Chapter 7: Normalization

Database System Concepts, 7th Ed.

Outline

- Features of Good Relational Design
- Functional Dependencies
- Decomposition Using Functional Dependencies
- Normal Forms
- Functional Dependency Theory
- Algorithms for Decomposition using Functional Dependencies

Features of Good Relational Designs

- Suppose we combine *instructor* and *department* into *in_dep,* which represents the natural join of the relations *instructor* and *department*

ID	*name*	*salary*	*dept_name*	*building*	*budget*
22222	Einstein	95000	Physics	Watson	70000
12121	Wu	90000	Finance	Painter	120000
32343	El Said	60000	History	Painter	50000
45565	Katz	75000	Comp. Sci.	Taylor	100000
98345	Kim	80000	Elec. Eng.	Taylor	85000
76766	Crick	72000	Biology	Watson	90000
10101	Srinivasan	65000	Comp. Sci.	Taylor	100000
58583	Califieri	62000	History	Painter	50000
83821	Brandt	92000	Comp. Sci.	Taylor	100000
15151	Mozart	40000	Music	Packard	80000
33456	Gold	87000	Physics	Watson	70000
76543	Singh	80000	Finance	Painter	120000

- There is a repetition of information
- Need to use null values (if we add a new department with no instructors)

Decomposition

- The only way to avoid the repetition-of-information problem in the i*n_dep* schema is to decompose it into two schemas – instructor and *department* schemas.
- Not all decompositions are good. Suppose we decompose

  employee(ID, name, street, city, salary)

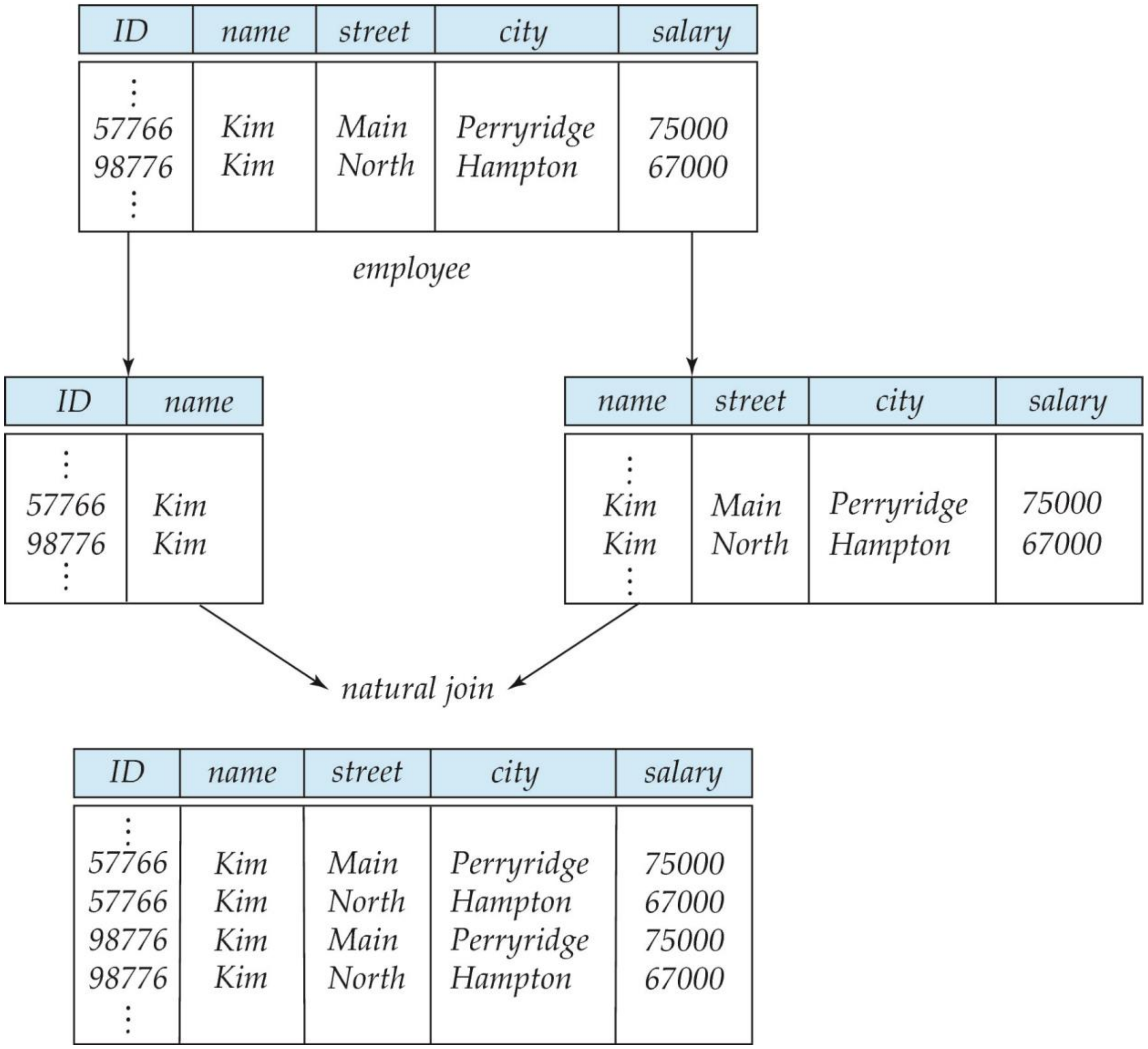
  into

  employee1 (*ID*, *name*)

  employee2 (*name*, *street, city, salary*)

  The problem arises when we have two employees with the same name
- The next slide shows how we lose information -- we cannot reconstruct the original *employee* relation -- and so, this is a **lossy decomposition**.

A Lossy Decomposition



ID	name	street	city	salary
⋮				
57766	Kim	Main	Perryridge	75000
98776	Kim	North	Hampton	67000
⋮				

employee

ID	name
⋮	
57766	Kim
98776	Kim
⋮	

name	street	city	salary
⋮			
Kim	Main	Perryridge	75000
Kim	North	Hampton	67000
⋮			

natural join

ID	name	street	city	salary
⋮				
57766	Kim	Main	Perryridge	75000
57766	Kim	North	Hampton	67000
98776	Kim	Main	Perryridge	75000
98776	Kim	North	Hampton	67000
⋮				

Lossless Decomposition

- Let *R* be a relation schema and let R_1 and R_2 form a decomposition of *R*. That is $R = R_1 \cup R_2$
- We say that the decomposition is a **lossless decomposition** if there is no loss of information by replacing *R* with the two relation schemas $R_1 \cup R_2$
- Formally:

$$\Pi_{R_1}(r) \bowtie \Pi_{R_2}(r) = r$$

- Conversely, a decomposition is lossy if:

$$r \subset \Pi_{R_1}(r) \bowtie \Pi_{R_2}(r)$$

Example of Lossless Decomposition

- Decomposition of $R = (A, B, C)$

  $R_1 = (A, B)$ $R_2 = (B, C)$

A	B	C
α	1	A
β	2	B

r

A	B
α	1
β	2

$\Pi_{A,B}(r)$

B	C
1	A
2	B

$\Pi_{B,C}(r)$

$\Pi_A(r) \bowtie \Pi_B(r)$

A	B	C
α	1	A
β	2	B

Normalization Theory

- Decide whether a particular relation *R* is in “good” form.
- In the case that a relation *R* is not in “good” form, decompose it into set of relations $\{R_1, R_2, ..., R_n\}$ such that
  - Each relation is in good form
  - The decomposition is the lossless decomposition
- Our theory is based on:
  - Functional dependencies

Functional Dependencies

- There are usually a variety of constraints (rules) valid on the data in the real world.
- For example, some of the constraints that are expected to hold in a university database are:
  - Students and instructors are uniquely identified by their ID.
  - Each student and instructor has only one name.
  - Each instructor and student is (primarily) associated with only one department.
  - Each department has only one value for its budget, and only one associated building.

Functional Dependencies (Cont.)

- An instance of a relation that satisfies all such real-world constraints is a **legal instance** of the relation;
- A legal instance of a database requires all the relation instances to be legal instances
- Constraints on the set of legal relations require that the value for a certain set of attributes determines uniquely the value for another set of attributes.
- A functional dependency is a generalization of the notion of a *key.*

Functional Dependencies Definition

- Let R be a relation schema

$$\alpha \subseteq R \;\; and \;\; \beta \subseteq R$$

- The **functional dependency**

$$\alpha \rightarrow \beta$$

holds on R if and only if for any legal relations $r(R)$, whenever any two tuples t_1 and t_2 of r agree on the attributes α, they also agree on the attributes β. That is,

$$t_1[\alpha] = t_2[\alpha] \;\; \Rightarrow \;\; t_1[\beta] = t_2[\beta]$$

- Example: Consider $r(A,B)$ with the following instance of r.

1	4
1	5
3	7

- On this instance, $B \rightarrow A$ hold; $A \rightarrow B$ does **NOT** hold,

Closure of a Set of Functional Dependencies

- Given a set *F* of functional dependencies, there are certain other functional dependencies that are logically implied by *F*.
  - If $A \rightarrow B$ and $B \rightarrow C$, then we can infer that $A \rightarrow C$
  - etc.
- The set of **all** functional dependencies logically implied by *F* is the **closure** of *F*.
- We denote the *closure* of *F* by $\boldsymbol{F}^+$.

Keys and Functional Dependencies

- K is a superkey for relation schema R if and only if $K \rightarrow R$
- K is a candidate key for R if and only if
  - $K \rightarrow R$, and
  - for no $\alpha \subset K$, $\alpha \rightarrow R$
- Functional dependencies allow us to express constraints that cannot be expressed using superkeys. Consider the schema:

  in_dep (*ID, name, salary, dept_name, building, budget*).

  We expect the following functional dependencies to hold:

  $$dept\_name \rightarrow building$$

  $$ID \rightarrow building$$

  but would not expect the following to hold:

  $$dept\_name \rightarrow salary$$

Use of Functional Dependencies

- We use functional dependencies to:
  - To test relations to see if they are legal under a given set of functional dependencies.
    - If a relation *r* is legal under a set *F* of functional dependencies, we say that *r* **satisfies** *F.*
  - To specify constraints on the set of legal relations
    - We say that *F* **holds on** *R* if all legal relations on *R* satisfy the set of functional dependencies *F.*
- Note: A specific instance of a relation may satisfy a functional dependency even if the functional dependency does not hold on all legal instances.
  - For example, a specific instance of *instructor* may, by chance, satisfy
    $name \rightarrow ID.$

Trivial Functional Dependencies

- *A* functional dependency is **trivial** if it is satisfied by all instances of a relation
- Example*:*
  - *ID, name* $\rightarrow$ *ID*
  - *name* $\rightarrow$ *name*
- In general, $\alpha \rightarrow \beta$ is trivial if $\beta \subseteq \alpha$

Lossless Decomposition

- We can use functional dependencies to show when certain decompositions are lossless.
- For the case of $R = (R_1, R_2)$, we require that for all possible relations r on schema R

$$r = \Pi_{R1}(r) \bowtie \Pi_{R2}(r)$$

- A decomposition of R into R_1 and R_2 is lossless decomposition if at least one of the following dependencies is in F^+:
  - $R_1 \cap R_2 \rightarrow R_1$
  - $R_1 \cap R_2 \rightarrow R_2$
- The above functional dependencies are a sufficient condition for lossless join decomposition; the dependencies are a necessary condition only if all constraints are functional dependencies

Example

- $R = (A, B, C)$
  $F = \{A \rightarrow B, B \rightarrow C\}$
- $R_1 = (A, B), \quad R_2 = (B, C)$
  - Lossless decomposition:

    $$R_1 \cap R_2 = \{B\} \text{ and } B \rightarrow BC$$
- $R_1 = (A, B), \quad R_2 = (A, C)$
  - Lossless decomposition:

    $$R_1 \cap R_2 = \{A\} \text{ and } A \rightarrow AB$$
- *Note:*
  - $B \rightarrow BC$

    is a shorthand notation for
  - $B \rightarrow \{B, C\}$

Dependency Preservation

- Testing functional dependency constraints each time the database is updated can be costly
- It is useful to design the database in a way that constraints can be tested efficiently.
- If testing a functional dependency can be done by considering just one relation, then the cost of testing this constraint is low
- When decomposing a relation, it may no longer be possible testing the dependence without performing a Cartesian Product (Join)
- A decomposition that makes it computationally hard to enforce functional dependency is said to be NOT **dependency preserving**.

Dependency Preservation Example

- Consider a schema:

  dept_advisor(s_ID, i_ID, dept_name)
- With function dependencies:

  $i\_ID \rightarrow dept\_name$

  $s\_ID, dept\_name \rightarrow i\_ID$
- In the above design we are forced to repeat the department name once for each time an instructor participates in a *dept_advisor* relationship.
- To fix this, we need to decompose *dept_advisor*
- Any decomposition will not include all the attributes in

  $s\_ID, dept\_name \rightarrow i\_ID$
- Thus, any decomposition is NOT dependency preserving

Boyce-Codd Normal Form

- A relation schema *R* is in BCNF with respect to a set *F* of functional dependencies if for all functional dependencies in F^+ of the form

  $$\alpha \rightarrow \beta$$

  where $\alpha \subseteq R$ and $\beta \subseteq R$, at least one of the following holds:
  - $\alpha \rightarrow \beta$ is trivial (i.e., $\beta \subseteq \alpha$)
  - α is a superkey for *R*

Boyce-Codd Normal Form (Cont.)

- Example schema that is ***not*** in BCNF:

  in_dep (*ID, name, salary, dept_name, building, budget*)

  because :

  - *dept_name*$\rightarrow$ *building, budget*
    - holds on *in_dep*
    - but
  - *dept_name* is not a superkey
- When decompose *in_dept* into *instructor* and *department*
  - *instructor* is in BCNF
  - *department* is in BCNF

Decomposing a Schema into BCNF

- Let *R* be a schema that is not in BCNF. Let $\alpha \rightarrow \beta$ be the FD that causes a violation of BCNF.
- We decompose *R* into:
  - $(\alpha \cup \beta)$
  - $(R - (\beta - \alpha))$
- In our example of *in_dep*,
  - α = *dept_name*
  - β = *building, budget*

  and *in_dep* is replaced by
  - $(\alpha \cup \beta)$ = (*dept_name, building, budget*)
  - $(R - (\beta - \alpha))$ = (*ID, name, dept_name, salary*)

Example

- $R = (A, B, C)$
  $F = \{A \rightarrow B, B \rightarrow C)$
- $R_1 = (A, B), \quad R_2 = (B, C)$
  - Lossless-join decomposition:

$$R_1 \cap R_2 = \{B\} \quad \text{and } B \rightarrow BC$$

  - Dependency preserving
- $R_1 = (A, B), \quad R_2 = (A, C)$
  - Lossless-join decomposition:

$$R_1 \cap R_2 = \{A\} \text{ and } A \rightarrow AB$$

  - Not dependency preserving
    (cannot check $B \rightarrow C$ without computing $R_1 \bowtie R_2$)

BCNF and Dependency Preservation

- It is not always possible to achieve both BCNF and dependency preservation
- Consider a schema:

  dept_advisor(s_ID, i_ID, department_name)
- With function dependencies:

  $i\_ID \rightarrow dept\_name$

  $s\_ID, dept\_name \rightarrow i\_ID$
- *dept_advisor* is not in BCNF
  - *i_ID* is not a superkey.
- Any decomposition of *dept_advisor* will not include all the attributes in

  $s\_ID, dept\_name \rightarrow i\_ID$
- Thus, the composition is NOT dependency preserving

Third Normal Form

- A relation schema *R* is in the **third normal form (3NF)** if for all:

  $$\alpha \rightarrow \beta \text{ in } F^+$$

  at least one of the following holds:

  - $\alpha \rightarrow \beta$ is trivial (i.e., $\beta \in \alpha$)
  - α is a superkey for *R*
  - Each attribute *A* in $\beta - \alpha$ is contained in a candidate key for *R.*

  (**NOTE***:* each attribute may be in a different candidate key)
- If a relation is in BCNF it is in 3NF (since in BCNF one of the first two conditions above must hold).
- Third condition is the minimal relaxation of BCNF to ensure dependency preservation.

3NF Example

- Consider a schema:

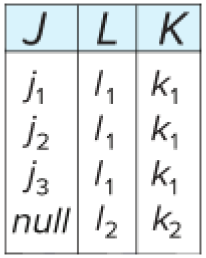
  dept_advisor(s_ID, i_ID, dept_name)
- With function dependencies:

  $i\_ID \rightarrow dept\_name$

  $s\_ID, dept\_name \rightarrow i\_ID$
- Two candidate keys = {*s_ID, dept_name*}, {*s_ID, i_ID* }
- We have seen before that *dept_advisor* is not in BCNF
- *R,* however, is in 3NF
  - *s_ID, dept_name* is a superkey
  - $i\_ID \rightarrow dept\_name$ *and* *i_ID* is NOT a superkey, but:
    - { *dept_name*} – {*i_ID* } = {*dept_name* } and
    - *dept_name* is contained in a candidate key

Redundancy in 3NF

- Consider the schema R below, which is in 3NF
  - $R = (J, K, L)$
  - $F = \{JK \rightarrow L, L \rightarrow K\}$
  - And an instance table:



J	L	K
j_1	l_1	k_1
j_2	l_1	k_1
j_3	l_1	k_1
null	l_2	k_2

- What is wrong with the table?
  - Repetition of information
  - Need to use null values (e.g., to represent the relationship l_2, k_2 where there is no corresponding value for J)

Comparison of BCNF and 3NF

- Advantages of 3NF over BCNF. It is always possible to obtain a 3NF design without sacrificing losslessness or dependency preservation.
- Disadvantages to 3NF.
  - We may have to use null values to represent some of the possible meaningful relationships among data items.
  - There is the problem of repetition of information.

Goals of Normalization

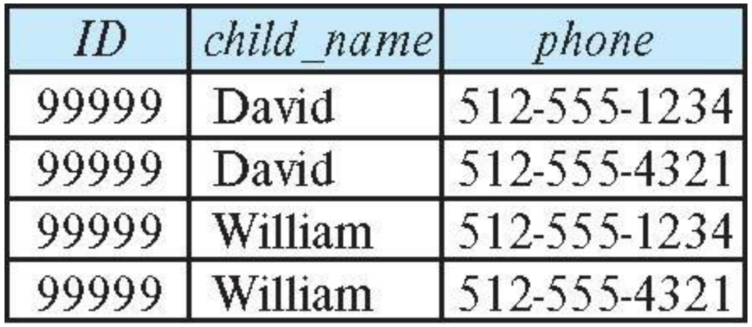
- Let R be a relation scheme with a set F of functional dependencies.
- Decide whether a relation scheme R is in "good" form.
- In the case that a relation scheme R is not in "good" form, decompose R into a set of relation schemes $\{R_1, R_2, ..., R_n\}$ such that:
  - Each relation scheme is in good form
  - The decomposition is a lossless decomposition
  - Preferably, the decomposition should be dependency preserving.

How good is BCNF?

- There are database schemas in BCNF that do not seem to be sufficiently normalized
- Consider a relation

  inst_info (ID, child_name, phone)

  - where an instructor may have more than one phone and can have multiple children
  - Instance of *inst_info*



ID	*child_name*	*phone*
99999	David	512-555-1234
99999	David	512-555-4321
99999	William	512-555-1234
99999	William	512-555-4321

How good is BCNF? (Cont.)

- There are no non-trivial functional dependencies and therefore the relation is in BCNF
- Insertion anomalies – i.e., if we add a phone 981-992-3443 to 99999, we need to add two tuples

  (99999, David, 981-992-3443)
  (99999, William, 981-992-3443)

Higher Normal Forms

- It is better to decompose *inst_info* into:
  - *inst_child*:

ID	*child_name*
99999	David
99999	William

  - *inst_phone:*

ID	*phone*
99999	512-555-1234
99999	512-555-4321

- This suggests the need for higher normal forms, such as Fourth Normal Form (4NF).

Functional-Dependency Theory Roadmap

- We now consider a formal theory that tells us which functional dependencies are implied logically by a given set of functional dependencies.
- We then develop algorithms to generate lossless decompositions into BCNF and 3NF
- We then develop algorithms to test if a decomposition is dependency-preserving

Closure of a Set of Functional Dependencies

- Given a set *F* set of functional dependencies, there are certain other functional dependencies that are logically implied by *F*.
  - If $A \rightarrow B$ and $B \rightarrow C$, then we can infer that $A \rightarrow C$
  - etc.
- The set of **all** functional dependencies logically implied by *F* is the **closure** of *F*.
- We denote the *closure* of *F* by F^+.

Closure of a Set of Functional Dependencies

- We can compute F^+, the closure of F, by repeatedly applying **Armstrong's Axioms:**
  - **Reflexive rule:** if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
  - **Augmentation rule:** if $\alpha \rightarrow \beta$, then $\gamma\, \alpha \rightarrow \gamma\, \beta$
  - **Transitivity rule:** if $\alpha \rightarrow \beta$, and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$
- These rules are
  - **Sound** -- generate only functional dependencies that actually hold, and
  - **Complete** -- generate all functional dependencies that hold.

Example of F^+

- $R = (A, B, C, G, H, I)$
  $F = \{ A \rightarrow B$
  $A \rightarrow C$
  $CG \rightarrow H$
  $CG \rightarrow I$
  $B \rightarrow H\}$
- Some members of F^+
  - $A \rightarrow H$
    - by transitivity from $A \rightarrow B$ *and* $B \rightarrow H$
  - $AG \rightarrow I$
    - by augmenting $A \rightarrow C$ with G, to get $AG \rightarrow CG$
      and then transitivity with $CG \rightarrow I$
  - $CG \rightarrow HI$
    - by augmenting $CG \rightarrow I$ to infer $CG \rightarrow CGI$,
      and augmenting $CG \rightarrow H$ to infer $CGI \rightarrow HI$,
      and then transitivity

Closure of Functional Dependencies (Cont.)

- Additional rules:
  - **Union rule**: If $\alpha \rightarrow \beta$ holds and $\alpha \rightarrow \gamma$ holds, then $\alpha \rightarrow \beta\gamma$ holds.
  - **Decomposition rule**: If $\alpha \rightarrow \beta\gamma$ holds, then $\alpha \rightarrow \beta$ holds and $\alpha \rightarrow \gamma$ holds.
  - **Pseudotransitivity rule**:If $\alpha \rightarrow \beta$ holds and $\gamma\beta \rightarrow \delta$ holds, then $\alpha\gamma \rightarrow \delta$ holds**.**
- The above rules can be inferred from Armstrong's axioms.

Procedure for Computing F^+

- To compute the closure of a set of functional dependencies *F*:

```
$F^+ = F$
repeat
    for each functional dependency $f$ in $F^+$
        apply reflexivity and augmentation rules on $f$
        add the resulting functional dependencies to $F^+$
    for each pair of functional dependencies $f_1$ and $f_2$ in $F^+$
        if $f_1$ and $f_2$ can be combined using transitivity
            then add the resulting functional dependency to $F^+$
until $F^+$ does not change any further
```

- **NOTE**: We shall see an alternative procedure for this task later

Closure of Attribute Sets

- Given a set of attributes α, define the ***closure*** of α **under** *F* (denoted by α^+) as the set of attributes that are functionally determined by α under *F*
- Algorithm to compute α^+, the closure of α under *F*

```
result := $\alpha$;
while (changes to result) do
    for each $\beta \rightarrow \gamma$ in F do
        begin
            if $\beta \subseteq$ result then result := result $\cup \gamma$
        end
```

Example of Attribute Set Closure

- $R = (A, B, C, G, H, I)$
- $F = \{A \rightarrow B$
  $A \rightarrow C$
  $CG \rightarrow H$
  $CG \rightarrow I$
  $B \rightarrow H\}$
- $(AG)^+$
  1. $result = AG$
  2. $result = ABCG$ $(A \rightarrow C$ and $A \rightarrow B)$
  3. $result = ABCGH$ $(CG \rightarrow H$ and $CG \subseteq AGBC)$
  4. $result = ABCGHI$ $(CG \rightarrow I$ and $CG \subseteq AGBCH)$
- Is AG a candidate key?
  1. Is AG a super key?
     1. Does $AG \rightarrow R$? == Is R $\supseteq$ $(AG)^+$
  2. Is any subset of AG a superkey?
     1. Does $A \rightarrow R$? == Is R $\supseteq$ $(A)^+$
     2. Does $G \rightarrow R$? == Is R $\supseteq$ $(G)^+$
     3. In general: check for each subset of size *n-1*

Uses of Attribute Closure

There are several uses of the attribute closure algorithm:

- Testing for superkey:
  - To test if α is a superkey, we compute $\alpha^{+,}$ and check if α^{+} contains all attributes of *R*.
- Testing functional dependencies
  - To check if a functional dependency $\alpha \rightarrow \beta$ holds (or, in other words, is in F^{+}), just check if $\beta \subseteq \alpha^{+}$.
  - That is, we compute α^{+} by using attribute closure, and then check if it contains β.
  - Is a simple and cheap test, and very useful
- Computing closure of F
  - For each $\gamma \subseteq R$, we find the closure γ^{+}, and for each $S \subseteq \gamma^{+}$, we output a functional dependency $\gamma \rightarrow S$.

Canonical Cover

- Suppose that we have a set of functional dependencies *F* on a relation schema. Whenever a user performs an update on the relation, the database system must ensure that the update does not violate any functional dependencies; that is, all the functional dependencies in *F* are satisfied in the new database state.
- If an update violates any functional dependencies in set *F,* the system must roll back the update.
- We can reduce the effort spent in checking for violations by testing a simplified set of functional dependencies that has the same closure as the given set.
- This simplified set is termed the **canonical cover**
- To define canonical cover we must first define **extraneous attributes.**
  - An attribute of a functional dependency in *F* is **extraneous** if we can remove it without changing F^+

Extraneous Attributes

- Removing an attribute from the left side of a functional dependency could make it a stronger constraint.
  - For example, if we have AB $\rightarrow$ C and remove B, we get the possibly stronger result A $\rightarrow$ C. It may be stronger because A $\rightarrow$ C logically implies AB $\rightarrow$ C, but AB $\rightarrow$ C does not, on its own, logically imply A $\rightarrow$ C
- But, depending on what our set *F* of functional dependencies happens to be, we may be able to remove B from AB $\rightarrow$ C safely.
  - For example, suppose that
  - *F* = {AB $\rightarrow$ C, A $\rightarrow$ D, D $\rightarrow$ C}
  - Then we can show that *F* logically implies A $\rightarrow$ C, making B extraneous in AB $\rightarrow$ C.

Extraneous Attributes

- An attribute of a functional dependency in F is **extraneous** if we can remove it without changing F^+
- Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F.
  - **Remove from the left side**: Attribute A is **extraneous** in α if
    - $A \in \alpha$ and
    - F logically implies $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$.
  - **Remove from the right side**: Attribute A is **extraneous** in β if
    - $A \in \beta$ and
    - The set of functional dependencies

      $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ logically implies F.
- *Note:* implication in the opposite direction is trivial in each of the cases above since a “stronger” functional dependency always implies a weaker one

Testing if an Attribute is Extraneous

- Let *R* be a relation schema and let *F* be a set of functional dependencies that hold on *R* . Consider an attribute in the functional dependency $\alpha \rightarrow \beta$.
- To test if attribute $A \in \beta$ is extraneous in β
  - Consider the set:

    $$F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\},$$
  - check that α^+ contains *A;* if it does, *A* is extraneous in β
- To test if attribute $A \in \alpha$ is extraneous in α
  - Let $\gamma = \alpha - \{A\}$. Check if $\gamma \rightarrow \beta$ can be inferred from *F.*
    - Compute γ^+ using the dependencies in *F*
    - If γ^+ includes all attributes in β then, *A* is extraneous in α

Examples of Extraneous Attributes

- Let $F = \{AB \rightarrow CD, A \rightarrow E, E \rightarrow C\}$
- To check if C is extraneous in $AB \rightarrow CD$, we:
  - Compute the attribute closure of AB under $F' = \{AB \rightarrow D, A \rightarrow E, E \rightarrow C\}$
  - The closure is *ABCDE,* which includes *CD*
  - This implies that *C* is extraneous

Canonical Cover

A **canonical cover** for *F* is a set of dependencies F_c such that

- *F* logically implies all dependencies in F_c, and
- F_c logically implies all dependencies in *F*, and
- No functional dependency in F_c contains an extraneous attribute, and
- Each left side of functional dependency in F_c is unique. That is, there are no two dependencies in F_c
  - $\alpha_1 \rightarrow \beta_1$ and $\alpha_2 \rightarrow \beta_2$ such that
  - $\alpha_1 = \alpha_2$

Canonical Cover

- To compute a canonical cover for *F*:

```
repeat
        Use the union rule to replace any dependencies in F of the form
                $\alpha_1 \rightarrow \beta_1$ and $\alpha_1 \rightarrow \beta_2$ with $\alpha_1 \rightarrow \beta_1 \beta_2$
        Find a functional dependency $\alpha \rightarrow \beta$ in $F_c$ with an extraneous
        attribute either in $\alpha$ or in $\beta$
            /* Note: test for extraneous attributes done using $F_{c,}$ not $F^+$
        If an extraneous attribute is found, delete it from $\alpha \rightarrow \beta$
until $F_c$ not change
```

- Note: Union rule may become applicable after some extraneous attributes have been deleted, so it must be re-applied

Example: Computing a Canonical Cover

- $R = (A, B, C)$
  $F = \{A \rightarrow BC$
  $B \rightarrow C$
  $A \rightarrow B$
  $AB \rightarrow C\}$
- Combine $A \rightarrow BC$ and $A \rightarrow B$ into $A \rightarrow BC$
  - Set is now $\{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$
- A is extraneous in $AB \rightarrow C$
  - Check if the result of deleting A from $AB \rightarrow C$ is implied by the other dependencies
    - Yes: in fact, $B \rightarrow C$ is already present!
  - Set is now $\{A \rightarrow BC, B \rightarrow C\}$
- C is extraneous in $A \rightarrow BC$
  - Check if $A \rightarrow C$ is logically implied by $A \rightarrow B$ and the other dependencies
    - Yes*:* using transitivity on $A \rightarrow B$ *and* $B \rightarrow$ C.
      - Can use attribute closure of *A* in more complex cases
- The canonical cover is: $A \rightarrow B$
  $B \rightarrow C$

Dependency Preservation

- Let F_i be a subset of dependencies F^+ that include only attributes in R_i.
  - A decomposition is **dependency preserving**, if

$$(F_1 \cup F_2 \cup \ldots \cup F_n)^+ = F^+$$

- Using the above definition, testing for dependency preservation takes exponential time.
- Note that if decomposition is NOT dependency preserving then checking updates for violation of functional dependencies may require computing joins, which is expensive.

Dependency Preservation (Cont.)

- Let F be the set of dependencies on schema R and let R_1, R_2, .., R_n be a decomposition of R.
- The restriction of F to R_i is the set F_i of all functional dependencies in F^+ that include **only** attributes of R_i.
- Since all functional dependencies in a restriction involve attributes of only one relation schema, it is possible to test such a dependency for satisfaction by checking only one relation.
- Note that the definition of restriction uses all dependencies in F^+, not just those in F.
- The set of restrictions F_1, F_2, .. , F_n is the set of functional dependencies that can be checked efficiently.

Testing for Dependency Preservation

- To check if a dependency $\alpha \rightarrow \beta$ is preserved in a decomposition of R into $R_1, R_2, \ldots, R_n$, we apply the following test (with attribute closure done with respect to F)

  -
    ```
    result = α
    repeat
      for each R_i in the decomposition
              t = (result ∩ R_i)^+ ∩ R_i
              result  =  result  ∪ t
       until  (result does not change)
    ```
  - If the *result* contains all attributes in β, then the functional dependency $\alpha \rightarrow \beta$ is preserved.
- We apply the test on all dependencies in F to check if a decomposition is a dependency preserving
- This procedure takes polynomial time, instead of the exponential time required to compute F^+ and $(F_1 \cup F_2 \cup \ldots \cup F_n)^+$

Design Goals - Summary

- Goal for a relational database design is:
  - BCNF.
  - Lossless join.
  - Dependency preservation.
- If we cannot achieve this, we accept one of
  - Lack of dependency preservation
  - Redundancy due to the use of 3NF
- Interestingly, SQL does not provide a direct way of specifying functional dependencies other than superkeys.

  Can specify FDs using assertions, but they are expensive to test, (and currently not supported by any of the widely used databases!)
- Even if we had a dependency preserving decomposition, using SQL we would not be able to efficiently test a functional dependency whose left-hand side is not a key.

End of Chapter 7