

## Chapter 4

# Kenneth Arrow and Social Choice Theory

**Overview:** In the two previous chapters I have examined party behaviour in two-party and multi-party democracies. This chapter takes a largely normative rather than positive form in so far as it looks more generally at the efficacy of voting and democracy. Two key texts are examined. The first, Kenneth Arrow's (1951) *Social Choice and Individual Values*, has been interpreted as showing that it is impossible to provide a normative rationale for making social decisions when individual members of a society have different interests or opinions. The second, William Riker's (1982a) *Liberalism Against Populism*, uses Arrow's theorem and the work in social choice theory it inspired to defend a 'liberal' rather than 'populist' theory of democracy. Arrow and Riker's work has cast a 'very long, dark shadow over democratic politics' (Cain, 2001: 111). In the final section of this chapter I argue that this work does, however, need to be placed in context. Social choice theory may show that democracy is problematic and imperfect; it does not, however, show that democracy is impossible or necessarily undesirable.

### Setting the stage: democracy and the public will

Samuel Huntington (1991: 7) defines a country as democratic if its most powerful collective decision-makers are selected through fair, honest and regular elections in which candidates freely compete for votes and in which virtually the entire adult population is eligible to vote. Using this definition Huntington identifies three historical 'waves' of democratization. In the first long wave between 1828 and 1926, around 30 countries including Australia, the UK, the USA and New Zealand underwent the transition to democracy. In a second and shorter wave between 1943 and 1962 around 25 countries including Brazil, Sri Lanka and Jamaica democratized. In a third wave which started in the mid-1970s and has yet to dissipate, more than 30 countries including Spain, Portugal and Greece, Chile, Argentina and

Uruguay, Hungary, Poland and the Czech and Slovak republics, Estonia, Lithuania and Latvia, and, more recently and problematically, Iraq and the Ukraine have embraced democracy. There are of course still countries out there, North Korea, Saudi Arabia and Libya amongst them, which remain rigidly authoritarian. There are also democratic countries which may one day experience an anti-democratic 'backlash' of the sort afflicting Germany in the 1930s and Brazil, India and South Korea in the late 1960s and early 1970s. In Pakistan, for example, the 'war against terror' has recently been used to justify the withdrawal of democratic freedoms. Yet, for the moment, and as Albert Weale (1999: 1) observes, it would seem that democracy has 'ceased to be a matter of contention and become a matter of convention'.

Political theorists have often expressed a determined cynicism about or outright hostility to democracy (Mackie, 2003: 2; Dahl, 1989). Plato argues that democracy leads to social anarchy. Marxists argue that representative democracy is a sham and that real power is monopolized by business (see Elster, 1985: 408–11 for a discussion). Elitists like Pareto and Michels argue that democratic politicians pander to the lowest common policy denominator. More recently, political scientists have argued that democracy causes government 'overload' (King, 1975; Brittan, 1975) and that voters have extremely fickle and ill-informed preferences (Converse, 1970; Page and Shapiro, 1992: 3–15 and references therein). In recent years, a sub-branch of rational choice theory known as social choice theory has joined this chorus of dissent.

Voting, whether it involves voting directly for some policy in, for example, a referendum, or voting indirectly for a candidate in a legislature, is a defining part of the democratic process. Voting reveals the 'public will' or 'collective interest'. The critique of democracy offered by the two social-choice theorists examined here, Kenneth Arrow and William Riker, is a critique of voting. Arrow and Riker argue that there is no method of counting or, as they prefer, aggregating individual preferences that is fair and accurate:

Outcomes of voting cannot, in general, be regarded as accurate amalgamations of voters' values. Sometimes they may be accurate, sometimes not; but since we seldom know which situation exists, we cannot, in general, expect accuracy. Hence we cannot expect fairness either. (Riker, 1982a: 236)

It is worth emphasizing that this is *not* an argument about any particular method of voting. The target social choice theorists have in their

sights is not plurality voting, proportional representation or some other specific way of counting votes. It is, rather, and more generally, the very idea of being able to aggregate the preferences of individual voters in such a way as to generate a fair and accurate decision or 'social choice'. Stated in this way, the concerns of social choice theorists appear both mysterious and fantastical. Mysterious because it is unclear what, precisely, such a sweeping claim might involve. Fantastical because the notion that voting might be an inherently flawed way of reaching social decisions seems, in this democratic age, a counter-intuitive one. Yet, as we will see, the basic argument involved here, although routinely presented in a technical way, is actually quite simple.

### The precursors of social choice theory

Social choice theory did not emerge as a recognized subject area until the publication of Arrow's *Social Choice and Individual Values* in 1951. Yet as Iain McLean and Arnold Urken (1995) carefully demonstrate, many of the puzzles and inconsistencies associated with voting that Arrow carefully dissects were actually discovered and discarded in the eighteenth century. Of particular significance here is the work of two French mathematicians and members of the Paris Academy of Sciences, Jean-Charles Borda (1733–99) and the Marquis de Condorcet (1743–94). Borda, whose obituary records that he 'worked much and published little', delivered a paper to the Paris Academy in 1770 showing that the winner of a plurality or 'first-past-the-post' election might be opposed by a majority of voters (McLean and Urken, 1995: 25). Consider, for example, the preferences of the seven (1–7) voters in Table 4.1 who must select between three options (x–z). The first, second and third preferences of the voters are shown. In a plurality election y will receive the votes of the third and fourth voters, z the votes of the fifth and seventh voters and the winner, x, the votes of the first, second and sixth voters. Yet a majority composed of the third, fourth, fifth and seventh voters clearly prefer both y and z to x.

The obvious problem with the plurality voting system is that it fails to take account of the full range of voters' preferences and, in this case, of the fact that x is the last choice of four voters. The solution, Borda argued, was to devise a method of voting taking account of this information. The result, the eponymous Borda count, works as follows. For any given number of options ( $n$ ), assign  $n - 1$  points

Table 4.1 Majority voting generates a majority-opposed decision

Voters	Preferences			Borda count points score			
	1st	2nd	3rd	x	y	z	
1	x	y	z	2	1	0	
2	x	z	y	2	0	1	
3	y	z	x	0	2	1	
4	y	z	x	0	2	1	
5	z	y	x	0	1	2	
6	x	y	z	2	1	0	
7	z	y	x	0	1	2	
				Total	6	8	7

to the option the voter ranks first on their ballot paper,  $n - 2$  to the option they rank second and so on; the winner being the option receiving the most points. The right-hand part of Table 4.1 shows how this works by translating the preferences of the seven voters into point tallies. It shows that y is the Borda winner and that x receives the lowest score.

In an essay published in 1784, the Marquis de Condorcet, who was the Permanent Secretary of the Paris Academy until he fell to the Jacobin terror, showed that the Borda count is, however, vulnerable to exactly the same problem as plurality voting. It will sometimes select as the social choice an option a majority of the electorate oppose. Table 4.2 shows the preferences of five (1–5) voters over five (a–e) options. As the right-hand part of the table shows, e wins the Borda count with 16 points. Yet a series of pair-wise comparisons shows that a majority composed of the first, third and fourth voters prefer d to e. The claim that e ought to be the social choice because it is the Borda winner therefore appears questionable.

It is a defect of both the plurality and Borda methods that they sometimes fail to select the majority-preferred option. It was therefore Condorcet's suggestion that the social choice ought to be determined by making a series of pair-wise comparisons between *all* the options and selecting that one (the 'Condorcet winner') which is majority-preferred to all the others. In Table 4.1 we have already seen that a majority prefers d to e. By looking more carefully at this table, it can be seen that a clear majority also prefer d to either a, b or c. There is,

Table 4.2 Borda count winner is majority-opposed

Voters	Preferences					Borda count points score					
	1st	2nd	3rd	4th	5th	d	e	a	b	c	
1	d	e	a	b	c	4	3	2	1	0	
2	e	a	c	b	d	0	4	3	1	2	
3	c	d	e	a	b	3	2	1	0	4	
4	d	e	b	c	a	4	3	0	2	1	
5	e	b	a	d	c	1	4	2	3	0	
						Total	12	16	8	7	7

Note: The left columns of this table again show each voter's preference-ranking, whilst the right-hand columns translate this into point scores. With five options ( $n = 5$ ) the first choice of each voter is assigned four points, the second-choice three points and so on.

however, a problem here which Condorcet himself recognized and that has preoccupied social choice theorists ever since. In certain situations, the preferences of voters can be such that whilst one option is always majority-preferred to another, no one option is majority-preferred to *all* the others. This is the so-called paradox of voting (not to be confused with the paradox of not voting discussed in Box 2.1). Table 4.3 shows the preferences of three voters (1–3) over three options (a–c). Pair-wise comparisons reveal that a is majority-preferred to b (by virtue of the support of the first and third voters), that b is majority-preferred to c (by virtue of the support of the first and second voters) but that c is majority-preferred to a (by virtue of the second and third voters). There is therefore a 'cycle' such that  $a > b$  and  $b > c$  but  $c > a$ .

In the 1870s the existence of voting cycles was independently rediscovered by an Oxford mathematician, Charles Dodgson, better known as Lewis Carroll, the author of *Alice's Adventures in Wonderland*. Yet although he apparently 'felt sure that . . . the paradox of voting was known', Arrow (1951: 93) was unaware of Condorcet or Dodgson's work. One possible inspiration for his own argument can, however, be identified. In Chapter 2 we saw how Anthony Downs (1995: 197) regards Joseph Schumpeter's (1942) *Capitalism, Socialism and Democracy* as providing the 'foundation' for *An Economic Theory of Democracy*. Schumpeter argues that most voters have very little

Table 4.3 The paradox of voting

Voters	Preferences		
	1st	2nd	3rd
1	a	b	c
2	b	c	a
3	c	a	b

knowledge about or interest in politics, that they are prone to irrational prejudices and impulses, and that they are susceptible to capture by special interest groups. The notion of there being a settled and reasonable public will which it is the duty of politicians to discern and respect is, he therefore argues, nonsense:

There is no such thing as a uniquely determined common good that all people could agree on . . . this is due not primarily to the fact that some people may want things other than the common good but to the much more fundamental fact that to different individuals and groups the common good is bound to mean different things. (Schumpeter, 1942: 251)

Having previously been Austria's Minister of Finance during a period of hyperinflation and the President of a bank which subsequently collapsed, Schumpeter left Austria for Harvard University in 1932 where he went on to establish himself as one of America's leading economists. This does not necessarily mean that Arrow would have encountered Schumpeter's work as an undergraduate student. There is, however, a closer connection between the two. Arrow was Downs' doctoral supervisor. Although it was another professor at Stanford, Julius Margolis, who initially suggested to Downs that he look at *Capitalism, Socialism and Democracy*, Arrow's willingness to act as Downs' supervisor suggests that he may have already had some basic knowledge of Schumpeter's arguments. Either way, Arrow's *Social Choice and Individual Values* can be interpreted as providing a more detailed account than that offered by Schumpeter of the mechanisms of preference aggregation, and thereby an explanation of why there can be no such thing as a uniquely determined common good.

### Arrow: social choice and individual values

Kenneth Arrow (1921– ) was educated in New York and has spent most of his working life at Stanford University. Awarded the Nobel Prize in 1972, Arrow has worked on general equilibrium theory and the economics of uncertainty, risk-taking, teamwork and the environment, as well as social choice theory. Arrow wrote *Social Choice and Individual Values* as his doctoral thesis following a brief spell working at the Rand Corporation in the late 1940s. Economists and mathematicians at Rand were, at this time, pioneering the use of game theory to analyse international relations. When applied in this way, game theorists assume that it is possible to talk about the pay-offs or utility accruing to countries from 'playing' different strategies. Given the commitment made to methodological individualism within rational choice in general and game theory in particular, the utility being invoked here can only refer to the combined utility of the individual citizens of those countries. But how can individual preferences be aggregated in such a way that we might talk about the utility of the citizens of a country? The conclusion Arrow reached is that it may not be possible to do so:

For any method of deriving social choices by aggregating individual preference patterns which satisfies certain natural conditions, it is possible to find individual preference patterns which give rise to a social choice pattern which is not a linear [or transitive] ordering. (Arrow, 1950: 7; original emphasis)

This finding, which has subsequently become known as the 'impossibility' theorem, has been interpreted as showing that 'majority rule is fatally flawed' (Wolff, 1970: 59) and that 'strict democracy is impossible' (Runciman, 1963: 133).

Arrow (1951: 51) constructs his argument by identifying a number of 'unanimously acceptable' canons of democratic fairness, listed below, which any reasonable method of aggregating preferences must, he argues, satisfy. He goes on to show, given the existence of a particular set of preference-rankings (or profiles), that no possible method of aggregating preferences can simultaneously satisfy all these principles:

- 1 *Unrestricted domain or 'free triple'*. Assume that there are three options (a–c) between which voters must choose. The condition of unrestricted domain simply requires that individuals should be free to rank these options in any order they want.

- 2 *Transitivity or 'linearity'*. Transitivity requires that the social choice be consistent. Consistency requires that if there are three options (a–c) and  $a > b$  and  $b > c$  that  $a > c$ .
- 3 *Connectivity or decisiveness*. This condition simply requires that when comparing options, one option is either preferred to the other or that those making the choice are indifferent between them.
- 4 *Independence of irrelevant alternatives*. As it was initially presented by Arrow, this condition requires that the social choice between a set of options, a, b and c, is not affected by the existence of or changes in preferences over other infeasible and so irrelevant alternatives outside of the choice set. This condition, Arrow (1951: 93) argues, precludes any attempts to make interpersonal comparisons of utility (Box 4.1).
- 5 *Citizens' sovereignty*. This condition requires that an option be the social choice because of the preferences of those making the choice. This means that if there are, for example, three individuals and all three rank the available options  $a > b > c$  that b and c should not be the social choice.
- 6 *Non-dictatorship*. This condition requires that there is no individual whose preferences are automatically the social choice independent of the preferences of all other individuals.

The crucial point to grasp in understanding the nature and significance of Arrow's impossibility theorem is that it is really just a generalization of the paradox of voting discovered by Condorcet (Riker, 1982a: 16; Weale, 1999: 139; Mackie, 2003: 37; Arrow, 1967: 227). What Arrow shows is that *if* individuals' preferences are such that there is a cycle of the sort shown in Table 3.3 then 'no method of voting' will generate a social choice satisfying all six conditions. To see this assume that three voters have the following preference-schedule (for a formal proof of the theorem see Abrams, 1980: 53–60 or Sen, 1970: 41–6).

Voter 1	$a > b > c$
Voter 2	$b > c > a$
Voter 3	$c > a > b$

What ought to be the social choice here? We cannot simply declare that the preference-ranking of the third voter is, for whatever reason, impermissible and ought to be changed to read  $b > a > c$ . Doing this would make b the obvious candidate to be the social choice but would violate the *first* condition of unrestricted domain. Neither can we

#### Box 4.1 Interpersonal comparisons of utility

Assume that there is just one person, Ben, and two goods, x and y. How can we determine whether Ben derives more utility from x than from y? If Ben is given a choice between x and y and chooses x we can infer that he has 'revealed' a preference for x and so expects to derive more utility from its consumption. But can we know how much more utility? At this point, we could simply invite Ben to reflect upon the strength of his preference for x over y. Alternatively, we could rig the choice between x and y in such a way that it revealed something about his preferences. Assume, for example, that we give Ben the choice between getting y with absolute certainty or a 50 per cent chance of getting x and a 50 per cent chance of getting nothing. If Ben chooses x we can infer that he expects to derive at least twice as much utility from x. In this way, we can, in principle at least, derive a cardinal rather than ordinal preference-ranking for Ben.

Assume now that there are two people, Alice and Ben, and only one good, x. How can we tell whether Ben or Alice will derive more utility from consuming x? In *An Essay on the Nature and Significance of Economic Science*, Lionel Robbins argues that there is no rigorous, scientific way of making these kind of interpersonal comparisons of utility. 'There is no means of testing the magnitude of [Alice's] satisfactions compared' with [Ben's] and so 'no way of comparing the satisfactions of different people' (Robbins, 1935: 124). There are two problems involved in making interpersonal comparisons. (1) We cannot see inside other people's minds and read their thoughts in a way that would allow us to directly compare utility streams. We can, Robbins (1932: 139) observes, test people's bloodstream and we can now measure endorphins and record the firing of neurons within the brain. But these do not provide us with the direct measure of utility necessary to show that Alice derives more satisfaction from consuming x than Ben. (2) We cannot rig the choice between Alice and Ben in such a way that their behaviour reveals information about their utility. We might find that Alice is prepared to spend £10 on x whilst Ben is only prepared to spend £5, or that Alice is prepared to queue for two hours to get x and Ben only one. But we cannot infer from this that Alice derives twice as much utility from x because we cannot know that Alice and Ben place an equal value upon particular amounts of money or particular lengths of time spent queuing (for more details see Elster and Roemer, 1991).

Much to his apparent surprise, this argument about the impossibility of making interpersonal comparisons soon became economic orthodoxy (Robbins, 1971: 147–9). But Robbins' argument remains controversial. Some economists and many ordinary people would argue that it is a matter of 'plain common sense' (Cole, 1936: 149) that we can and do use language as a guide to making interpersonal comparisons of utility. Imagine, when asked, Alice were to say that she 'really wanted' x and Ben that he was 'not all that bothered'. Could we not take this to indicate something about the intensity of their preferences?

simply declare that  $a > b$  (by virtue of the preferences of the first and third voters),  $b > c$  (by virtue of the preferences of the first and second voters) and that  $c > a$  (by virtue of the preferences of the second and third voters) because this would violate the *second* condition of transitivity. One way in which we might escape this cycle is by declaring that whilst  $a > b$  and  $b > c$  that no judgement can be reached about the relationship between  $a$  and  $c$ . This would however mean breaking the *third* condition of decisiveness.

Another way in which we might try to break the cycle is by asking voters to state their preferences over a fourth and hypothetical option in order to try and gauge the strength of their preferences over  $a$ ,  $b$  and  $c$ . This would however violate the *fourth* requirement that that the social choice not be affected by the existence of infeasible and so irrelevant alternatives. On the grounds that it was more just than the others or that it would better preserve the environment and so protect the intrinsic value of nature, we might simply declare that  $b$  ought to be the social choice. But this would be inconsistent with rational choice theory's commitment to political individualism and to the *fifth* condition of citizen's sovereignty. Finally, and perhaps most obviously, we could simply declare that one person ought to be given the exclusive right to determine the social choice. This would however violate the *sixth* condition of non-dictatorship.

### Riker: liberalism against populism

Although Arrow's work on social choice theory was lauded by economists, it was, initially at least, largely ignored by most political scientists and philosophers. This is no doubt partly because Arrow's presentation in *Social Choice and Individual Values* makes relatively few concessions to non-mathematicians. It is also because Arrow, who devoted himself almost exclusively to the study of general equilibrium theory in the years following the publication of his thesis, made little effort to relate his argument to existing debates about the nature and limitations of democracy (but see Arrow, 1967). One of the few political scientists who immediately recognized the importance of Arrow's work was, however, William Riker (1958, 1961, 1965) who, in *Liberalism Against Populism* (1982a), eventually succeeded in establishing the significance of Arrow's work for a wider social science audience.

The first few chapters of *Liberalism against Populism* carefully

document the perversities of a series of voting methods including plurality voting, proportional representation and the Borda count. The next few chapters then use Arrow's theorem to generalize these findings and show that *any* method of voting is flawed. But it would be a mistake to regard Riker as *simply* popularizing Arrow's work. For in two important respects, Riker's argument actually differs from that offered by Arrow. In the first place, Riker is at pains to emphasize the practical significance of social choice theory. Arrow's work constitutes a theorem rather than a theory. It seeks to show that *if* voters have a particular set of preferences it will not be possible to generate a social choice without violating one of the previously listed conditions. Arrow does not attempt to demonstrate that any actual set of voters have or have had these preferences. On the basis of a set of detailed case-studies in which he reconstructs the preference-rankings of assorted politicians and voters, Riker claims that the cycles which give rise to the paradox of voting are actually quite common and that they disfigure democratic politics (Riker, 1982a: 197–202; Riker and Weingest, 1988).

The existence of voting cycles means that elections often generate intransitive results. This, in turn, means that elections 'fail to make sense' because they result in the selection of candidates or policies to which there are majority-preferred alternatives (Riker, 1982a: 115). How do we know this? Arrow's theorem tells us that when there is a voting cycle one or more of the previously listed conditions have been broken in the process of aggregating preferences. It would not appear to be the case that the conditions of unrestricted domain, citizen's sovereignty or non-dictatorship are being routinely broken. The problem, Riker concludes, must therefore be that elections are generating intransitive results. The argument at this point may still seem a little fanciful. We do not, after all, regularly encounter newspaper reports that in some election the socialists were majority-preferred to the centre party, the centre party to the conservatives and the conservatives to the socialists. But, as Riker emphasizes, this is only because we do not usually test whether the winner of an election *is* majority-preferred to *all* the other alternatives. We cannot conclude that intransitives do not exist because we do not look for them. Furthermore, and because we do not know which election results would, if we checked, generate intransitive results, we cannot have confidence in the results of *any* election.

Riker's argument differs from Arrow's in a second way. Arrow simply looks at the problem of preference aggregation posed by voting cycles. Riker, however, wants to show how self-interested political

actors can exploit those cycles through a mixture of agenda control, strategic manipulation and heresthetics in order to achieve their preferred outcomes.

### Agenda control

In most democratic bodies, whether they are committees, legislatures or executive cabinets, one person is usually given control of the agenda. Political scientists have previously recognized that this person may be able to secure their preferred outcome by excluding certain options from formal consideration (Lukes, 1974; Barach and Baratz, 1970). Riker (1982a: 169–92) shows that the agenda-setter may also be able to secure their preferred outcome by determining the *sequence* in which options are considered. Assume that there are four options ( $w$ – $z$ ) and three political parties (1–3) with the following preferences:

Party 1	$w > x > y > z$
Party 2	$x > y > z > w$
Party 3	$y > z > w > x$

There is a cycle here such that  $z > w$ ,  $w > x$ , and  $x > y$  but  $y > z$ . But assume that the leader of the third party controls the agenda. They can arrange for an initial contest between  $x$  and  $w$  (which  $w$  will win), then one between  $w$  and  $y$  (which  $y$  will win) and, finally, one between  $y$  and  $z$  (which  $y$  will win). In doing so, they can secure the eventual triumph of their preferred option.

### Strategic manipulation

Drawing on the work of Allan Gibbard (1973) and Mark Satterthwaite (1975), Riker (1982a: 137–62) demonstrates that any method of voting is vulnerable to strategic manipulation by voters who, by misrepresenting their preferences, can try to secure their preferred outcome. To see what this might entail, look again at the preference-rankings of the three parties above. If the leader of the first party can spot what it is that the leader of the third party is trying to achieve in sequencing the votes in a particular way, they can thwart their ambitions by voting strategically for  $x$  during the initial contest between  $x$  and  $w$ , so ensuring the victory of  $x$ . During the subsequent contests between  $x$  and  $y$  and  $x$  and  $z$ ,  $x$  will again triumph. In this way, the leader of the second party can secure their second rather than, as would otherwise be the case, third choice.

### Heresthetics

At times, the preferences voters have are such that there will be a stable equilibrium manifesting itself in a clear and enduring majority for one particular party. Such equilibria are, however, extremely fragile (Riker 1982a: 136–9; 1984, 1986, 1996). By either reframing or reigniting a previously dormant policy issue, politicians can undermine existing equilibrium by introducing new issue dimensions. In Chapter 2 we saw how parties must sometimes compete in a multi-dimensional setting and how this can engender instability. At that time, the existence and number of dimensions was treated as being exogenous to the process of competition itself. Riker shows why parties sometimes compete *by* trying to create new issue dimensions. In the case of, for example, the American Civil War, Riker argues that the Republicans, led by Lincoln, were able to overcome their political isolation by introducing a new and previously dormant issue, slavery, that split the existing and winning coalition between the north-western states that were against the admission of new slave states to the union and the south, which argued that this was a matter for the states themselves.

Democracy is supposed to embody a certain notion of political equality; every person has one vote and that vote is supposed to count equally. Riker (1982a: 200–1) argues that the existence of agenda-setting, strategic manipulation and heresthetics shows that this equality is a chimera:

The absence of political equilibria means that outcomes depend not simply on participants' values and constitutional structures, but also on matters such as whether some people have the will or the wit to vote strategically, whether some leader has the skill, energy, and resources to manipulate the agenda, or whether some backbencher – in a committee or out – has the imagination and determination to generate a cyclical majority by introducing new alternatives and new issues. These are matters of perception and personality and understanding and character.

Such is the ferocity of Riker's denunciation it might seem that he must be opposed to democracy itself. This is not, however, the case. Riker distinguishes between what he calls a 'populist' and a 'liberal' theory of democracy. The populist theory, which he associates (rather unfairly) with Rousseau as a political theorist and Britain as a country, interprets democracy as requiring the translation of the public will into public

policy. The liberal theory, which he associates with Madison, is understood as requiring, firstly, that voters be given the opportunity to remove from office an incumbent government which has offended their sensibilities, and, secondly, the existence of constitutional checks and balances such as a bill of rights, the separation of powers, federalism, a multicameral legislature and fixed terms of office, limiting the power of the executive.

Social choice theory shows that the populist theory of democracy is flawed 'not because it is morally wrong, but because it is empty' (Riker, 1982a: 236). There is no reliable way in which voters' preferences can be aggregated and because any election is vulnerable to strategic manipulation, we simply cannot equate democratic rule with the translation of the public will into public policy. The liberal theory of democracy is not however compromised in the same way. Because instead of demanding that a government embody the public will, it requires only that it be possible for voters to remove a government from office (Riker, 1982a: 243). Furthermore, the constitutional constraints liberals demand can actually be justified in terms of the demonstration that incumbent governments may not represent the popular will and so should not be allowed to govern as 'elective dictatorships' (Hailsham, 1978).

### Reining-in social choice theory

In the previous sections I have shown why social choice theorists' arguments about democracy deserve to be taken seriously. In this section, however, I will seek reasons to temper Arrow and Riker's pessimism. I start here by identifying possible objections to two of Arrow's 'unanimously acceptable' conditions, and then critically examine Riker's claim that voting cycles are a common feature of political life before reassessing the implications of social choice theory not only for the 'liberal' theory of democracy but for the market.

#### Transitivity

It may well be reasonable to require that rational individuals have transitive preferences. It is less clear that it is always reasonable to require of a method of voting that it generate transitive results. Consider the following example offered by Fishburn (1970). There are 21 voters whose preferences over three candidates ( $x-z$ ) are as follows:

Voters 1-10	$x > z > y$
Voters 11-20	$y > x > z$
Voter 21	$z > x = y$

Which candidate should be declared to be the winner? There is an intuitively obvious case here for declaring that  $x$  ought to be the winner. For whilst  $x$  and  $y$  are each the first choice of ten voters,  $x$  is also the second choice of 10 voters. If we were to conduct a Borda count here,  $x$  would be the clear winner. Making a series of pair wise comparisons, we can see that  $x > z$  (by 20 votes to 1) and  $z > y$  (by 11 votes to 10). By extension we might therefore expect that  $x > y$ . But this is not actually the case as  $x$  and  $y$  are tied (10 votes each). There is a further complication here. If  $y$  and  $x$  are equal, this would imply that they could be substituted for each other without it making any difference to the other comparisons. Because it is the case that  $x > z$ , it should therefore also be the case that  $y > z$ . Yet  $z$  is actually majority-preferred to  $y$  (by 11 votes to 10).

The lesson Arrow would draw here is that it is not possible to identify a meaningful and fair social choice from this preference profile. But there are, as I have already said, good reasons for regarding  $x$  as being a reasonable social choice. To therefore require by definitional fiat that a candidate needs to defeat every other candidate in a pair-wise comparison so as to ensure a transitive social ordering may be thought arbitrary. Riker (1982a: 100) states that when 'an alternative opposed by a majority wins, quite clearly the votes of some people are not being counted the same as other people'. But whilst it is certainly unfortunate in this instance that  $x$  cannot defeat  $y$ , it is simply not true that if  $x$  were declared the winner on the basis of a previously-agreed constitutional rule that second preferences ought to be taken into consideration, that some people's votes would have been treated differently.

#### Independence of irrelevant alternatives

The requirement that a social choice be made independently of irrelevant alternatives has proven to be the most controversial of Arrow's conditions. As it was previously defined, the independence of irrelevant alternatives means that the social choice ought not to be affected by the existence of or changes in voters' preferences over other infeasible options outside of the choice set. By way of an illustration, Arrow invites us to imagine a city whose inhabitants are asked to choose between various alternative transport systems: rapid transit, underground, roads,



buses and so on. What, he asks, if someone suggests that the inhabitants ought to be canvassed about a system which would, at the touch of the button, dissolve them into molecules and instantly reform them elsewhere in the city? The independence of irrelevant alternatives means that 'such preferences ought to have no bearing upon the choice to be made' (Arrow, 1967: 226).

This example is so far-fetched it is tempting to think no more about the argument it is defending. Yet as Gerry Mackie (2003: 133) shows, it may sometimes make good sense to take preferences over infeasible options into account. Suppose a reception is to be held and the caterers will only provide one drink, either beer or coffee. To save time, the organizer copies a form from the previous year's event which asks people to rank their preferences over beer, coffee, water, tea, milk and fruit juice. Assume that only two families reply and that they indicate the following preferences:

Family 1 (5 people) beer > coffee > water > tea > milk > fruit juice  
 Family 2 (4 people) coffee > beer > water > tea > milk > fruit juice

Given the caterer's requirements, there are only two feasible (relevant) alternatives and beer is majority-preferred to coffee. Assume now that at the last moment the second family pulls out and a third family with the following preferences decides to attend in its place:

Family 3 (4 people) coffee > water > tea > milk > fruit juice > beer

What should the organizer now do? In terms of the simple pair-wise comparison, beer is still majority-preferred to coffee and so nothing has therefore changed. If the other infeasible alternatives are to be considered as simply irrelevant, the organizer will have to choose beer. But surely it is relevant that the third family rank beer last? Now we need to be careful here. The difficulties involved in making interpersonal comparisons of utility are such that we cannot necessarily assume that the members of the third family derive less utility from the consumption of beer than the first family simply because they rank it lower. It may simply be that the members of the third family derive exceptional amounts of utility from all the other drinks. But given the limited amount of information available, there is nevertheless a good case for making coffee the social choice. Consider what would happen if we were to conduct a Borda count here. There are six options ( $n = 6$ ), so five points should be awarded to the first-ranked option, four points

to the second and so on. Beer is the first-ranked option of five people ( $5 \times 5 = 25$ ) and the last-ranked option of 4 ( $4 \times 0 = 0$ ) and so scores 25 points. Coffee is the first-ranked option of four people ( $4 \times 5 = 20$ ) and the second-ranked option of five ( $5 \times 4 = 20$ ) and so scores 40 points.

There is a further problem with this condition. The way in which Arrow defines the independence of irrelevant alternatives is very different from the way in which he actually uses it in practice. In theory, the condition is supposed to exclude consideration of options outside the actual choice set. In practice, Arrow uses the condition to ensure that a social choice is made exclusively on the basis of pair-wise comparisons between the options *within* the choice set. As Mackie (2003: 137) observes, the independence condition 'would be better named the pair-wise comparison condition, as it requires that choices among several alternatives be carried out only with information about choices between pairs'. This is important because the reliance upon pair-wise comparisons precludes the use of 'positional' methods of counting votes like the single transferable vote or the Borda count.

The Borda count is not an infallible method of aggregating preferences. As Condorcet first recognized, it sometimes fails to select the majority-preferred alternative. But a great deal of comparative research by social choice theorists into the relative merits of different methods of voting has shown that the Borda count most frequently delivers fair and defensible election results (Dummett, 1998; Saari, 2000). In particular, and given the sort of preference-profile which sustains the paradox of voting, the Borda count has the great advantage of reporting a tied result rather than an intransitive cycle. This is important because there is, in principle, no difficulty in dealing with tied results by agreeing, at a prior constitutional stage, to flip a coin or giving a casting vote to the speaker or longest-serving member.

### The frequency of cycles

Riker argues that voting cycles are quite common and that they disfigure democratic politics. Yet the available evidence here suggests that although cycles do sometimes occur, they are actually quite rare. We can start here by assuming that voters' preferences are random in the sense that people are as likely to rank any given set of options in one way as they are in any other. The proportion of possible preference-profiles resulting in cycles can then be calculated for any number of options and voters. The results are shown in Table 4.4. The first point to note here is that when the number of options and the number of

Table 4.4 Proportion of all possible profiles displaying the paradox of voting

Number of options	Number of voters				
	3	5	7	9	11
3	.056	.069	.075	.078	.080
4	.111	.139	.150	.156	.160
5	.160	.20	.215		
6	.202				

Source: Data from Riker (1982a), *Liberalism Against Populism*, p. 122.

voters increase, cycles become more likely. The second point to note is that even when the number of options is quite low that there is a significant chance of cycles arising. If there are 4 options and 11 voters, cycles will arise 16 per cent of the time (.160).

The assumption employed here that any one preference-profile is as likely as any other may not, however, be a good one to make. Imagine now that there are three parties: a left-wing party (lw), a right-wing party (rw) and a centre party (ce). If voters have as their first-choice the party 'closest' to them in political space and as their last-choice the party furthest away from them, they will either rank the parties:

- (i)  $rw > ce > lw$ ,
- (ii)  $lw > ce > rw$ ,
- (iii)  $ce > lw > rw$ , or
- (iv)  $ce > rw > lw$ .

Nobody will rank the parties:

- (v)  $rw > lw > ce$ , or
- (vi)  $lw > rw > ce$ ,

for if spatial proximity is what counts how could a person who most-preferred the right-wing party prefer the left-wing party to the centre party? This is significant because when voters agree in this way about the criteria by which to rank options, cycles and the paradox of voting cannot arise. A cycle would arise if, for example, the three voters ranked the three parties in the following way:

- Voter 1  $lw > ce > rw$
- Voter 2  $rw > lw > ce$
- Voter 3  $ce > rw > lw$

Here, the left-wing party is majority-preferred to the centre party (by virtue of the first and second voters) and the centre party is majority-preferred to the right-wing party (by virtue of the first and third voters) but the right-wing party is majority-preferred to the left-wing party (by virtue of the second and third voters). But if voters judge parties in terms of spatial proximity, the second voter could not have the preference-ranking attributed to them here because if they most prefer the right-wing party they must prefer the centre party to the left-wing one. Assume that this voter were to instead rank the parties  $rw > ce > lw$ . In this case, the right-wing party would be majority-preferred to both the left-wing party and the centre party and no cycle could exist.

Of course it is implausible to assume that every voter will employ the same criteria in judging candidates or policies. Yet when even a minority of voters agree upon the criteria, the chances of a cycle arising dramatically fall (Niemi, 1969, 1983). There are two important and practical lessons to be drawn from this. The first is that democracy may best work in those situations where voters already share a common political culture: a point made many decades ago by behaviouralists like Gabriel Almond (1963). Attempts to export democracy to either supra-national bodies like the European Union where there is currently no shared political culture or to countries where there are multiple cleavages and little agreement about which of these cleavages are the most significant, may simply generate intransitive results. The second is that democracy might work best where democratic institutions encourage deliberation amongst participants prior to voting and in which there are therefore more opportunities to agree upon the criteria by which issues ought to be judged (Box 4.2).

As I have already noted, Riker (1982a, 1986) illustrates and defends his claim that voting cycles are extremely common by way of a series of historical case-studies. These include the American Civil War, the introduction of a school-construction bill in the US House of Representatives in 1956, the adoption of the Seventeenth Amendment to the US Constitution (on the direct election of senators), the trial of the servants believed to have killed Afranius Dexter in about 100 AD and C.P. Snow's fictional account of a Cambridge University college election in *The Masters*. Yet as Mackie (2003: 197–309) demonstrates, these accounts depend upon contestable interpretations of politicians'

#### Box 4.2 Deliberation and social choice

In recent years the theory and practice of democracy is said to have taken a 'deliberative turn' (Dryzek and Braithwaite, 2000: 241). Deliberative democrats argue that legitimate decision-making requires not simply the aggregation of preferences but a period of careful reflection upon and debate about those preferences prior to voting. During this period of deliberation, participants, it is argued, should not simply assert their own claims and viewpoints; they ought instead to frame their arguments in terms of common interests whilst responding to the force of the better argument. Such theoretical arguments have inspired interest in and the greater use of 'deliberative' opinion polls (Fishkin, 1991; Luskin, Fishkin and Jowell, 2002) and 'Citizen's Juries' (Crosby *et al.*, 1986). They have also led political scientists to extol mechanisms which facilitate deliberation within existing legislatures (Sunstein, 1993; Uhr, 1998; Steiner *et al.*, 2004).

Drawing upon this literature, some theorists have argued that deliberation may ameliorate some of the problems associated with aggregating preferences identified by social choice theorists (Miller, 1992; List and Pettit, 2002; Dryzek and List, 2003). The claim most often made here is that deliberation may increase 'structuration': that is the degree to which individual preferences are aligned along the same shared set of underlying dimensions. This is important because increased structuration reduces the chances that voters' preferences will generate a cycle. Deliberation may increase preference structuration in two main ways (Farrar *et al.*, 2003). Firstly, as people talk and learn from each other, they may come to adopt criteria for judging alternatives that they recognize it is conventional to use. Secondly, through careful deliberation they may influence each other's thinking and acquire more of a shared understanding of what an issue involves.

Deliberation may assist decision-making in a number of other ways. (1) As we have seen, the chances of a cycle occurring are positively related to the number of issue alternatives. A process of public deliberation may make it extremely difficult for participant's to defend proposals that are obviously intended to serve their self-interest (Elster, 1998). In this way, deliberation may lower the total number of issue alternatives to eventually be voted upon. (2) Deliberation gives participants an opportunity to examine and challenge the arguments and motives of those with opposing views. It may therefore make it easier to expose those who are strategically misrepresenting their preferences in an attempt to secure a preferred outcome. (3) Deliberation gives participants an opportunity not only to state their case but to express the intensity of their preferences over an issue. It therefore offers those making a decision an alternative to simply counting equal votes.

preference-rankings and ignore alternative and usually more obvious explanations of the same event. Furthermore, and even if we accept that these accounts do actually illustrate cases of voting cycles, agenda-control, strategic manipulation and heresthetics, it is unclear how representative they are. The very obscurity of at least some of Riker's stories suggests not only that he is extremely erudite but that he had to search long and hard for some of his examples.

In recent years, social choice theorists have searched for the existence of voting cycles in, for example, Presidential elections (Niemi and Wright, 1987), Dutch Parliamentary elections (Van Deemen and Vergunst, 1988), and legislatures (Stratmann, 1997). The results uniformly indicate that voting cycles are actually extremely rare (Feld and Grofman, 1986; Krehbiel and Rivers, 1990). One possible explanation of this is, as we have seen, that whilst voters and legislatures have different preferences many of them nevertheless use the same criteria to judge alternatives. In their study of Congressional voting, for example, Keith Poole and Howard Rosenthal (1997: 227) show that 85 per cent of all the roll-call votes held between 1789 and 1995 can be accounted for in terms of just two dimensions, and that 'except for two [relatively] short periods in American history when race was prominent on the agenda, whenever voting could be captured by the spatial model, a one-dimensional model does all the work'.

#### The liberal theory of democracy and markets

Riker interprets the findings of social choice theory as constituting an indictment of the 'populist' theory of democracy and a defence of its 'liberal' variant. Yet there are good reasons for believing that *if* social choice theory damns populism, it damns liberalism as well. It is an important part of the liberal theory that the prospect of having to fight and win future elections should encourage incumbent politicians to abide by their election promises and so refrain from pursuing their own interests in office. In this way, elections ensure that politicians' 'interests coincide with their duty' (Alexander Hamilton, *Federalist*, 72). But if, as Riker maintains, elections are essentially arbitrary, it is unclear what reason incumbent politicians have to pursue their constituents' interests. As Mackie (2003: 412) quotes a younger Riker (1953: 110) as observing

the process of government can be controlled by citizens only when elections are a transmission belt of ideas and decisions from the

### Box 4.3 Pareto superiority and Pareto optimality

If we cannot make interpersonal comparisons of utility, can we say anything about the potential welfare attractiveness of different distributions? The Italian economist Vilfredo Pareto (1909) argues that we can. Suppose we compare two distributions and find that at least one person regards themselves as being better-off in the second distribution and none regard themselves as being worse-off. Without having to make any interpersonal comparison of utility, we can say that the second state is preferable, or Pareto-superior, to the first.

In the diagram opposite, Alice's utility from the consumption of  $x$  is measured along the horizontal and Ben's on the vertical axis. The curve  $M-M'$  shows the maximum possible utility levels each can achieve given a fixed supply of the good.  $M$  shows that point where Ben consumes all of the good and  $M'$  that point at which Alice consumes all of the good.

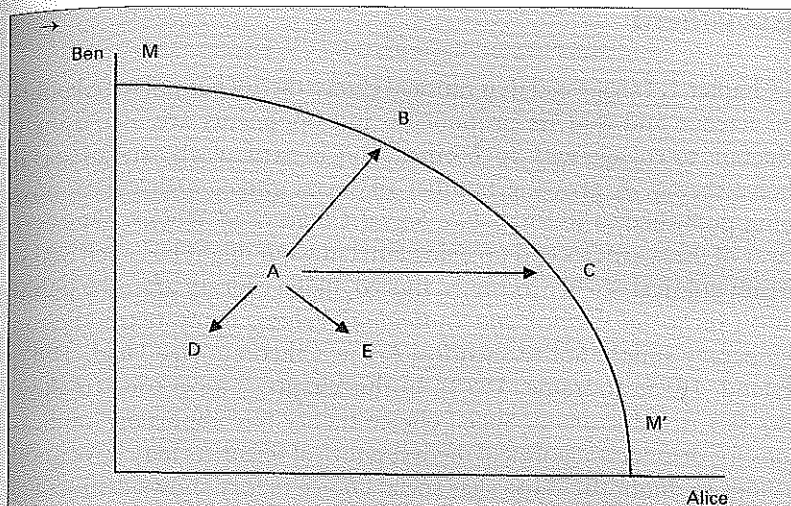
The move from  $A$  to  $B$  is Pareto-superior because Alice and Ben are both better-off in  $B$ .

The move from  $A$  to  $C$  is also Pareto-superior because Alice is better-off in  $C$  and Ben no worse-off.

The move from  $A$  to  $D$  is Pareto-inferior because both are worse-off in  $D$ .

The move from  $A$  to  $E$  is Pareto-incomparable because Alice is better-off and Ben worse-off. It may well be that *total* utility at  $E$  is higher than it is at  $A$  but the impossibility of making interpersonal comparisons of utility is such that, Pareto argued, we cannot know. We simply cannot say anything about the relative attractiveness of  $A$  and  $E$  using the Pareto criterion.

We can also identify a set of Pareto-optimal points running from  $M$  to  $M'$  from which it is not possible to make any Pareto-superior moves. This point of Pareto optimality or, as he preferred, maximum



opportunity, is a point 'from which it is impossible to move a very small distance, in such a way that the opportunities of the individuals, except for some which remain constant, all increase' (Pareto, quoted in Feldman, 1998: 6). This is important because the first fundamental theorem of welfare economics holds that the outcome of any perfectly competitive market process will be a Pareto optimum (see Stiglitz, 1996: 27-44). Although described as only requiring the acceptance of some 'very weak' ethical precepts by Buchanan and Tullock (1962: 92), the application of the Pareto principle would actually rule out any government redistribution. It is difficult to think of a more politically charged welfare principle (Dowding and Hindmoor, 1998).

voters to the rules. If elections have no relevance to public policy, then the policy makers need not respect the electoral sanction.

If democracy in both its populist and liberal variants is flawed, does this mean that market decision-making ought to be substituted for collective and democratic decision-making? This is the conclusion drawn by Charles Rowley (1993: xiii) who suggests that Arrow's theorem 'provides incontrovertible support for market processes and encouragement for those who seek . . . the minimal state'. Yet as Arrow (1997) has himself emphasized, the impossibility theorem applies as much to statements about the welfare implications of market decision-making as it does to democracy. It is often claimed

that competitive markets are valuable and ought to be promoted because they maximize consumer welfare. To take just one example, Adam Smith (1776, Vol. 1: 12) argues that the 'propensity to truck, barter and exchange' are inherent features in human nature and that individuals will be led, as if by an 'invisible hand', to promote the common good whilst pursuing their self-interest. Yet whilst it might still be possible to defend competitive markets as resulting in Pareto-optimal distributions (Box 4.3), or as maximizing individual freedom, Arrow's theorem suggests that we cannot say whether markets maximize welfare. For how can we aggregate individual utility in such a way as to be able to then say that one outcome maximizes welfare but another does not?

## Assessment

In Chapter 1, I suggested that rational choice theory has polarized political science and that its proponents and opponents are increasingly reluctant to engage with each other. This is particularly true of social choice theory. A growing number of social choice articles are now being published in journals like *Social Choice and Welfare*, but ever fewer of these articles end up being cited in what might be called mainstream political science journals. Indeed, half a century after the publication of *Social Choice and Individual Values*, many textbooks on democracy simply fail to mention Arrow's theorem (see Held, 1987; Hyland, 1995; Touraine, 1998). In this chapter I have tried to show why this is unfortunate. Social choice theory has important implications for democratic political theory; political scientists may have good reasons to doubt the claim that social choice theory shows democracy to be 'impossible', but this is a claim that they ought to address themselves to and use to sharpen their own definitions and discussions of democracy. As we have already seen, if democracy is understood to require not only the aggregation of preferences but deliberation and reflection about those preferences then the force of Arrow's argument is blunted.

Social choice theorists might also benefit from drawing more systematically on existing democratic theory in order to provide a context for their own work. Arrow and Riker have each been *interpreted* as arguing not only that all methods of voting are flawed, but that democracy itself is 'impossible'. This is an exaggeration. As we have seen, Arrow carefully avoids drawing too many implications from his work whilst Riker argues that social choice theory only poses problems for the 'populist' theory of democracy. It is also misleading. There is more to social choice theory than arguments about whether or not democracy is impossible (Box 4.4). The earlier discussion of the relative merits of the Borda method demonstrate that social choice theory has a potentially practical pay-off for those designing electoral systems. Yet it is the argument that democracy is impossible which inevitably grabs the academic headlines. Looked at in another way, this argument is, however, far from being original. As I observed at the very start of the chapter, theorists have for many centuries delighted in finding faults with democracy, but you do not have to believe that democracy is a perfect system of decision-making in order to believe that it is sometimes the most appropriate system. Winston Churchill once famously said that democracy 'is the worst form of Government,

### Box 4.4 Liberalism and social choice

There is at least one other area of social choice theory worth drawing attention to here. Using the same nomenclature as Arrow, Amartya Sen (1970) argues that the liberal principle that people ought to be allowed to be decisive over particular social issues cannot be reconciled with the Pareto principle. Sen's argument and much of the subsequent literature revolves around the once controversial publication of D.H. Lawrence's *Lady Chatterley's Lover*. There are two people, A (the prude) and B (the lewd). Depending on whether or not A and B read the book there are four possible outcomes:

- 1 A and B read the book;
- 2 A reads the book and B does not read the book;
- 3 B reads the book and A does not read the book; or
- 4 Neither reads the book.

A has the following preference ordering:  $4 > 2$  (A would rather that he have to read the book than see B wallow in its depravity)  $> 3 > 1$ .

B has the following preference ordering:  $1 > 2$  (B would take pleasure from A having to read the book and from, perhaps, secretly enjoying it)  $> 3 > 4$ .

It is obvious from this preference-ranking that outcome 2 is Pareto-superior to outcome 3. Both A and B, that is, prefer an outcome in which A reads the book and B does not read the book to one in which B reads the book and A does not read the book. Yet, according to Sen, liberalism demands that both A and B be allowed to choose whether or not to read the book. In this case, this means that A should not read the book and B should read the book. Yet this outcome, 3, is Pareto-inferior to 2.

Sen's 'liberal paradox' has generated a great deal of discussion and argument (Buchanan, 1996; or Sen, 1976, for an early review). The most powerful rejoinder to Sen is, however, that offered by Brian Barry (1989: 81). He argues that Sen simply misconstrues the requirements of liberalism. 'Liberal principles do not say in a context like the *Lady Chatterley* case who should read what; rather, liberalism is a doctrine about who should have a right to decide who reads what'. If the prude freely chooses to read the book and the lewd chooses not to – perhaps as part of a Pareto-efficient deal between them – the notion that liberalism has been violated is, Barry concludes, 'pure fantasy'.

except for all those other forms of Government which have been tried'. As a one-line riposte to Riker's (1982a) 300-page book this takes some beating.