Algebra I – autumn 2024 – Homework 6

Let $(G, \cdot)$ be a group and n arbitrary positive integer.

1. Prove that if
$$H_n = \{g \in G; g^n = 1_G\}$$
is a subgroup of G, then H is a normal subgroup.

2. Give an example of a group G such that H is **not** a subgroup, and explain why it is not a subgroup.

3. Give an example of a noncommutative group G such that H_3 **is** a nontrivial subgroup, and prove that H_3 is a subgroup that is normal.