PensionMetrics I: Stochastic Pension Plan Design during the Accumulation Phase

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Motivation

- Millions of dollars spent on car design
  - Why?
  - Customer can value product immediately
  - Choices of consumer v producer
- Nothing spent on pension plan design
  - Why?
  - Customer cannot value product for 70 years

Risks with DC plans

- Contribution risk
  - Unemployment risk
- Asset price/return risk
- Interest rate risk

Aims

- Assess how closely DC can match Defined Benefit (DB) pension programmes
  - in terms of pension ratio
- Design DC plan to target final salary pension with specified probability
- Estimate Values-at-Risk (VaRs) implied by alternative DC plans

Stochastic Simulation

- Given assumptions about asset returns, annuity rate, mortality risks, etc.
- Choose investment strategy
  - ◆ Contribution rate: eg. 10% of earnings
  - ◆ Asset allocation: eg. 60% equities, 40% bonds
- What is the probability of beating target pension?

No DC plan can guarantee target pension with 100% probability

- Earnings growth is a **non-hedgeable** risk
- **But** possible to use design to determine contribution rate and asset allocation
- Chosen strategy is **satisfactory** if projected DC pension exceeds target at specified VaR confidence level (eg. 75%)

Pension Plan Model

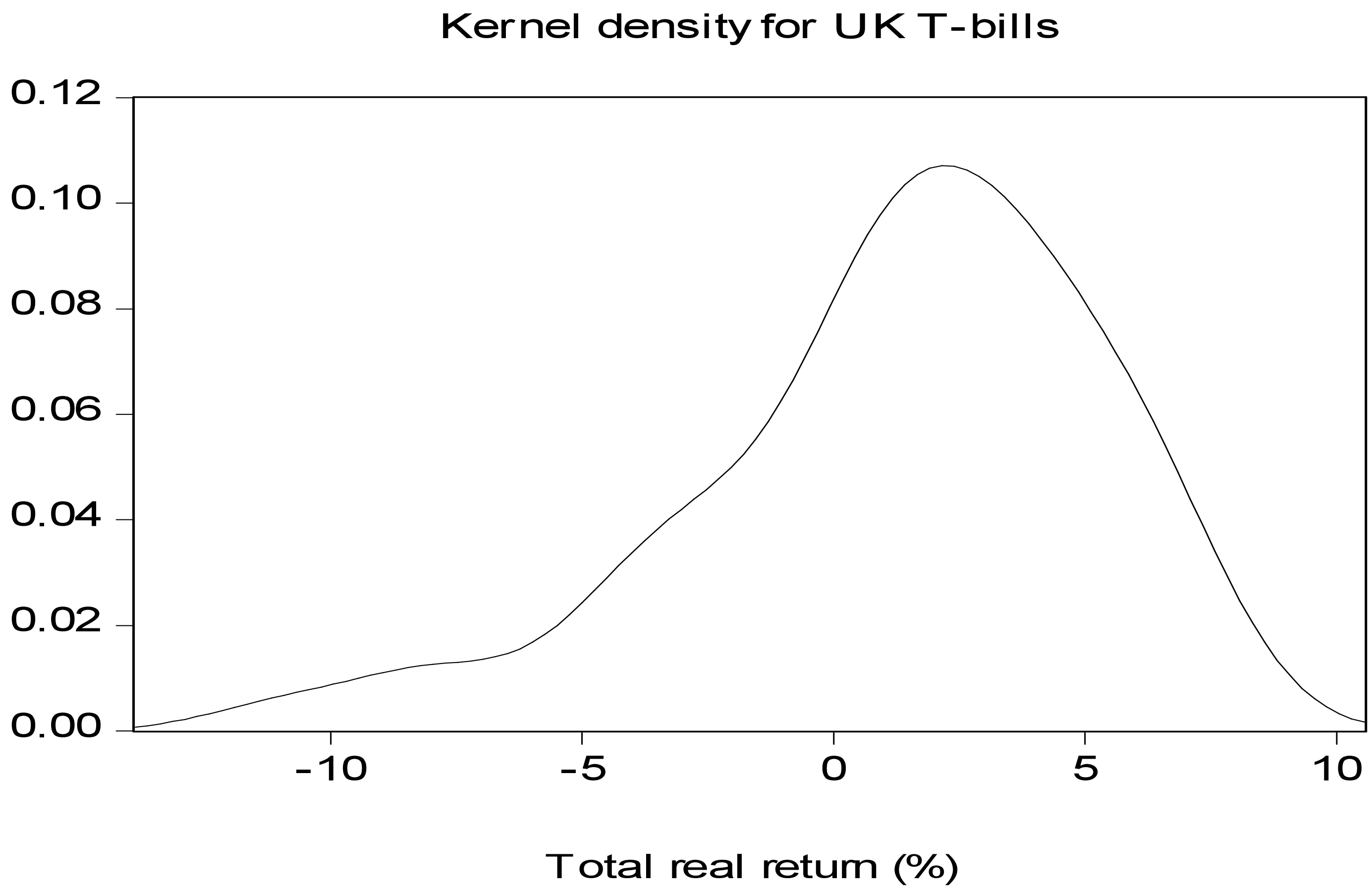
- Asset returns model
- Interest rate model
- Liability model
- Asset allocation strategy
- But the higher the required probability
  - the higher the contribution rate,and/or
  - the higher the relative investment in more capital-certain investments, e.g., bonds

Stylised facts on asset returns

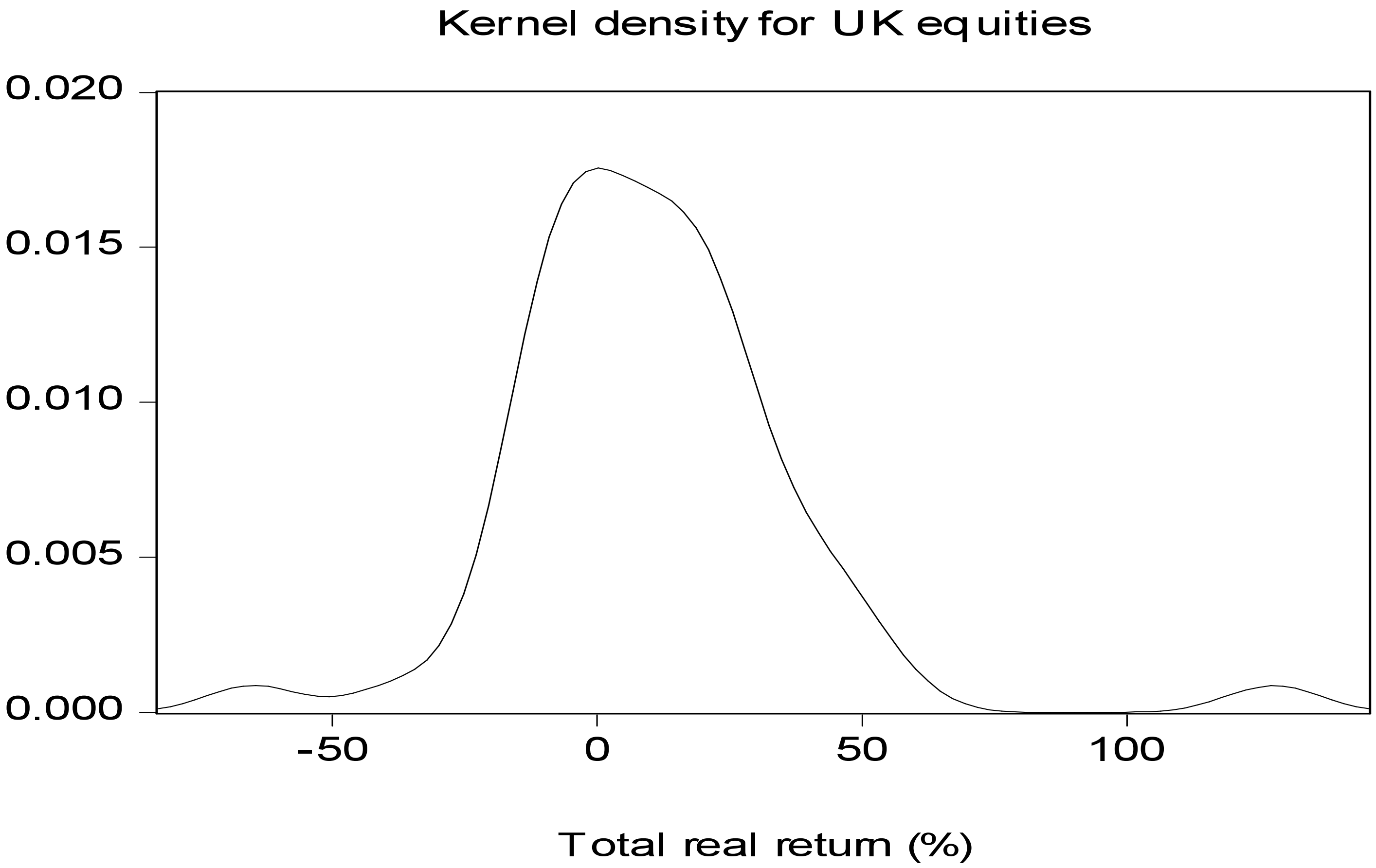
- Mean reversion for long-term assets
- Positive long-term equity risk premium
- Leptokurtosis (fat tails): large positive and negative shocks to equities
- Timing of shocks important:
  disaster if near end of career

$\Rightarrow$ Support for lifestyling

Some typical returns: UK T-bills



Some typical returns: UK equities



Lifestyling

- Equities are best asset early in investment programme
  - risks outweighed by risk premium over the long term
- Can also use index bonds
- Later switch to less volatile assets
  - e.g. conventional fixed income bonds

Moments models of returns

- Fixed moments models
  - Multivariate normal model
  - Mixed multivariate normal model
  - Multivariate *t* model
  - Non-central *t* model
  - Stationary bootstrap model
- Time-varying moments models
  - ARDL: autoregressive distributed lag
  - ARCH: autoregressive conditional heteroskedasticity
  - ARDL-ARCH model

Other models of returns

- Regime switching models
  - Markov switching model

- Fundamentals models
  - Return-on-yield model
  - Return-on-yield-ratio model
  - Excess-return-on-yield model
  - Wilkie model

Interest rate model

- Need to simulate interest rates for retirement date to price annuities
- Must be consistent with asset returns model
  - if bond returns high, yields low

Liability model

- Career type
- Lifetime earnings profile
- Unemployment, illness, child care during accumulation phase
- Mortality during distribution phase
    - affects annuity price

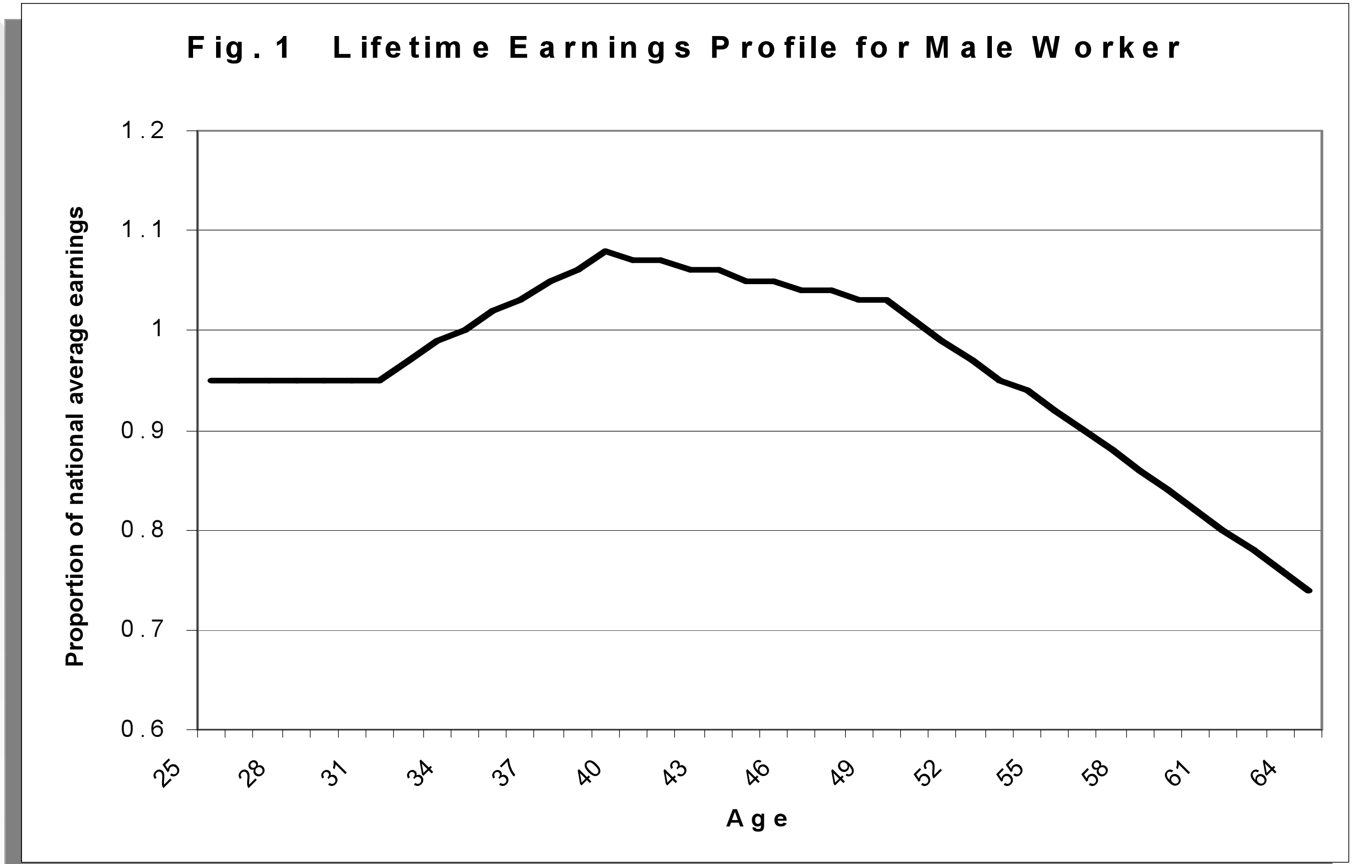
Liability model

- Accrued benefit:
- $L(t) = \alpha(t)W(t)R(t,T)\ddot{a}(T)D(t,T)MVA$
- $\alpha(t)$ – accrual factor at t (e.g., 5/60th)
- $W(t)$ – pensionable salary at t
- $R(t,T)$ – revaluation factor for earnings
- $\ddot{a}(T)$ – annuity factor at retirement T
- $D(t,T)$ – discount factor
- MVA – market value adjustment
  - affects annuity price

Liability model

- Current unit method (discontinuance):
  - $R(t,T) = (1 + \pi)^{(T-t)}$
  - entitlement today (e.g. if transfer)
- Projected unit method (ongoing):
  - $R(t,T) = (1 + \omega)^{(T-t)}$
  - expectation at retirement date

Lifetime earnings – typical example



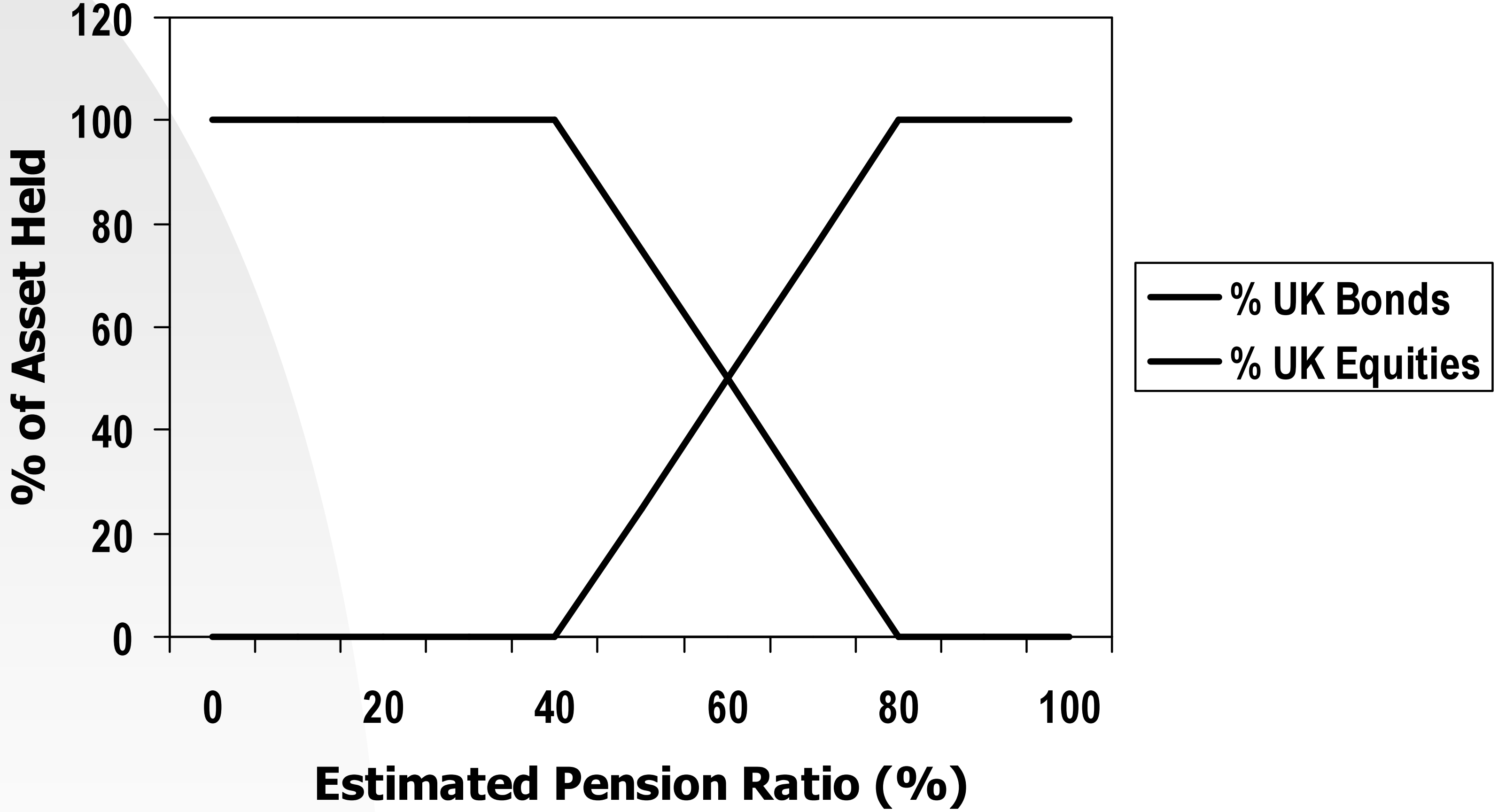
Static asset allocation strategies

- UK pension fund average (e.g., company DB)
  - ☞ 5% Cash
  - ☞ 15% UK bonds
  - ☞ 51% UK equities
  - ☞ 5% UK property
  - ☞ 20% International equities
  - ☞ 4% International bonds
- 100% UK equities

Dynamic asset allocation strategies

- Either with or without feedback control
- Lifestyle
  - 100% UK equities for first 20 years
  - switch 5% p.a. into UK bonds over last 20 years
- Threshold (or ‘funded status’)
  - 100% equities if pension ratio < T_L (0.4)
  - 100% bonds if pension ratio > T_H (0.8)

Threshold Strategy



Dynamic asset allocation strategies

- Constant proportion portfolio insurance
  - Weight in PFA=C_M(1- C_F(Liabilities/Fund))
  - with C_F =0.5, C_M=2
  - Weight in Cash= 1 - Weight in PFA

Simulation Case Study

- Contribution rate = 10% of earnings
- Duration 40 years (age 25 $\rightarrow$ 65)
- 5,000 Monte Carlo simulations
- 7 asset return models
- 5 asset-allocation strategies
  - Static: UK pension fund average
  - Static: 100% UK equities
  - Dynamic: Lifestyle
  - Dynamic: Threshold
  - Dynamic: CPPI

Simulation output

- Asset-return model not significant

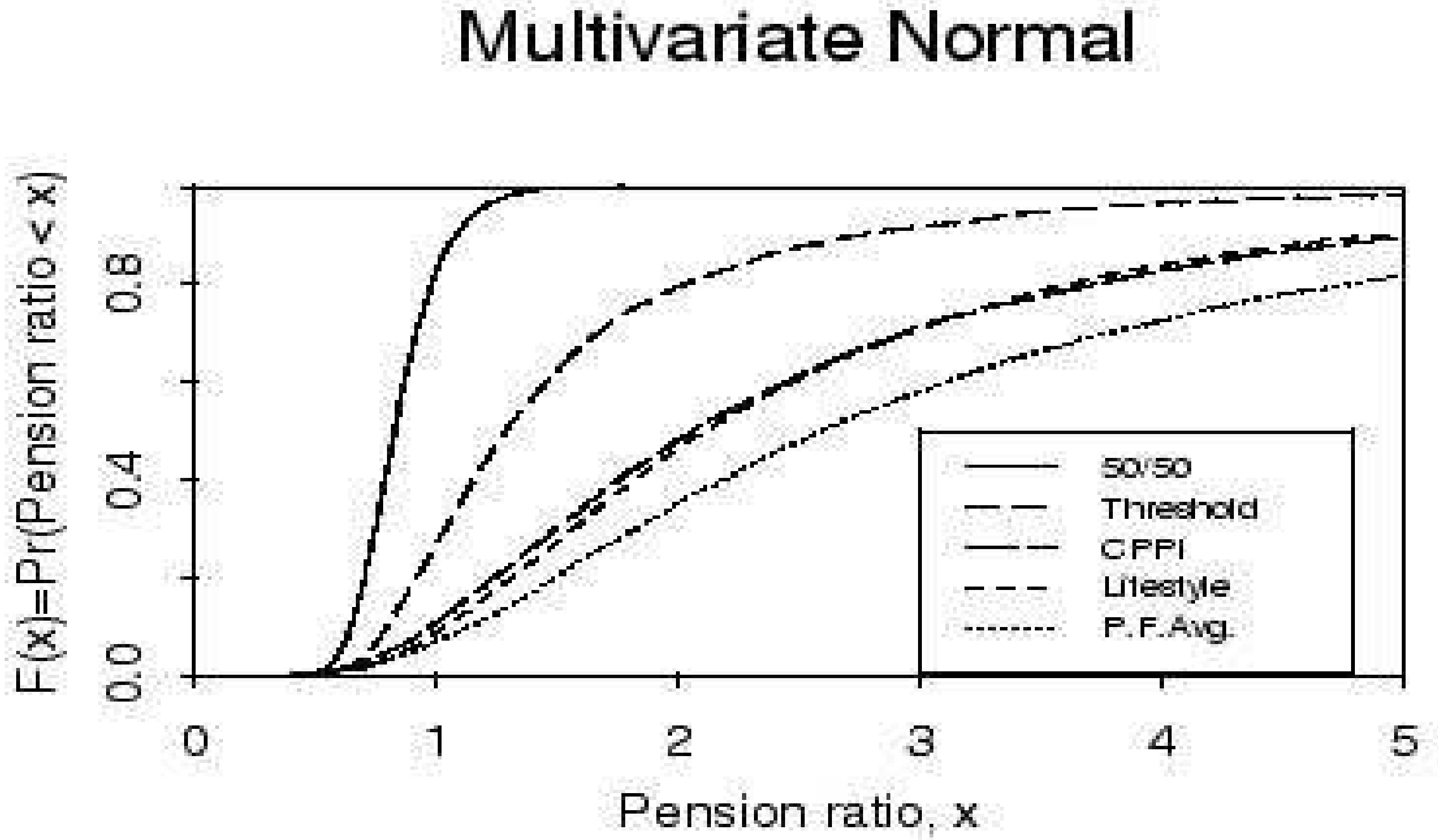
Table 4.1 Model Uncertainty for the PFA and Threshold Strategies						
	Pension ratios for:					
	PFA strategy			Threshold Strategy		
	Mean	SD	5% VaR	Mean	SD	5% VaR
Multivariate normal	3.41	2.65	0.95	1.61	1.10	0.74
Mixed multivariate normal	3.56	2.73	1.03	1.62	1.03	0.75
Multivariate *t*	3.51	3.35	0.87	1.66	1.23	0.72
Non-central *t*	3.69	2.69	1.17	1.67	1.02	0.81
Stationary bootstrap	3.48	2.83	0.97	1.65	1.22	0.73
Markov	3.74	2.71	1.13	1.65	1.29	0.65
Wilkie	2.71	1.54	1.04	1.51	1.15	0.52

Simulation output

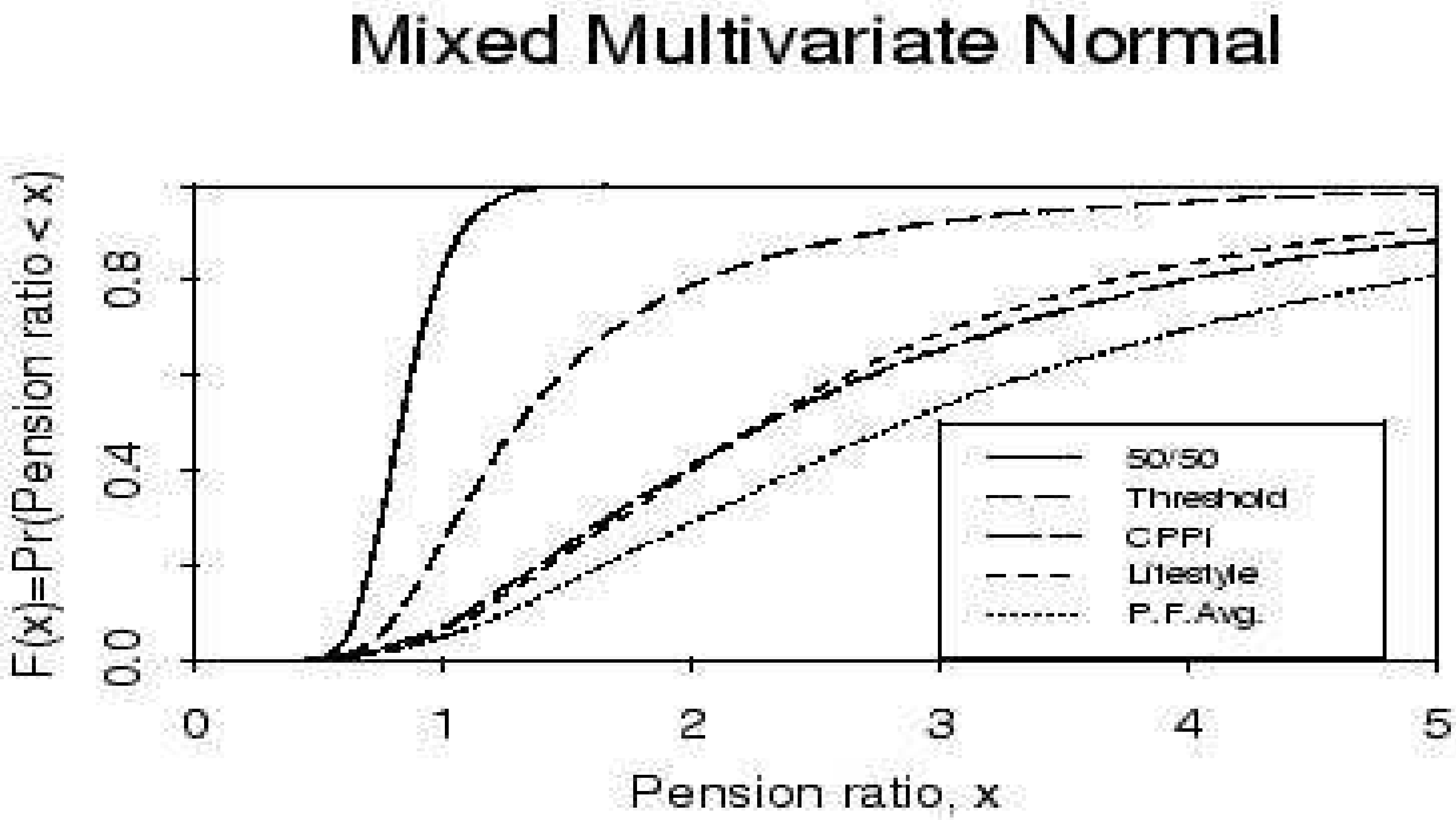
- Static PFA dominates

Table 4.2 Asset Allocation Strategies with Multivariate Normal Model				
	Pension ratios:			
	Median	SD	5% VaR	20% VaR
50/50	0.84	0.18	0.61	0.71
PFA	2.63	2.65	0.95	1.53
Lifestyle	1.74	1.21	0.87	1.19
Threshold	1.30	1.10	0.74	0.93
CPPI	2.10	2.06	0.83	1.27

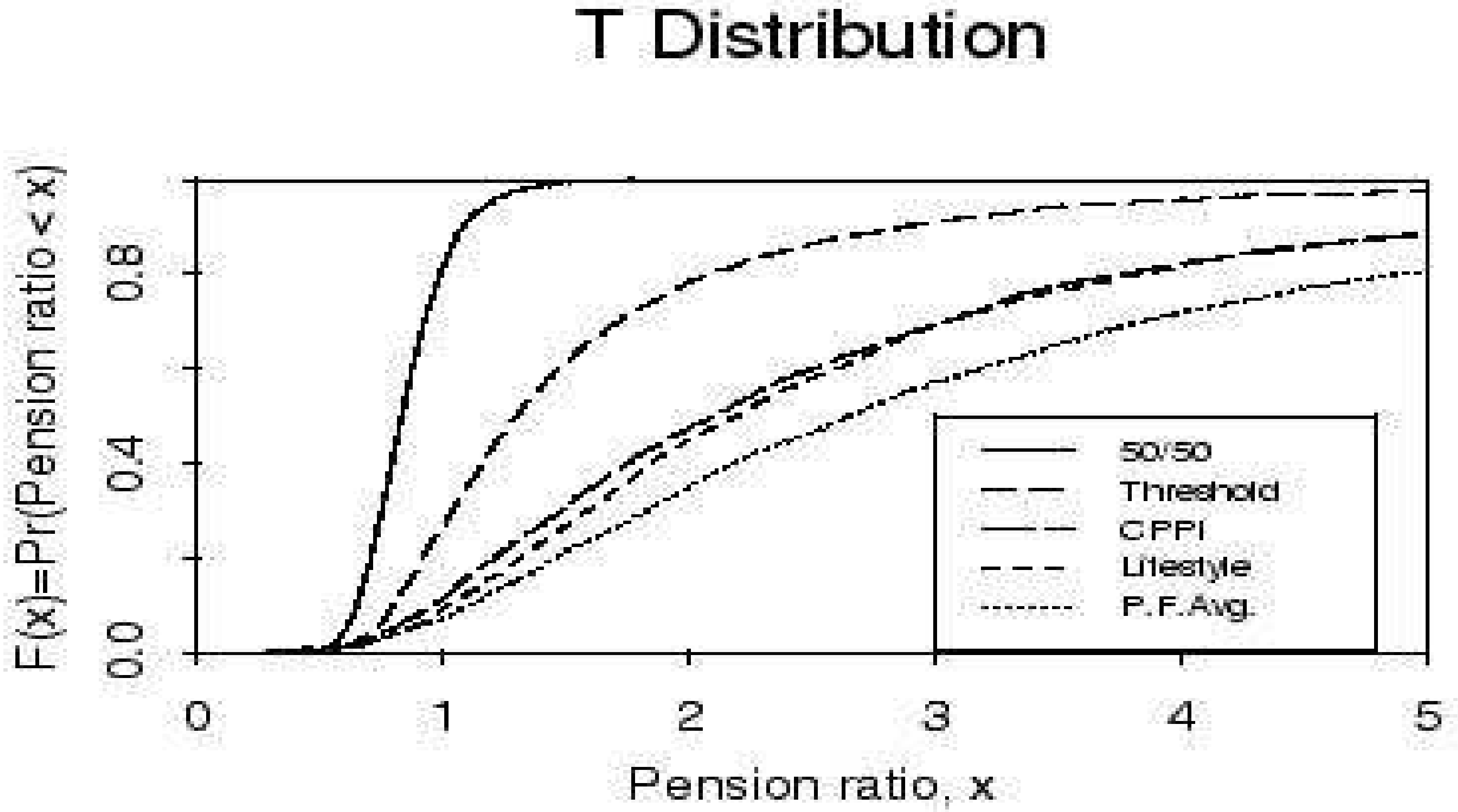
Cumulative return distribution



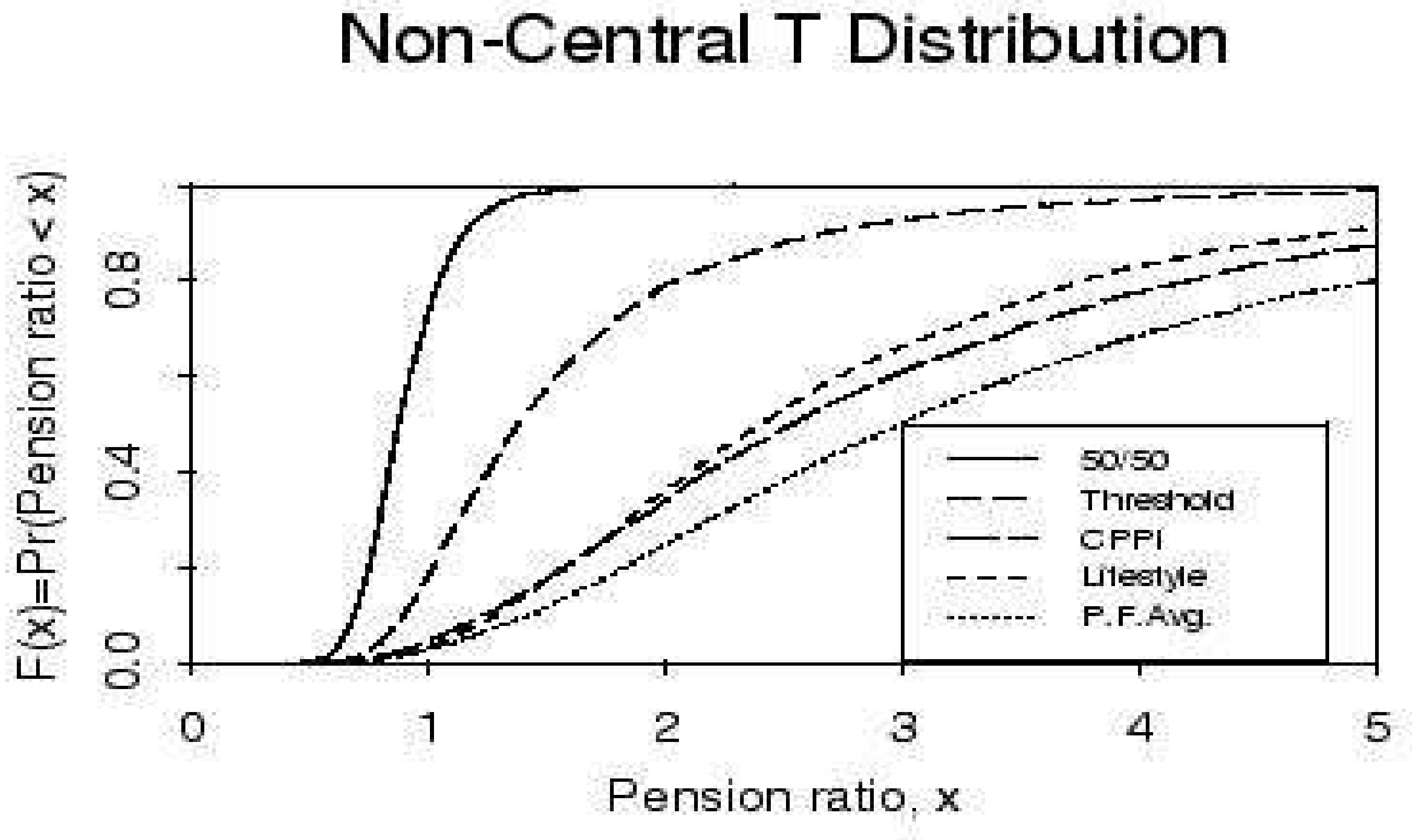
Cumulative return distribution



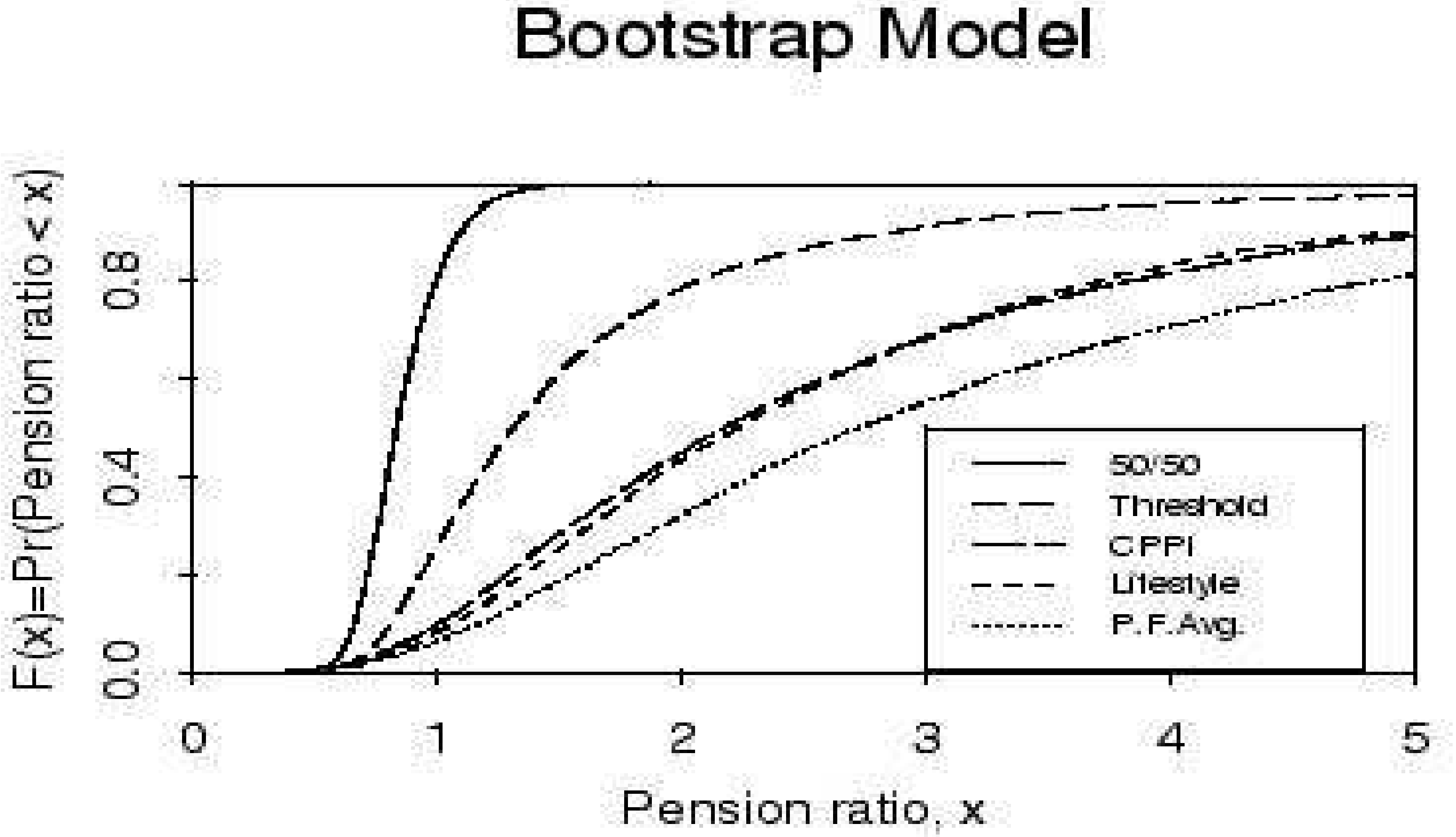
Cumulative return distribution



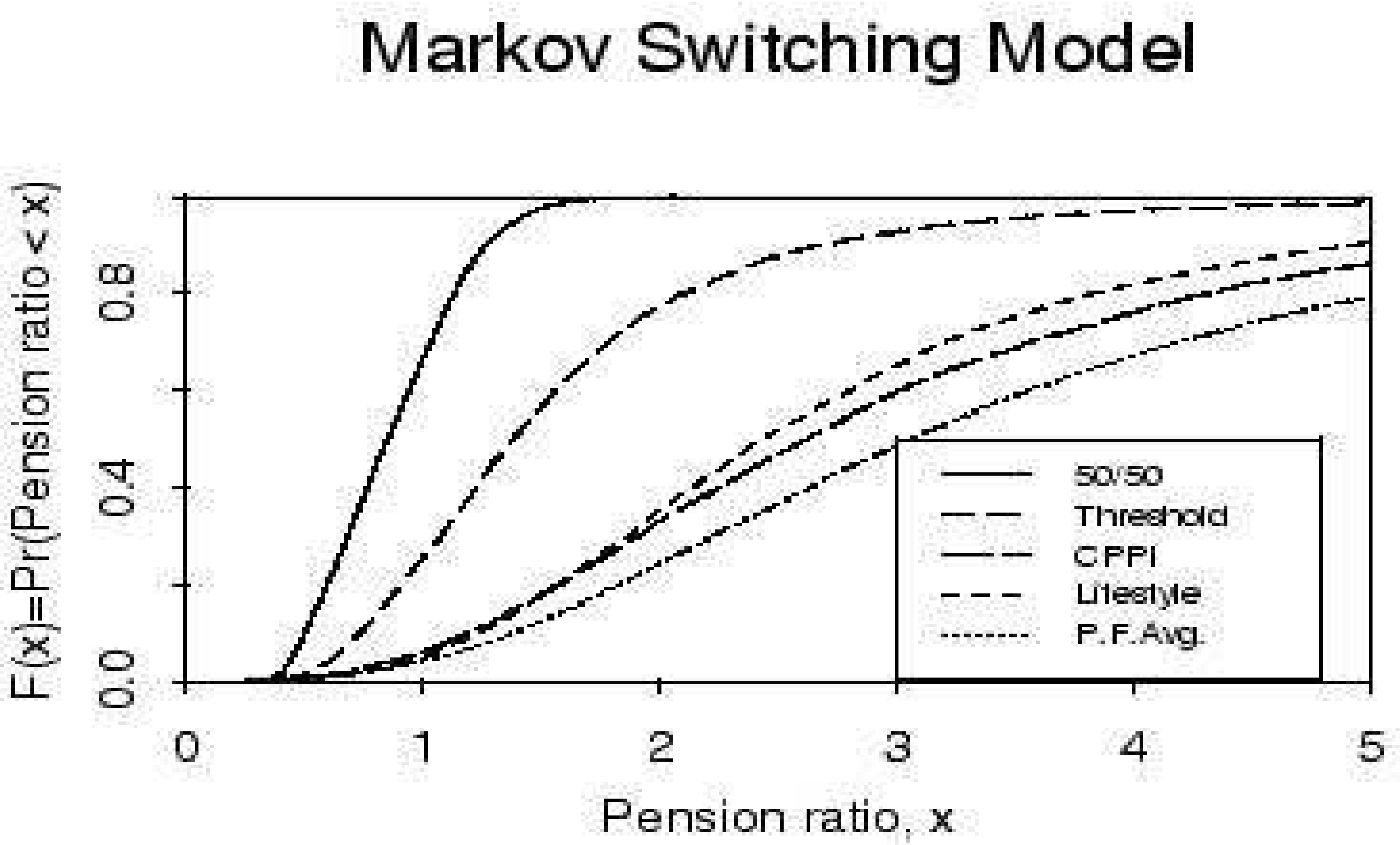
Cumulative return distribution



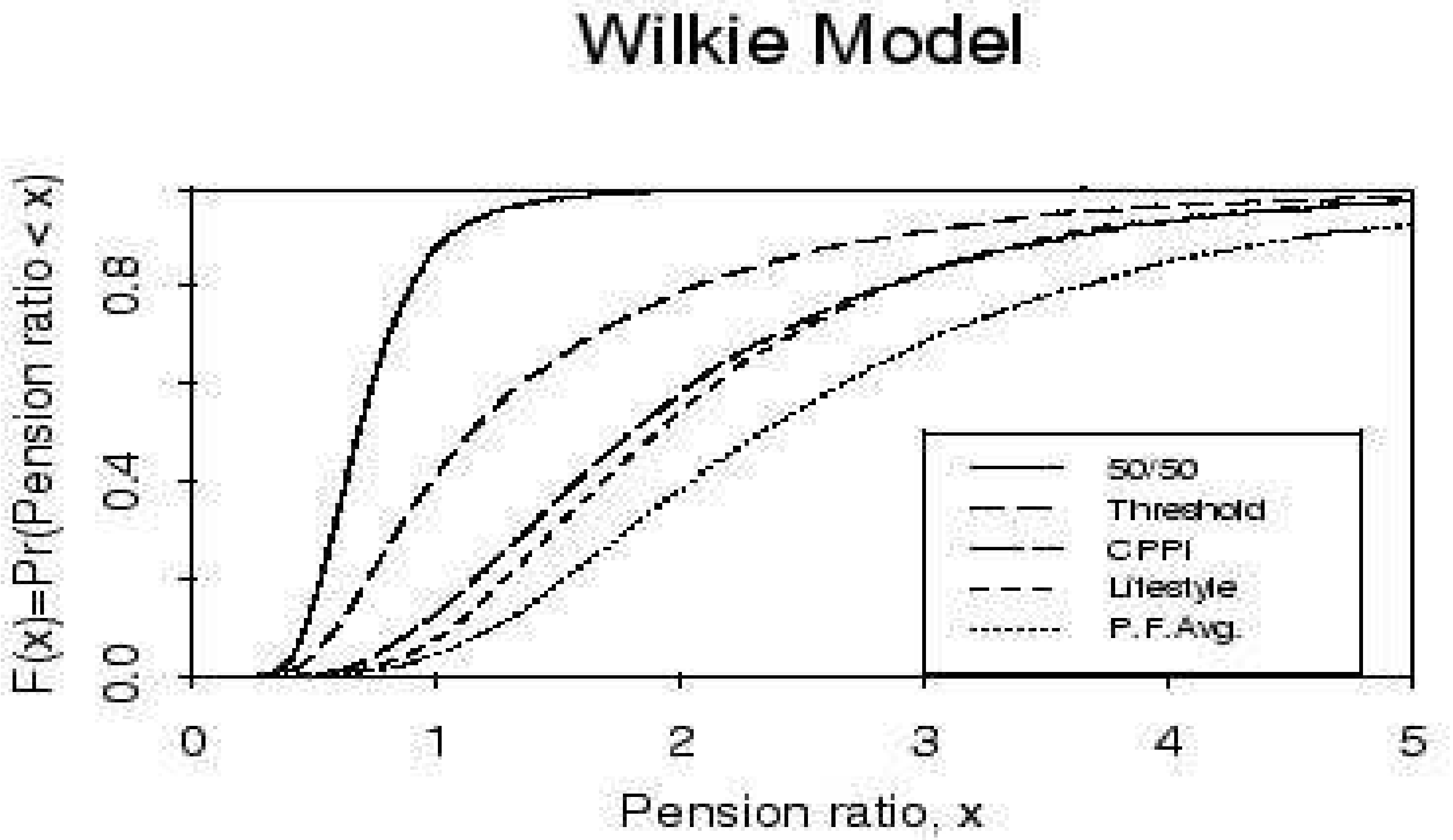
Cumulative return distribution



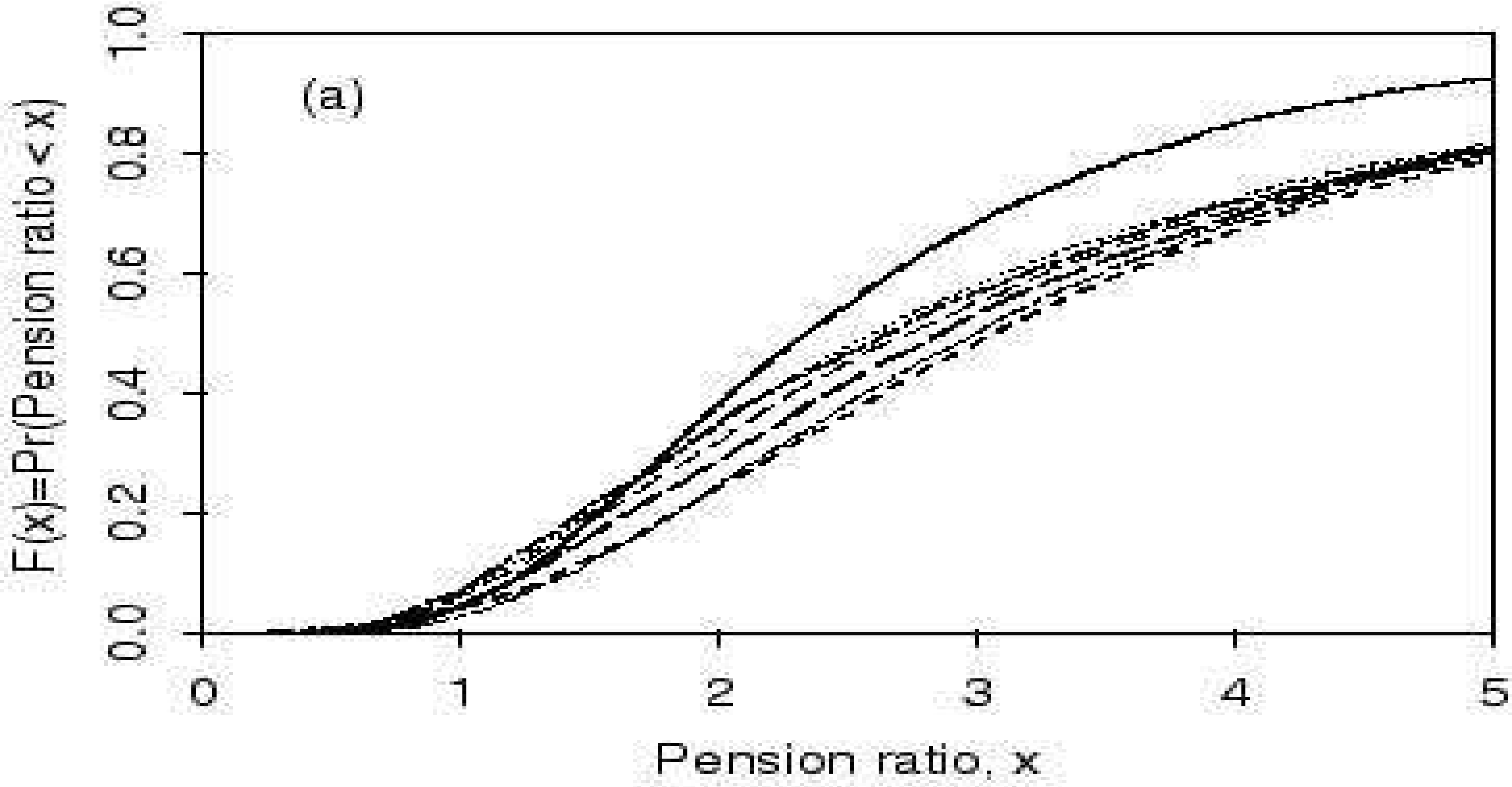
Cumulative return distribution



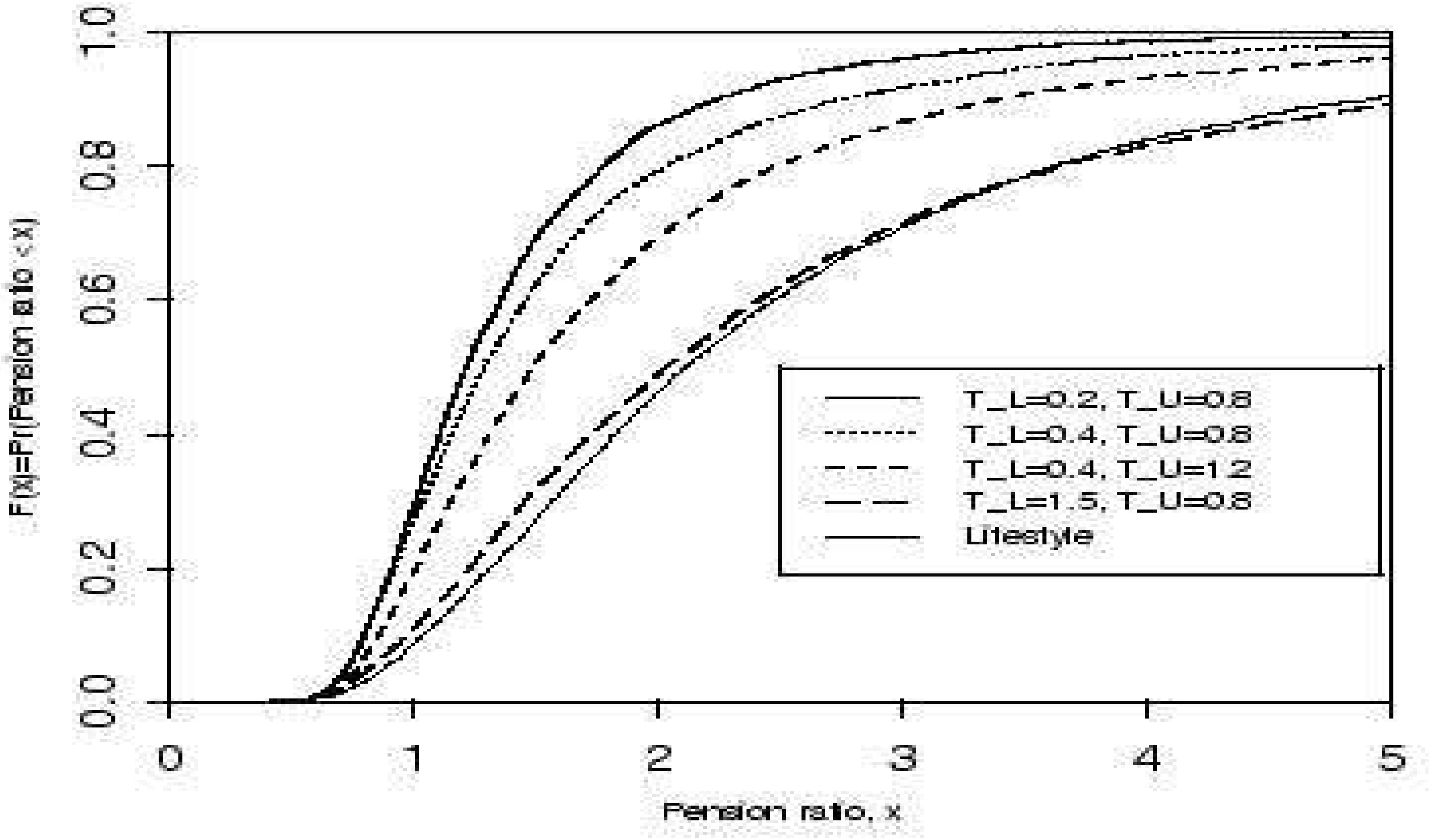
Cumulative return distribution



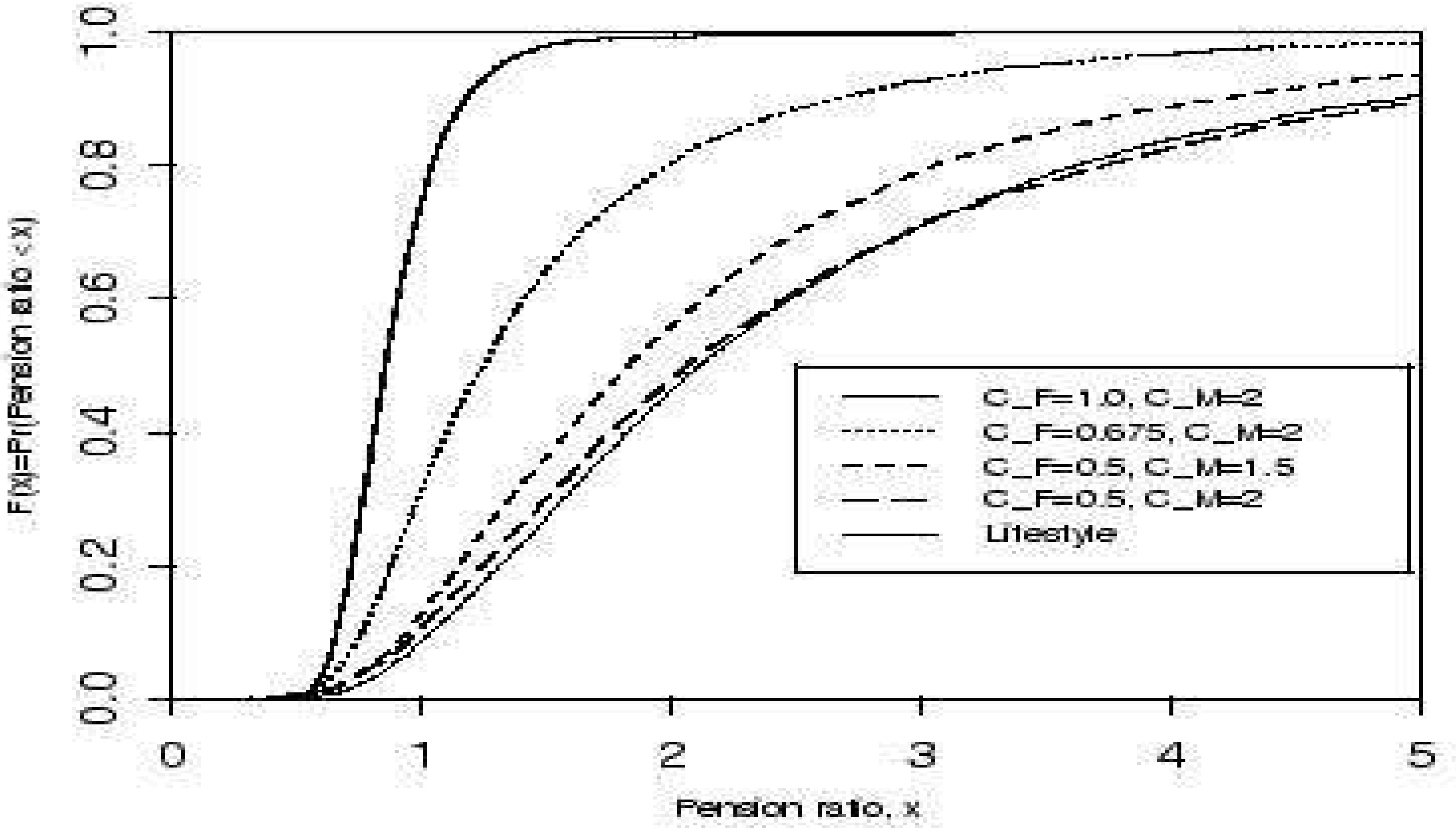
Model risk with PFA strategy



Different threshold strategies



Different CPPI strategies



Simulation output

- DC accepted at 50% VaR-CL but not at 80% or 95%

Table 4.3 Retirement VaRs for the Threshold Strategy

	VaR confidence level			Critical
	50%	80%	95%	VaR-CL
Multivariate normal	1.30	0.93	0.74	73%
Mixed multivariate normal	1.32	0.95	0.75	75%
Multivariate *t*	1.31	0.93	0.73	74%
Non-central *t*	1.39	1.03	0.81	82%
Stationary bootstrap	1.32	0.94	0.74	74%
Markov	1.39	0.92	0.64	75%
Wilkie	1.15	0.73	0.52	59%

Simulation output

- Low risk strategies are expensive
- CPPI is cheapest after PFA

Table 4.4 Required Contribution Rates for the Multivariate Normal Model

VaR confidence level for the PFA	1%	5%	50%	95%
VaR for the PFA	0.62	0.95	2.63	8.47
	Contribution rates (%)			
50/50	11.6	15.4	31.4	72.2
Lifestyle	9.0	10.9	15.1	19.3
Threshold	9.6	12.8	20.2	23.7
CPPI	10.4	11.4	12.5	12.9

Conclusions

- DC plans risky relative to DB benchmark
- Asset allocation strategy more important than asset returns model
- Static PFA delivers better results than lifestyle or other dynamic strategies
- Bond strategies require higher contributions than equity strategies

Conclusions

- UK pension fund average looks best overall
- 100% equities
  - can perform very well
  - can perform very badly
- Lifestyle
  - very poor: switch from equities into bonds $\Rightarrow$ often locking into poor equity returns
- Threshold much better than lifestyle
  - shift to bonds normally after good equity returns
  - downside risks much lower
  - but restricted upside potential

PensionMetrics 2: Stochastic Pension Plan Design during the Decumulation Phase

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Motivation

- What to do with defined contribution (DC) pension fund when policyholder retires?
- BIG cultural differences $\Rightarrow$
- Traditional solution A: (e.g. UK)
  - ☞ Purchase an annuity
  - ◆ Requirement in many countries to purchase annuities
  - ◆ Pension income related to interest rates at retirement and forecasts of longevity
  - ◆ Policyholder switches suddenly from equity exposure to interest-rate exposure

Motivation

- Traditional solution B: (e.g. USA)
  - ☞ Do not annuitise
  - ◆ Continued equity participation:
    - ☞ continuity of pre-retirement investment strategy
  - ◆ Versus expensive annuities
  - ◆ Allows for bequest motive
  - ◆ Concerns:
    - ☞ exposed to market risk
    - ☞ risk from poor diversification
    - ☞ risk of outliving assets
    - ☞ no pooling of longevity risk

Motivation

- Are either of the ‘extreme’ traditional solutions appropriate?
  - Is annuity best?
  - Is 100% equities best?
- Maybe neither
- Individual plan member needs to weigh up:
  - current bond yields
  - versus equity risk premium and risk level
  - attitude to risk of plan member
  - longevity prospects/state of health
  - desire to leave bequest

Aims

- To compare various retirement income programmes:
  - with/without equity participation
  - with/without pooling of longevity risk
- Alternative programmes:
  - Purchased Life Annuity (PLA)
  - Equity-Linked Annuity (ELA)
  - Equity-Linked Income Drawdown (ELID)
- Analysis based on:
  - male retiree, now aged 65, with investment options until age 75

Purchased Life Annuity - PLA

- Provides benchmark for other programmes
- At age 65:
  - convert fund into level annuity
- Pooling of longevity risk:
  - protection against outliving resources
- Risks to policyholder:
  - low interest rates at retirement
  - high inflation after retirement
- Risks to insurer:
  - reinvestment risk
  - mortality improvements

Purchased Life Annuity - PLA

- Annuity factor at retirement date:

$$\ddot{a}(T) = \frac{(1+\pi_{T+1})p_{T+1}}{(1+f_{T+1})} + \frac{(1+\pi_{T+1})(1+\pi_{T+2})p_{T+2}}{(1+f_{T+1})(1+f_{T+2})} + \dots$$

$$= \frac{p_{T+1}}{(1+r_{T+1})} + \frac{p_{T+2}}{(1+r_{T+1})(1+r_{T+2})} + \dots$$

Equity-Linked Annuity - ELA

- Income linked to performance of mixed equity/bond fund

- Annuity $\Rightarrow$ pooling of longevity risk:
  - insurer pays plan member annually in advance:
    - ☞ a SURVIVAL CREDIT
  - in return plan member's assets pass to insurer on death $\Rightarrow$
    - ☞ NO BEQUEST

Equity-Linked Annuity - ELA

- Payments linked to current fund size $\Rightarrow$
  - no risk of outliving assets
  - but variable annual income
- Compulsory conversion to FIXED annuity at age 75
  - fixed equity/bond mix for 10 years
  - level annuity from age 75
- NO BEQUESTS
- Note: ELA with 0% equities $\equiv$ PLA

Equity-Linked Income Drawdown - ELID

- Income linked to performance of mixed equity/bond fund
- NO pooling of longevity risk ⇒
  - BEQUEST on death to dependants
- Payments linked to current fund size ⇒
  - no risk of outliving assets
  - but variable annual income
- Compulsory conversion to FIXED annuity at age 75 ⇒
  - no bequests after 75

Method of comparison

- For given plan member:
  - what is attitude to risk?
  - what degree of bequest motive?
  - what mortality prospects?
- Combine this into plan member's <u>expected discounted utility</u> for each:
  - income programme
  - bond/equity mix (ELA, ELID only)
- Choose programme with <u>highest expected discounted utility</u>

Method of comparison

- Aim - we seek:

- practical solution:
  - easy to implement

- rather than theoretically superior solution:
  - which is difficult to understand or implement

Modelling assumptions

- Equity returns follow Geometric Brownian Motion
- Constant interest rate
- Returns on equities consistent with past UK past experience
- Simple asset model $\Rightarrow$
  - giving straightforward numerical evaluation of utility
  - allowing clear qualitative conclusions

Policy holder’s utility

- Expected discounted utility
  = utility from retirement income (U_1)
  + utility from bequests on death (U_2)

- What are U_1 and U_2?

Policy holder's utility

- P_B = benchmark annuity (PLA)
- $P(t)$ =actual pension at t (ELA/ELID)
- 1-γ = relative risk aversion (RRA) = 4
- K = completed years before death
- $D(K+1)$=amount of bequest:
  - benchmark bequest D_B = 0
- d_2 = other assets, e.g. house (10,000, similar results at 50,000)
- Retire at 65
- Compulsory annuitisation at 75

Policy holder's utility

- Power utility

$$U_1 = E\left[\sum_{t=1}^{K} e^{-\beta t} h_1(\gamma)\left(\frac{P(t)}{P_B}\right)^{\gamma}\right]$$

$$U_2 = k_2 E\left[e^{-\beta(K+1)} h_2(\gamma)\left\{\left(\frac{D(K+1)+d_2}{D_B+d_2}\right)^{\gamma}-1\right\}\right]$$

$k_2 \Rightarrow$ balance between income & bequest

No bequest (PLA / ELA) $\Rightarrow U_2 = 0, D_B = 0$

Policy holder's utility

- Functional forms chosen to avoid strongly dichotomous preferences:
  - Members with low RRA preferring ELA with no bequest
  - Members with high RRA preferring ELID with bequests

$$h_1(\gamma) = \frac{1}{1 - d_1^{\gamma}} \text{ not } \frac{1}{\gamma} \text{ with } 0 \le d_1 \le 1$$

$$h_2(\gamma) = \frac{1}{\left[\frac{F(0) + d_2}{d_2}\right]^{\gamma} - 1} \text{ not } \frac{1}{\gamma}$$

Results - moderate bequest motive

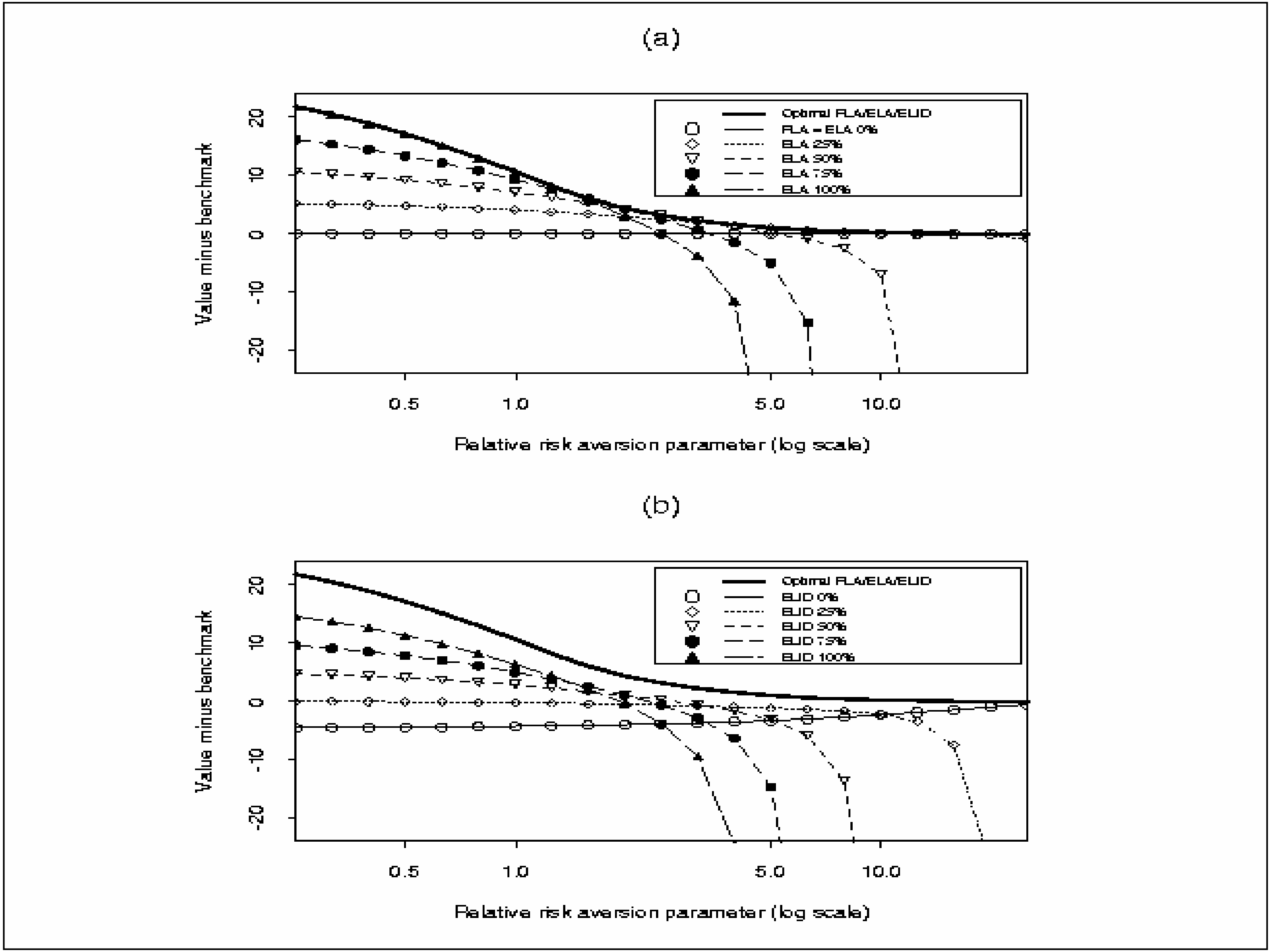
- RRA = 4, k_2 = 5 (moderate bequest motive)

Programme	Equity %	Expected utility
PLA	0	-8.82
ELA	0	-8.82
ELA	**25**	**-6.96**
ELA	50	-7.35
ELA	75	-9.99
ELA	100	-19.99
ELID	0	-11.98
ELID	25	-9.42
ELID	50	-10.10
ELID	75	-14.79
ELID	100	-33.11

Results - moderate bequest motive

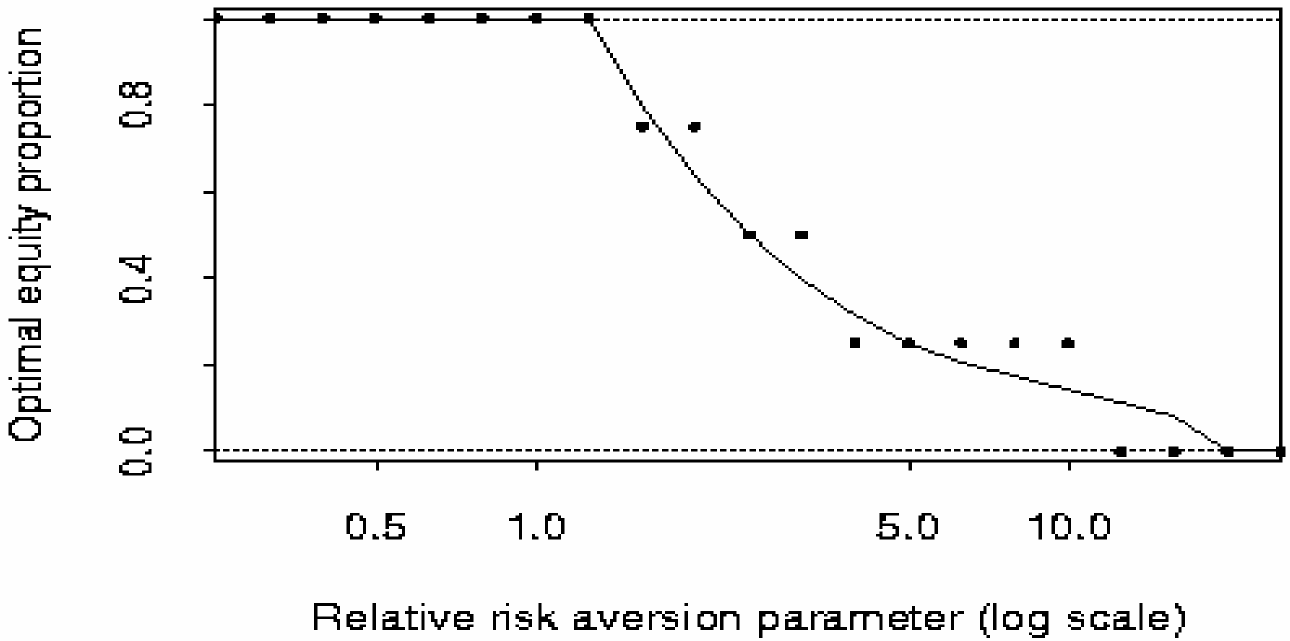
- For $k_2 = 5$, optimal policies use:
  - survival credits
  - rather than bequests

- Best programme is:
  - ELA (100% equity) if RRA < 1.25
  - ELA with declining equity weight for RRA < 10
  - PLA for RRA > 10


(a)
Optimal FLA/ELA/ELID
FLA = ELA 0%
ELA 25%
ELA 50%
ELA 75%
ELA 100%
Value minus benchmark
20
10
0
-10
-20
0.5
1.0
5.0
10.0
Relative risk aversion parameter (log scale)
(b)
Optimal FLA/ELA/ELID
ELID 0%
ELID 25%
ELID 50%
ELID 75%
ELID 100%
Value minus benchmark
20
10
0
-10
-20
0.5
1.0
5.0
10.0
Relative risk aversion parameter (log scale)

Results - moderate bequest motive

- What is the optimal equity proportion out of 0%, 25%, 50%, 75%, 100% for the ELA?


Optimal equity proportion
0.0
0.4
0.8
0.5
1.0
5.0
10.0
Relative risk aversion parameter (log scale)

Results - moderate bequest motive

- However, only 5% of UK investors have RRA < 10:
  - only a small % of investors likely to choose ELA

Wealth range (£91-92 values)	% in range	Relative risk aversion parameter
50-454	25	47.60
455-3,499	25	34.33
3,500-7,904	15	28.82
7,905-14,999	5	17.90
15,000-36,799	4	12.44
36,800+	1	7.88

Results - moderate bequest motive

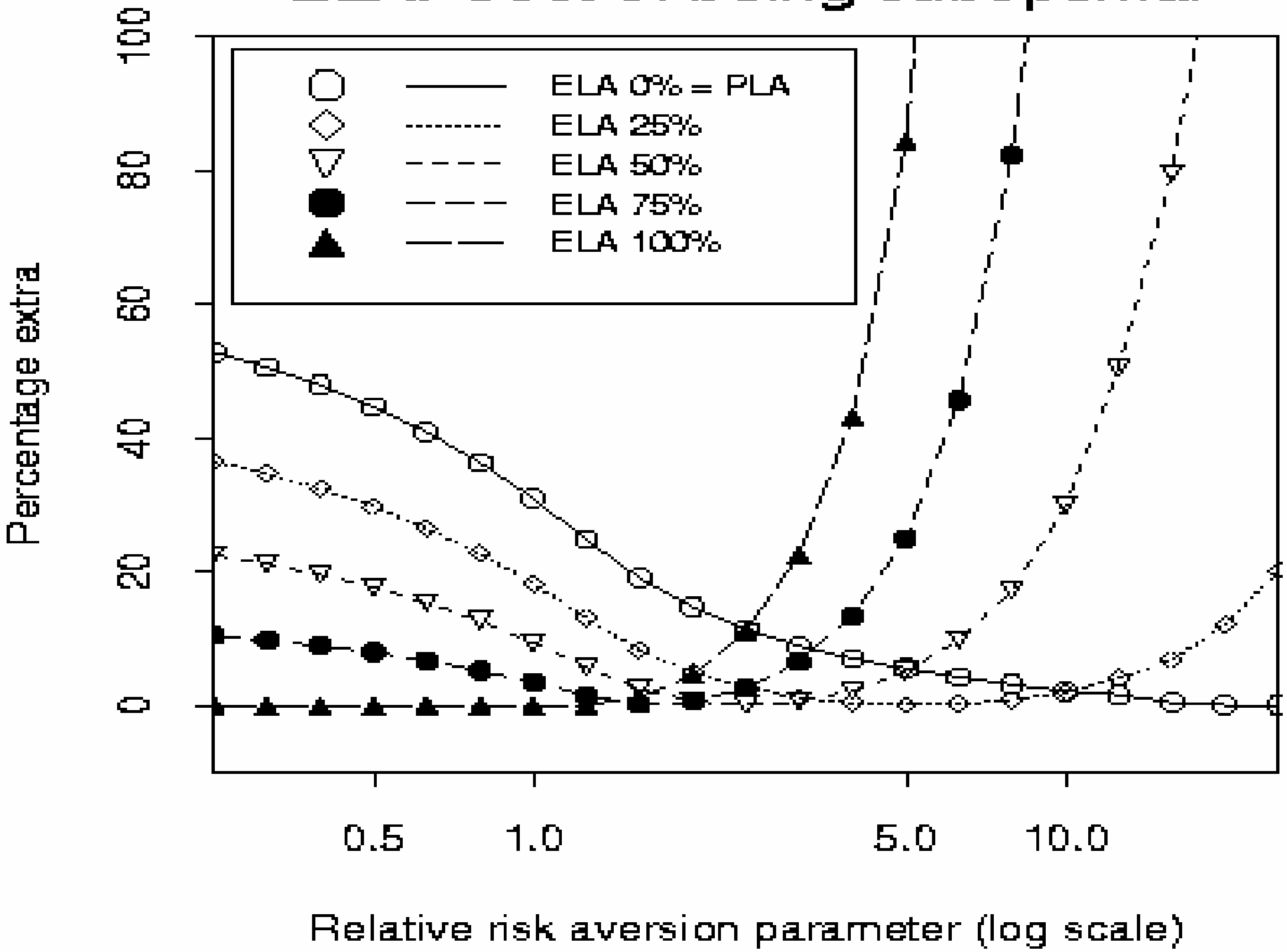
- For RRA = 4, how much cash do we need in order to match the optimal utility?
  - Eg, enforced PLA at 65 costs 7%

Programme	Equity %	Extra cash required (%)
PLA	0	7
ELA	0	7
ELA	**25**	**0**
ELA	50	2
ELA	75	13
ELA	100	43
ELID	0	18
ELID	25	10
ELID	50	12
ELID	75	27
ELID	100	65

Results - moderate bequest motive

- We can repeat this over a range of values for the RRA
- "20% extra" means:
  - For this plan member "How much extra cash do we require in order to match the expected discounted utility of the optimal programme?"

Figure 4.2.3(a)
ELA: Cost of being suboptimal



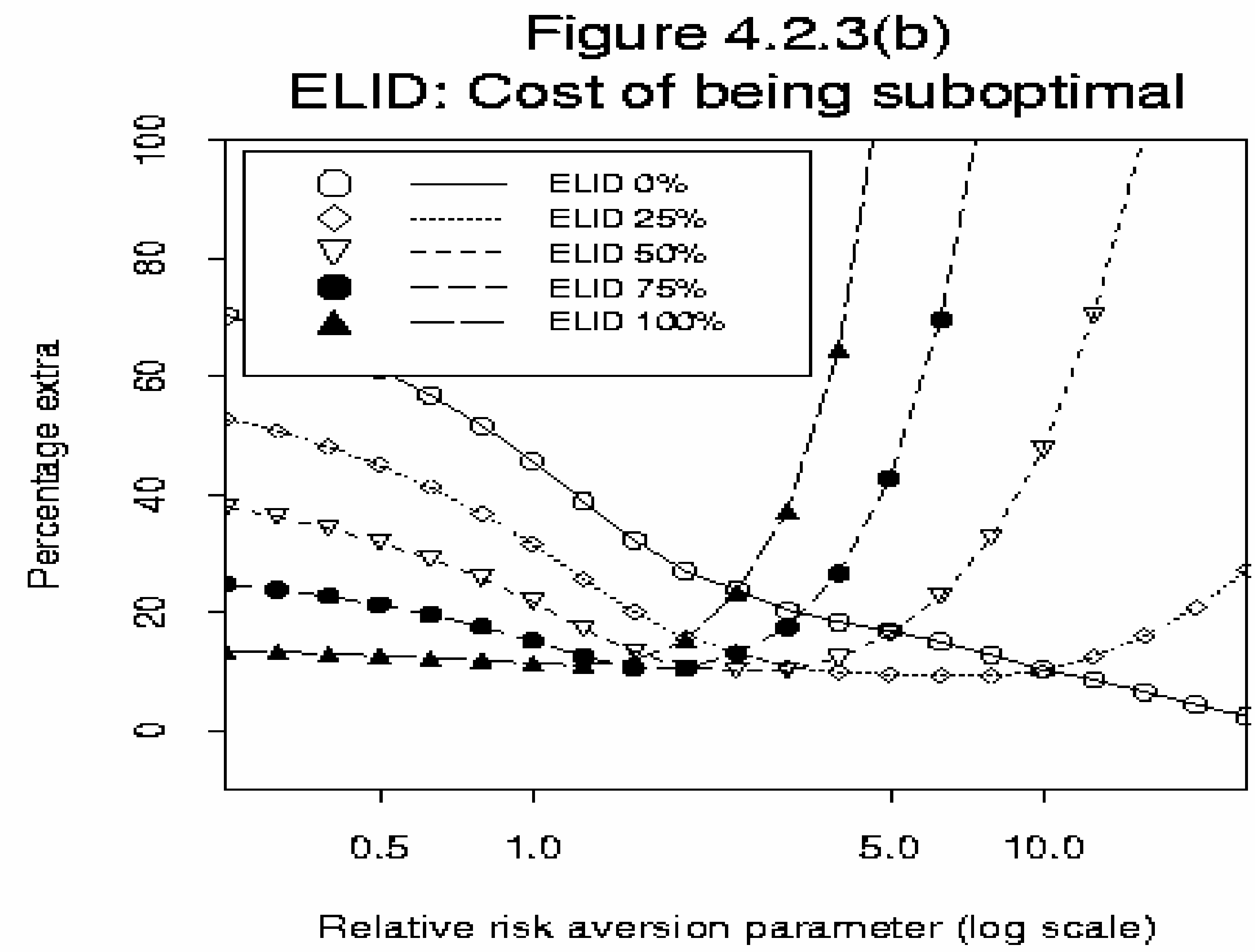
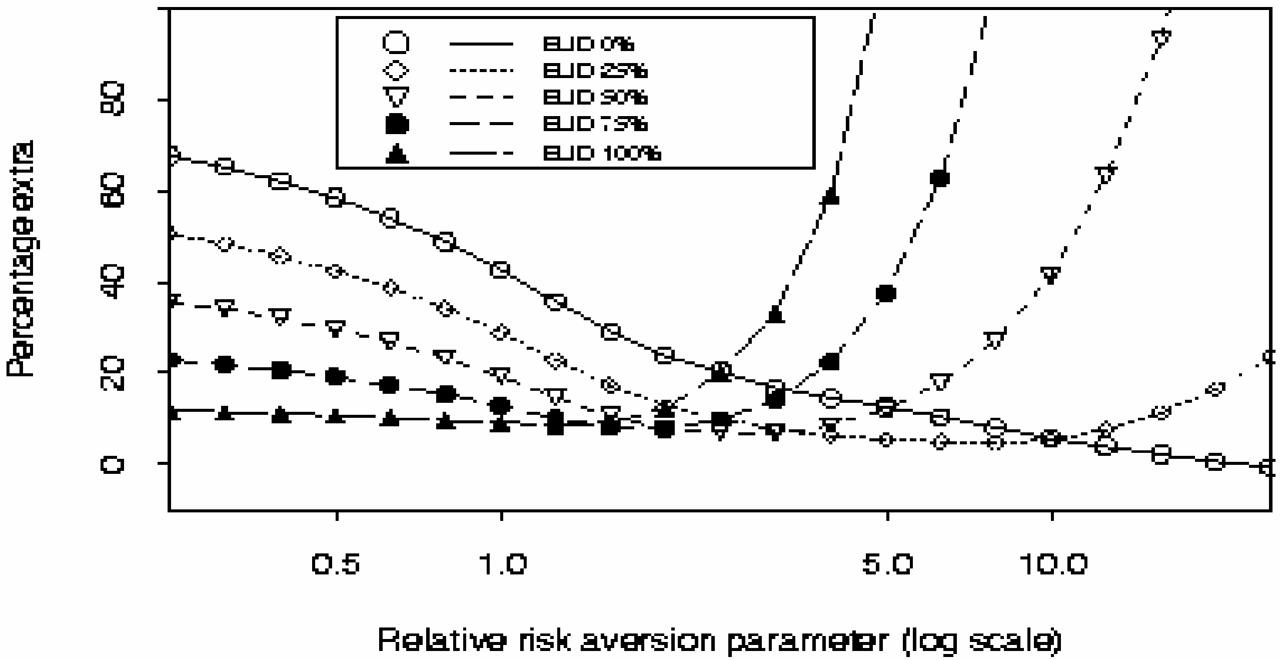


Figure 4.2.3(b)
ELID: Cost of being suboptimal
ELID 0%
ELID 25%
ELID 50%
ELID 75%
ELID 100%
Percentage extra
0
20
40
60
80
100
0.5
1.0
5.0
10.0
Relative risk aversion parameter (log scale)

Results - stronger bequest motive

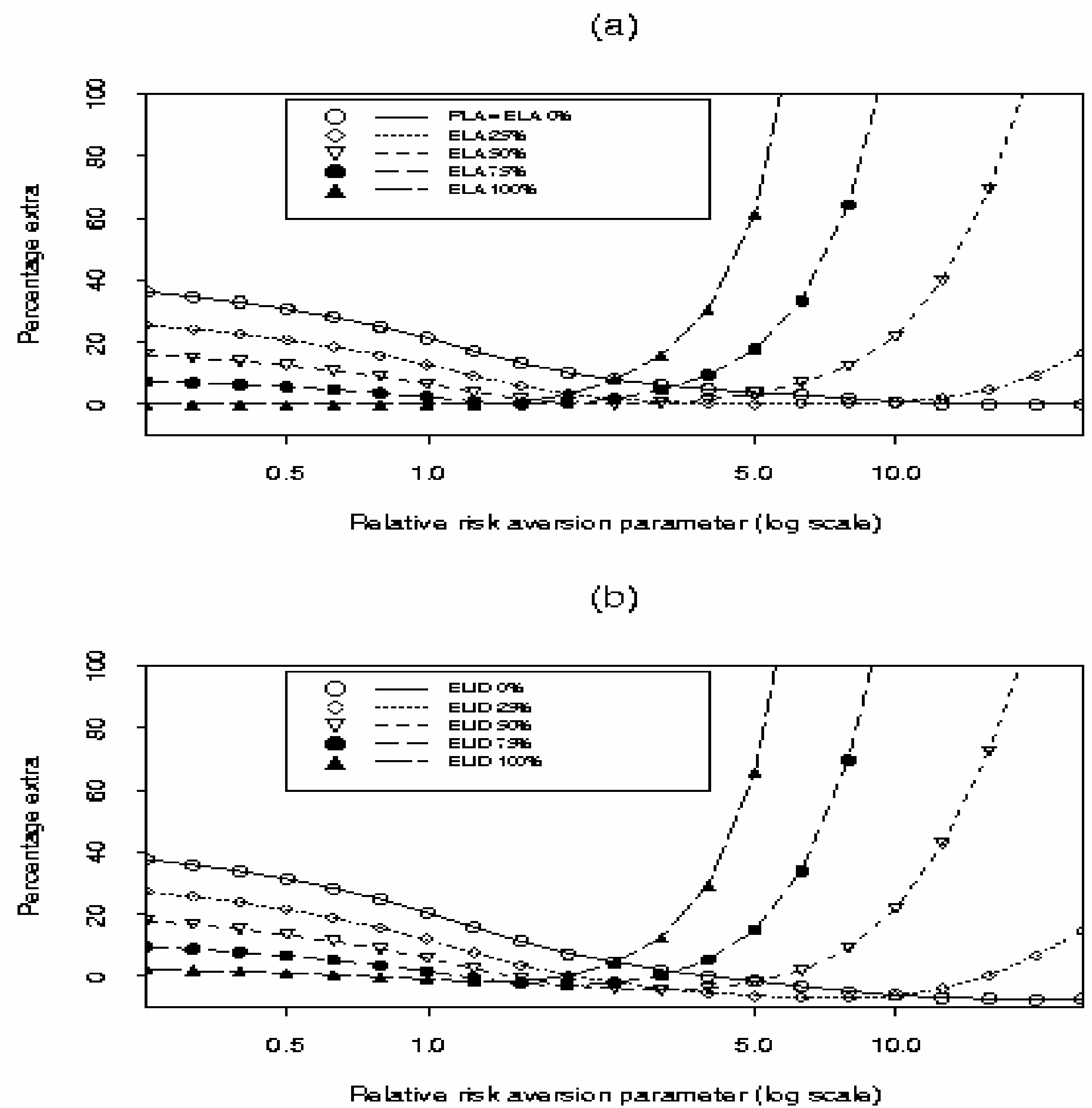
- For $k_2 = 10$, optimal policies still use survival credits rather than bequests

- Gap between ELA and ELID is reduced, BUT not by very much.


EUD 0%
EUD 25%
EUD 50%
EUD 75%
EUD 100%
Percentage extra
0
20
40
60
80
0.5
1.0
5.0
10.0
Relative risk aversion parameter (log scale)

Adverse mortality - Impaired lives

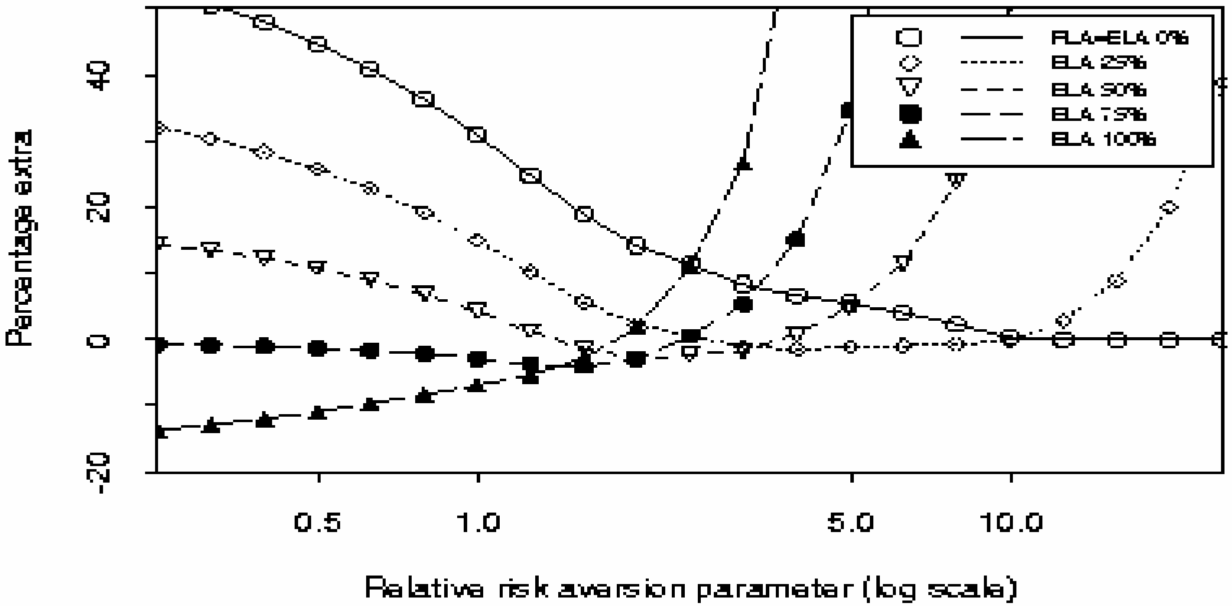
- Annuities/ survival credits:
  - POOR VALUE FOR MONEY
- For programmes considered and for standard parameterisation:
  - Depends upon bequest motive
  - ELA still likely to be optimal
  - payment of unfair survival credit is better than none at all
- Require VERY poor health to switch from ELA to ELID:
  - i.e. standard mortality rates x 4



(a)
PLA = ELA 0%
ELA 25%
ELA 50%
ELA 75%
ELA 100%
Percentage extra
0
20
40
60
80
100
0.5
1.0
5.0
10.0
Relative risk aversion parameter (log scale)
(b)
EUD 0%
EUD 25%
EUD 50%
EUD 75%
EUD 100%
Percentage extra
0
20
40
60
80
100
0.5
1.0
5.0
10.0
Relative risk aversion parameter (log scale)

Timing of annuitisation - cost of regulation

- For ELA:
  - Compare compulsory annuitisation at 85 with benchmark age of 75
- Members with low RRA benefit:
  - RRA=0 $\Rightarrow$ 15% less cash needed at 65 to achieve same expected utility at 65
  - $\Rightarrow$ optimal to annuitise at 85
  - $\Rightarrow$ Regulations CAN cost money
- With higher RRA the cost is smaller and in the limit zero for those with 0% equity weighting ($\equiv$ PLA)
  - $\Rightarrow$ optimal to annuitise at 75



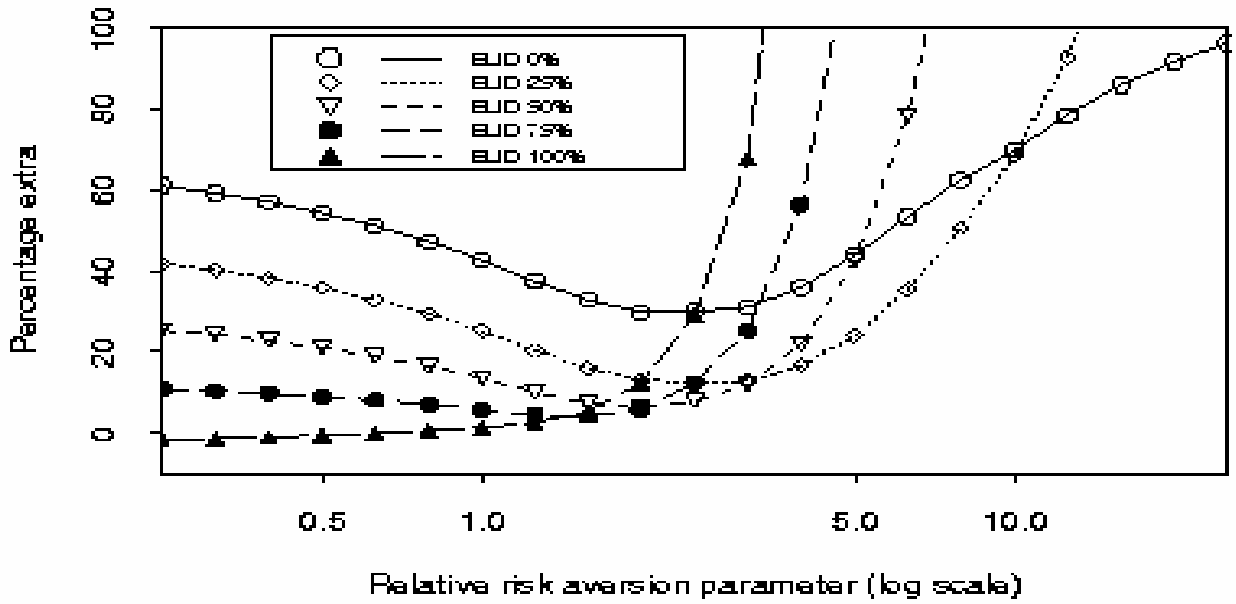
FLA=ELA 0%
ELA 25%
ELA 50%
ELA 75%
ELA 100%
Percentage extra
-20
0
20
40
0.5
1.0
5.0
10.0
Relative risk aversion parameter (log scale)

Timing of annuitisation

- Under ELID: when is it optimal to annuitise?
- Greater bequest motive with ELID
  - $\Rightarrow$ annuitise later than ELA
- If no bequest motive & RRA = 0
  - $\Rightarrow$ annuitise when equity risk premium = survival credit (cf. Milevsky, 1998)
- In general, annuitisation decision depends on:
  - plan member's risk aversion
  - asset mix
  - fund performance prior to annuitisation

Timing of annuitisation

RRA	Optimal equity mix	Optimal age to ann.	Cost of ann. by 75
0.25	100	80	2.4 (%)
0.31	100	80	2.1
0.40	100	80	1.9
0.50	100	80	1.5
0.63	100	79	1.2
0.79	100	78	0.7
1.00	100	77	0.3
1.25	100	76	0
1.58	75	74	0
1.99	75	72	0
2.50	50	70	0
3.15	50	68	0
3.96	25	67	0



ELID 0%
ELID 25%
ELID 50%
ELID 75%
ELID 100%
Percentage extra
0
20
40
60
80
100
0.5
1.0
5.0
10.0
Relative risk aversion parameter (log scale)

Dynamic stochastic optimisation

- Take into account fund performance when making annuitisation decision

- Each year decide to:
  - annuitise immediately, or
  - wait one more year, taking into account:
    - expected return on fund
    - probability of survival
    - possible bequest if die during year

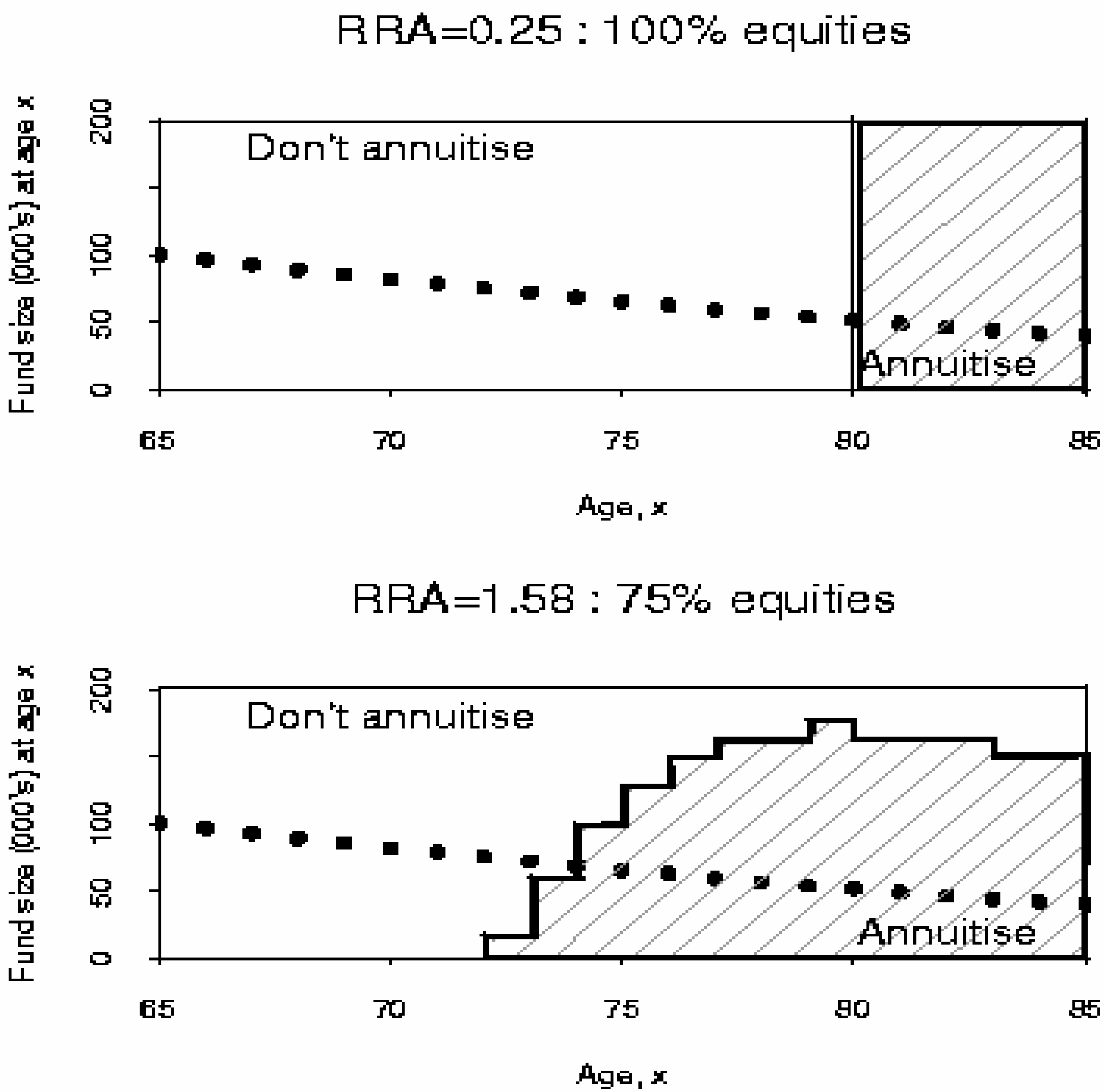
Dynamic stochastic optimisation

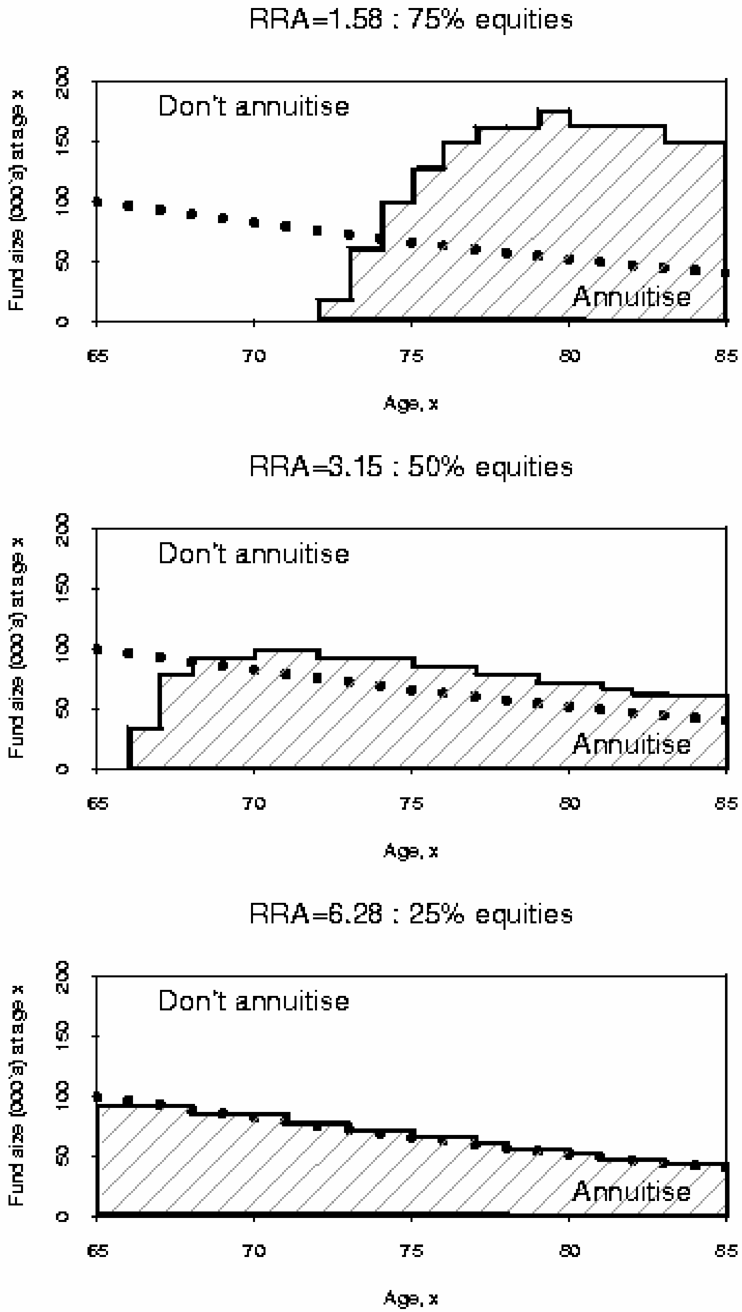
- More likely to delay annuitisation if investments are performing well

- However, more likely to annuitise if fund size is small because:
  - bequest value is small
  - begin to enjoy survival credits

Dynamic stochastic optimisation

- RRA = 0.25 : 75% equities:
  - $\Rightarrow$ annuitise at age 80
- RRA = 1.58 : 75% equities:
  - $\Rightarrow$ annuitise between 72 and 80+
  - depending on fund performance
- Dotted line shows value of fund with PLA


RRA=0.25 : 100% equities
Don't annuitise
Annuitise
Fund size (000's) at age x
0
50
100
200
65
70
75
80
85
Age, x
RRA=1.58 : 75% equities
Don't annuitise
Annuitise
Fund size (000's) at age x
0
50
100
200
65
70
75
80
85
Age, x


RRA=1.58 : 75% equities
Don't annuitise
Annuitise
Fund size (000's) at age x
Age, x
RRA=3.15 : 50% equities
Don't annuitise
Annuitise
Fund size (000's) at age x
Age, x
RRA=6.28 : 25% equities
Don't annuitise
Annuitise
Fund size (000's) at age x
Age, x

Conclusion

- Equity-linking can be very valuable option
- Equity % is MOST IMPORTANT factor
- Bequest motive important:
  - BUT bequest weight has to be high
- Results not very sensitive to health status
- Forced annuitisation at 75 can be costly for plan member
- Full flexibility $\Rightarrow$ timing of annuitisation depends on:
  - plan member's risk aversion
  - fund performance

Main findings-3 programmes

- Purchased life annuity (PLA) from 65:
  - payable until death
  - implicit survival credits
  - no bequest
- Equity-linked annuity (ELA) until 75 then PLA:
  - payment adjusted in light of equity performance
  - implicit survival credits
- Equity-linked income drawdown (ELID) until 75 then PLA:
  - no survival credits
  - residual fund bequested on death before 75

Main findings

- **Best programme**:
- equity-linked annuity
- does not involve bequest
- pays survival credits
- has 100% in equities for RRA < 1.25
- has 0% in equities for RRA > 10
- based on typical UK RRAs, only about 5% of plan members would choose a significant equity weighting

Main findings

- Weight attached to bequest (in plan member's utility function) would have to be very high for equity-linked income drawdown programme to be optimal:
  - suggests members value pension plan's ability to provide retirement income security for however long they live over and above possibility of being able to use fund to make bequests to their children

Main findings

- The equity proportion chosen for distribution programme has considerably more important effect on plan member's welfare than particular distribution programme chosen:
  - poor choice lowers expected discounted utility.
- However, limiting equity proportions to 0%, 25%, 50%, 75%, and 100% does not significantly reduce expected discounted utility:
  - justifies pension providers having simple product range

Main findings

- Plan members in poor health relative to average may still prefer ELA programme paying standard-rate survival credits to ELID programme paying bequests:
  - impaired mortality rates would need to be four times standard rates before ELID preferred.
- Those in extremely poor health and attaching some weight to a bequest are rather more likely to prefer ELID.

Main findings

- Forcing members of ELA programmes to annuitise at 75 rather than 85 can significantly lower expected discounted utility for those with low degrees of risk aversion:
  - equivalent to 15% of initial fund value for risk-neutral plan members.
- Forcing members of ELID programmes desiring to make bequest to annuitise at 75 rather than 85 is rather less costly:
  - at most 2% at low RRAs and nothing at all at RRAs above unity.

Main findings

- Optimal age to annuitise very sensitive to plan member's degree of risk aversion.
- Where no value attached to bequest, optimal age ranges from 79 for plan member with very low RRA to immediate annuitisation at 65 if RRA exceeds about 4.
- Switching rule based solely on comparison between the equity risk premium and mortality drag overestimates optimal switching age if member is risk averse.
- Value attached to bequest delays anuitisation

Main findings

- Annuitisation-timing decision depends upon fund size.
- The larger the fund size, the more likely it is that plan member will delay annuitisation.
- Arises from dynamic element in annuitisation decision that depends on size of fund as well as age.

Main findings

- Once plan member's degree of relative risk aversion (RRA) has been assessed, optimal choice of programme is not overly sensitive to the form of utility function (for example, power or exponential).

- Again helps to simplify product design.

Future work

- Allowing for stochastic risk-free rate of interest
- Allowing for mortality improvements:
  - investigate flexible unit-linked programmes where income received and survival credits payable fall in response to mortality improvements.
- Investigating utility gains (if any) from optimal solutions based on stochastic dynamic programming.