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Capitalism's growth imperative

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A capitalist firm operating in a competitive market is subject to a growth imperative, because uncertainty about the profit rate under a no-growth policy makes the firm's prospects highly unattractive in finite time and bankruptcy practically certain in the long run. A no-growth policy determines consumption and investment so that they and capital would remain constant over time if the latter's expected return were realised with certainty. Simulation is used to arrive at the probability of bankruptcy by the end of *t* periods and the expected values of capital and money, for relevant combinations of time and uncertainty under successively more realistic models of a no-growth firm in a competitive market. The sensitivity of the results to variation in the parameters in each of the models is evaluated. Finally, we establish that a plausible growth policy may achieve growth, but the problem of bankruptcy is not resolved.

Key words: Growth, Bankruptcy, Capitalist system, Investment
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Introduction

The primary purpose of this paper is to establish that a capitalist enterprise operating in a competitive environment, be it a proprietorship or a corporation, is subject to a growth imperative. By a growth imperative, we mean that the enterprise requires the expectation of a positive growth rate, probably one that is well above the physically feasible rate of growth for an actual closed competitive capitalist system. A positive and probably high mean rate of growth is necessitated by the facts that the actual profit rate is uncertain, and its realisation varies over a very wide range. This high variability makes an enterprise with a zero or negative mean expected value for its growth rate face a future in which bankruptcy is practically certain in the long run, and has an intolerably high probability in the short run, while providing little or no compensating benefits by way of growth in income or wealth until bankruptcy takes place. In fact, income and wealth can be expected to fall over time. By a competitive capitalist enterprise, we mean one in which the enterprise buys and sells in markets in which the government or other regulatory authority plays no role in determining the prices and quantities for what it buys and sells.

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The reliance on competitive markets for specialisation and exchange and for the determination of each capitalist's profit is the source of the high variability in the profit rate and the necessity of a positive and probably high mean expected rate of growth.

It is widely if not universally accepted that growth is desirable. Keynes gave rise to a macroeconomic theory under which investment is not only desirable for growth in output in the long run, but necessary to avoid stagnation and unemployment in the short run. We make the stronger claim here, that growth is necessary for tolerable prospects for future survival for each capitalist and perhaps for the system as a whole. Some business leaders make the same claim, but they argue that innovations in technology, marketing, etc. by their competitors force them to participate in the quest for competitive advantage. To our knowledge, no theory of a capitalist firm or individual has been advanced under which a policy intended to maintain it at a stationary level is certain or even highly likely to lead to its collapse in the long run and offer highly unattractive prospects in the short run.

Neoclassical theory, mainstream economic theory, offers a radically different view of a capitalist firm. Nothing is easier for it to do than remain in a stationary state, and for a real person, growth is a matter of taste. It is no more than preference between present and future consumption.

We say it is capitalism that is subject to a growth imperative, because the same is not true of a pure socialist system. Regardless of what other problems are faced by a socialist system, its own operation does not subject it to a growth imperative. It will be seen that, notwithstanding a common technology of production, the difference between how the firm is administered in the two systems gives rise to a growth imperative in one and its absence in the other. The propositions that a pure socialist system offers security and stagnation, while a pure (competitive) capitalist system offers insecurity and growth have been recognised, but they have not been investigated rigorously. What we shall do here is explain how a capitalist system generates insecurity and growth, and then consider what capitalists do to deal with the insecurity.

The next section presents a simple model of a capitalist firm and establishes what happens to its probability of survival and its wealth over time under a no-growth policy, both conditional on its survival and without that condition. We find that it is certain to go bankrupt in the long run, and simulation of the model reveals that the firm does poorly in finite time *regardless* of its profitability and other relevant variables. Section 2 reviews the treatment of growth, uncertainty and bankruptcy in the economics literature. Prior to Keynes, the problem of uncertainty about the future was resolved by assuming that the uncertain future value of a variable could be represented by its mean. Keynes's dissatisfaction with this solution to the investment problem motivated a post-World War II literature on investment under uncertainty and risk aversion. The mainstream theoretical literature established that the utopian properties of a perfectly competitive capitalist system are preserved in the face of uncertainty and risk aversion. This was accomplished and the feasibility of a stationary state was maintained, we shall see, by ignoring or trivialising the problem of bankruptcy.

Section 3 introduces a number of refinements in the model that make it more realistic. Bankruptcy in the long run is not shown to be certain in every case, but its probability is very high for combinations of uncertainty and time that are large. Finally, Section 4 establishes that investment and expenditure policies to achieve growth realise that objective, but they do not increase the probability of survival. Far more is needed.

1. A simple model of capital

This section will present a model of a capitalist, i.e., a proprietor or a corporation, that is simple, powerful, and, we believe, useful for understanding their behaviour on the micro level and the system's macro behaviour. The model incorporates physical and nominal capital, uncertainty about the profit rate on capital, consumption, investment in both forms of capital, and bankruptcy, all in a plausible manner. There are also important omissions, as we shall see, among them labour, government and monetary policy.

1.1 The model

The wealth of a capitalist, also called a firm in what follows, at the start of period t is comprised of its capital valued at cost, $K(t)$, and the nominal amount of its net monetary assets or money, $M(t)$. Their sum is the capitalist's wealth,

$$W(t) = K(t) + M(t) \tag{1}$$

The initial value of $K(t)$, $K(1)$, is given, and the change in $K(t)$, from one period to the next is given by

$$K(t + 1) = K(t)[1-\lambda] + I(t) \tag{2}$$

where λ is the rate at which capital depreciates in productivity from one period to the next, and $I(t)$ is the expenditure of money on additions to the gross stock of capital.

The firm's quantity of net monetary assets is its cash, receivables and bonds, less its payables and debt. $M(1)$ is given and $M(t + 1)$ is raised above $M(t)$ by $M(t)r$, the interest on its initial balance and by $P(t)$, the gross profit on production. $M(t + 1)$ falls as a consequence of the period's investment and $C(t)$, the consumption or dividend plus overhead costs incurred by the firm. Hence,

$$M(t + 1) = M(t)[1 + r] + P(t)-C(t)-I(t) \tag{3}$$

The actual gross profit on production during t is

$$P(t) = \alpha(t)K(t) \tag{4}$$

with $\alpha(t)$ assumed for simplicity to be a normally distributed random variable with mean μ and standard deviation σ.

The capitalist's consumption plus administration cost is the fraction η of $K(t)$ or

$$C(t) = \eta K(t) \tag{5}$$

and the gross investment in capital is

$$I(t) = g\lambda K(t) \tag{6}$$

Here, g is the ratio of the investment expenditure to the depreciation on the capital, so that $g = 0$ means that gross investment is zero and $g = 2$ means that investment in t is twice the depreciation on the existing stock.[1]

The economic security of a firm increases with the ratio $M(t)/K(t)$, and insecurity rises as $M(t)/K(t)$ falls below zero. There is a lower limit to the ratio; that limit is no lower than $M/K = -1$, and when that limit is reached, the firm is bankrupt.[2] The creditors then take

[1] Capital here includes only 'fixed' assets subject to depreciation. Inventory has different properties, and a possible extension of the model (that will not be undertaken here) is to include both types of capital.

[2] A widely used measure of financial position is the debt–equity ratio, $-M(t)/[K(t)+M(t)]$. We could just as well have used it as our bankruptcy condition.

and liquidate the firm's assets for whatever can be recovered of the money owed to them. The corporation ceases to exist, and the proprietor ceases to be a capitalist. The upper limit on $-M/K$ is in fact less than one, owing to the loss on and costs of taking over and liquidating the assets of a bankrupt firm. This condition for bankruptcy may seem to be excessively harsh and, today, bankruptcy law is far kinder to firms in financial distress, as will be seen later. However, this seemingly harsh policy would be the practice in a competitive capitalist system, where the law does not overrule private contracts.

1.2 The no-growth policy

A firm operating in competitive markets is subject to a growth imperative because of the variability in its profit rate. The gross profit on production, that is, earnings before deducting administration or overhead expenses, depreciation and interest, expressed as a fraction of capital employed by the firm, can fluctuate over a wide range from one period to the next. This is captured by the mean and variance of $\alpha(t)$ in equation (4).

The basis for our claim that a capitalist enterprise operating in competitive markets is subject to a growth imperative is that a no-growth policy makes bankruptcy practically certain in the long run and has an intolerably high probability in the short run, while providing little or no compensating benefits. We define a no-growth policy as one under which C and I are determined so that the values of K and M would remain constant over time if the expected value of the profit rate were realised with certainty. That policy is realised under the model represented by the above equations when the expenditure rate

$$q = g\lambda + \eta = \mu, \text{ and } g = 1 \text{ and } r = 0 \qquad (7)$$

The condition that $g = 1$ guarantees that K remains constant over time by making investment equal to depreciation. The condition that $q = \mu$ makes the expenditure in each period equal to the average profit rate. Hence, with $r = 0$, we have C, I and M as well as K constant over time when the future is certain, that is, when $\alpha(t) = \mu$ for all t.

With the equations (7) true, the model satisfies a martingale property, so that the firm is *certain* to go bankrupt in the long run. This is demonstrated in the Appendix (Theorems 3 and 4).[1] An explanation of why bankruptcy is inevitable sooner or later that is an over-simplification may nonetheless help understand what is going on. Let bankruptcy take place when $M/K = -1$ or less and ignore for the moment the investment and consumption–administration expenditures. Note that the operating profit or loss for a period can be a loss of any magnitude, such as 300% of K or more. If M/K at the period's start is low enough so that a loss of 300% or more makes $M/K = -1$ or less, the firm is bankrupt in that period. The likelihood of bankruptcy increases with the variance of the profit rate and inversely with M/K. Also, the likelihood it will be sooner rather than later is increased by the size of the periodic investment and consumption–administration expenditures as a fraction of K.

1.3 Simulation of model

The proof that a capitalist enterprise is certain to go bankrupt in the long run may remind some readers of Keynes's response to the proof that the long-run tendency of a perfectly competitive capitalist system is full employment: 'In the long run we are all dead.' An

[1] The appendix establishes that bankruptcy is certain in the long run under weaker assumptions than those stated above. In the course of what follows, we shall develop more realistic models of the firm which will recognise $r > 0$ among other things, and we shall consider their consequences for the no-growth policy and the firm's long-run survival.

actual capitalist is more concerned with the probability of bankruptcy over a reasonable, finite time horizon.

To arrive at some idea of the consequences of a no-growth policy in finite time for a firm described by the above model, we simulated its fortunes. We arrived at the probability of bankruptcy by the end of t periods, and the expected values of K and M, both conditional on survival for t periods and without that condition. For each simulation, we calculated the probability of bankruptcy by the end of t periods for $t = 2, 10, 25, 50$ and 100. Bankruptcy was assumed to take place when the debt reaches $-M(t) = 0{\cdot}5K(t)$.[1] The calculations were made for $\sigma = 0{\cdot}0, 0{\cdot}10, 0{\cdot}20, 0{\cdot}40$ and $0{\cdot}60$. The simulation was carried out for a system with 10,000 firms, each one initially identical and with subsequent fortunes that differed only with respect to the random outcome with respect to $\alpha(t)$.

Table 1 presents the results of a simulation under which $K(1)$ has the arbitrary value of \$1,000 and $M(1) = 0$. Regardless of the initial $K(1)$ value, all that matters is $M(1)$ relative to $K(1)$. Reasonable values assigned to the relevant variables that satisfy the no-growth condition were depreciation rate $\lambda = 0{\cdot}10$, consumption–administration rate $\eta = 0{\cdot}15$, mean gross profit rate $\mu = 0{\cdot}25$, and investment ratio $g = 1$. The expenditure rate is $q = g\lambda + \eta$ and with $r = 0$, the no-growth condition is satisfied.

We see in Table 1 that with $\sigma = 0$ and the future certain, $K = 1{,}000$ and $M = 0$ in every future period. Also, gross investment $\lambda K = 100$ and consumption $\eta K = 150$ in every period. With $\sigma > 0$, the investment policy keeps the mean $K = \$1{,}000$ as long as the firm survives, but that is only true conditional upon the firm surviving for the t periods. Most important, the probability of bankruptcy rises as σ rises above zero and as we go further into the future. For example, with $\sigma = 0{\cdot}40$, the probability of bankruptcy rises above one-half by $t = 10$, and with $\sigma = 0{\cdot}60$, the probability of bankruptcy rises to $0{\cdot}84$ by $t = 50$.

With the mean values of K and M over time conditional on survival, M rises sharply with σ as well as t, because the surviving firms are profitable and the no-growth policy limits them to putting the excess profits in the bank. The unconditional expected value of M also rises with both σ and time, but the unconditional values of wealth, that is $W = K + M$ fall for each value of $\sigma > 0$ as t rises. The important attributes of W are that it falls as t rises for each value of σ and it never rises above its initial value of $W(1) = 1{,}000$. Note how sharply the unconditional expected value of K falls with time.

We simulated the model represented by equations (1)–(6) for a number of other combinations of the parameters that satisfy the no-growth conditions of equation (7). We were surprised to find that all of these produced exactly the same output that we see in Table 1.[2] For instance, $\lambda = \eta = \mu = 0$ also satisfy the no-growth condition, and that simulation also results in exactly the same survival probabilities and K and M values as in Table 1. We then found that to be no coincidence: Theorem 1 in the Appendix proves that it must take place.

[1] It may be wondered why bankruptcy takes place when $-M(t)$ reaches $0{\cdot}5K(t)$ instead of $K(t)$. When $-M(t)$ reaches $0{\cdot}5K(t)$, the debt covenant allows the creditors to call their loans. The market value of the firm's capital will then be below K, its book value, so that the firm is unable to meet their demand or refinance the debt elsewhere, and it is wiped out. Making bankruptcy take place when—$M(t)$ is closer to $K(t)$ would not result in any substantive change in the conclusions reached. The critical assumption is that government does not intervene in credit markets to govern or overrule the terms of debt contracts in the interest of debtors.

[2] This takes place when the random number generator that determines the sequence of profit rates for each firm starts at the same place as we go from one combination of the parameters to another. Starting a simulation with a new random number produces different numerical results that are not statistically significant.

Table 1. *Survival probabilities and expected values of capital* t *periods in future for a capitalist firm that follows a no-growth policy. Expenditures depend solely on capital, as in equations (5) and (6) with parameter values:* $\lambda = 0{\cdot}10$, $g = 1{\cdot}0$, $\eta = 0{\cdot}15$, $\mu = 0{\cdot}25$, ivr $= 0{\cdot}10$, $q = 0{\cdot}25$

t	Fraction bankrupt	Conditional on survival		Unconditional	
		K	*M*	*K*	*M*
Uncertainty $\sigma = 0{\cdot}00$					
2	0·0000	1,000	0	1,000	0
5	0·0000	1,000	0	1,000	0
10	0·0000	1,000	0	1,000	0
25	0·0000	1,000	0	1,000	0
50	0·0000	1,000	0	1,000	0
100	0·0000	1,000	0	1,000	0
Uncertainty $\sigma = 0{\cdot}10$					
2	0·0000	1,000	1	1,000	1
5	0·0054	1,000	3	995	3
10	0·0663	1,000	42	934	39
25	0·2504	1,000	196	750	147
50	0·4230	1,000	422	577	243
100	0·5669	1,000	752	433	326
Uncertainty $\sigma = 0{\cdot}20$					
2	0·0069	1,000	7	993	7
5	0·1303	1,000	92	870	80
10	0·3086	1,000	284	691	196
25	0·5272	1,000	717	473	339
50	0·6572	1,000	1,207	343	414
100	0·7503	1,000	1,922	250	480
Uncertainty $\sigma = 0{\cdot}40$					
2	0·1004	1,000	84	900	75
5	0·3735	1,000	427	627	268
10	0·5499	1,000	895	450	403
25	0·7084	1,000	1,823	292	532
50	0·7907	1,000	2,854	209	597
100	0·8487	1,000	4,373	151	662
Uncertainty $\sigma = 0{\cdot}60$					
2	0·1929	1,000	210	807	170
5	0·4858	1,000	802	514	412
10	0·6425	1,000	1,551	358	554
25	0·7707	1,000	2,953	229	677
50	0·8358	1,000	4,522	164	742
100	0·8813	1,000	6,769	119	804

1.4 Uncertainty of the profit rate

Table 1 makes clear that the probability of bankruptcy in finite time depends materially on σ, the uncertainty of the profit rate on capital. It would therefore be most desirable to have some idea as to the plausible range of σ. We have practically no direct data on that statistic, but the price of a company's stock moves more or less with its earnings, and a statistic on which considerable data exists is the standard deviation of the return on the US stock market in the past. The capitalisation weighted real return on all stocks traded on the NYSE over the years 1926–90, had an arithmetic mean of 8·6%, a geometric mean of 6·4%, and a standard deviation of 21·1% (see Siegel, 1992, p. 31). Individual stocks had mean returns that were larger or smaller than 8·6%, and their standard deviations were on average larger, since the return on the stock market as a whole benefited from diversification. Also, firms traded on the NYSE did not operate in competitive markets as described earlier. They all enjoyed varying degrees of monopoly power, acquired in order to raise gross return and reduce risk. Consequently, we may speculate that $\sigma = 0{\cdot}20$ is very low for an average firm that buys and sells in competitive markets. They may well have returns with a standard deviation as high as or higher than 0·60, and 0·40 is a quite conservative figure, in our judgement.

A further consideration in reaching a judgement on our value of σ is that the upper limit on the loss on a share of stock over some time period is 100%,[1] but there is no upper limit on a capitalist loss expressed as a fraction of the capital employed. To see why, recall that the rate of profit (or loss) on a share of stock is the dividend plus the rise (fall) in price divided by the *initial price*. Since the dividend and price cannot fall below zero, the largest possible loss is 100%. The rate of profit (loss) for an enterprise is the operating profit (loss) divided by the capital employed valued at its cost. Subject to the ability of the enterprise to draw on a cash reserve or borrow from a bank, the loss in any period can be a few hundred per cent. One of the advantages attributed to private enterprise by comparison with government enterprise is that there is a limit on the ability of the former 'to throw good money after bad money' or get others to join in doing so.

1.5 Consequences

The consequences of a no-growth policy for a capitalist described by equations (1)–(6) are quite unattractive. As stated earlier, bankruptcy is certain in the long run. In finite time, consumption and the stock of capital conditional on survival remain constant: only the stock of money grows, but its sole purpose is to defer bankruptcy, and it is not very effective for that purpose. *The unconditional expected values of total wealth decline over time.* The absence of growth in consumption and capital are an expected consequence of the no-growth policy, but there is no compensatory benefit in a high probability of survival in finite time as well as the long run. Recall for instance that with $\sigma = 0{\cdot}40$, the probability of bankruptcy rises above one-half by $t = 10$. These dismal consequences for a no-growth policy represented by equations (1)–(6) are true regardless of the values the capitalist assigns to or enjoys for the investment, consumption, profit and depreciation rates as long as they satisfy the no-growth conditions of equation (7).

The firm referred to in the presentation of the above model is a capitalist firm for good reason. The fundamental distinction between a capitalist and socialist firm is with regard to its control, which is exercised most simply and effectively through ownership. In a capitalist economy, the control is private, so that each firm has an owner who enjoys or suffers the firm's profit or loss. That is the objective of control. The profit, wealth etc. of each capitalist fluctuated more or less from one period to the next due to the vagaries of operating in a competitive market. In aggregate over the large number of capitalists, the

[1] This property of a stock has generated much interest in the comparative features of the arithmetic and geometric mean rates of return on a security. The arithmetic mean is an average of N simultaneous returns, while the geometric mean is the average rate at which wealth will grow over N periods. It is the annual return that one can expect to earn in the long run – as time goes to infinity. The geometric mean is below the arithmetic mean to a degree that depends on the variance of the return, and if there is a non-zero probability of a return of –100%, a portfolio that consists solely of this security will go to zero in the long run. This property of the geometric mean return has generated interest in Kelly strategies, that is, portfolios that maximise the geometric mean and combining them with a risk-free asset to limit the probability of bankruptcy (see Bernouilli, 1954; Latané and Tuttle, 1967; Markowitz, 1976; MacLean and Ziemba, 1999). There will be more on bankruptcy in the next section.

aggregate values of K, M and the other variables are practically constant from one period to the next. In a socialist economy, the manager of each firm does not enjoy the firm's profit. The profits of all firms flow to the state, and the constant aggregate is distributed in some way by the central authority. The insecurity and bankruptcy in our simple competitive capitalist system is completely absent in pure simple socialist system. Its hallmark is security. Of course, this socialist system might well have real-world problems of motivation and growth. We shall not deal with that subject here beyond noting that actual socialist systems that were fairly close to a pure socialist system, such as Stalin's Soviet Union and Mao's China, were forced to take extraordinary measures to deal with the problems of motivation and growth. Mao tried the Great Leap Forward and the Cultural Revolution.

2. Growth, uncertainty and bankruptcy

Before going on to extensions of our model that make it a more realistic representation of a capitalist and then considering the consequences of pursuing growth, we shall attempt to review its relation to the relevant literature. Our objective is to establish where our model departs from or advances existing theory. That takes place for the most part with respect to growth, uncertainty and bankruptcy, and these topics, bankruptcy in particular, will be emphasised in the review. The treatment of bankruptcy in mainstream (neoclassical) theory has much in common with the treatment of death in religion. We shall see that bankruptcy is ignored, denied, glorified or passed over quickly in the neoclassical theory of the firm or individual.

2.1 Smith and Marx on growth

As indicated by the title of his great work, Adam Smith's objective was maximising the wealth of nations. To realise that objective, he argued that: (1) there should be specialisation and exchange in markets free of government and other restrictions on free competition among capitalists, and (2) each capitalist should maximise the share of operating profit devoted to the growth of capital. Operating profit is value added less the wages of production workers, and to the extent possible, it should be devoted to the further accumulation of capital. Operating profit devoted to consumption, management, the arts, general welfare etc. should be minimised regardless of how useful such expenditures might be (Smith, 1822, vol. 2, bk. 2, ch. 3). By implication, Smith disapproved of feudal lords who devoted their entire surplus to their pleasures, war, the church and other non-productive purposes.

Marx argued that the entire surplus was expropriated from the working class, and he attacked that expropriation, while occasionally expressing admiration for the progress made possible by its use. To our knowledge, Marx was the first person to state that capitalists were subject to a growth imperative. He wrote:

> Moreover, the development of capitalist production makes it constantly necessary to keep increasing the amount of the capital laid out in a given industrial undertaking, and competition makes the immanent laws of capitalist production to be felt by each individual capitalist, as external coercive laws. It compels him to keep constantly extending his capital, in order to preserve it, but extend it he cannot, except by means of progressive accumulation. (Marx, 1906, p. 649)

For the most part, Marx attributed the behaviour of capitalists to the nature or personality of the people who become capitalists. It was not made clear what, if anything, in their circumstances drives them to subordinate all else to the further accumulation of wealth.

Luxemburg (1968) is the Marxist whose main thesis is that capitalism is subject to a growth imperative. She relied on history to prove that capitalism has always relied on predatory relations with non-capitalist systems at home and abroad to survive. She wrote 'real life has never known a self-sufficient capitalist society under the exclusive domination of the capitalist mode of production' (p. 348). Her effort at a more rigorous proof of her thesis was flawed, and that resulted in its rejection by other Marxists (see Sweezy, 1942, pp. 202–7). The most favourable treatment of Luxemburg's thesis was by Joan Robinson in an introduction to the edition cited (see also Gordon, 1987).

2.2 Neoclassical and Keynesian theory

Prior to World War II, the theory of investment was based on the assumption that the future payoffs on an investment are known with certainty, or what commonly amounts to the same thing, their uncertain future values can be represented by their expected values. It could then be reasoned that the capitalist maximised the market value of wealth with the investment or disinvestment that equates the marginal rate of return on investment with the interest rate. The utopian properties of this investment decision were explored at great length in the development of the theory of interest and capital. No reason apart from taste and technology was found why a firm could not remain in a stationary state, and the same was true of the allocation of income between current and future consumption on the part of real persons.

The combination of the Great Depression, Keynes's reputation and his literary skills finally made underconsumption and underinvestment theories of aggregate demand respectable. Kalecki (1954) developed a similar theory. Previously, the instability of actual capitalist systems was attributed mainly to monetary policy, labour or 'political' events. Keynesian economics made it acceptable to use fiscal as well as monetary policy to manage the economy. From a theoretical viewpoint, the Keynesian consumption function made private investment the critical economic variable, and Keynes railed against the inadequacy of the prevailing theory of investment, both its failure to deal with uncertainty about the future and the desire for security (see Keynes, 1936, bk. IV, esp. ch. 12).

Modigliani and Miller (1958) established the conditions under which the neoclassical theory of valuation and investment under certainty holds in the presence of uncertainty and risk aversion. To achieve this, they implicitly assumed that the corporation is nothing more or less than a collection of separable assets and investment opportunities. That makes it possible to value each asset or opportunity at a discount rate that is greater than the interest rate by an amount that depends solely on *its* risk. Consequently, the value of a firm is completely independent (apart from taxes and other market imperfections) of its capital structure, its dividend policy and even its investment decision, the last because the value of each investment opportunity may be captured as easily by selling it as by undertaking it. Gordon (1994, ch. 2, 4, 5 and 6) summarised the widely recognised failure of the theory to explain corporate practice. The theory was equally useless in explaining how stocks are valued and in estimating the all-important cost of equity capital (see Brigham and Gordon, 1968; Malkiel and Cragg, 1970; Fazzari *et al.*, 1988; Gordon and Gordon, 1997).[1]

[1] Gordon and Shapiro (1956) and Gordon (1962) developed an alternative theory called the dividend growth model, under which the value of a firm is the present value of its dividend expectation, and its cost of capital is the dividend yield plus the expected growth rate in the value of its shares. The theory is widely used in the practice of finance, and it is widely taught in business school finance courses. It has been banished, however, from the theoretical literature on finance, probably because it recognises that growth is risky.

Markowitz (1959) arrived at the set of portfolios that are mean-variance efficient in one period returns, and Sharpe (1964) made the further assumptions needed to arrive at the Capital Asset Pricing Model (CAPM) for shares. The separability of the shares in a portfolio made it easy for Hamada (1969) to relate the CAPM to the Modigliani–Miller theory of corporate finance. However, for the CAPM to be true, the efficient market hypothesis must also be true: the expected return on a share must be equal to the average realised returns. Unfortunately, 25 years of testing led Fama and French (1992), Lakonishok and Shapiro (1986) and others to the conclusion that there is no evidence to support the joint hypothesis that CAPM and EMH are true (see Frankfurter and McGoun, 1999).

Papers by Samuelson (1969) and Merton (1969, 1971) established the consumption and portfolio policy (allocation of wealth between a risk-free asset and a diversified portfolio of risky shares) over time of an individual whose objective is the maximisation of *expected* utility of future consumption. The individual is assumed to have constant relative risk aversion and no minimum consumption, so the allocation is independent of the level of wealth. The fraction of net worth invested in the risky share portfolio is found to be

$$\pi = (\mu - r)/\beta\sigma^2 \tag{8}$$

with μ the mean rate of return on the portfolio, r the risk-free interest rate, β the investor's degree of relative risk aversion and σ^2 the variance of the portfolio's rate of return. On average, over all investors, π should be equal to one in a closed system without government and π should be below one if investors own but do not owe the public debt. The data are consistent with this conclusion, since μ–r is about 0·06, and σ^2 is about 0·045. An investor who puts all wealth in the risky portfolio has $\pi = 1$, and that implies $\beta = 1{\cdot}33$, which is a reasonable degree of risk aversion. As β and σ^2 rise above these values or the risk premium $\mu - r$ falls below 0·06, π falls below one.[1]

2.3 Bankruptcy

There was no explicit recognition of bankruptcy in the Samuelson–Merton model. That in effect made it an infinitely terrible event that was never allowed to take place. Gordon and Sethi (1997, 1998), recognised that there is life after bankruptcy by imposing a minimum level on consumption. This extension of the Samuelson–Merton model produces intuitively attractive results. Notwithstanding the assumption of constant relative risk aversion, consumption expressed as a fraction of wealth and the fraction of wealth invested in the risky asset both decline as wealth increases. As wealth falls toward zero, the individual adopts a go-for-broke investment policy. They invest to the limit allowed by creditors, since a conservative financial policy makes bankruptcy certain within a few periods

[1] Robert Lucas and his followers have developed a macroeconomic model in which the risk of holding the market portfolio is the *variance in the growth rate of per capita consumption*, and that has given rise to the 'equity premium puzzle' (see Mehra and Prescott, 1985; Kocherlakata, 1996). The latter reports on p. 47 that the variance in the growth rate in aggregate consumption per capita is about 0·0013. That figure, combined with an equity premium of about 0·06 would require a degree of relative risk aversion that is well beyond all reason, if we are to have π=1 in equation (8). We do not understand the measure of risk used in the Lucas model, unless the model is intended to represent no more than a pure exchange economy with the sole asset an *endowment* of grain that is consumed, stored or loaned, and with *the mean return on storage positive*. In an economy that employs land to produce grain, land would be the dominant asset; its value would be the discounted value of its expected future output, and the variance in its return would be the change in the discounted value of that expectation from one period to the next. Its risk would be far greater than the variance in the growth rate in grain consumption. The Lucas model is so lacking in empirical relevance as to make one wonder what makes its empirical results a puzzle.

anyway. The fraction of wealth consumed also rises as wealth falls toward zero, for the simple reason that consumption will certainly not be allowed to fall below what it would be in the event that bankruptcy takes place.

The theoretical literature that recognises the existence of bankruptcy also has treated it in a curious way. Two theories of bankruptcy exist: the absolute priority rule and the relative priority rule. Under the former, bankruptcy takes place when the value of the firm falls to the amount of its debt, in which case the corporation is reorganised with all creditors paid in full and the equity holders left with nothing. Neoclassical economists advocate the absolute priority rule, claiming that a bankruptcy would then be a relatively costless and unimportant event, like the refinancing of an outstanding debt issue by a strong corporation (see, for example, Senbet and Seward, 1995).

In fact, the question of bankruptcy does not arise with a determination that the market value of a corporation's assets has fallen below its outstanding debt. The question arises when a corporation violates one of its debt contracts and the creditor demands payment. What the law provides for in that event is the basis for the historical development of the relative priority rule. In order to ensure an orderly disposition of the firm's assets, the corporation may and usually does obtain the protection of the courts when a creditor demands payment. The court then may and usually does arrange a reorganisation of the corporation in which the claims of the investors in the corporation, including stockholders, are recognised on the basis of their priority, the weakest claims being scaled down the most. Practically all business bankruptcies take place under the relative priority rule, because the law makes it available to corporations. The costs of bankruptcy can be substantial, according to neoclassical economists, only because the misguided intervention of government has resulted in litigation, uncertainty and other costs.[1]

It may be wondered why we use the book instead of the market value of capital in deciding when *M*/*K* puts the firm in bankruptcy. The use of market value instead of book value for *K*, the real assets of a corporation, is widely recommended in the finance literature. The practice is to use book value, because the alternative is more easily advocated than carried out, for a number of reasons that will not be elaborated here. Perhaps more importantly, *M*/*K* at market is far more volatile than a cost-based ratio, so that it is quite impossible for a firm to maintain some debt–capital ratio at market value, and fairly possible to do so at book value.

It may be argued that bankruptcy is of no concern on the level of an industry or an entire economy, because we have birth as well as death: new firms replace old firms and life goes on. However, the knowledge that they will be replaced rarely if ever makes death or bankruptcy of no concern for real and corporate persons. What they do to avoid or delay that possibility is what makes death or bankruptcy important for understanding the behaviour of both real and corporate persons.

3. Extensions of the model

The model represented by the simulation in Table 1 is unrealistic in a number of ways. For instance, the closed competitive capitalist system it is intended to represent has existed

[1] For the development of this reasoning, see Senbet and Seward (1995), Baird (1996) and numerous other essays in Bhandari and Weiss (1996). We could not find a defence of the relative priority rule other than Warren (1996), who argued that its widespread use makes it deserving of consideration. One of the reasons why corporations elect to declare bankruptcy is that the relative priority rule eliminates the obligations to workers created by union contracts.

only in theory: its use may be justified only as a starting point for the consideration of more realistic models. What we shall do now is explain or improve on what we consider to be the most glaring or objectionable of the assumptions incorporated in the model.

3.1 Dependence on profitability

Perhaps the most obvious objection to the plausibility of the model represented in Table 1 is the assumption that consumption and investment are independent of income or profitability. They both move only with K, and it changes little from one period to the next. In fact, a policy of making investment equal to depreciation makes K remain constant at \$1,000 over time for the surviving capitalists. Clearly, consumption and investment should in part respond to income or profitability, and changing our model to realise that objective is accomplished by replacing equation (5) with

$$C(t) = \eta K(t) + \max[ySP(t) \text{ or zero}] \tag{5a}$$

and by replacing equation (6) with

$$I(t) = g\lambda K(t) + \max[hSP(t) \text{ or zero}] \tag{6a}$$

$SP(t)$, the smoothed profit, is an exponential average of its previous value and current profit.

$$SP(t + 1) = \gamma SP(t) + (1-\gamma)P(t) \tag{9}$$

and the initial value of SP is $SP(1) = \mu K(1)$.

Equation (5a) is a Keynesian consumption function. It makes the reasonable statement that in each period there is a 'minimum' consumption and administrative expense. It is represented by $\eta K(t)$, so that the minimum changes slowly over time with capital valued at cost. In addition, the expenditure varies with *expected* income, but $C(t)$ cannot fall below $\eta K(t)$, even though $SP(t) < 0$ is possible. Equation (6a) states that with $g < 1$ there is a minimum expenditure on asset replacement regardless of profitability that is less than λK. The actual level of investment depends also on profitability, but this component of investment cannot be negative, so that gross investment cannot fall below $g\lambda K$. Finally, the sensitivity of C, I and SP to current profit increases as γ falls toward zero.

Is a no-growth policy violated by making C and I partially dependent on profitability? No. We now replace the conditions $q = g\lambda + \eta = \mu$ and $g = 1$ with the conditions that the expenditure rate and the investment rate,

$$q = g\lambda + \eta + \mu(y + h) = \mu \text{ and } ivr = g\lambda + \mu h = \lambda \tag{10}$$

With these conditions satisfied, and with $0 \leq \gamma \leq 1$, K and M, as well as the decision variables, remain in a stationary state when μ is certain, which is our condition for a no-growth policy.

The Appendix proves that when $r = 0$ and either the 'max' constraints are removed from equations (5a) and (6a) (Theorem 5), or $\gamma = 1$ (Theorem 6) or γ is near 1 (Theorem 7), the above model satisfies a martingale property, which means that the firm is certain to go bankrupt in the long run. The proof is difficult under the max conditions of equations (5a), (6a) but simulations with and without the max conditions under a wide range of circumstances revealed that for every combination of σ and t, the probability of bankruptcy was greater with the max conditions than without them. We therefore can say with great confidence that bankruptcy is certain in the long run for the system represented by the above model.

To see what happens in finite time, we simulated the model under a range of values for the parameters that seemed of interest. Of particular interest is what happens to the probability of bankruptcy as the dependence of C and I on profitability is raised by moving γ from 1 towards 0 and by moving $y + h$ from 0 towards 1, the latter with compensating reductions in $g\lambda$ and η to satisfy the conditions that $q = \mu$ and $\lambda = g\lambda + \mu h$. Either when $\gamma = 1$ or when $y = h = 0$ is true, there is no dependence of C and I on profitability, and we are back with the model of Section 1 that produced the output in Table 1. When $\gamma = 0$ and $y + h = 1$ with $\eta = g\lambda = 0$, there is complete dependence on profitability, since $\gamma = 0$ makes SP equal to current profits, and the minimum levels of both C and I are zero. The simulations revealed that the probabilities of bankruptcy in finite time are substantially unchanged.

Table 2 presents the simulation results when C and I are made dependent on both capital and profitability by setting $\gamma = 0{\cdot}5$, with g and η made positive and h and y taking the values needed to satisfy equations (10). We kept $\mu = 0{\cdot}25$ and $\lambda = 0{\cdot}10$, and the values of g, h, η and y were set to arrive at a reasonable distribution between the 'fixed' and 'variable' components of C and I, while keeping the expenditure rate $q = \mu$ and the investment rate equal to the depreciation rate. Comparing the statistics in Tables 1 and 2 reveals that partial dependence of C and I on profitability provides less attractive prospects than no dependence.

We simulated the profitability model over a wide range of values for the above parameters without violating the conditions in equations (10). In particular, we tried μ above and below $0{\cdot}25$, raised and lowered the relative importance of C and I, and raised or lowered the fixed and variable components of C and I. Unlike the model of Section 1, where the probability of bankruptcy and other statistics were independent of these parameter values, here the statistics change in one way or another. However, as long as the no-growth conditions of equations (10) are satisfied and $r = 0$, the probability of bankruptcy for each combination of t and σ varied over a limited range from one simulation to the next. There are seemingly small changes in the parameters that produce ridiculously large or small changes in K or M over time. We shall not burden the text with a discussion of how reasonable they are.

Our conclusion is that the extension of our no-growth policy to make expenditure partially dependent on profitability does not eliminate the unattractive features of the policy. The opposite is true. Bankruptcy is still inevitable in the long run. Furthermore, the probability of bankruptcy in finite time is commonly raised for each value of t, and the unconditional expected value of the growth in wealth is still negative. The lack of growth in a no-growth policy is still not compensated for by a high probability of survival.

3.2 Other refinements

The assumption that the interest rate $r = 0$ was not made solely for convenience, in that for $r > 0$ the theorems proved in the Appendix either could not have been proven, or they would require a far more complicated argument. However, we established by simulation that recognising interest paid on loans has a negligible impact on the survival probabilities and on the expected value of K and M over a wide range of the values for r. Of course, a very large initial wealth, a very high rate of interest on money loaned, very low values for C/K etc., might make it possible and attractive for a capitalist to become a rentier and live forever on M. However, these conditions are exceptional, and need not concern us.[1]

[1] In the real world, we have rentiers who live solely on interest income, but that is possible only through a redistribution of income through government and business debt. Real income is produced by K and not by M.

Table 2. *Survival probabilities and expected values of capital* t *periods in future for a capitalist firm that follows a no-growth policy. Expenditures depend on capital and profits, as in equations (5a), (6a) and (9) with parameter values:* $\lambda = 0{\cdot}10$, $g = 0{\cdot}50$, $h = 0{\cdot}20$, $\eta = 0{\cdot}05$, $y = 0{\cdot}50$, $\mu = 0{\cdot}30$, $\gamma = 0{\cdot}50$, $ivr = 0{\cdot}10$, $q = 0{\cdot}30$

t	Fraction bankrupt	Conditional on survival		Unconditional	
		K	*M*	*K*	*M*
Uncertainty $\sigma = 0{\cdot}00$					
2	0·0000	1,000	0	1,000	0
5	0·0000	1,000	0	1,000	0
10	0·0000	1,000	0	1,000	0
25	0·0000	1,000	0	1,000	0
50	0·0000	1,000	0	1,000	0
100	0·0000	1,000	0	1,000	0
Uncertainty $\sigma = 0{\cdot}10$					
2	0·0000	1,000	1	1,000	1
5	0·0004	1,000	0	1,000	0
10	0·0044	1,001	3	996	3
25	0·0550	1,008	24	953	23
50	0·1734	1,030	79	851	66
100	0·3392	1,073	193	709	128
Uncertainty $\sigma = 0{\cdot}20$					
2	0·0069	1,000	7	993	7
5	0·0708	1,005	33	934	30
10	0·1917	1,024	89	828	72
25	0·4174	1,085	238	632	139
50	0·5857	1,172	451	485	187
100	0·7147	1,303	795	372	227
Uncertainty $\sigma = 0{\cdot}40$					
2	0·1004	1,000	84	900	75
5	0·3510	1,039	239	675	155
10	0·5607	1,125	438	494	192
25	0·7696	1,345	918	310	211
50	0·8728	1,676	1,558	213	198
100	0·9408	2,430	3,036	144	180
Uncertainty $\sigma = 0{\cdot}60$					
2	0·1929	1,000	210	807	170
5	0·5050	1,084	505	537	250
10	0·7078	1,251	877	365	256
25	0·8770	1,730	1,843	213	227
50	0·9493	2,705	3,501	137	178
100	0·9881	6,405	10,380	76	124

Another possible objection to the models represented by Tables 1 and 2 is the growth in M/K over time, conditional on the firm's survival for the t periods. M grows beyond all reason in relation to K in Table 1, and the growth in M/K is still large in Table 2, where excess profits are absorbed in part by C and I. $M = 0$ is a strong financial position, and having $M>K$, as takes place in these simulations, provides exceptional security. It is quite possible that any particular capitalist is so obsessed with the fear of bankruptcy that all excess cash is held as money. There are two polar alternatives for capitalist policy. One is to spend all excess M on additional consumption, and the other is to devote it to the further accumulation of capital. Under both policies with regard to the disposition of excess M, its expected value is kept within reason, but the probability of bankruptcy rises more sharply with σ and t. Hence, putting excess cash into some combination of consumption and capital does not improve the prospects for survival. Quite the contrary, they are reduced materially, the only compensation being that, conditional on surviving for t years, the intervening years are somewhat more satisfying.

3.3 Serial correlation and mean reversion

An interesting and reasonable departure from our assumption that the expected rate of profit on capital is μ regardless of its realised value is the assumption that realised values of $\alpha(t)$ convey information about future values of $\mu(t)$. This dependence of the expected return on capital on its realised values and a tendency for the expected return to revert towards some long run value is captured by the expression:

$$\mu(t+1) = c(1)\mu(t) + c(2)\alpha(t) + [1 - c(1) - c(2)]\mu(1) \tag{11}$$

with the three coefficients all positive and their sum equal to one. Bankruptcy in the long run cannot be proven under the serial correlation and mean reversion that we have in equation (11) (see the Remark just before Theorem 6 in the Appendix).

What happens in finite time would seem to depend materially on the relative values of the three coefficients of equation (11), and we simulated the model described in Table 2 with a limited range of values for the coefficients of equation (11). Table 3 presents the simulation results with $\mu(1) = 0{\cdot}25$, $c(1) = 0{\cdot}5$ and $c(2) = 0{\cdot}4$, and with the other parameters the same as in Table 2. Comparison of Tables 2 and 3 reveals that in Table 3 the probability of bankruptcy rises much more rapidly with t and σ except for combinations of t and σ that are both very large. The explanation is that in Table 2 at time $t > 0$, the variation in K and W are due solely to σ. In Table 3, serial correlation makes μ vary among firms over time, so that μ as well as σ contribute to the variation K and W as t increases. When t and σ are both very large, the few firms that remain are very wealthy and profitable. The important conclusions are that serial correlation makes the probability of bankruptcy rise more rapidly in the short run, and its continued rise in the long run would seem to make it inevitable unless another policy is adopted.

We repeated the simulation in Table 3 with a few other combinations of the coefficients of equation (11). For $c(1) = 0{\cdot}4$ and $c(2) = 0{\cdot}5$, for $c(1) = 0{\cdot}6$ and $c(2) = 0{\cdot}3$, and for $c(1) = 0{\cdot}4$ and $c(2) = 0{\cdot}3$, the rates of growth in the probability of bankruptcy with σ and t were remarkably close to the values in Table 3. On the other hand, the rates of growth in K and M were raised, particularly for high values of σ and t, as $c(2)$ was raised relative to $c(1)$. The rates of growth in K and M were reduced materially as the mean reversion was raised by reducing the sum of $c(1)$ and $c(2)$. We did not carry out simulations for radically different relative values for the coefficients of equation (11), or for other values of the other parameters in Table 3.

4. Growth policies

What might a firm do to reduce materially the high probability of bankruptcy that it faces under the no-growth policies examined above? We shall examine three alternative policies, with each policy taking as its starting point the parameters under the no-growth policies in Tables 2 and 3. The three alternatives are: (1) reduce the expenditure rate below the profit rate, while leaving the investment rate equal to the depreciation rate; (2) raise the investment rate above the depreciation rate while leaving the expenditure rate

Table 3. *Survival probabilities and expected values of capital* t *periods in future for a capitalist firm that follows a no-growth policy. Expenditures depend on capital and profits, with profits influenced by serial correlation as in equations (5a), (6a), (9) and (11) with parameter values:* $\lambda = 0{\cdot}10$, $g = 0{\cdot}50$, $h = 0{\cdot}20$, $\eta = 0{\cdot}05$, $y = 0{\cdot}50$, $\mu = 0{\cdot}30$, $\gamma = 0{\cdot}50$, $ivr = 0{\cdot}10$, $q = 0{\cdot}30$, $c(1) = 0{\cdot}5$, $c(2) = 0{\cdot}4$

t	Fraction bankrupt	Conditional on survival		Unconditional	
		K	*M*	*K*	*M*
Uncertainty σ = 0·00					
2	0·0000	1,000	0	1,000	0
5	0·0000	1,000	0	1,000	0
10	0·0000	1,000	0	1,000	0
25	0·0000	1,000	0	1,000	0
50	0·0000	1,000	0	1,000	0
100	0·0000	1,000	0	1,000	0
Uncertainty σ = 0·10					
2	0·0000	1,000	1	1,000	1
5	0·0145	1,001	9	987	9
10	0·1304	1,021	102	888	89
25	0·3668	1,166	516	738	327
50	0·5250	1,442	1,220	685	579
100	0·6583	1,971	2,600	674	888
Uncertainty σ = 0·20					
2	0·0069	1,000	7	993	7
5	0·1565	1,014	123	856	103
10	0·3552	1,100	461	710	297
25	0·5621	1,540	1,697	675	743
50	0·6909	2,533	4,273	783	1,321
100	0·8010	5,751	12,671	1,144	2,522
Uncertainty σ = 0·40					
2	0·1004	1,000	84	900	75
5	0·3827	1,064	511	657	315
10	0·5529	1,313	1,423	587	636
25	0·7151	2,805	5,957	799	1,697
50	0·8277	9,646	25,667	1,662	4,422
100	0·9311	102,775	297,828	7,081	20,520
Uncertainty σ = 0·60					
2	0·1929	1,000	210	807	170
5	0·4828	1,120	947	579	490
10	0·6342	1,568	2,659	574	973
25	0·7815	5,156	14,636	1,127	3,198
50	0·8874	40,349	130,603	4,543	14,706
100	0·9707	2,222,473	8,154,835	65,118	238,937

equal to the profit rate; and (3) do both. The no-growth policies in Tables 2 and 3 are identical in their dependence of expenditures on capital and profits, and they differ only in that there is serial correlation and mean reversion in Table 3. Their presence is more realistic than their absence: so we shall confine our comparison of the three alternatives with the no-growth policy to the simulations in which there is serial correlation and mean reversion.

We could not establish theoretically whether bankruptcy is inevitable in the long run under these growth policies. But in the simulations with the levels of μ and *ivr* and with their excess over q and λg reasonable, the rise in the probability of bankruptcy with σ and t

are rapid enough to suggest that it is certain sooner or later. When the levels of μ and *ivr* and their excess over q and λg are both large, this conclusion is in some doubt.

Our first alternative policy, which is the reduction in the expenditure rate below the profit rate, was achieved by making $q = 0{\cdot}25$ and $y = 1/3$, while leaving $\mu = 0{\cdot}30$ and $h = 1/6$ at their values in Table 3. As expected, the probability of bankruptcy was reduced somewhat, and unconditional expected values of K and M were raised, but the latter only slightly. In short, the increased probability of survival was achieved at the cost of very little growth in capital.

Under the second alternative policy, we made $ivr = 0{\cdot}15$ and $h = 1/3$, while leaving $q = \mu$ at $0{\cdot}30$. Here, the probability of bankruptcy rose somewhat more rapidly with t and σ than in Table 3, while the unconditional growth rate in K was much greater and quite fantastic for high combinations of t and σ, where serial correlation has eliminated all but the most profitable firms.

The simulation of both departures from the no-growth policy appear in Table 4, where the parameter changes from Table 3 are stated. Comparison of the tables makes clear that the survival and growth statistics realise the best of both of the previous departures from a no-growth policy. Here again, the elimination of the unprofitable firms by serial correlation results in fantastic growth rates for the few firms remaining at high σ and t.

Clearly, the growth policy represented in Table 4 offers far more attractive prospects than the no-growth policies examined in the previous section. There are, however, two problems with this growth policy. Any one capitalist can follow a policy of making the expenditure rate less than the profit rate, but that policy cannot be followed widely in a closed capitalist system without government. In aggregate, we must have the expenditure rate equal to the gross profit rate, unless there is lending to, or investment in, other sectors—government, workers or the foreign sector. The other problem with the policy is that it is Keynesian economics: with the limited exception of loans to government to finance military expenditures, loans as well as gifts to others beyond charity are not popular with capitalists. Welfare state policies were adopted in response to the horrendous economic and political failures of capitalism over the years 1929–45, and they met with great success over the years 1950 to 1973.Cornwall and Cornwall (2001) called the period 'the golden age' of capitalism, but, for various reasons, the period was followed by stagflation and the glorification of the market and corporate rule.

What do real firms actually do to avoid the dismal prospects offered by a no-growth policy? Real firms are radically different from their representations in neoclassical theory, where they engage only in production; they are completely represented by a production function; and they passively accept domination by a perfectly competitive market. Real firms engage in a wide range of non-production activities in the pursuit of monopoly power. These non-production activities range from research and development, advertising and marketing, to bribing government officials and overthrowing governments if necessary. For high technology corporations such as Microsoft and Merck, expenditures in the pursuit of monopoly power are much greater than their expenditures for production. Nike is reported to engage in everything but the production of its products. A simple and powerful measure of the monopoly power real firms enjoy is Kalecki's Degree of

Table 4. *Survival probabilities and expected values of capital* t *periods in future for a capitalist firm that follows a growth policy. Expenditures depend on capital and profits, with profits influenced by serial correlation as in equations (5a), (6a), (9) and (11) with parameter values:* $\lambda = 0{\cdot}10$, $g = 0{\cdot}5$, $h = 0{\cdot}33$, $\eta = 0{\cdot}05$, $y = 1/6$, $\mu = 0{\cdot}30$, $\gamma = 0{\cdot}5$, $ivr = 0{\cdot}15$, $q = 0{\cdot}25$, $c(1) = 0{\cdot}5$, $c(2) = 0{\cdot}4$

t	Fraction bankrupt	Conditional on survival		Unconditional	
		K	*M*	*K*	*M*
Uncertainty σ = 0·00					
2	0·0000	1,050	50	1,050	50
5	0·0000	1,194	246	1,194	246
10	0·0000	1,469	646	1,469	646
25	0·0000	2,731	2,490	2,731	2,490
50	0·0000	7,677	9,717	7,677	9,717
100	0·0000	60,680	87,158	60,680	87,158
Uncertainty σ = 0·10					
2	0·0000	1,050	51	1,050	51
5	0·0017	1,195	249	1,193	249
10	0·0210	1,485	706	1,454	691
25	0·0721	3,040	3,190	2,821	2,960
50	0·1060	10,250	14,598	9,164	13,051
100	0·1369	112,496	176,300	97,095	152,164
Uncertainty σ = 0·20					
2	0·0025	1,050	54	1,047	54
5	0·0818	1,212	338	1,113	310
10	0·1969	1,629	1,144	1,309	919
25	0·3425	4,595	6,653	3,021	4,374
50	0·4676	27,134	47,647	14,446	25,367
100	0·6244	879,127	1,618,435	330,200	607,884
Uncertainty σ = 0·40					
2	0·0709	1,050	114	976	106
5	0·2993	1,303	764	913	535
10	0·4534	2,155	2,765	1,178	1,511
25	0·6260	12,857	26,986	4,809	10,093
50	0·7757	302,365	688,523	67,820	154,436
100	0·9189	155,449,265	382,219,535	12,606,935	30,998,004
Uncertainty σ = 0·60					
2	0·1612	1,050	232	881	195
5	0·4231	1,418	1,306	818	754
10	0·5699	2,852	5,080	1,227	2,185
25	0·7373	34,627	88,469	9,096	23,241
50	0·873	3,375,768	9,189,769	428,722	1,167,101
100	0·9716	19,177,053,613	59,514,142,750	544,628,323	1,690,201,654

Monopoly Power, which can be expressed as the ratio of value added to the wages of production workers. Their difference is the cost of the firm's non-production activities and the gross profit on its capital. With a DMP = 1 meaning that there is no monopoly power, the DMP in the US manufacturing sector fluctuated in a narrow range, around 2·5 from 1899 to 1949, and then rose to 5.25 by 1996 (see Gordon, 1998). The purpose of these expenditures in the pursuit of monopoly power is to raise the mean and reduce the variance of the return on capital.

The contrast between this theory of the firm and mainstream theory is also reflected in the literature that deals with growth. Solow (1970) has explored many interesting questions involved in going beyond a two-factor model in which labour and capital are the only factors of production, the growth rate in wages, profits and output converge, and the excess of the growth rate in output over the growth rates in labour and capital is due to exogenous technological progress. The current new idea in growth theory is the distinction between exogenous and endogenous growth, the former (latter) being generated by institutions and policies outside (inside) the economic system. However, how this endogenous growth takes place is far from clear, apart from being the inevitable fruit of government support for education and research. It does not take place in firms where agents implement or subvert technologically given production functions (see Romer, 1994; Solow, 1994; Nelson, 1997). The contrast between the two theories of the firm is further illustrated by the literature on globalisation, where the anti-establishment literature sees corporations ruling the world with terrible consequences for people and the environment (see Barnet and Cavanaugh, 1994; Greider, 1997).

We have seen in Table 4 how a firm can achieve both survival and growth well into the future. It must engage in a high rate of net investment, make the gross profit on production greater than the sum of the expenditures on administration, other non-production activities, investment and dividends. And it must also enjoy a low variance in the rate of return on capital. Successful expenditures in the pursuit of monopoly power make all this possible for any one firm, and for a sub-group of firms in a system. But how is it achieved for an entire system? Expanding the role of government is one means of achieving the goal, but the objective of capitalist growth is not to finance a welfare state. In fact, the elimination of the welfare state is now widely seen as the ideal growth policy.

Perhaps we should close with a puzzle that starts with the same statistic as the 'equity premium' puzzle discussed in footnote 1 on p. 34. All corporations traded on the NYSE over the long period 1926–90 had a real arithmetic mean rate of return on their shares of 8·6%. The rate of growth in GNP over these years was more like 3%. Technological progress and improved training of the labour force help explain the growth in GNP and the stock market, but not the higher growth in the stock market. Is the Luxemburg thesis mentioned earlier part of the explanation? What else is at work?

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Appendix

In this Appendix, we prove various results claimed in the text.
We begin with a general result which says that our model only depends on seven particular combinations of parameter values.

Theorem 1

Consider the model presented in the text as equations (1)–(4), (5a), (6a) and (9). The distributions of $K(t)$ and $M(t)$, for all $t \geq 1$, depend on the various parameter values only through the seven quantities σ, r, h, y, γ, $(g-1)\lambda$, and $\mu - \eta - g\lambda$. That is, if the individual parameter values are changed in such a way that these seven quantities are unchanged, then the distribution of $K(t)$ and $M(t)$ is unchanged for all $t > 1$.

Proof

We can write $\alpha(t) = \mu + \sigma N(t)$, where $N(t)$ are standard normal random variables (i.e., with mean 0 and variance 1).
In terms of this, we compute algebraically that

$$K(t+1) = K(t)[1 + (g-1)\lambda] + h \max[SP(t),0] \tag{A1}$$

and

$$M(t+1) = M(t)[1 + r] + K(t)[\sigma N(t) + \mu - \eta - g\lambda] - (y + h) \max[SP(t),0] \tag{A2}$$

We thus see explicitly that these equations can be written entirely in terms of the seven stated quantities, which completes the proof. ■
We now turn to issues of bankruptcy. We require some preliminaries.
We shall have occasion to consider continuous-time versions of the processes $K(t)$ and $M(t)$, denoted $KK(s)$ and $MM(s)$, respectively. To define them, fix all the model parameter values and an integer $t \geq 1$. Then, given values of $K(t)$ and $M(t)$, find the (deterministic) value $K(t+1)$ and the distribution of $M(t+1)$ of the form Normal(m,v), both specified by our model. In terms of all of this, we define $KK(s)$ and $MM(s)$ for $t \leq s \leq t+1$, in differential form, by

$$KK(t) = K(t)$$

$$MM(t) = M(t)$$

$$\mathrm{d}KK(s) = [K(t+1) - K(t)]\mathrm{d}s$$

$$\mathrm{d}MM(s) = [m - M(t)]\mathrm{d}s + \mathrm{Sqrt}[v]\mathrm{d}B_s$$

here $\{B_s\}$ is standard one-dimensional Brownian motion.
These continuous-time processes $KK(s)$ and $MM(s)$ have been defined precisely so that $KK(t) = K(t)$ and $MM(t) = M(t)$ for every integer t. Indeed, these processes simply *extend* our discrete-time processes to continuous time. (In the language of stochastic processes, we have *embedded* our discrete-time processes into continuous-time processes.)

Using this continuous-time embedding, we give a precise definition of 'bankruptcy' as we shall prove it.

Definition

We shall say that 'bankruptcy is certain' for a model, if the following holds: With probability 1, either $KK(s) + MM(s)$ is negative for some $s > 1$, or else $KK(s)$ [and hence also $K(t)$] converges to 0 as s (and t) go to infinity.
In terms of this definition, we have the following.

Theorem 2

Consider the model presented in the text as equations (1)–(4), (5a), (6a) and (9).
Suppose that the model parameters are such that for all $t \geq 1$, the conditional expected values of $K(t)$ and $M(t)$ satisfy that

$$E\{[K(t+1) + M(t+1)] - [K(t) + M(t)] \mid K(t), M(t)\} \leq 0 \tag{A3}$$

Suppose further that $r = 0$ and $\sigma > 0$. Then bankruptcy is certain [regardless of the initial values $K(1)$ and $M(1)$].

Proof

Let $KK(s)$ and $MM(s)$ be the continuous-time versions of $K(t)$ and $M(t)$, as above. Let

$$\tau = \inf\{s > 1;\ KK(s) + MM(s) \leq -1\}$$

Define a new process $\{X_s\}$ by

$$X_s = \begin{cases} KK(s) + MM(S), \tau < s \\ -1, \tau \leq s \end{cases}$$

(In particular, note that $X_t = K(t) + M(t)$, for any integer $t \geq 1$.) That is, X_s is the total wealth of the player at time s, except that $\{X_s\}$ gets 'stopped' at –1 as soon as the player's total wealth hits this value.
In terms of $\{X_t\}$, equation (A3) says that $E[X_{t+1} - X_t \mid K(t), M(t)] \leq 0$. Because of the construction of $\{KK(s)\}$ and $\{MM(s)\}$, it follows from this that $\mathrm{E}[X_{s+r} - X_s \mid X_u\ (u \leq s)] \leq 0$ for any $r > 0$. That is, the process $\{X_s\}$ is a *supermartingale*, i.e., on average it will stay constant or get smaller.
In particular, $E[X_s] \leq X_1 = K(1) + M(1)$ for all s. On the other hand, since $\{X_s\}$ was 'stopped' as soon as it hits –1, we see that $X_s \geq -1$ for all s > 1.
We conclude that $\{1 + X_s\}$ is a non-negative supermartingale with bounded expectation. It then follows from the standard Martingale Convergence Theorem (see e.g., Theorem 14.2.1 of Rosenthal, 2000) that with probability 1, the sequence $\{X_t\}$ must converge pointwise to some limiting random variable, say X.
Now, if $X = -1$, then in particular $X_s < 0$ for some s. This implies that $MM(s) + KK(s) < 0$. Hence, in this case the player was bankrupt before time s.
On the other hand, if $X > -1$, then $\{MM(s) + KK(s)\}$ converges to X, hence also $\{M(t) + K(t)\}$ converges to X. In this case, $(K_{t+1} + M_{t+1}) - (K_t + M_t)$ converges to 0. But from equation (A2) above, we see that conditional on K_t and M_t, the difference $(K_{t+1} + M_{t+1}) - (K_t + M_t)$ has conditional variance $[\sigma K(t)]^2$. If the difference converges to 0, then this variance $[\sigma K(t)]^2$ must converge to 0. Since we are assuming that $\sigma > 0$, then this implies that $K(t)$ converges to 0. Hence, if $X > -1$ then $K(t)$ converges to 0.
Since we always have either $X > -1$ or $X = -1$, this completes the proof of the theorem. ■

Theorem 3

Consider the model presented in the text as equations (1)–(4), (5a), (6a) and (9).
Suppose that $r = 0$, $\sigma > 0$, and that $\mu \leq \lambda + \eta$. Then bankruptcy is certain [regardless of $K(1)$ and $M(1)$, and regardless of the values of g, h, η, y, and γ].

Proof

Again let $X_t = K(t) + M(t)$ be the total wealth of a given player. Then

$$X_{t+1}-X_t = -\lambda K(t) + \alpha(t)K(t)-\eta K(t)-y\max[0,SP(t)] = [\alpha(t)-\lambda-\eta]K(t)-y\max[0,SP(t)].$$

Recall now that $\alpha(t)$ has mean μ, so that $E[\alpha(t)-\lambda-\eta] = \mu-\lambda-\eta \leq 0$. Furthermore $\max[0,SP(t)] \geq 0$. Therefore,

$$E[X_{t+1}-X_t \mid K(t),M(t)] \leq 0$$

i.e., $\{X_t\}$ is a supermartingale.

The result now follows from Theorem 2. ■

Remark: Note that this Theorem remains valid for any value $\beta \leq 1$, not just $\beta = 0{\cdot}5$.

Remark: Suppose $\mu = g\lambda + \eta$. In this case, if $g \leq 1$, then the conditions of Theorem 3 are satisfied and eventual bankruptcy is certain. However, if $g > 1$, then the conditions of the theorem are not satisfied, and it may be possible to never go bankrupt, or at least to postpone bankruptcy longer.

Remark: This theorem can be interpreted informally as saying that, if the parameters satisfy the given conditions, then bankruptcy is certain in the long run. Strictly speaking, our conclusion of bankruptcy must also allow for the possibility that a player will keep all their wealth in cash $M(t) > 0$ even as their capital wealth $K(t)$ goes to 0. However, this possibility can be ruled out of $g \geq 1$, as the following theorem shows.

Theorem 4

If the hypotheses of Theorem 3 are satisfied, and if furthermore $g \geq 1$, then there is $s > 1$ with $MM(s) < -\beta KK(s)$ for any $\beta \leq 1$.

Proof

If $g \geq 1$, then $K(t) \geq K(1)$ for all $t \geq 1$. It is therefore not possible that $K(t)$ converges to 0 in this case. According to Theorem 3, the only other possibility is that there is $s > 1$ with $MM(s) + KK(s) < 0$. The result follows. ■

We now consider further the 'no growth condition' discussed in the text, namely

$$(g-1)\lambda + h\mu = \mu - \eta - g\lambda - \mu(y+h) = 0 \tag{10}$$

If $y = h = 0$ then this equation implies that $\mu = g\lambda + \eta$; hence, if $g \leq 1$, then the conditions of Theorem 3 are satisfied. However, if y and/or h are positive, the situation is less clear. We have the following.

Theorem 5

Consider the model presented in the text as equations (1)–(4), (5a), (6a) and (9).

Suppose we replace $\max[SP(t),0]$ with $SP(t)$ in equations (5a), (6a) leaving equations (1)–(4) and (9) unchanged.

Suppose further that (10) holds and that $r = 0$. Then for this modified model, $E[K(t)] = K(1)$ and $E[M(t)] = M(1)$ for all $t > 1$ (i.e., on average there is zero growth). Hence, if $\sigma > 0$, then Theorem 2 applies, and bankruptcy is certain.

Proof

We prove by induction that $E[K(t)] = K(1)$ and $E[SP(t)] = \mu K(1)$. Recall that $SP(1) = \mu K(1)$, hence the statement is obviously true for $t = 1$.

Now assume it is true for some $t \geq 1$. Then from equation (A1) above, with $\max[SP(t),0]$ replaced by $SP(t)$, since $(g-1)\lambda + h\mu = 0$ and $E[SP(t)] = \mu K(1)$, we see that $E[K(t+1)] = K(1)$. Now, since $P(t) = \alpha(t)K(t)$, and since $\alpha(t)$ and $K(t)$ are independent, it follows that $E[P(t)] = \mu E[K(t)] = \mu K(1)$. But then since

$$SP(t+1) = \sigma\gamma SP(t) + (1-\gamma)P(t)$$

it follows that $E[SP(t+1)] = \mu K(1)$. Hence, it follows by induction that $E[K(t)] = K(1)$ and $E[SP(t)] = \mu K(1)$ for all $t > 1$.

But once we know that $E[SP(t)] = \mu K(1)$ and (10) holds, then it follows from equation (A2) that $E[M(t+1)] = E[M(t)]$ for all $t \geq 1$. Hence, by induction again, $E[M(t)] = M(1)$ for all $t > 1$. This completes the proof. ■

We conclude from Theorem 5 that, if (10) holds, then any possible escape from eventual bankruptcy must occur entirely due to the difference between max[$SP(t)$,0] and $SP(t)$ in our model.
Remark: If we do not replace max[$SP(t)$,0] with $SP(t)$ in equations (5a), (6a) then Theorem 5 does not apply. However, it is not clear in this case that the probability of bankruptcy is reduced. Indeed, the simulations presented in the text indicate that eventual bankruptcy is certain in this case as well. (Mathematically speaking, the 'max' in equations (5a) and (6a) slightly increases both I and C. The increase in C can only lower the player's wealth. However, the increase in I is more subtle, which prevents a clear mathematical analysis.)
Remark: Theorem 5 assumes (as do all other results in the Appendix) that the mean profit rate, μ, is held constant. If we instead assume mean reversion for μ, as in equation (12) of the text, then $\alpha(t)$ and $K(t)$ are no longer independent. Hence, Theorem 5 does not apply in this case.

Theorem 6

Consider the model presented in the text as equations (1)–(4), (5a), (6a) and (9).
If (10) holds and $r = 0$ and $\mu \geq 0$ and $\gamma = 1$, then $E[K(t)] = K(1)$ and $E[M(t)] = M(1)$ for all $t > 1$. Hence again, if $\sigma > 0$, then bankruptcy is certain.

Proof

In this case, $SP(t) = \mu K(1) > 0$ for all t, so that max[$SP(t)$,0] $= SP(t)$, and Theorem 5 applies directly. This completes the proof. ■

Theorem 7

Consider the model presented in the text as equations (1)–(4), (5a), (6a) and (9).
If (10) holds and $r = 0$ and $\mu > 0$, then for fixed $\sigma > 0$ and fixed $\varepsilon > 0$, the probability that the player goes bankrupt or $\inf_t K(t)$ is less than ε goes to 1 as γ goes to 1. In symbols,

$$\lim P[\text{bankrupt, or } \inf_t K(t) < \varepsilon] = 1, \qquad \gamma \to 1$$

Proof

As γ goes to 1, the variance of $SP(t)$ goes to 0. But $SP(t)$ has positive mean. Hence, max[$SP(t)$,0] becomes a closer and closer approximation to $SP(t)$. Consequently, the no-growth conditions of Theorem 5 get closer and closer to being satisfied.
Now, write $p(\gamma,\varepsilon,t)$ for the probability that, for a given γ, the player goes bankrupt by time t, or $K(t)$ is less than ε.
From Theorem 5, for any $\delta > 0$, we can find large enough t that $p(1,\varepsilon,t) > 1-\delta$. But then from the above observation, we can find $a < 1$ which is close enough to 1 that $|p(1,\varepsilon,t)-p(\gamma,\varepsilon,t)| < \delta$ whenever $\gamma > a$.
It follows from the triangle inequality that $p(\gamma,\varepsilon,t) > 1 - 2\delta$ whenever $\gamma > a$. In particular, for $\gamma > a$,

$$P[\text{bankrupt, or } \inf_t K(t) < \varepsilon] \geq 1-2\delta$$

Since $\delta > 0$ was arbitrary, the result follows. ■
Remark: Even if (10) holds with $r = 0$ and $\mu > 0$, then for large enough σ and small enough γ, there could be a positive probability of avoiding eventual bankruptcy. However, this probability would usually be extremely small, since (by Theorem 5) it arises solely because of the very subtle difference between max[$SP(t)$,0] and $SP(t)$.