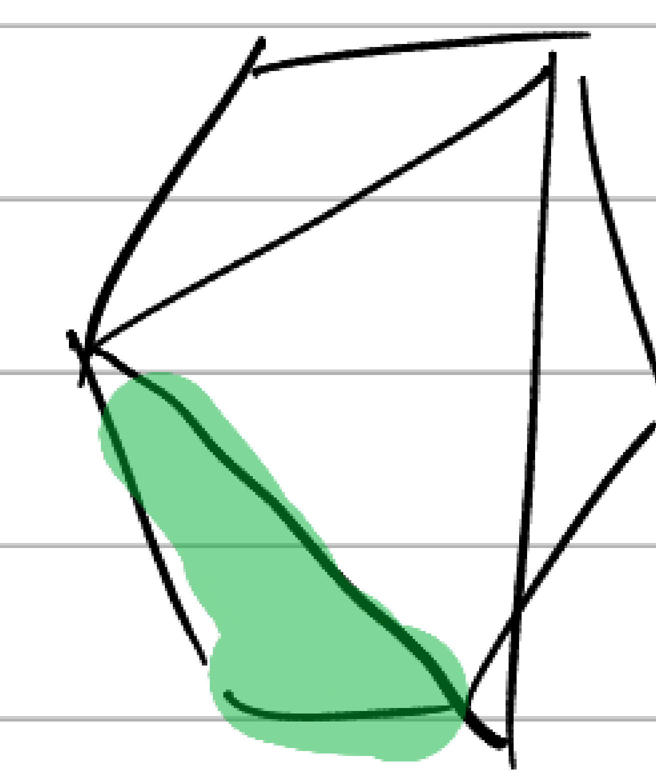


$(\underbrace{()()})\,(\underbrace{()})\,\underbrace{()}$



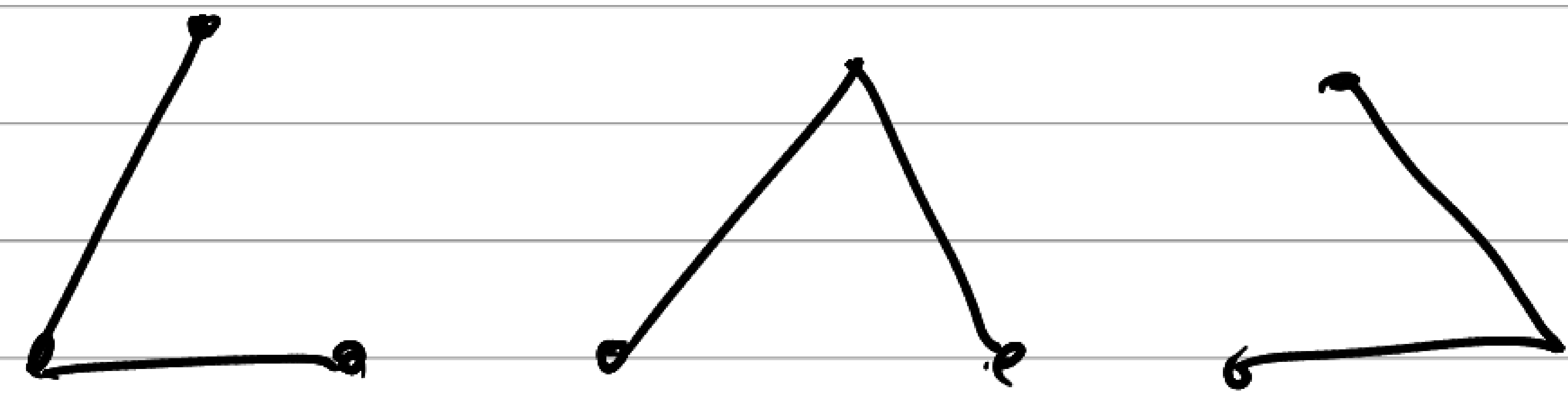
triang. $(n+2)$-úh. má n trojúhelníků

$(i+2)$-úhelník $\longleftrightarrow$ $i \times \Delta$



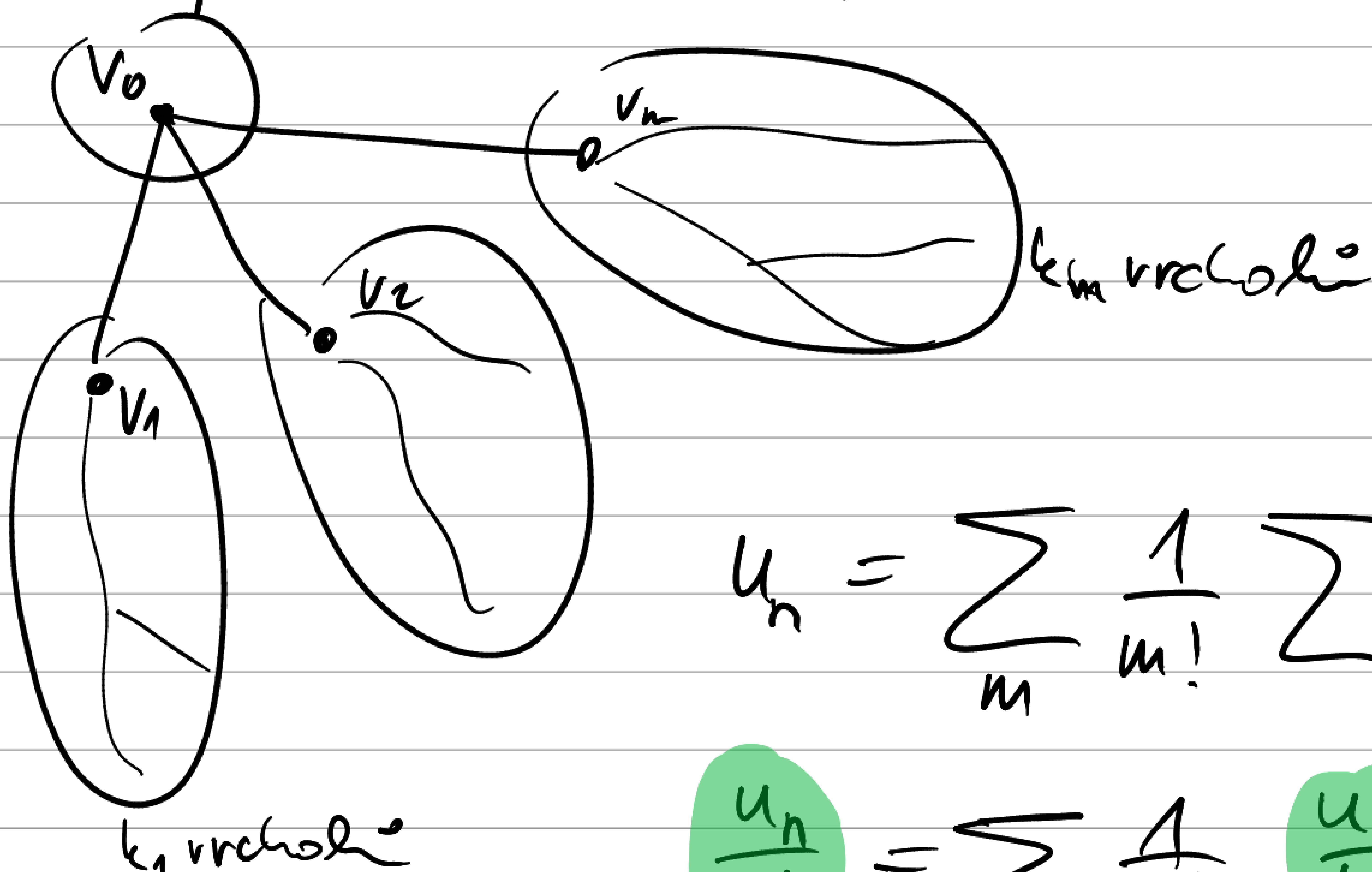
$(j+2)$-úhelník $\longleftrightarrow$ $j \times \Delta$

$$C_n = \sum_{i+j=n-1} C_i \cdot C_j$$



t_n = počet stromů

u_n = počet kořenových stromů $\quad u_n = t_n \cdot n$



$$u_n = \sum_m \frac{1}{m!} \sum \frac{n!}{1!\, k_1! \cdots k_m!} u_{k_1} \cdots u_{k_m}$$

$$\frac{u_n}{n!} = \sum_m \frac{1}{m!} \frac{u_{k_1}}{k_1!} \cdots \frac{u_{k_m}}{k_m!}$$

$$\frac{u_n}{n!} = \sum_{k_1+\dots+k_m=n-1} \frac{1}{m!} \frac{u_{k_1}}{k_1!} \cdots \frac{u_{k_m}}{k_m!} \qquad \sum \frac{1}{m!} x^m = e^x$$

$$\hat{u}(x) = \sum \frac{1}{m!} \underbrace{\hat{u}(x) \cdots \hat{u}(x)}_{m\times} \cdot x$$

$$= x \cdot e^{\hat{u}(x)}$$

$$f(u) = a_1 u + a_2 u^2 + \cdots$$

$$g(x) = b_1 x + b_2 x^2 + \cdots$$

$$\underline{x} = f(g(x)) = a_1 (b_1 x + b_2 x^2 + \cdots)$$
$$+ a_2 (b_1 x + b_2 x^2 + \cdots)^2$$
$$+ \cdots$$

$b_1 = \cdots$
$b_2 = \cdots$
$b_3 = \cdots$

$$\underline{1 = a_1 b_1} \qquad 0 = a_1 b_2 + a_2 b_1^2 \qquad 0 = a_1 b_3 + a_2 (2 b_1 b_2) + a_3 b_1^3$$

$$\hat{u}(x) = x\, e^{\hat{u}(x)}$$

$$\hat{u}(x) / e^{\hat{u}(x)} = x \qquad \text{pro} \quad f(u) = u / e^{u}$$

$$\| $$

$$f(\hat{u}(x))$$

n-tý koef. $\hat{u}(x)$

$$\|$$

$$\frac{u_n}{n!} = \frac{1}{n} [u^{n-1}] \Bigg(\frac{u}{u/e^u} \Bigg)^{n} \qquad \Bigg(\underbrace{}_{(e^u)^n} \Bigg)$$

$$e^{nu} = \sum \frac{1}{k!} (nu)^k$$

$$= \sum \frac{1}{k!} n^k \cdot u^k \qquad k = n-1$$

$$= \frac{1}{n} \frac{1}{(n-1)!} n^{n-1}$$

$$= \frac{1}{n!} \cdot n^{n-1}$$

$$\Rightarrow u_n = n^{n-1} \xRightarrow{/:n} \boxed{t_n = n^{n-2}}$$

$$B(x) = x\,B(x)^2 + 1$$

$$B(x) - 1 = x\,B(x)^2$$

$$\boxed{C(x) \overset{\text{def}}{=} B(x) - 1}$$

$$C(x) = x\,(C(x)+1)^2$$

$$k = n-1$$

$$\frac{C(x)}{(C(x)+1)^2} = x$$

$$\sum \binom{2n}{k} u^k$$

$$f(C(x)) = x \qquad \text{pro } f(u) = \frac{u}{(u+1)^2}$$

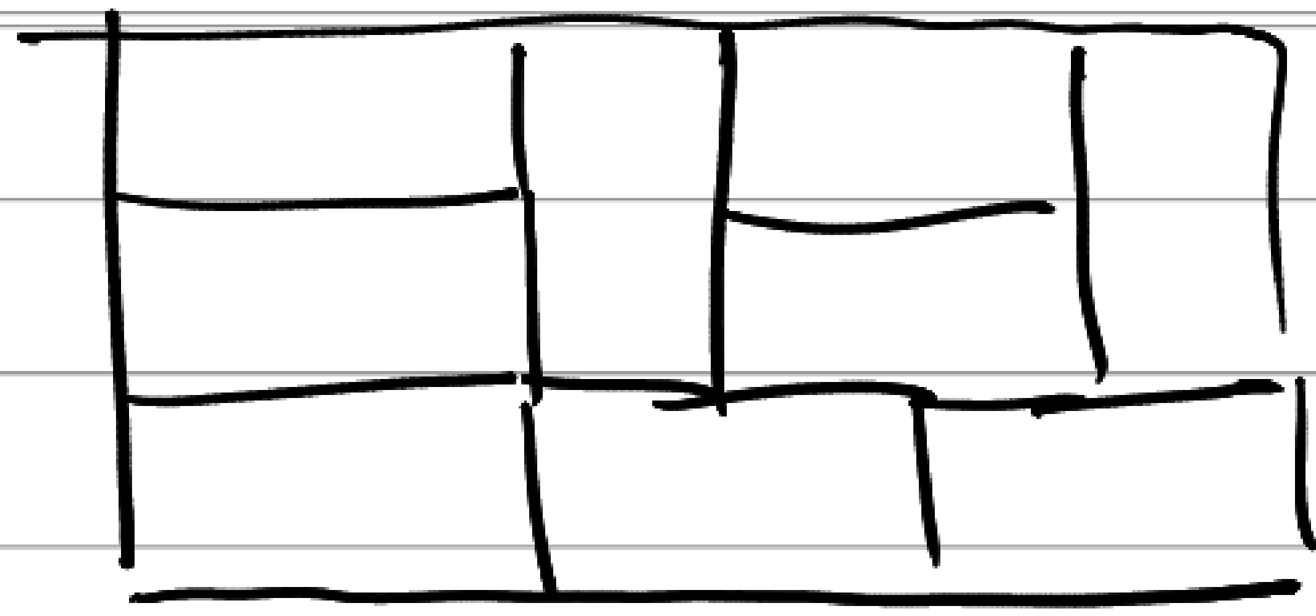
$$\sum \binom{2n}{k} u^k = (1+u)^{2n}$$

$$c_n = \frac{1}{n}[u^{n-1}]\left(\frac{u}{u/(u+1)^2}\right)^n = \frac{1}{n}[u^{n-1}]\left((u+1)^2\right)^n$$

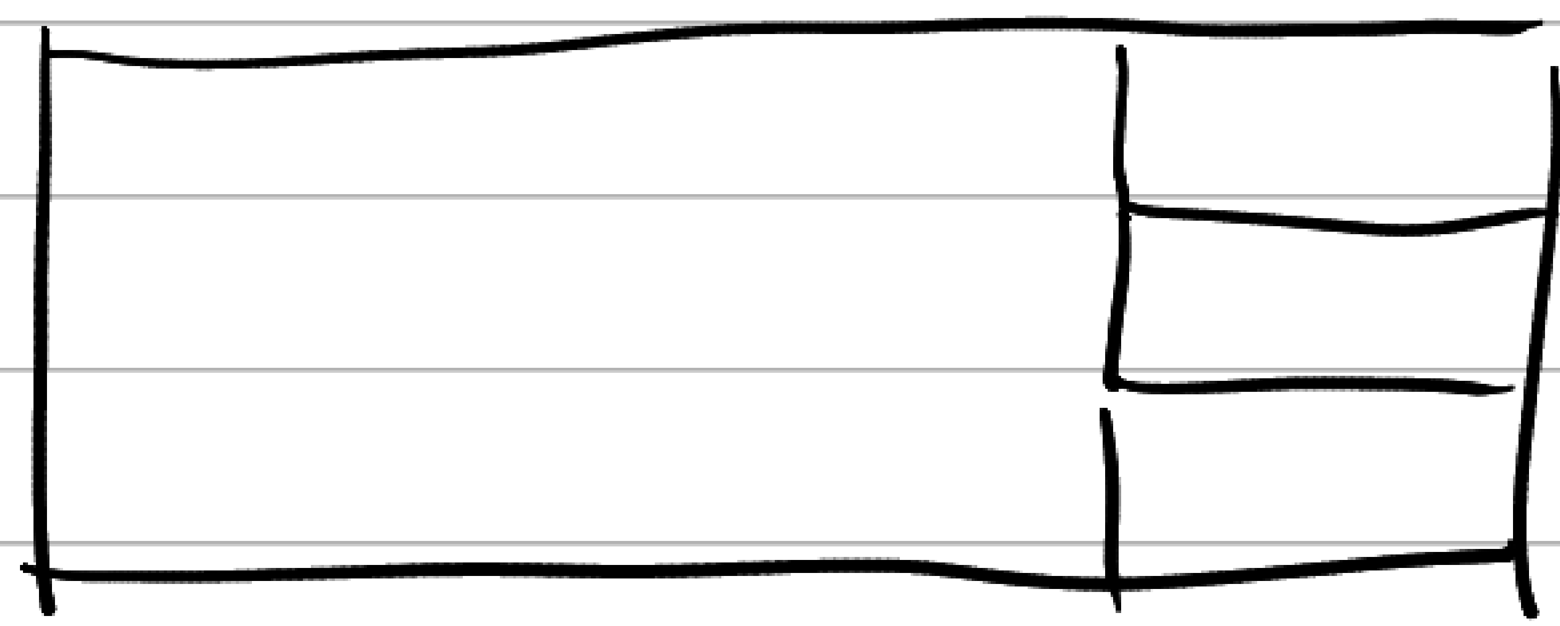
$$= \frac{1}{n}\binom{2n}{n-1} = \frac{1}{n+1}\binom{2n}{n}$$

$$\frac{1}{n}\,\frac{(2n)!}{(n-1)!\,(n+1)!} = \frac{1}{(n+1)}\,\frac{(2n)!}{n!\,n!}$$

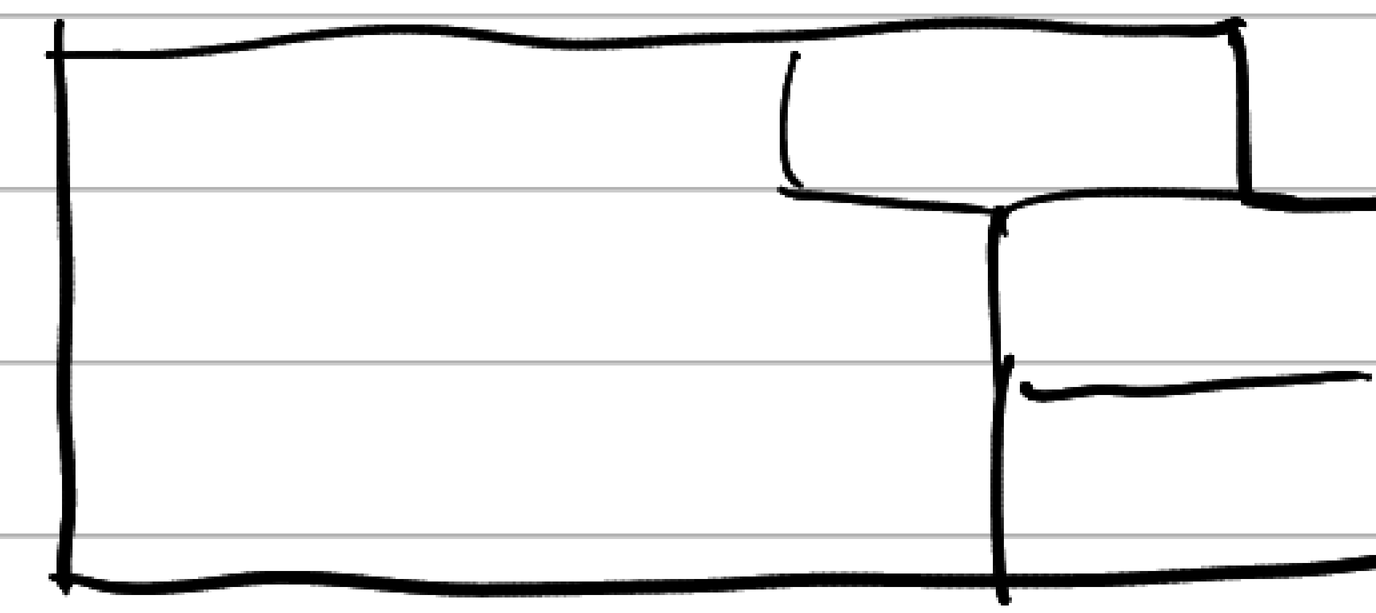
$$= (n+1)\cdot n!$$



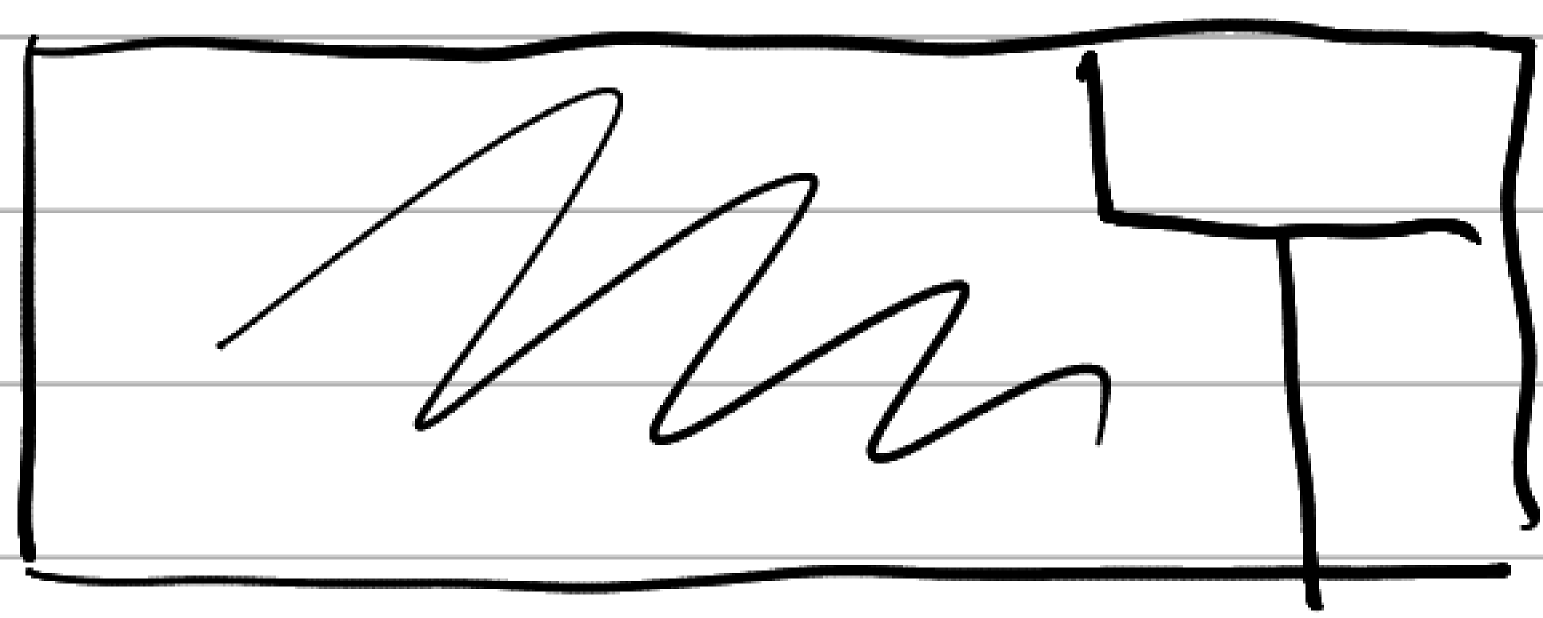
c_n



a) c_{n-2}



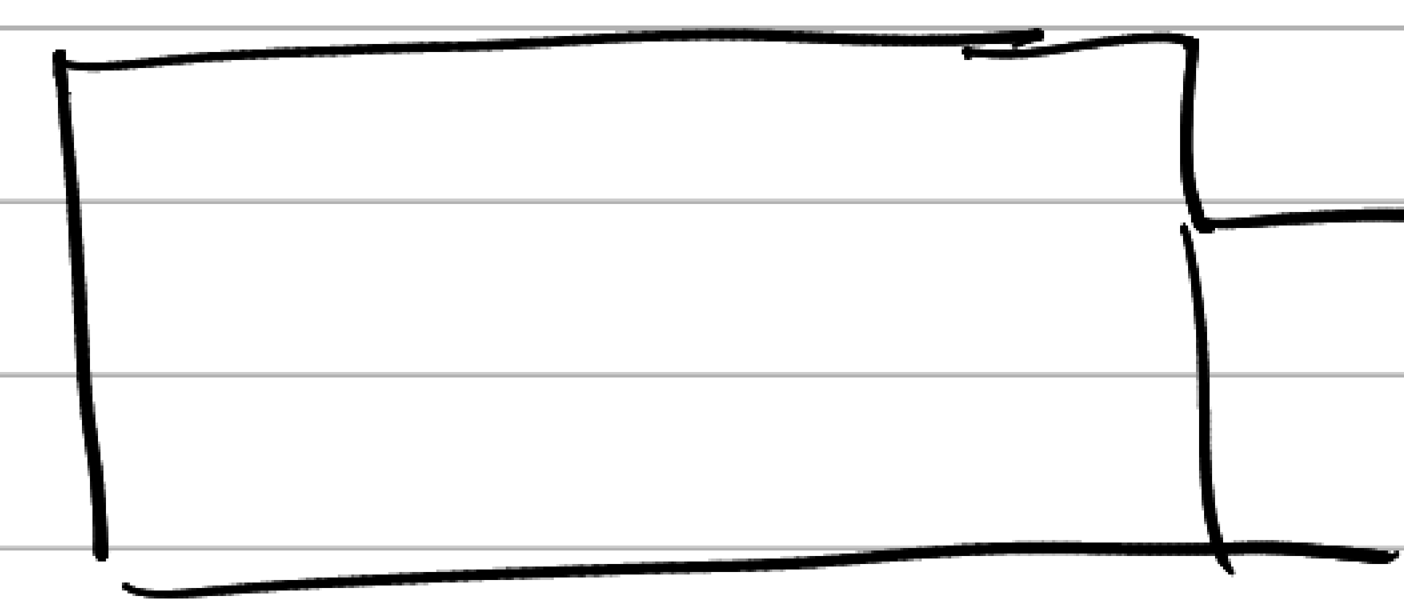
r_{n-2}



b) r_{n-1}

sym.

c) r_{n-1}



c_{n-1}

$c_n = 2r_{n-1} + c_{n-2}$

$r_n = c_{n-1} + r_{n-2}$

$$(1-x^2)\, C(x) - 2x\, R(x) = 1$$
$$-x\, C(x) - (1-x^2)\, R(x) = 0$$

$$\left(\begin{array}{cc|c} 1-x^2 & -2x & 1 \\ -x & 1-x^2 & 0 \end{array}\right)$$

$$D(z) = \frac{1-z}{1-4z+z^2} \qquad C(x) = D(x^2)$$

$$= \text{std. rozv.} \qquad 1-4z+z^2 = (1-z\cdot(2+\sqrt{3}))\cdot$$
$$\cdot(1-z\cdot(2-\sqrt{3}))$$

$$= \frac{A}{1-z(2+\sqrt{3})} + \frac{B}{1-z\cdot(2-\sqrt{3})}$$

$$= A\cdot \sum (2+\sqrt{3})^n z^n + B\cdot \sum (2-\sqrt{3})^n z^n$$

$$C_{2n} = A\cdot(2+\sqrt{3})^n + B(2-\sqrt{3})^n$$

$$\begin{array}{ll} & \overset{1}{\overset{\shortparallel}{}} \\ n=0: & C_0 = \,\text{--} \\ n=1: & C_2 = \,\text{--} \\ & \underset{3}{\underset{\shortparallel}{}} \end{array}$$