8., 13., 22., 30. 1.

$$(a_0, a_1, a_2, \ldots) \doteq a_0 + a_1 x + a_2 x^2 + \ldots$$

Σ Taylor. rozv v $x = 0$

$a(x)$

$$\frac{x^2+x+1}{x-1} = x+2+\frac{3}{x-1}$$

$$(x^2+x+1) : (x-1) = x+2$$

$$\begin{array}{l} -(x^2-x) \\ \hline \quad 2x+1 \\ \quad -(2x-2) \\ \hline \qquad 3 \end{array}$$

$$a_k = a_{k-1} + 2a_{k-2} + (-1)^k [k \geq 0] + [k=1]$$

$k=1$: $1 = 1 \qquad + (-1)^1 \qquad + 1$

$k=0$: $1 = \qquad 1$

$| \cdot x^k \quad | \sum$

$$\sum a_k x^k = \sum a_{k-1} x^k + 2 \sum a_{k-2} x^k + \sum (-1)^k [k \geq 0] x^k + \sum [k=1] x^k$$

$\| \text{def}$

$$a(x) = x \cdot a(x) + 2x^2 a(x) + \frac{1}{1+x} + x$$

$$a(x)(1 - x - 2x^2) = \frac{1+x+x^2}{1+x}$$

$$a(x) = \frac{1+x+x^2}{(1+x)(1+x)(1-2x)} =$$

$$= \frac{7/9}{1-2x} + \frac{3/9}{(1+x)^2} - \frac{4/9}{1+x}$$

$$= \sum 7/9 \cdot 2^k x^k + \sum 3/9 \, (k+1)(-1)^k x^k$$

$$- \sum 1/9 \, (-1)^k x^k$$

$$a_k = \frac{7}{9} 2^k + \frac{3}{9}(k+1)(-1)^k - \frac{1}{9}(-1)^k \qquad \checkmark$$

$$F_k = F_{k-1} + F_{k-2} + [k=1]$$

$$\Big\downarrow \text{def} \qquad \Big\downarrow \qquad \Big\downarrow \qquad \Big\downarrow$$

$$F(x) = xF(x) + x^2F(x) + x$$

$$F(x)\left(1-x-x^2\right) = x$$

$$F(x) = \frac{x}{1-x-x^2}$$

$$t^2-t-1 = (t-\lambda)(t-\mu)$$

$$\lambda, \mu = \frac{1 \pm \sqrt{5}}{2}$$

$$F(x) = \frac{x}{(1-\lambda x)(1-\mu x)}$$

$$= \frac{A}{1-\lambda x} + \frac{B}{1-\mu x}$$

$$= \sum A \cdot \lambda^k x^k + \sum B \cdot \mu^k x^k$$

$$F_k = A \cdot \lambda^k + B \cdot \mu^k$$

$$\frac{a}{a+b} = \frac{b}{a}$$

$$k C_k = k(k-1) + 2(C_0 + \dots + C_{k-1})$$

$$C_k \longleftrightarrow C(x)$$

$$\boxed{\left(\sum C_k x^k\right)' = \sum k C_k \cdot x^{k-1}}$$

$$C_0 + \dots + C_k \longleftrightarrow \frac{1}{1-x} C(x)$$

$$C_0 + \dots + C_{k-1} \longleftrightarrow x \cdot \frac{1}{1-x} C(x)$$

$$(k+1) C_{k+1} \longleftrightarrow C'(x)$$

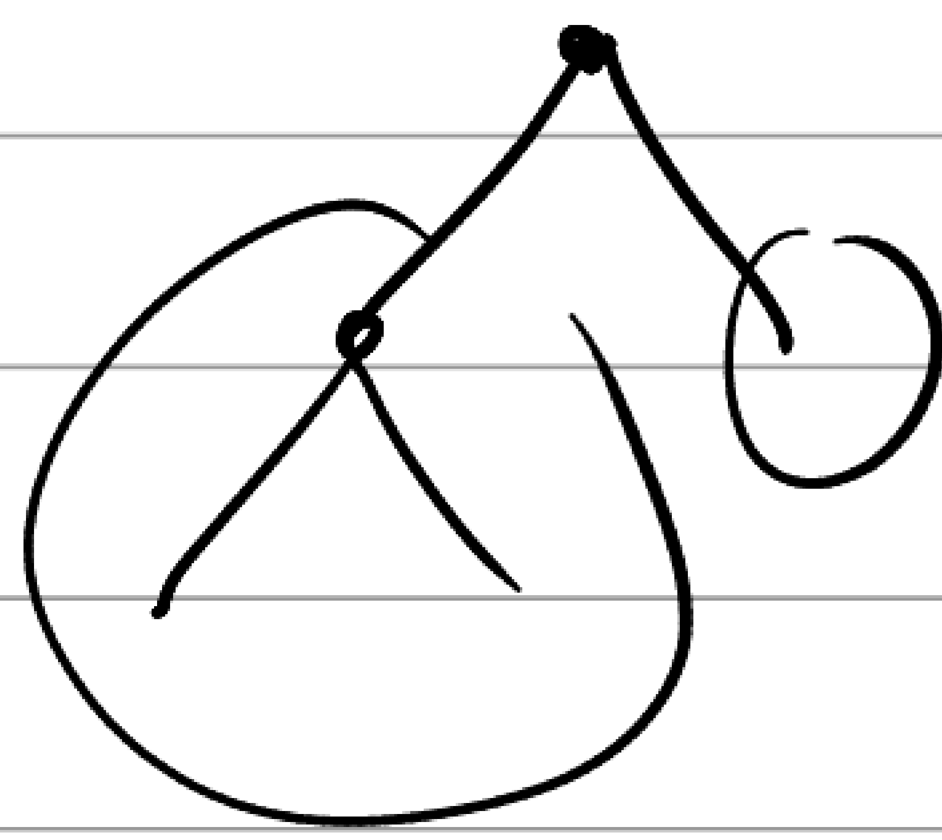
$$k C_k \longleftrightarrow x \cdot C'(x)$$

$$\frac{(k+2)(k+1)}{2 \cdot 1} \longleftrightarrow \frac{1}{(1-x)^3}$$

$$k(k-1) \longleftrightarrow \frac{2x^2}{(1-x)^3}$$

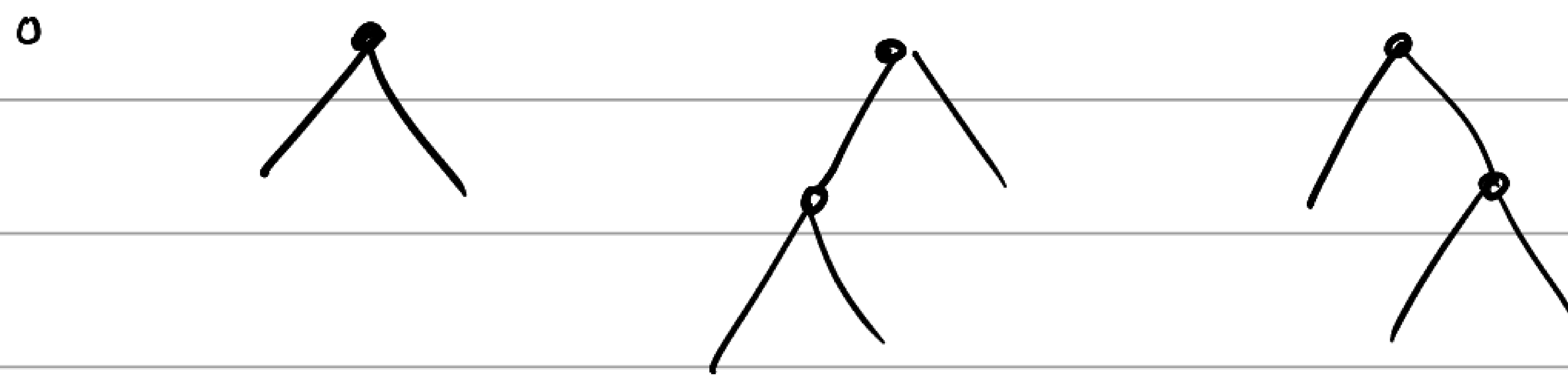
$$(a_k) * (b_k) = (c_k)$$

$$c_k = a_0 b_k + a_1 b_{k-1} + \dots + a_k b_0$$

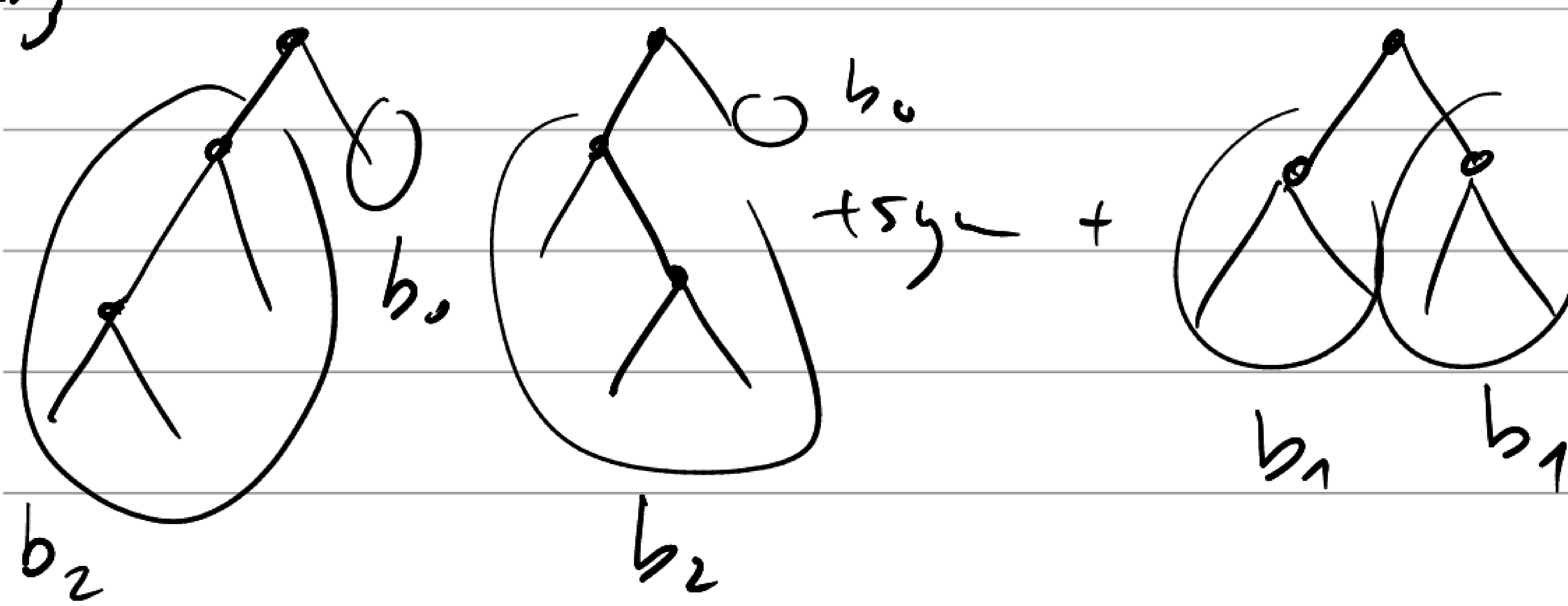


pravý podstrom

levý podstrom



b_3:



$$b_k = b_0 b_{k-1} + b_1 b_{k-2} + \dots + b_{k-1} b_0 + [k=0]$$

$\updownarrow$ def

$$B(x) = \underbrace{x \cdot}_{\text{posunutí}} \overbrace{B(x) \cdot B(x)}^{\text{konvoluce}} + 1$$

$$xB(x)^2 - B(x) + 1 = 0$$

$$\sqrt{1+x} \doteq 1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2 + \dots$$

$$B(x) = \frac{1 \pm \sqrt{1 - 4 \cdot x \cdot 1}}{2x}$$

$$= \frac{1 - \sqrt{1-4x}}{2x} \qquad \sqrt{1-4x} \doteq 1 - 2x$$

$$(1+x)^r = \sum \binom{r}{k} x^k$$

$$(1-4x)^{1/2} = \sum \binom{1/2}{k} (-4x)^k$$

např. $\binom{1/2}{4} = \frac{1/2 \cdot (-1/2) \cdot (-3/2) \cdot (-5/2)}{4 \cdot 3 \cdot 2 \cdot 1}$

$$10 \cdot 8 \cdot 6 \cdot 4 \cdot 2 = 2^5 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$= 2^5 \cdot 5!$$

$$\binom{-1/2}{n} = \binom{2n}{n} \cdot \left(-\frac{1}{4}\right)^n$$

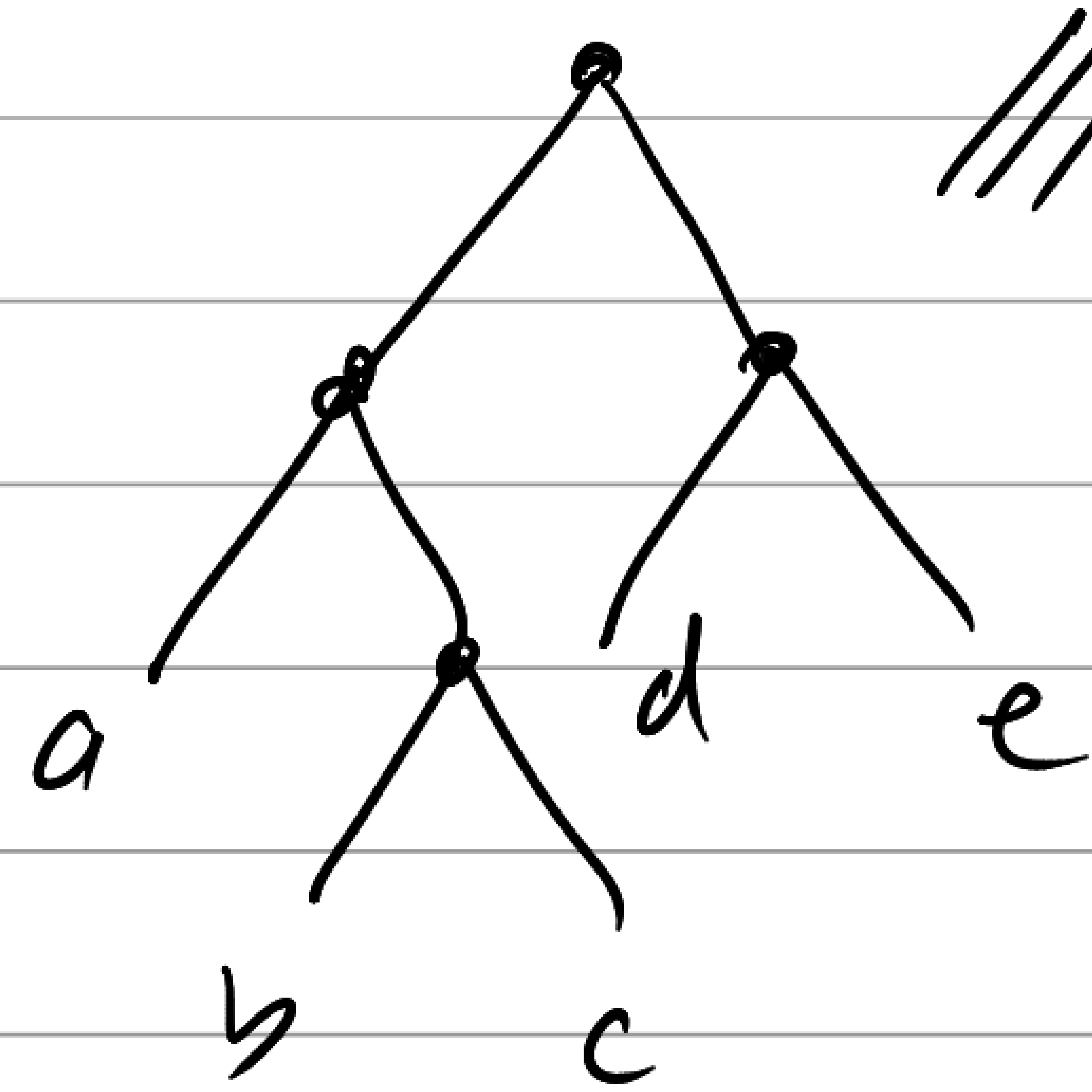
$$b_k = \frac{1}{k+1} \binom{2k}{k}$$

$$b_k = b_0 b_{k-1} + \dots + b_{k-1} b_0, \quad b_0 = 1$$

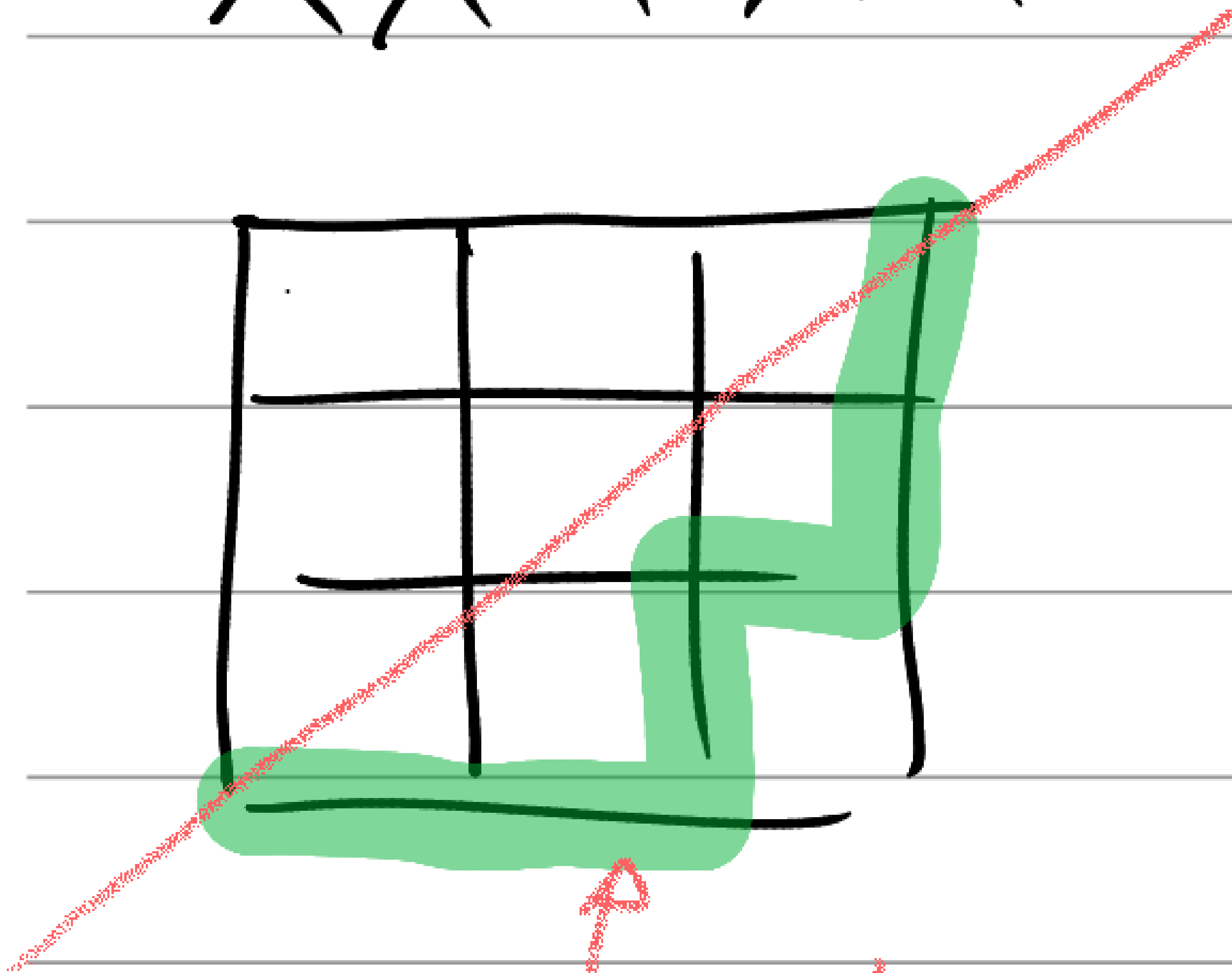
$$\Rightarrow \quad b_k = \frac{1}{k+1}\binom{2k}{k} \quad \text{Catalanova čísla}$$

uzávorkování součinu
s k operacemi

$k=4$: $\quad (a\cdot(b\cdot c))\cdot(d\cdot e)$

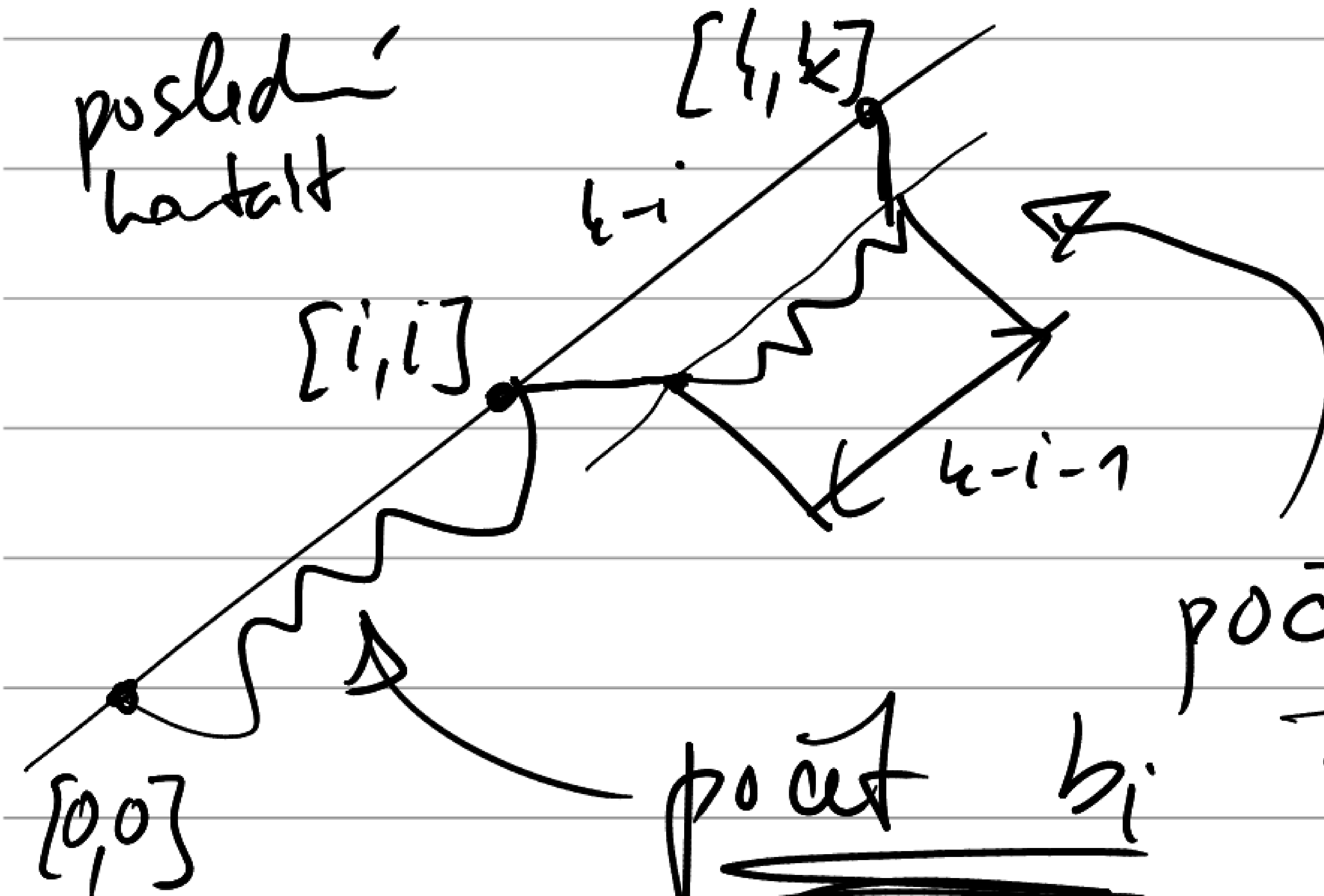


X X Y X Y Y



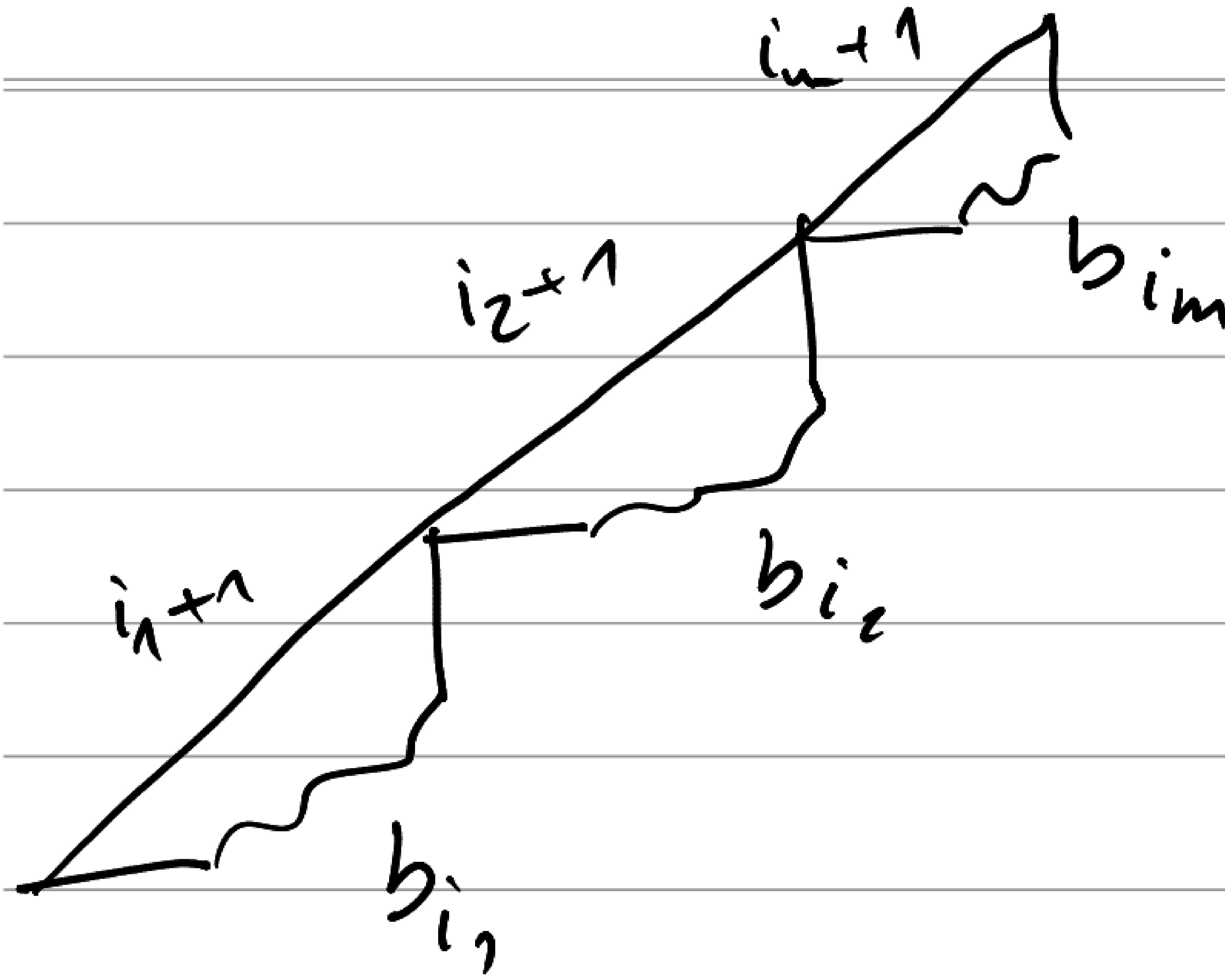
X = doprava
Y = nahoru

cesta pod diagonálou



i libovolné

$\underbrace{X \ldots Y}\, X \underbrace{\ldots\ldots}\, Y$



$$b_k = \sum_m b_{i_1} \cdots b_{i_m}$$

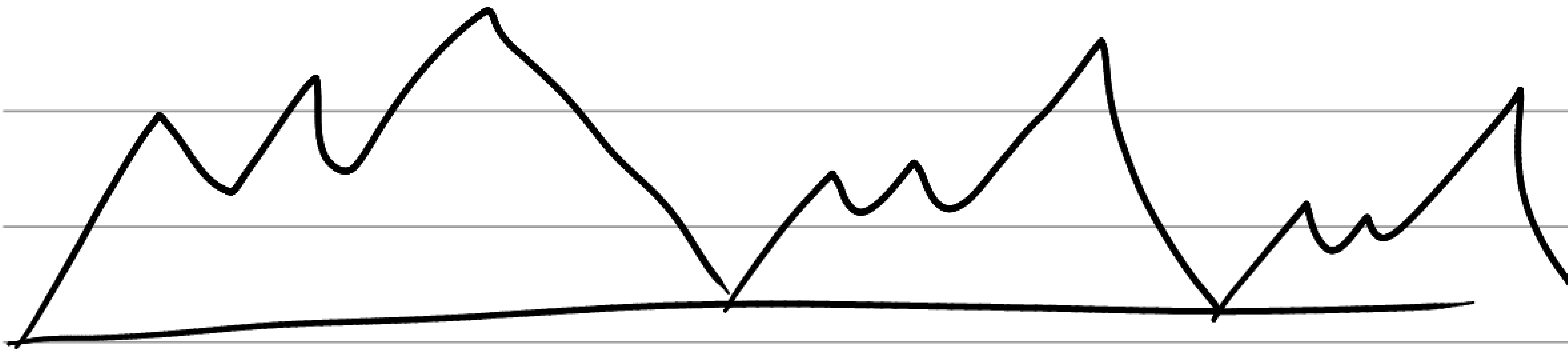
$$(i_1+1) + \cdots + (i_m+1) = k$$

$$\boxed{b_k = \sum_{m;\, i_1 + \cdots + i_m = k-m} b_{i_1} \cdots b_{i_m}}$$

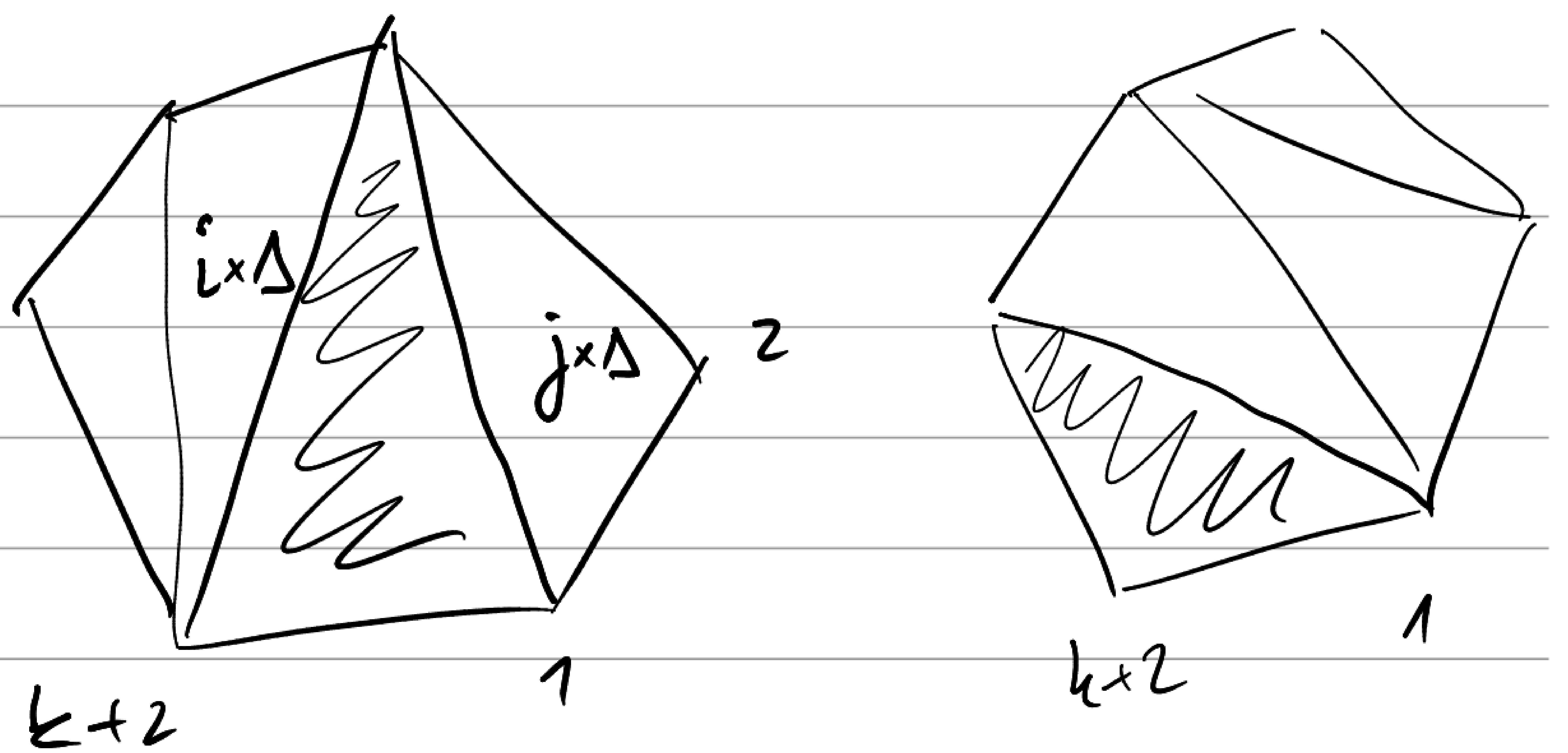
$$\Rightarrow b_k = \frac{1}{k+1}\binom{2k}{k}$$

$$b_k = b_0 b_{k-2} + b_1 b_{k-3} + \cdots + b_{k-2} b_0$$

$$+ b_0 b_0 b_{k-3} + \cdots$$



počet triangulací konv. $(k+2)$-úh.



$$i+1+j=k$$

$$b_k=\sum_{i+j=k-1} b_i\cdot b_j=b_0b_{k-1}+\dots+b_{k-1}b_0$$

$$(()())(())()\qquad X\ Y$$

$$\sum_i [i=3]\, x^i$$

$$\underbrace{[0=3]}_{0} x^0 + \underbrace{[1=3]}_{0} x^1 + \dots + \underbrace{[3=3]}_{1} x^3 + \dots$$

$$= x^3$$