11. Introduction to Discounted Cash Flow Analysis and Financial Functions in Excel

John Herbohn and Steve Harrison

The financial and economic analysis of investment projects is typically carried out using the technique of discounted cash flow (DCF) analysis. This module introduces concepts of discounting and DCF analysis for the derivation of project performance criteria such as net present value (NPV), internal rate of return (IRR) and benefit to cost (B/C) ratios. These concepts and criteria are introduced with respect to a simple example, for which calculations using MicroSoft Excel are demonstrated.

1. CASH FLOWS, COMPOUNDING AND DISCOUNTING

Discounted Cash Flow (DCF) analysis is the technique used to derive economic and financial performance criteria for investment projects. It is important to review some of the basic concepts of DCF analysis before proceeding to topics such as cost0benefit analysis (CBA), financial analysis (FA), linear programming and the estimation of non-market benefits.

Cash flow analysis is simply the process of identifying and categories of cash flows associated with a project or proposed course of action, and making estimates of their values. For example, when considering establishment of a forestry plantation, this would involve identifying and making estimates of the cash outflows associated with establishing the trees (e.g. the cost of buying or leasing the land, purchasing seedlings, and planting the seedlings), maintaining the plantation (such as cost of fertilizer, labour, pruning and thinning) and harvesting. As well, it would be necessary to estimate the cash inflows from the plantation through sales of thinnings and timber at final harvest.

Discounted cash flow analysis is an extension of simple cash flow analysis and takes into account the time value of money and the risks of investing in a project. A number of criteria are used in DCF to estimate project performance including Net Present Value (NPV), Internal Rate of Return (IRR) and Benefit-Cost (B/C) ratios.

Before discussing criteria to measure project performance, it is necessary to introduce some concepts and procedures with respect to compounding and discounting. Let us begin with the concepts of simple and compound interest. For the moment, consider the interest rate as the cost of capital for the project.

Suppose a person has to choose between receiving $1000 now, or a guaranteed $1000 in 12 months time. A rational person will naturally choose the former, because during the intervening period he or she could use the $1000 for profitable investment (e.g. earning interest in the bank) or desired consumption. If the $1000 were invested at an annual interest rate of 8%, then over the year it would earn $80 in interest. That is, a principal of $1000 invested for one year at an interest rate of 8% would have a future value of $1000 (1.08) or $1080.

The $1000 may be invested for a second year, in which case it will earn further interest. If the interest again accrues on the principal of $1000 only, it is known as *simple interest*. In this case the *future value* after two years will be $1160. On the other hand, if interest in the second year accrues on the whole $1080, known as *compound interest*, the future value will be $1080 (1.08) or $1166.40. Most investment and borrowing situations involve compound interest, although the timing of interest payments may be such that all interest is paid before further interest accrues.

The future value of the $1000 after two years may also be derived as

$$\$1000 \ (1.08)^2 = \$1166.40$$

In general, the future value of an amount $a, invested for n years at an interest rate of i, is $a $(1+i)^n$, where it it to be noted that the interest rate i is expressed as a decimal (e.g. 0.08 and not 8 for an 8% rate).

The reverse of compounding – finding the present-day equivalent to a future sum – is known as *discounting*. Because $1000 invested for one year at an interest rate of 8% would have a value of $1080 in one year, the *present value* of $1080 one year from now, when the interest rate is 8%, is $1080/1.08 = $1000. Similarly, the present value of $1000 to be received one year from now, when the interest rate is 8%, is

$$\$1000/1.08 = \$925.93$$

In general, if an amount $a is to be received after n years, and the annual interest rate is i, then the present value is

$$\$a\,/\,(1+i)^n$$

The above discussion has been in terms of amounts in a single year. Investments usually incur costs and generate income in each of a number of years. Suppose the amount of $1000 is to be received at the end of each of the next four years. If not discounted, the sum of these amounts would be $4000. But suppose the interest rate is 8%. What is the present value of this stream of amounts? This is obtained by discounting the amount at the end of each year by the appropriate discount factor then summing:

$1000/1.08 + $1000/1.08^2 + $1000/1.08^3 + $1000/1.08^4

= $1000/1.08 + $1000/1.1664 + $1000/1.2597 + $1000/1.3605

= $925.93 + $857.34 + $793.83 + $735.03

= $3312.13

The *discount factors* – $1/1.08^t$ for t = 1 to 4 – may be calculated for each year or read from published tables. It is to be noted that the present value of the annual amounts is progressively reduced for each year further into the future (from $925.93 after one year to $735.03 after four years), and the sum is approximately $700 less than if no discounting (a zero discount rate) had been applied.

2. DEFINITION OF ANNUAL NET CASH FLOWS

DCF analysis is applied to the evaluation of investment *projects*. Such a project may involve creation of a terminating asset (such as a forestry plantation), infrastructure (such as a road or plywood plant) or research (including scientific and socio-economic research). Any project may be regarded as generating cash flows. The term *cash flow* refers to any movement of money to or away from an investor (an individual, firm, industry or government). Projects require payments in the form of capital outlays and annual operating costs, referred to as cash outflows. They give rise to receipts or revenues, referred to as project benefits or cash inflows. For each year, the difference between project benefits and capital plus operating costs is known as the net cash flow for that year. The net cash flow in any year may be defined as

$$a_t = b_t - (k_t + c_t)$$

where b_t are project benefits in year t
k_t are capital outlays in year t
c_t are operating costs in year t.

It is to be noted that when determining these net cash flows, expenditure items and income items are timed for the point at which the transactions takes place, rather than the time at which they are used. Thus for example expenditure on purchase of an item of machinery rather than annual allowances for depreciation would enter the cash flows. It is to be further noted that cash flows should not include interest payments. The discounting procedure in a sense simulates interest payments, so to include these in the operating costs would be to double-count them.

Example 1

A project involves an immediate outlay of $25,000, with annual expenditures in each of three years of $4000, and generates revenue in each of three years of $15,000. These cash flows data may be set out, and annual net cash flows derived, as in Table 1.

Table 1. Annual cash flows for a hypothetical project

Year	Project benefits ($)	Capital outlays ($)	Operating costs ($)	Net cash flow ($)
0	0	25000	2000	-27000
1	15000	0	4000	11000
2	15000	0	4000	11000
3	15000	0	2000	13000

Two points may be noted about these cash flows. First, the capital outlay is timed for Year 0. By convention this is the beginning of the first year (i.e. right now). On the other hand, only half of the first year's operating costs are scheduled for the Year 0 (the beginning of the first year). The remaining half of the first year's operating costs plus the first half of the second year's operating costs are scheduled for the end of the first year (or, equivalently, the beginning of the second year). In this way, operating costs are spread equally between the beginning and the end of each year. (The final half of the third year's operating costs are scheduled for the end of Year 3.) In the case of project benefits, these are assumed to accrue at the end of each year, which would be consistent with lags in production or payments. These within-year timing issues are unlikely to make a large difference to overall project profitability, but it is useful to make these timing assumptions clear.

A second point to note about Table 1 is that net cash flows (second column less third plus fourth column) are at first negative, but then become positive and increase over time. This is a typical pattern of well-behaved cash flows, for which performance criteria can usually be derived without computational difficulties.

3. PROJECT PERFORMANCE CRITERIA

Let us now consider a number of project performance criteria which can be obtained by discounted cash flow analysis. These criteria will be defined, then derived for the cash flow data of Example 1.

Net present value

The net present value (NPV) is the sum of the discounted annual cash flows. For the example, taking an interest rate of 8%, this is

$$NPV = a_0 + a_1/(1+i) + a_2/(1+i)^2 + a_3/(1+i)^3$$

A project is regarded as economically desirable if the NPV is positive. The project can then bear the cost of capital (the interest rate) and still leave a surplus or profit. For the example,

$$NPV = -27000 + 11000/(1.08) + 11000/(1.08)^2 + 13000/(1.08)^3$$

$$= -27000 + 11000/1.1664 + 11000/1.2597 + 13000/1.3605$$

$$= \$2935.73$$

The interpretation of this figure is that the project can suppport an 8% interest rate and still generate a surplus of benefits over costs, after allowing for timing differences in these, of approximately $3000.

Net future value

An alternative to the net present value is the net future value (NFV), for which annual cash flows are compounded forward to their value at the end of the project's life. Once the NPV is known, the NFV may be obtained indirectly by compounding forward the NPV by the number of years of the project life. For Example 1, the net future value is

$$NFV = NPV (1.08)^3 = \$2935.73 \times 1.3605 = 3994.06$$

Internal rate of return

The internal rate of return (IRR) is the interest rate such that the discounted sum

of net cash flows is zero. If the interest rate were equal to the IRR, the net present value would be exactly zero. The IRR cannot be determined by an algebraic formula, but rather has to be approximated by trial and error methods. For the above example, we know that the IRR is somewhere above 8%. Deriving the NPV with a range of discount rates would reveal that the IRR falls between 13% and 14%, but closer to the latter. In practice, a financial function can be called up to perform the trial-and-error calculations. It would be found in this case that the IRR is about 13.8%.[1]

The IRR is the highest interest rate which the project can support and still break even. A project is judged to be worthwhile in economic terms if the internal rate of return is greater than the cost of capital. If this is the case, the project could have supported a higher rate of interest than was actually experienced, and still made a positive payoff. In the above case, the project would be profitable provided the cost of capital was less than 13.8%.

The IRR as a criterion of project profitability suffers from a number of theoretical and practical limitations. On the theoretical side, it assumes that the same rate of return is appropriate when the project is in surplus and when it is in deficit. However, the cost of borrowed funds may be quite different to the earning rate of the firm. It could be more appropriate to use two rates when determining the IRR. The actual cost of capital could be used when the project is in deficit, and the earning rate (unknown, to be determined by trial-and-error) could be applied when the project is in surplus. This would give a better indication of the earning rate of the project to the firm or government.

From a practical viewpoint, the IRR may not exist or it may not be unique. This problem may be examined in terms of the *NPV profile*, a graph of NPV versus the rate of interest. When the IRR is well behaved, this profile takes the form as in Figure 1. As the interest rate increases the NPV falls, being zero where the NPV curve crosses the interest rate axis; the IRR corresponds with this discount rate.

Consider a project for which the net cash flow in each year (including Year 0) is positive. Regardless of the interest rate, the NPV will never be zero, so it will not be possible to determine an IRR. Similarly, a project with a large initial capital outlay and for which future benefits are relatively small or negative may not have a positive NPV regardless of the interest rate, so again the curve for the NPV profile may never cross the interest rate axis.

If a project generates runs of positive and negative net cash flows, the NPV profile may take the form of a roller-coaster curve, crossing the interest rate axis in several places.[2] This indicates multiple internal rates of return, one at each interest rate where NPV is zero. It is then by no means clear which if any of the rates we should choose to call the IRR. Further, for some sections of the NPV profile (those that are upward sloping), the NPV is increasing as the interest rate increases. This implies that the greater the cost of capital the more profitable the project. Clearly, multiple internal rates of return and perverse relationships between the NPV and the discount rate are not very satisfactory.

Benefit-to-cost ratios

A number of benefit-cost ratio concepts have been developed. For simplicity, we will consider only two concepts, referred to as the gross and net B/C ratio and defined respectively as

Gross B/C ratio = PV of benefits/(PV of capital costs + PV of operating costs)

Net B/C ratio = (PV of benefits – PV of operating costs)/PV of capital costs

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[1] Calculation of internal rate of return in fact involves solving a polynomial equation, and efficient solution methods such as Newton's approximation are used in computer packages.

[2] Mathematically, the polynomial equation defining the NPV can have up as many 'roots' or solutions as there are turning points in NPV values (changes from positive to negative or negative to positive). A project which has alternating runs of positive and negative cash flows is a candidate for problems with estimation of the IRR.

For the above project, the present value of capital outlays is $25,000, since outlays are made immediately and as a single amount. The present values of project benefits and operating costs are:

PV of benefits = $15000/1.08 + $15000/1.08^2 + $15000/1.08^3 = $38656.45

PV of operating costs = $2000 + $4000/1.08 + $4000/1.08^2 + $2000/1.08^3 = $9418.38

Hence the benefit-to-cost ratios are

$$\text{gross B/C ratio} = \frac{38,656.45}{25,000 + 9418.38} = 1.12$$

$$\text{net B/C ratio} = \frac{38,656.45 - 9418.38}{25,000} = 1.45$$

A project is judged to be worthwhile in economic terms if it has a B/C ratio is greater than unity, i.e. if the present value of benefits exceeds the present value of costs (in gross or net terms). If one of the above ratios is greater than unity, then the other will be greater than unity also. In the above example, the ratios are greater than unity, indicating that the project is worthwhile on economic grounds. It is not clear on logical grounds which of the ratios is the most useful.
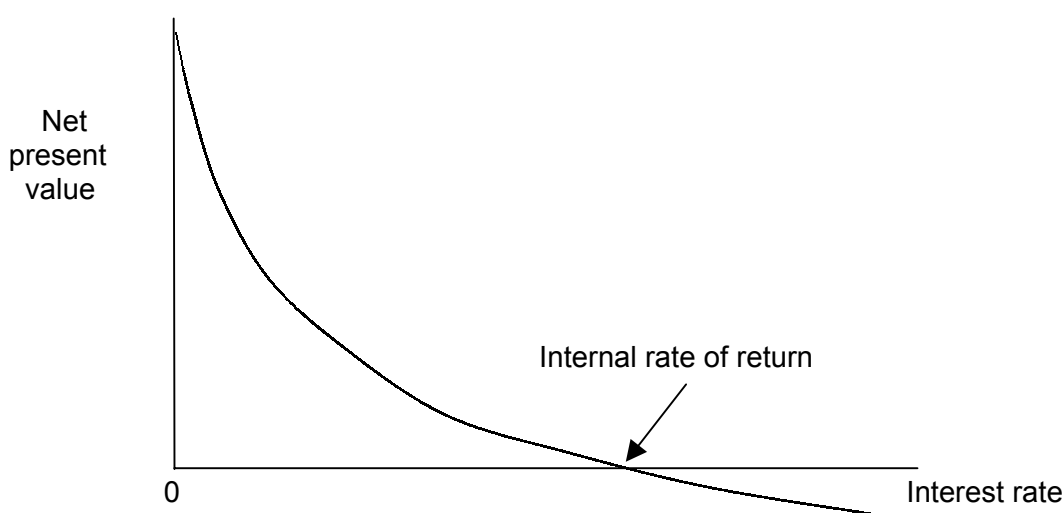

Figure 1. The NPV profile for a project

The payback period

The payback period (PP) is the number of years for the projects to break even, i.e. the number of years for which discounted annual net cash flows must be summed before the sum becomes positive (and remains positive for the remainder of the project's life). The payback period for a project with the above net cash flows can be determined as in the following table. From this table, it is apparent that the sum of discounted net cash flows does not become positive until Year 3, so the payback period is three years.

The payback period indicates the number of years until the investment in a project is recovered. It is a useful criterion for a firm with a short planning horizon, but does not take account of all the information available, i.e. the net cash flows for years beyond the payback period.

The peak deficit

This is a measure of the greatest amount that the project 'owes' the firm or government, i.e. the furtherest 'in the red' it goes. In the above table, the largest negative value is -$27,000, so this is the peak deficit. Peak deficit is a useful measure in terms of financing a project, since it indicates the total amount of finance that will be required.

Table 2. Derivation of 'project balances' and payback period

Year	Net cash flow ($)	PV of net cash flow ($)	Cumulative PV of net cash flow (or project balance) ($)
0	-27000	-27000.00	-27000
1	11000	10185.19	-27000 + 10185.19 = -16814.81
2	11000	9430.73	-16814.81 + 9340.73 = -7384.09
3	13000	10319.82	-7384.09 + 10319.82 = 2935.73

Review of DCF performance criteria

The most commonly used discounted cash flow performance criteria – NPV, IRR, B/C ratios and payback period – may be summarised as in Table 3. The various criteria are closely related, but measure slightly different things. In this respect, they tend to complement one another, so that it is common to estimate and report more than one of the measures.

Perhaps the most useful measure, and the one most often reported, is the net present value. This tells the total payoff from a project. A limitation of the NPV is that it is not related to the size of the project. If one project has a slightly lower NPV than another, but the capital outlays required are much lower, then the second project will probably be the preferred one. In this sense, a rate of return measure such as the IRR is also useful.

The payback period and peak deficit are useful supplementary project information for decision-makers. They have greater relevance for private sector investments, for firms which cannot afford long delays in recouping expenditure, and where careful attention must be paid to the total amount of funds that will need to be committed to the project to remain solvent.

Table 3. Summary of definitions of main DFC performance criteria

Net present value (NPV)	$\sum_{t=1}^{p} a_t / (1+i)^t$, where t is time, c_t is the annual net cash flow, i is the discount rate p is the planning horizon
Internal rate of return (IRR)	The value of r such that $\sum_{t=1}^{p} a_t / (1+r)^t = 0$
Benefit to cost ratio (B/C)	Present value of project benefits / present value of project costs
Payback period	Number of periods until NPV becomes (and remains) positive

4. USING EXCEL FINANCIAL FUNCTIONS

It is common to use a spreadsheet program such as Excel to undertake financial and cost benefit analyses. Excel has a range of financial functions that can be useful (as listed in Apppendix 1). Two of the most useful and commonly used financial functions in Excel are those to calculate Net Present Value ('NPV') and Internal Rate of Return ('IRR'). This section illustrates the use of two of the NPV and IRR functions, applied to the data in Example 1, in an Excel worksheet:

Using the 'NPV' and 'IRR' functions

The 'NPV' function calculates the net present value of an investment (i.e. series of cash flows) by using a discount rate and a series of future payments (negative

values) and income (positive values). The syntax for this function is:

NPV(discount rate, value1, value2 ….)

It is important to remember that the NPV calculation is for cashflows starting at the end of period 1. If the first cash flow occurs at the beginning of period 1 (i.e. period 0), the first value must be added to the NPV result, not included in the value 'arguments' of the function.

Thus to value the NPV for example 1, the following formula would be inserted into a spreadsheet cell:

=NPV(8%, 11000, 11000, 13000) - 27000

Note that the cashflow sequence used in the NPV calculation is for years 1 to 3. The value for year 0 (-$27000) is added to the NPV result.

An alternative means of calculating NPV and IRR in the above example would be to replace the string of values with cell references to where the values are located within the workbook. This is illustrated in the worksheet below. This approach has the advantage of allowing changes to the made to the cashflows (in Column E) which then automatically flow through to the calculation of the NPV. This is particularly useful when it is desired to investigate how changes in cash flows affect NPV. Similarly, the discount rate given in the formula (in this case 8%) could be replaced with a cell reference. In this case the effects of a change in discount rate on NPV could be observed simply changing the cell value rather than the formula. In the worksheet below, the '8%' in the formula for NPV in cell B8 would be replaced by a reference to cell B12. The new formula would be '=NPV(B12, E3:E5)+E2'.

Another approach to calculating NPV is to simply calculate the present value of cash flows for each year using the discounting formula given in Equation 1 and then simply add these individual figures. This approach has a number of advantages in more complex applications – particularly those involved complex financial models with variable periods of cash flows and where discount rates change over the life of a project. This is illustrated in the workbook below:

	A	B	C	D	E	F
1	Year	Project benefits ($)	Capital outlays ($)	Operating costs ($)	Net cashflow ($)	
2	0	0	25000	2000	-27000	
3	1	15000	0	4000	11000	
4	2	15000	0	4000	11000	
5	3	15000	0	2000	13000	
6						
7						
8						
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15						

The formula in Cells F2, F3, F4 and F5 calculate the present value of the net cash flow in each of the years 0 through to 3 using the formula given in Equation 1. These present values for each of the years are then summed in cell F6 to give the Net Present Value of the series of cash flows associated with the project. The resulting NPV in cell F6 is identical to the NPV calculated in cell B8. Note that an absolute cell address for the discount rate has been used (B12) using '$'. This allows the formula in cell F2 to be simply copied into cells F3, F4 and F5.

APPENDIX 1: FINANCIAL FUNCTIONS IN EXCEL

ACCRINT   Returns the accrued interest for a security that pays periodic interest

ACCRINTM   Returns the accrued interest for a security that pays interest at maturity

AMORDEGRC   Returns the depreciation for each accounting period

AMORLINC   Returns the depreciation for each accounting period

COUPDAYBS   Returns the number of days from the beginning of the coupon period to the settlement date

COUPDAYS   Returns the number of days in the coupon period that contains the settlement date

COUPDAYSNC   Returns the number of days from the settlement date to the next coupon date

COUPNCD   Returns the next coupon date after the settlement date

COUPNUM   Returns the number of coupons payable between the settlement date and maturity date

COUPPCD   Returns the previous coupon date before the settlement date

CUMIPMT   Returns the cumulative interest paid between two periods

CUMPRINC   Returns the cumulative principal paid on a loan between two periods

DB   Returns the depreciation of an asset for a specified period using the fixed-declining balance method

DDB   Returns the depreciation of an asset for a specified period using the double-declining balance method or some other depreciation method

DISC   Returns the discount rate for a security

DOLLARDE   Converts a dollar price, expressed as a fraction, into a dollar price, expressed as a decimal number

DOLLARFR   Converts a dollar price, expressed as a decimal number, into a dollar price, expressed as a fraction

DURATION   Returns the annual duration of a security with periodic interest payments

EFFECT   Returns the effective annual interest rate

FV   Returns the future value of an investment

FVSCHEDULE   Returns the future value of an initial principal after applying a series of compound interest rates

INTRATE   Returns the interest rate for a fully invested security

IPMT   Returns the interest payment for an investment for a given period

IRR   Returns the internal rate of return for a series of cash flows

ISPMT   Calculates the interest paid during a specific period of an investment.

MDURATION   Returns the Macauley modified duration for a security with an assumed par value of $100

MIRR   Returns the internal rate of return where positive and negative cash flows are financed at different rates

NOMINAL   Returns the annual nominal interest rate

NPER   Returns the number of periods for an investment

NPV   Returns the net present value of an investment based on a series of periodic cash flows and a discount rate

ODDFPRICE   Returns the price per $100 face value of a security with an odd first period

ODDFYIELD   Returns the yield of a security with an odd first period

ODDLPRICE   Returns the price per $100 face value of a security with an odd last period

ODDLYIELD   Returns the yield of a security with an odd last period

PMT   Returns the periodic payment for an annuity

PPMT   Returns the payment on the principal for an investment for a given period

PRICE   Returns the price per $100 face value of a security that pays periodic interest

PRICEDISC   Returns the price per $100 face value of a discounted security

PRICEMAT Returns the price per $100 face value of a security that pays interest at maturity

PV Returns the present value of an investment

RATE Returns the interest rate per period of an annuity

RECEIVED Returns the amount received at maturity for a fully invested security

SLN Returns the straight-line depreciation of an asset for one period

SYD Returns the sum-of-years' digits depreciation of an asset for a specified period

TBILLEQ Returns the bond-equivalent yield for a Treasury bill

TBILLPRICE Returns the price per $100 face value for a Treasury bill

TBILLYIELD Returns the yield for a Treasury bill

VDB Returns the depreciation of an asset for a specified or partial period using a declining balance method

XIRR Returns the internal rate of return for a schedule of cash flows that is not necessarily periodic

XNPV Returns the net present value for a schedule of cash flows that is not necessarily periodic

YIELD Returns the yield on a security that pays periodic interest

YIELDDISC Returns the annual yield for a discounted security. For example, a Treasury bill

YIELDMAT Returns the annual yield of a security that pays interest at maturity