Algebra I – autumn 2024 – Homework 10

1. Prove that $\{\, a + b \cdot \sqrt{2} + c \cdot \mathrm{i} + d \cdot \sqrt{2} \cdot \mathrm{i} \mid a, b, c, d \in \mathbb{Q} \,\}$ is a subring of the ring $(\mathbb{C}, +, \cdot)$.
2. Find all ring homomorphisms from

   $$(\{\, a + b \cdot \sqrt{2} + c \cdot \mathrm{i} + d \cdot \sqrt{2} \cdot \mathrm{i} \mid a, b, c, d \in \mathbb{Q} \,\}, +, \cdot)$$

   to $(\mathbb{C}, +, \cdot)$. Decide which of them are injective.