Applied Categorical & Nonnormal Data Analysis

Review of OLS Models

In OLS regression, we use linear combinations of predictor (independent) variables to compute expected values of the response (dependent) variable.

$$\hat{\mu} = \mathrm{E}(y|\mathbf{x}) = \sum \mathbf{x}_j \beta_j = \mathbf{x}\beta$$

These expected values are conditional on the independent variables. The full model for OLS includes both the structural or systematic component, Σxβ, and a random component, ε.

$$\mathbf{y} = \sum x_j \beta_j + \varepsilon_i = \mathbf{x}\beta + \varepsilon_i$$

The matrix formulation for OLS regression looks like this

1) $\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}$
2) $\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{e}$
3) $\hat{\mathbf{y}} = \mathbf{X}\mathbf{b}$
4) $\mathbf{e} = \mathbf{y} - \hat{\mathbf{y}}$
5) $\mathbf{ss}_{\mathrm{resid}} = \mathbf{e}'\mathbf{e} = \mathbf{y}'\mathbf{y} - \mathbf{b}'\mathbf{X}'\mathbf{y}$

Equation 1) gives the formula for obtaining the least squares regression coefficients. Equation 2) is the regression equation in matrix form, while equation 3) is used to obtain the conditional expected (predicted) values. The residuals, 4), are the difference between the observed values and the predicted values. The values of the coefficients are such that the sum of squared residuals, ss_{resid} in 5), are a minimum.

Stata Program using Matrix Arithmetic

```
program define matreg2, eclass
  version 6.0

    syntax varlist(min=2 numeric) [if] [in] [, Level(integer $S_level)]
    marksample touse                        /* mark cases in the sample */
    tokenize "`varlist'"

    quietly matrix accum sscp = `varlist' if `touse'
    local nobs = r(N)
    local df = `nobs' - (rowsof(sscp) - 1) /* df residual */

    matrix XX = sscp[2...,2...]             /* X'X */
    matrix Xy = sscp[1,2...]                /* X'y */

    matrix b = Xy * syminv(XX)              /* (X'X)-1X'y */
    local k = colsof(b)                     /* number of coefs */
    matrix hat = Xy * b'
    matrix V = syminv(XX) * (sscp[1,1] - hat[1,1])/`df'
```

```
    estimates post b V, dof(`df') obs(`nobs') depname(`1') /*
      */ esample(`touse')
    est local depvar "`1'"
    est local cmd "matreg"

    display
    estimates display, level(`level')

  matrix drop sscp XX Xy hat
end
```

Example using **matreg2**

```
use http://www.gseis.ucla.edu/courses/data/hsb2

regress write read female

matreg2 write read female
```

Assumptions in OLS Regression

Linearity - The expected value of y is linearly related to the x's through the β parameters. Specification errors result when there is a nonlinear relationship.

Independence - The independence of the x's and ε is necessary in order to identify the unknown β parameters, that is, in order to be able to solve for the β's

ε are i.i.d. - The assumption is that the ε's are independent and identically distributed which implies there should be no heterogeneity of variance and no autocorrelation among the residuals.

All relevant variables are in the model - A specification error can occur when the model does not contain all of the relevant variables. As a corollary, a specification error can occur when irrelevant variables are included in the model.

x's are measured without error - The independent variables are measured without error.

Normality* - If we wish to draw statistical inferences we need to add the further assumption that the ε are normally distributed.

Example

```
use http://www.gseis.ucla.edu/courses/data/hsb2

describe

Contains data from http://www.gseis.ucla.edu/courses/data/hsb2.dta
  obs:           200                          highschool and beyond
(200
                                                cases)
 vars:            11                          21 Jun 2000 08:54
 size:         9,600 (99.8% of memory free)
-------------------------------------------------------------------
----------
```

```
   1. id        float  %9.0g
   2. female    float  %9.0g       fl
   3. race      float  %12.0g      rl
   4. ses       float  %9.0g       sl
   5. schtyp    float  %9.0g       scl        type of school
   6. prog      float  %9.0g       sel        type of program
   7. read      float  %9.0g                  reading score
   8. write     float  %9.0g                  writing score
   9. math      float  %9.0g                  math score
  10. science   float  %9.0g                  science score
  11. socst     float  %9.0g                  social studies score
------------------------------------------------------------------------------
----------
```

summarize

```
Variable |     Obs        Mean    Std. Dev.       Min        Max
---------+--------------------------------------------------------
      id |     200       100.5    57.87918          1        200
  female |     200        .545    .4992205          0          1
    race |     200        3.43    1.039472          1          4
     ses |     200       2.055    .7242914          1          3
  schtyp |     200        1.16     .367526          1          2
    prog |     200       2.025    .6904772          1          3
    read |     200       52.23    10.25294         28         76
   write |     200      52.775    9.478586         31         67
    math |     200      52.645    9.368448         33         75
 science |     200       51.85    9.900891         26         74
   socst |     200      52.405    10.73579         26         71
```

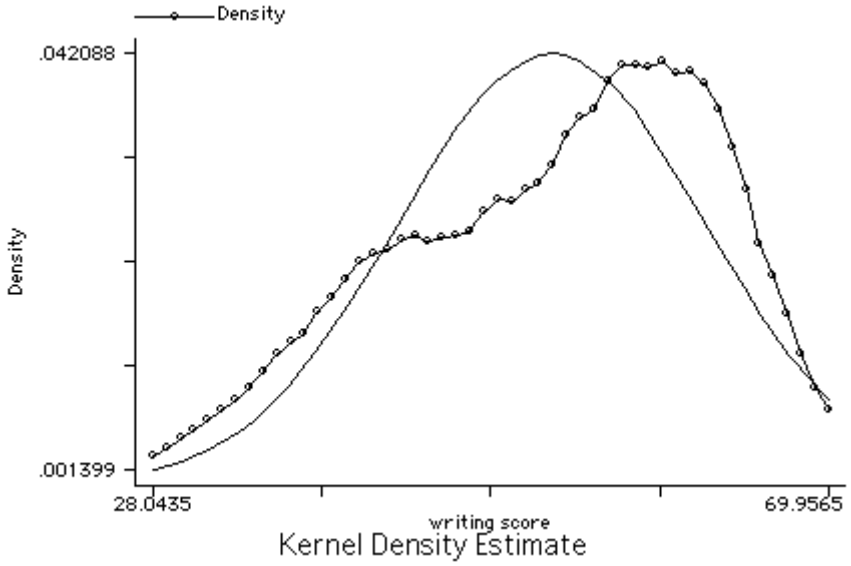
corr write read math science socst female

```
(obs=200)

             |    write     read     math  science    socst   female
-------------+------------------------------------------------------
       write |   1.0000
        read |   0.5968   1.0000
        math |   0.6174   0.6623   1.0000
     science |   0.5704   0.6302   0.6307   1.0000
       socst |   0.6048   0.6215   0.5445   0.4651   1.0000
      female |   0.2565  -0.0531  -0.0293  -0.1277   0.0524   1.0000
```

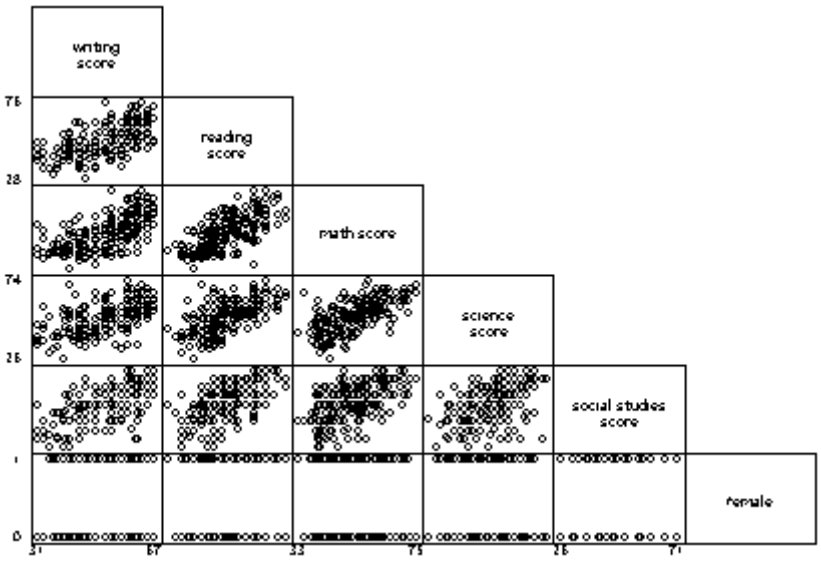
pcorr write read math science socst female

```
(obs=200)

Partial correlation of write with

    Variable |    Corr.     Sig.
-------------+------------------
        read |   0.1373    0.055
        math |   0.2468    0.000
     science |   0.2751    0.000
       socst |   0.2974    0.000
      female |   0.4107    0.000
```

kdensity write, normal



graph read math science socst female write, matrix half



tab1 female prog

```
-> tabulation of female

     female |      Freq.     Percent        Cum.
------------+-----------------------------------
       male |         91       45.50       45.50
     female |        109       54.50      100.00
------------+-----------------------------------
      Total |        200      100.00

-> tabulation of prog

    type of |
    program |      Freq.     Percent        Cum.
------------+-----------------------------------
    general |         45       22.50       22.50
   academic |        105       52.50       75.00
   vocation |         50       25.00      100.00
```

```
------------+-----------------------------------
      Total |        200      100.00
```

xi: regress write read math female i.prog

```
i.prog                Iprog_1-3    (naturally coded; Iprog_1 omitted)

  Source |       SS       df       MS                  Number of obs
=     200
---------+------------------------------               F(  5,   194)
=   45.01
   Model |  9602.28627     5  1920.45725               Prob > F
=  0.0000
Residual |  8276.58873   194  42.6628285               R-squared
=  0.5371
---------+------------------------------               Adj R-squared
=  0.5251
   Total |   17878.875   199   89.843593               Root MSE
=  6.5317

------------------------------------------------------------------------------
---------
   write |      Coef.   Std. Err.       t     P>|t|       [95% Conf.
Interval]
---------+--------------------------------------------------------------------
---------
    read |   .3069424   .0611262      5.021   0.000       .1863852
.4274996
    math |   .3603705   .0690064      5.222   0.000       .2242715
.4964695
  female |   5.384982    .929572      5.793   0.000       3.551617
7.218346
 Iprog_2 |    .436372   1.230379      0.355   0.723      -1.990265
2.863009
 Iprog_3 |  -2.219748   1.359353     -1.633   0.104      -4.900756
.4612602
   _cons |   15.16272   3.225088      4.701   0.000       8.801985
21.52346
------------------------------------------------------------------------------
---------
```

test Iprog_2 Iprog_3

```
 ( 1)  Iprog_2 = 0.0
 ( 2)  Iprog_3 = 0.0

       F(  2,   194) =    2.31
            Prob > F =    0.1022
regress write read math female

  Source |       SS       df       MS                  Number of obs
=     200
---------+------------------------------               F(  3,   196)
=   72.52
   Model |  9405.34864     3  3135.11621               Prob > F
=  0.0000
Residual |  8473.52636   196  43.2322773               R-squared
=  0.5261
---------+------------------------------               Adj R-squared
=  0.5188
```

```
   Total |   17878.875   199   89.843593               Root MSE
=  6.5751

------------------------------------------------------------------------
---------
   write |      Coef.   Std. Err.       t     P>|t|       [95% Conf.
Interval]
---------+--------------------------------------------------------------
---------
    read |   .3252389   .0607348      5.355   0.000       .2054613
.4450166
    math |   .3974826   .0664037      5.986   0.000        .266525
.5284401
  female |    5.44337   .9349987      5.822   0.000        3.59942
7.287319
   _cons |   11.89566   2.862845      4.155   0.000       6.249728
17.5416
------------------------------------------------------------------------
---------
```

listcoef /* from Long & Freese - findit spostado */

```
regress (N=200): Unstandardized and Standardized Estimates

 Observed SD: 9.478586
 SD of Error: 6.5751257

------------------------------------------------------------------------
------
   write |      b         t     P>|t|    bStdX    bStdY   bStdXY
SDofX
---------+--------------------------------------------------------------
------
    read |   0.32524    5.355   0.000   3.3347   0.0343   0.3518
10.2529
    math |   0.39748    5.986   0.000   3.7238   0.0419   0.3929
9.3684
  female |   5.44337    5.822   0.000   2.7174   0.5743   0.2867
0.4992
------------------------------------------------------------------------
------
```

linktest

```
  Source |       SS       df       MS                  Number of obs
=     200
---------+------------------------------               F(  2,   197)
=  116.16
   Model |  9674.70222     2  4837.35111               Prob > F
=  0.0000
Residual |  8204.17278   197  41.6455471               R-squared
=  0.5411
---------+------------------------------               Adj R-squared
=  0.5365
   Total |   17878.875   199   89.843593               Root MSE
=  6.4533

------------------------------------------------------------------------
---------
   write |      Coef.   Std. Err.       t     P>|t|       [95% Conf.
Interval]
```

```
---------+------------------------------------------------------------
---------
    _hat |   3.306865   .9095168      3.636   0.000       1.513226
5.100504
  _hatsq |  -.0215942    .008491     -2.543   0.012      -.0383392
-.0048492
   _cons |  -60.58511   24.08436     -2.516   0.013      -108.0814
-13.08885
------------------------------------------------------------------------
---------
```

ovtest

```
Ramsey RESET test using powers of the fitted values of write
       Ho:  model has no omitted variables
                 F(3, 193) =      3.06
                  Prob > F =      0.0295
```

whitetst /* downloaded via the Internet - findit whitetst */

```
White's general test statistic :  15.17126  Chi-sq( 8)  P-value =
.0559
```

regress write read math female science socst

```
  Source |       SS       df       MS                  Number of obs
=     200
---------+------------------------------               F(  5,   194)
=   58.60
   Model |  10756.9244     5  2151.38488               Prob > F
=  0.0000
Residual |   7121.9506   194  36.7110855               R-squared
=  0.6017
---------+------------------------------               Adj R-squared
=  0.5914
   Total |   17878.875   199   89.843593               Root MSE
=   6.059

------------------------------------------------------------------------
---------
   write |      Coef.   Std. Err.       t     P>|t|       [95% Conf.
Interval]
---------+--------------------------------------------------------------
---------
    read |   .1254123   .0649598      1.931   0.055      -.0027059
.2535304
    math |   .2380748   .0671266      3.547   0.000       .1056832
.3704665
  female |   5.492502   .8754227      6.274   0.000       3.765935
7.21907
 science |   .2419382   .0606997      3.986   0.000       .1222221
.3616542
   socst |   .2292644   .0528361      4.339   0.000       .1250575
.3334713
   _cons |   6.138759   2.808423      2.186   0.030        .599798
11.67772
------------------------------------------------------------------------
---------
```

linktest

```
  Source |       SS       df       MS                  Number of obs
=     200
---------+------------------------------               F(  2,   197)
=  155.20
   Model |  10937.2369     2  5468.61843               Prob > F
=  0.0000
Residual |  6941.63813   197  35.2367418               R-squared
=  0.6117
---------+------------------------------               Adj R-squared
=  0.6078
   Total |   17878.875   199   89.843593               Root MSE
=  5.9361

------------------------------------------------------------------------
---------
   write |      Coef.   Std. Err.       t     P>|t|       [95% Conf.
Interval]
---------+--------------------------------------------------------------
---------
    _hat |   2.577803   .6998344      3.683   0.000       1.197674
3.957931
  _hatsq |  -.0150213   .0066404     -2.262   0.025      -.0281166
-.0019259
   _cons |  -40.62334   18.21521     -2.230   0.027      -76.54518
-4.701504
------------------------------------------------------------------------
---------
```

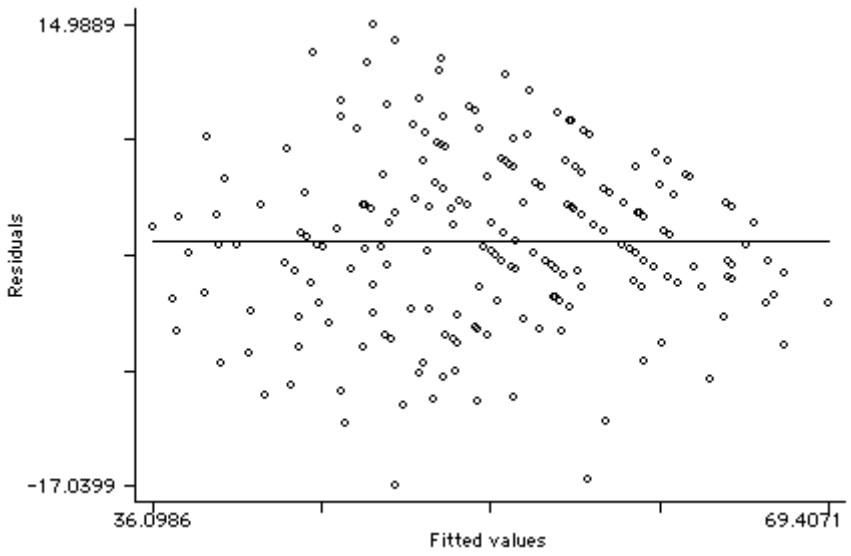
ovtest

```
Ramsey RESET test using powers of the fitted values of write
       Ho:  model has no omitted variables
                 F(3, 191) =      2.03
                  Prob > F =      0.1117
```

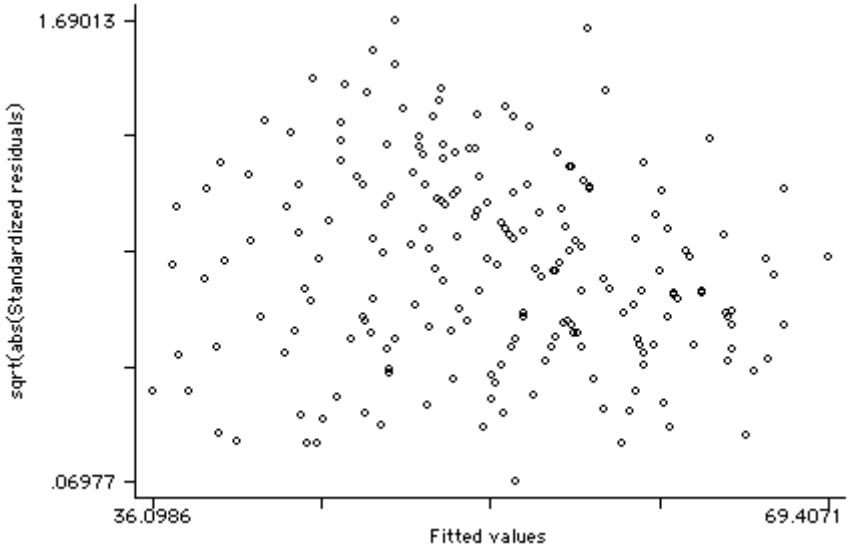
whitetst /* downloaded via the Internet - findit whitetst */

```
White's general test statistic :  23.69338  Chi-sq(19)  P-value =
.2082
```

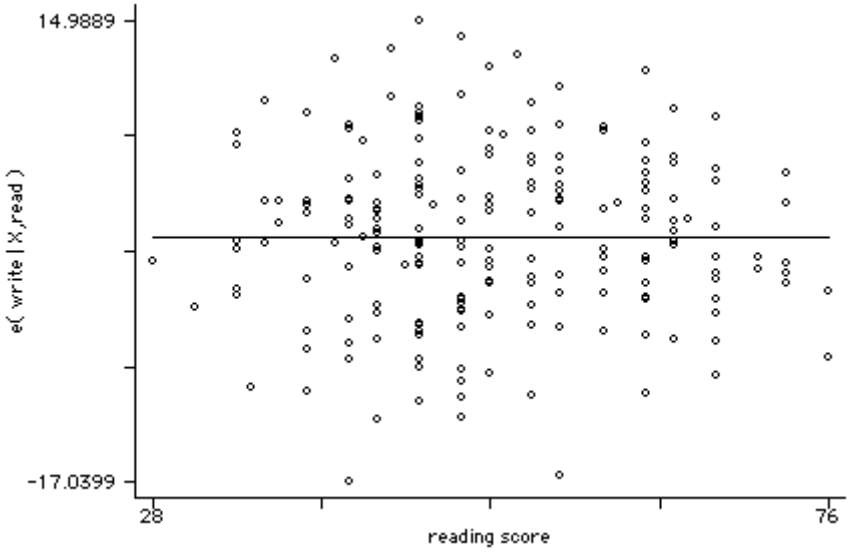
rvfplot, yline(0)



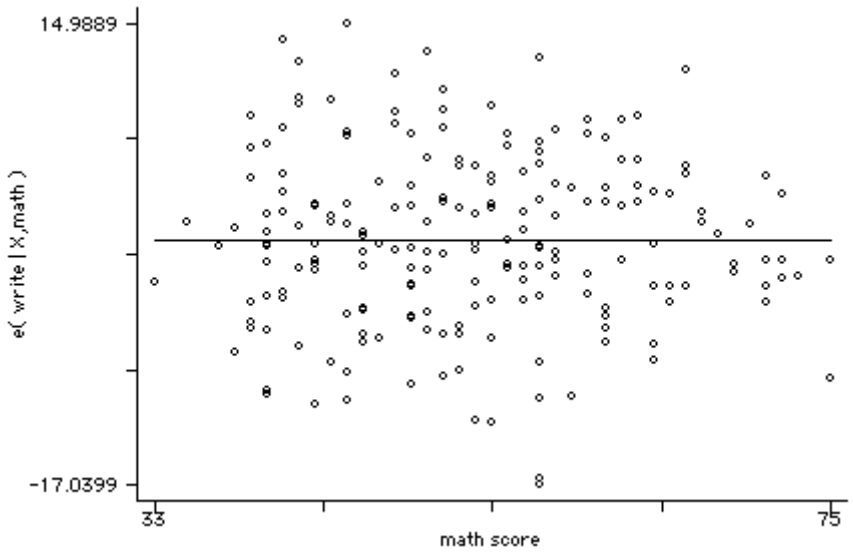
rvfplot2, rsta scale(sqrt(abs(X))) /* downloaded via the Internet - findit rvfplot2 */



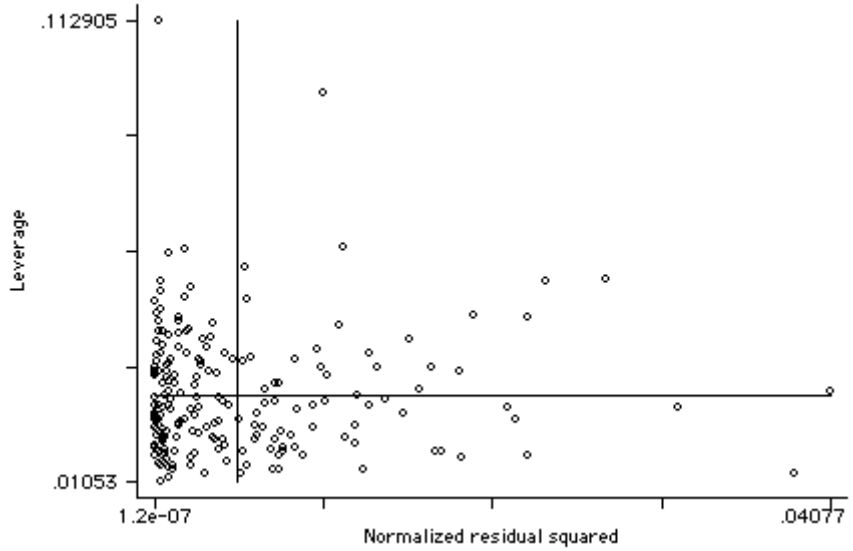
rvpplot read, yline(0)



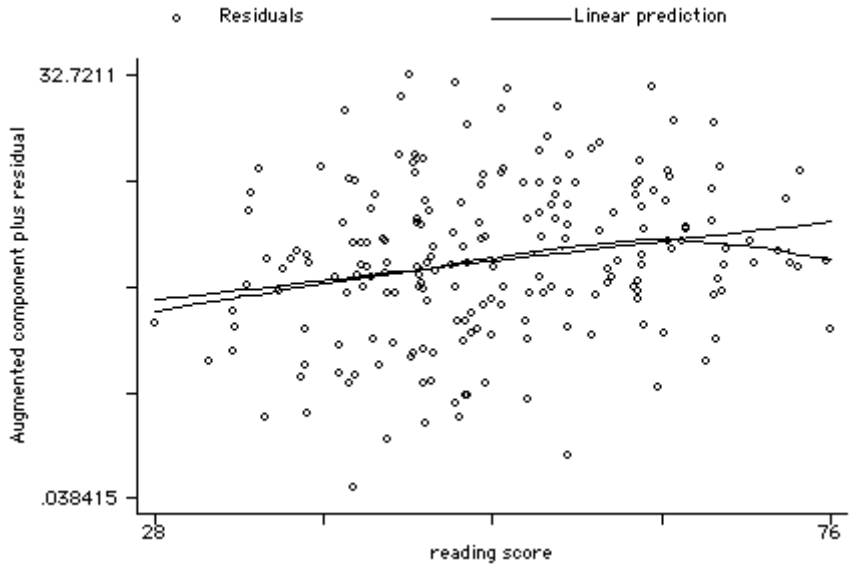
rvpplot math, yline(0)



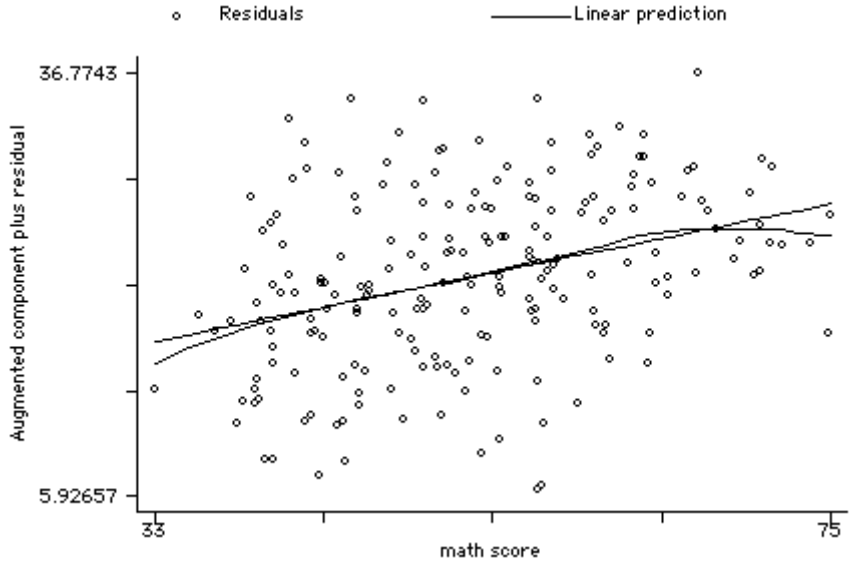
lvr2plot



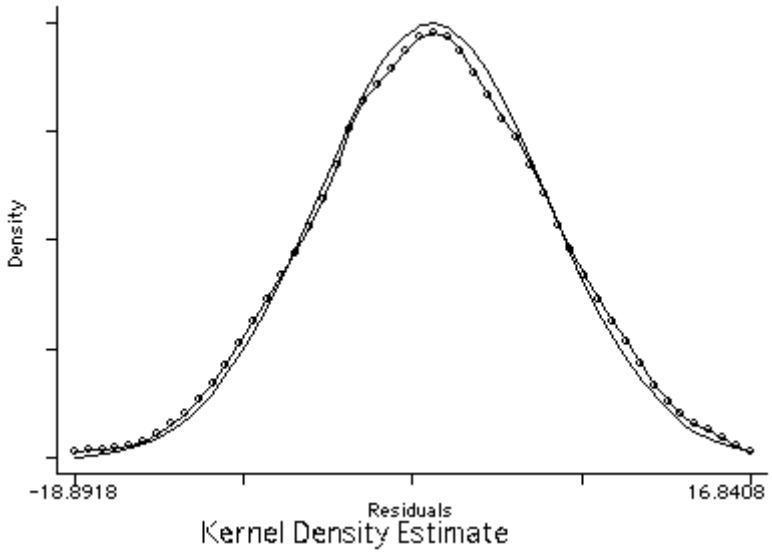
acprplot read, c(k) jit(2)




```
acprplot math, c(k) jit(2)
```




```
predict e, resid
predict rstu, rstu
predict h, hat
predict d, cooksd
dfbeta

kdensity e, normal
```



list id rstu h d if abs(rstu)>2

	id	rstu	h	d
6.	126	-2.911042	.0306996	.0430727
21.	86	-2.159355	.0471286	.0377245
42.	187	-2.801592	.012483	.0159722
71.	52	-2.102624	.0243733	.0180889
119.	38	2.225437	.0551778	.0472426
127.	104	2.12688	.0164202	.0123619
137.	30	2.077298	.0271825	.0197581
156.	44	2.543062	.0270288	.0291219
160.	83	2.393317	.0556316	.0549

list id rstu h d if h>(2*5+2)/200

	id	rstu	h	d
38.	167	.2202322	.1129052	.0010339
74.	198	1.536736	.0626366	.0261174
140.	170	.615899	.0624444	.0042243
174.	165	.4212941	.0611342	.0019344
190.	150	-1.481232	.096819	.0389597

list id rstu h d if d>4/200

	id	rstu	h	d
6.	126	-2.911042	.0306996	.0430727
21.	86	-2.159355	.0471286	.0377245
48.	24	1.995236	.0473758	.0324974
57.	3	1.937731	.0350953	.0224428
74.	198	1.536736	.0626366	.0261174
119.	38	2.225437	.0551778	.0472426
144.	81	-1.843365	.0358387	.020794
156.	44	2.543062	.0270288	.0291219
160.	83	2.393317	.0556316	.0549
190.	150	-1.481232	.096819	.0389597
196.	89	-1.771696	.0422013	.022799

global a = 2/sqrt(200)

list id DFread DFmath DFfemale DFscienc DFsocst if abs(DFread)>$a | abs(DFmat
> h)>$a | abs(DFfemale)>$a | abs(DFscienc)>$a | abs(DFsocst)>$a

	id	DFread	DFmath	DFfemale	DFscienc	DFsocst
3.	51	-.0515056	-.014398	-.0626913	.1468613	.0604603
6.	126	.3206731	-.3131511	.2656341	.1226514	-.0953587
9.	175	-.1430553	-.0606901	.1151235	.1475843	-.0274637
21.	86	.0980499	-.1335611	.1154265	-.1851264	.3323804
24.	62	.1088972	-.2401099	-.1083746	.1068583	.1222108
33.	50	-.0349264	-.2120363	-.1395483	.0761597	.1938863
42.	187	-.0829706	-.0537039	-.1999146	-.0223069	.1177308
48.	24	-.0347862	.3662628	-.1715901	-.2201185	-.1457788
55.	60	-.0166963	-.1514111	-.1222548	.1529634	.1088403
57.	3	.1622479	-.2598199	-.1198602	.1595348	.0001202
71.	52	.1520838	.0367512	-.1116373	-.0425734	-.2523727
74.	198	-.0183564	-.0090244	.1612648	.2468743	-.2791837
76.	186	.0420188	.0984112	.0757014	-.0094653	-.1424532
81.	103	-.2083498	.0167522	.0938213	.0255195	.0635244
102.	189	-.1159227	.2095263	-.1127493	-.0281062	-.0706801
109.	41	.0333516	.1437377	.1196449	-.0881504	-.1007374
110.	185	.1353671	-.012621	-.0607331	.0121166	-.1576565
113.	46	.081996	.0152051	.0863532	-.173799	-.0803784
119.	38	-.0400404	.1510486	-.2598652	-.4282386	.2007007
127.	104	.0250837	.0949613	-.147284	.0018263	-.1392599
134.	159	.0034894	.0056749	-.1422074	-.0914222	.1120808
137.	30	-.0398192	-.033048	.0875061	-.1889646	.1237977
139.	200	.0064968	-.2036626	.1237578	.0268331	-.0128401
144.	81	-.1431379	-.0059714	.0933692	-.1201475	.2423838
154.	133	-.1600672	.0340288	.0858443	.1343556	.1441613
155.	98	.1430875	-.0204046	.1143515	.0250712	-.2119574
156.	44	.1320848	.0129432	.1237565	-.3070052	-.0404682
160.	83	.2011815	-.2430923	.2346933	.2569073	-.3997001
162.	18	-.1112641	-.0427497	.1235689	.1017029	.191583
166.	117	-.143197	-.0619351	-.10833	.0033026	.1643665
169.	153	.0994511	.0801062	.1757641	.0828039	-.1559673
186.	16	-.110932	-.0121689	.1209114	.1604108	.1236713
190.	150	.1878171	-.0798048	.0538377	-.3412711	.2439169
192.	142	.0095487	-.086411	-.0723012	.1508893	-.0155141
196.	89	.1260335	.0847179	-.1482973	-.2127656	.1433555

diag /* downloaded via the Internet - findit diag */

Summary statistics for Leverage/Residuals (Panel 1) and dfbetas (Panel 2)
Signals lists the obs that warrant attention (criteria: see online help)

Variable	Obs	Mean	Std. Dev.	Min	Max	%Signals
_hat	200	.03	.0137395	.0105296	.1129052	0.0250
_rstu	200	5.13e-06	1.00769	-2.911042	2.543062	0.0800
_dfits	200	.0013422	.1790669	-.5180668	.5808853	0.0350
_cooksd	200	.0052647	.0085635	9.85e-08	.0549	0.0550
_welsch	200	.0194215	2.574987	-7.423061	8.432303	0.0100
_covrati	200	1.031876	.0446067	.8223104	1.161025	0.0250

```
---------+--------------------------------------------------------
---------
read     |     200    .0001225    .0663172   -.2083498    .3206731
0.0550
math     |     200   -.0001245    .0732815   -.3131511    .3662628
0.0550
female   |     200    .0000807    .0748938   -.2598652    .2656341
0.0500
science  |     200    -.000107    .0791052   -.4282386    .2569073
0.0800
socst    |     200    .0000599    .0793047   -.3997001    .3323804
0.0850
---------+--------------------------------------------------------
---------
```

Frequency distribution of #signals

_Signals	Freq.	Percent	Cum.
0	159	79.50	79.50
1	17	8.50	88.00
2	9	4.50	92.50
3	4	2.00	94.50
4	3	1.50	96.00
5	2	1.00	97.00
6	3	1.50	98.50
7	1	0.50	99.00
8	1	0.50	99.50
9	1	0.50	100.00
Total	200	100.00	

Observations with #signals >= 1

```
-------------------------------------------------------------------------
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_Signals  #signals for the observation
_diag     signals for _HAT _RSTU _DFITS _COOKSD _WELSCH _COVRATIO
_dfbeta   signals for dbetas of read math female science socst
```

	_Signals	_diag	_dfbeta
160.	1	000000	01000
161.	1	000000	00010
162.	1	010000	00000
163.	1	000000	00100
164.	1	010000	00000
165.	1	000000	00010
166.	1	000000	01000
167.	1	000000	01000
168.	1	000000	01000
169.	1	000000	00001
170.	1	100000	00000
171.	1	000000	00001
172.	1	000000	00001
173.	1	000000	00010
174.	1	000000	00010
175.	1	000000	10000
176.	1	010000	00000
177.	2	000000	10001
178.	2	000000	10010
179.	2	010000	00100
180.	2	100001	00000

181.	2	000000	01001
182.	2	100001	00000
183.	2	000000	10001
184.	2	000000	10001
185.	2	000000	01010
186.	3	011000	00010
187.	3	010000	10001
188.	3	010001	00100
189.	3	010000	00101
190.	4	000100	00111
191.	4	010100	00011
192.	4	010100	10001
193.	5	100100	10011
194.	5	011101	00010
195.	6	101100	00111
196.	6	011100	11010
197.	6	010101	11100
198.	7	011100	01111
199.	8	011110	01111
200.	9	011110	11111