5 FUNDAMENTALS OF HEAT TRANSFER

Abstract

Energy from most resources is extracted in the form of heat by combustion. The heat or thermal energy is then transferred to other media, such as water to generate steam or a gas to heat it up. Steam is used to run a steam turbine, and hot gas is used in a gas turbine. The heat transfer mechanism should be understood in order to design such a system. Various heat transfer mechanisms and factors that affect the transfer of heat from one medium to another are discussed in this chapter.

5.1 Introduction

The goal of heat transfer is to predict the rate at which energy is transferred between various bodies of material. This energy transfer is calculated in terms of heat. There are three modes of heat transfer: conduction, convection, and radiation. Generally all three modes of heat transfer are important when calculating energy transfer. In any normal situation, all three modes of heat transfer may not occur simultaneously, or one mode may dominate other modes of heat transfer. Unless the temperatures are high, the effect of radiative heat transfer will be minimal. We will first consider these three modes of heat transfer separately and then will integrate them for a system.

5.2 Thermal Conduction

The flow of heat by conduction occurs when atoms and molecules within a substance collide with each other resulting in a transfer of kinetic energy that manifest in the form of heat. This transfer of kinetic energy from the hot to the cold side is called a flow of heat through conduction. Therefore, the thermal conduction

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occurs between parts of a continuum. Different materials transfer heat by conduction at different rates, which is measured by the material's **thermal conductivity**.

The heat transfer rate is usually expressed by the symbol "q". We first look at conduction heat transfer rate and its mathematical representation. If a temperature gradient exists in a body, there will be energy transfer from the high-temperature region to the low temperature region. It should be noted that although we are interested in total heat transfer, heat is not a directly measurable quantity. It is estimated from the knowledge of temperature which can be measured directly using various instruments such as a thermocouple. The energy that is transferred by conduction per unit area is proportional to the temperature gradient.

$$\frac{q}{A} \propto \frac{dT}{dx} \tag{5.1}$$

When a constant is inserted in the above equation it becomes:

$$q = -kA\frac{dT}{dx} \tag{5.2}$$

where

q = Heat transfer rate, Btu/h or W
k = Thermal conductivity, Btu/(h ft F) or W/(m C)
A = Area normal to the heat flow, ft^2 or m^2
T = Temperature, °F or °C
$\frac{dT}{dx}$ = Temperature gradient, °F/ft or °C/m.

Equation (5.2) is best known as the Fourier equation that expresses steady state heat conduction in one dimension. Equation (5.2) incorporates a negative sign because q flows in the positive direction of x when dT/dx is negative.

Consider a wall of thickness L whose two sides are assumed to be at uniform temperatures T_1 and T_2 (see Fig. 5.1). If the thermal conductivity, the heat transfer rate, and the area are constant, Eq. (5.2) may be integrated to obtain the rate of heat flow across the wall as follows:

$$\int_{x_1}^{x_2} q\,dx = \int_{T_1}^{T_2} -k\,A\,dT \tag{5.3}$$

$$q(x_2 - x_1) = -k\,A\,(T_2 - T_1) \tag{5.4}$$

$$q = \frac{-k\,A\,(T_2 - T_1)}{(x_2 - x_1)} \tag{5.5}$$

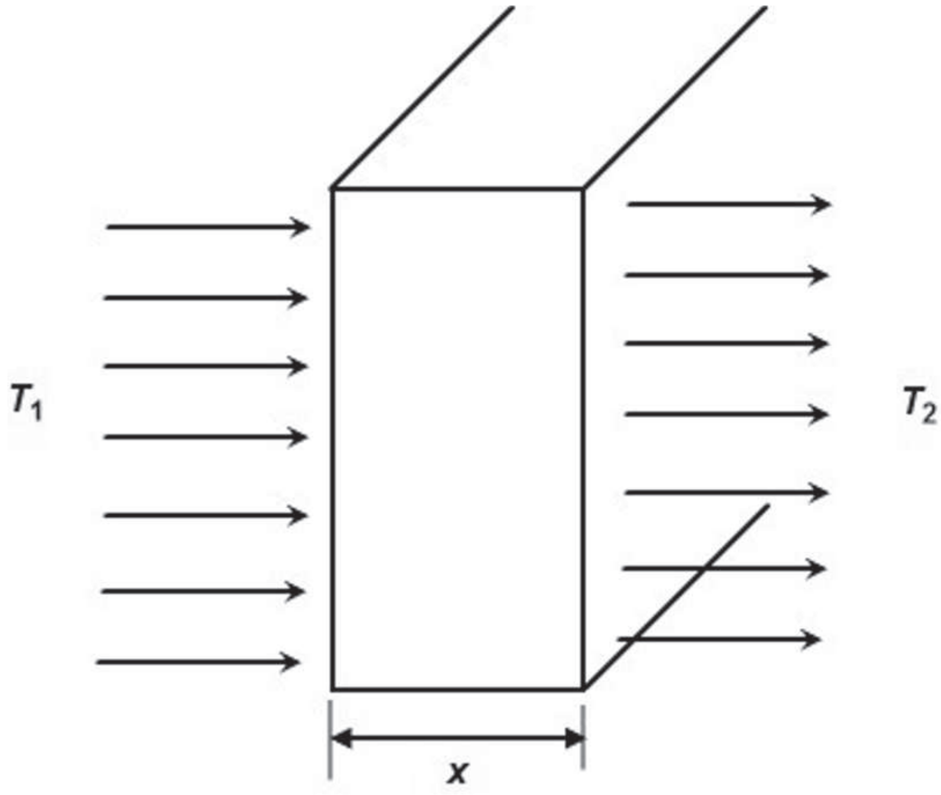

Fig. 5.1. Heat transfer through a solid object.

Example 5.1

What is the rate of heat loss per unit area of a 0.3 inch thick window if the room temperature is 20°C and the outside temperature is 0°C.

Solution

Assume that the problem is one dimensional and the window is made out of regular single pane glass. This is a good assumption since the window thickness is much less than the area of the window. In Table 5.1, the thermal conductivity of glass is given as 0.78 W/m °C. Thus the rate of heat loss per unit area is:

$$\frac{q}{A} = -k\frac{\partial T}{\partial x} \approx -k\frac{\Delta T}{\Delta x} = -0.78\, W/(mC)\frac{20-0}{0.00762} = -2047.24\frac{W}{m^2}$$

Table 5.1. Thermal conductivity of various materials.

Material	**k(W/m °C)**	**k(Btu/h ft °F)**
Aluminum	202	117
Copper	385	223
Iron	73	54
Lead	35	25
Nickel	93	54
Silver	410	237
Steel	43	20.3
Glass	0.78	0.45
Marble	2.08	1.2
Wood	0.17	0.096
Water	0.556	0.327
Air	0.0242	0.0139
CO_2	0.0146	0.00844
Helium	0.141	0.081
Hydrogen	0.175	0.101
H_2O (v)	0.0206	0.0119

Thermal conductivity for various building materials is given in Appendix V.

5.3 Modeling of Heat Transfer by Conduction

If a body of material has a temperature that is changing with time which may or may not be due to heat sources within the body, then one can model the system by using a detailed energy balance. The heat can flow in all three directions depending on the system. First, the energy balance equation is derived by considering heat flow in all three directions. This equation then can be simplified to address heat flow in a one-dimensional body. A rectangular parallelepiped volume element shown in Fig. 5.2 is considered for the energy balance. Assume that the solid thermal conductivity (k), specific heat (c), and the density (ρ) are independent of temperature. A small volume element of dimension $\Delta x \times \Delta y \times \Delta z$ is considered for energy balance. The energy balance equation of this element gives:

Rate of heat input + Rate of heat generation = Rate of heat output + Rate of accumulation of heat.

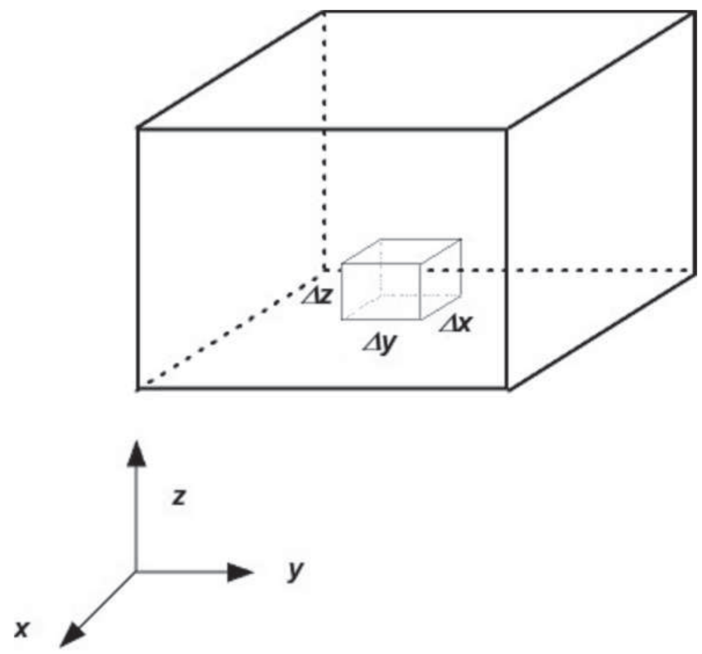

Fig. 5.2. Three dimensional heat transfers by conduction in a parallelogram.

The rate at which heat flows across an infinitesimal area in a direction normal to the area element is given by the Fourier equation (Eq. (5.2)). If the temperature gradient in the x direction is given by $\partial x/\partial T$, then the rate of heat flow in the direction of x across a plane of area $\Delta y \times \Delta z$ normal to the axis of x and passing through the mid-point of the volume element is given by:

$$q_{xx}\Big|_x = -k(\Delta y \times \Delta z)\frac{\partial T}{\partial x}\Big|_x \tag{5.6}$$

Similarly, the rates of heat flow in y and z directions are given by Eqs. (5.7) and (5.8), respectively.

$$q_{yy}\Big|_y = -k(\Delta z \times \Delta x)\frac{\partial T}{\partial y}\Big|_y \tag{5.7}$$

$$q_{zz}\Big|_z = -k(\Delta x \times \Delta y)\frac{\partial T}{\partial z}\Big|_z \tag{5.8}$$

The rate of heat flow out of the element in x, y, and z directions can be expressed by the following equations.

$$q_{xx}\Big|_{x+\Delta x} = -k(\Delta y \times \Delta z)\frac{\partial T}{\partial x}\Big|_{x+\Delta x} \tag{5.9}$$

$$q_{yy}\Big|_{y+\Delta y} = -k(\Delta z \times \Delta x)\frac{\partial T}{\partial y}\Big|_{y+\Delta y} \tag{5.10}$$

$$q_{zz}\Big|_{z+\Delta z} = -k(\Delta x \times \Delta y)\frac{\partial T}{\partial z}\Big|_{z+\Delta z} \tag{5.11}$$

If the heat generation is assumed to be $\ddot{q}$ per unit volume, then the total heat generation within the control element is given by

$$Heat\ generation = \ddot{q}\left(\Delta x \times \Delta y \times \Delta z\right) \tag{5.12}$$

The heat accumulation within the body is expressed as:

$$Accumulation = \rho\, c_p \Delta x\, \Delta y\, \Delta z \frac{\Delta T}{\Delta t} \tag{5.13}$$

After substitution of these terms into the energy balance equation, we get

$$\begin{aligned} &q_{xx}\Big|_{x} + q_{yy}\Big|_{y} + q_{zz}\Big|_{z} + \ddot{q}(\Delta x \times \Delta y \times \Delta z) = \\ &q_{xx}\Big|_{x+\Delta x} + q_{yy}\Big|_{y+\Delta y} + q_{zz}\Big|_{z+\Delta z} + \rho\, c_p\, \Delta x\, \Delta y\, \Delta z \frac{\Delta T}{\Delta t} \end{aligned} \tag{5.14}$$

Rearranging,

$$\begin{aligned} &\left(q_{xx}\Big|_{x} - q_{xx}\Big|_{x+\Delta x}\right) + \left(q_{yy}\Big|_{y} - q_{yy}\Big|_{y+\Delta y}\right) + \left(q_{zz}\Big|_{z} - q_{zz}\Big|_{z+\Delta z}\right) \\ &+ \ddot{q}(\Delta x \times \Delta y \times \Delta z) = \rho\, c_p\, \Delta x\, \Delta y\, \Delta z \frac{\Delta T}{\Delta t} \end{aligned} \tag{5.15}$$

Substitution of q by Eq. (5.2), and rearranging, the following expression is obtained.

$$k(\Delta y \times \Delta z)\left(\left.\frac{\partial T}{\partial x}\right|_{x+\Delta x} - \left.\frac{\partial T}{\partial x}\right|_{x} \right) + k(\Delta z \times \Delta x)\left(\left.\frac{\partial T}{\partial y}\right|_{y+\Delta y} - \left.\frac{\partial T}{\partial y}\right|_{y} \right)$$

$$+ k(\Delta x \times \Delta y)\left(\left.\frac{\partial T}{\partial z}\right|_{z+\Delta z} - \left.\frac{\partial T}{\partial z}\right|_{z} \right) + \ddot{q}(\Delta x \times \Delta y \times \Delta z) \tag{5.16}$$

$$= \rho\, c_p\, \Delta x\, \Delta y\, \Delta z \frac{\Delta T}{\Delta t}$$

Dividing both sides by $\Delta x \times \Delta y \times \Delta z$, Eq. (5.16) becomes

$$k\frac{\left(\left.\frac{\partial T}{\partial x}\right|_{x+\Delta x} - \left.\frac{\partial T}{\partial x}\right|_{x} \right)}{\Delta x} + k\frac{\left(\left.\frac{\partial T}{\partial y}\right|_{y+\Delta y} - \left.\frac{\partial T}{\partial y}\right|_{y} \right)}{\Delta y} + k\frac{\left(\left.\frac{\partial T}{\partial z}\right|_{z+\Delta z} - \left.\frac{\partial T}{\partial z}\right|_{z} \right)}{\Delta z} + \ddot{q} = \rho\, c_p \frac{\Delta T}{\Delta t} \tag{5.17}$$

Using the first principle of calculus and in the limit of $\Delta x \to 0$, $\Delta y \to 0$, $\Delta z \to 0$, and $\Delta t \to 0$, the above equation becomes

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) + \ddot{q} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{5.18}$$

where

$\alpha = \dfrac{k}{\rho\, c_p}$, and is called the thermal diffusivity.

Equation (5.18) can be further rewritten as:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} + \frac{\partial^2 z}{\partial z^2} + \ddot{q} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{5.19}$$

For one dimensional heat transfer, say in the direction of *x* and in the absence of any internal heat generation, Eq. (5.19) can be written as:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{5.20}$$

Solution of Eq. (5.20) under various initial and boundary conditions is given by a number of researchers [1–5].

5.4 Convective Heat Transfer

When a solid material is interfaced with a gaseous or liquid medium, the heat transfer at the boundary between these two materials differs from conduction. In the interface between two materials of different phases, such as a solid and gas, the rate of collisions between gas molecules in the solid surface determines the rate of heat transfer from the interface. In the very thin layer, generally known as a boundary layer, the heat transfer may be by conduction. However, in the bulk fluid, the mixing is the dominant energy transfer mechanism. The higher the velocity of the gas molecules, the larger is the rate of heat transfer. This mechanism of heat transfer is called convective heat transfer. The mathematical model for convective heat transfer is given by Newton's law of cooling.

$$q = hA(T_g - T_w) \tag{5.21}$$

where

q	=	Heat transfer rate from fluid to wall, Btu/h or W
h	=	Convective heat-transfer coefficient or film coefficient, Btu/(h-ft^2-F) or W/(m^2-s)
T_g	=	Bulk fluid or gas temperature, °F or °C
T_w	=	Wall temperature, °F or °C

The convective heat transfer coefficient (h) is generally obtained experimentally or from an empirical formula that is obtained by considering various factors that affect the heat transfer and fitting the experimental data [6, 7]. The values for convective heat transfer coefficient for various media are given in Table 5.2.

Example 5.2

What is the heat transfer rate of a container filled with boiling water? The area is 1 m^2. The wall temperature is 200^{o}C.

Solution

Here, T_w = 200^{o}C, and T_g (boiling point of water) = 100^{o}C.

$$q = -hA(T_w - T_g) = -2,500(200 - 100) = -250,000\,\text{W}\,[44,000\text{BTU}]$$

Table 5.2. Convective coefficient for various systems.

Material	k(W/m^2-^{o}C)	k(Btu/h-ft^2-^{o}F)
Free covection, ΔT = 30^{o}C, vertical plate, 0.3 m high in air	4.5	0.79
Horizontal cylinder, 5-cm diameter, in air	6.5	1.14
Free Convection Horizontal cylinder, 2 cm diameter, in water	890	157
Forced Convection, airflow at 2 m/s over 0.2 m square plate	12	2.1
Forced Convection, airflow 35 m/s over 0.75 m square plate	75	13.2
Forced Convection, air at 2 atm. Flowing in 2.5 cm diameter tube at 10 m/s	65	11.4
Forced convection, water at 0.5 kg/s flowing in 2.5 cm diameter tube	3500	616
Airflow across 5 cm diameter cylinder with velocity of 50 m/s	180	32
Boiling Water in a container	2,500–35,000	440–6200
Boiling Water in a tube	5,000–10,0000	880–17,600
Condensation of H_2O (v), 1 atm	4,000–11,300	700–2000

5.5 Radiative Heat Transfer

All matter emits electromagnetic radiation depending on its temperature. Thus, energy transfer can occur between two bodies with different temperatures through the exchange of photons. An ideal radiator has a net heat exchange rate that is proportional to the difference in T^4 and is given by

$$q = \sigma A(T_1^4 - T_2^4) \tag{5.22}$$

where,

q = Heat transfer rate

σ = Stefan-Boltzmann constant, 5.669×10^{-8} W/m^2-K^4

A = Area of heat transfer, m^2

T_1 = Temperature of the body 1, K

T_2 = Temperature of the body 2, K.

This equation is also known as the Stefan–Boltzmann law of thermal radiation.

This formula only applies to black bodies. A blackbody is a body which is in thermodynamic equilibrium and radiates energy according to the Stefan-Boltzmann law of thermal radiation. A body may not emit as much energy as a blackbody, but it may still follow the T^4 law. Such a body is called a “gray” body. Additionally, the emitting surface radiates in all directions. The body to which it radiates may only pick up a small fraction of the energy. This is due to geometrical factors in the line of site of the gray body to the receiving body (see Fig. 5.3).

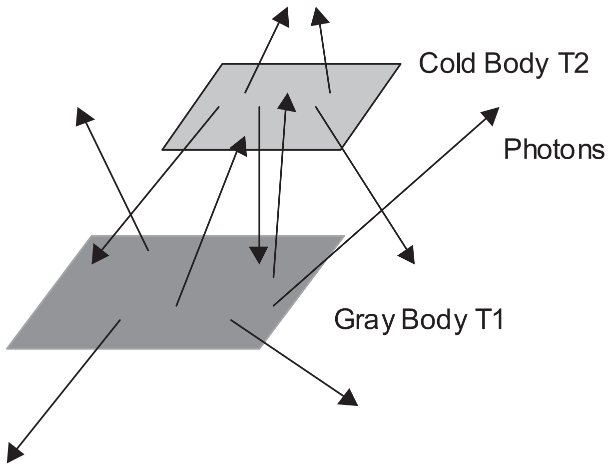

Fig. 5.3. Mechanism of radiative heat transfer. Photons travel in a straight line (the “ray” model), thus the number of photons which intersect the cold body depends upon the solid angle which the gray body sees. The geometrical view factor therefore plays an important role.

Introducing geometric view factor, the rate of heat transfer by radiation becomes:

$$q = F_e F_G \sigma A(T_1^4 - T_2^4) \tag{5.23}$$

where F_e is the emissivity function, F_G is the geometric “view factor.”

Example 5.3

Two infinite black plates in deep space at 800°C (1073 K) and 300°C (573 K) exchange heat by radiation. Calculate the heat transfer per unit area.

Solution:

$$\frac{q}{A} = \sigma(T_1^4 - T_2^4) = 5.669x10^{-8}(1073^4 - 573^4)$$

$$\frac{q}{A} = 69.03\frac{kW}{m^2}\left[21{,}884\frac{BTU}{h\,ft^2}\right]$$

Example 5.4

A 10% efficient, 1 kW_e nuclear battery is used in a deep space mission. This means that 90% of the energy produced by the battery must be transferred away from the battery. In deep space, the only means of transferring heat is by radiation. If the radiating surface is a 1 m^2 black body what is the equilibrium temperature of the nuclear battery? Deep space has a background temperature of 4 K.

Solution

Thermal energy rating = $1kW_e/0.1 = 10\ kW_{th}$

Energy that must be radiated = Δq = 10 kW-1 kW = 9 kW

Or, Δq = 9 kW

$$\frac{\Delta q}{A} = 9\frac{kW}{m^2} = \sigma(T_1^4 - T_2^4) = 5.669x10^{-8}(T^4 - 4^4)$$

$$T^4 - 256 = \frac{9}{5.660x10^{-8}}$$

$$T^4 = 1.5876x10^8 + 256 \approx 1.5876x10^8$$

$$T \approx 112.24\ \text{K}$$

5.6 Thermal Resistance

Materials with a large thermal conductivity will transfer large amounts of heat over time for a given temperature difference. Such materials include metals such as copper, aluminum, etc. and are called thermal conductors. Conversely, materials with low thermal conductivities will transfer small amounts of heat over time, such as ceramic materials, and are known as poor thermal conductors. Home insulation is thus a poor thermal conductor, which keeps as much heat in as possible. Instead of being rated in terms of thermal conductivity, insulation is therefore usually rated in terms of its **thermal resistance**.

In a steady-state conduction problem, it is possible to model heat transfer by a technique called thermal resistance. This method is analogous to electrical resistance.

The heat conduction Eq. (5.2) can be written in another form using the concept of thermal resistance.

$$q = -\frac{(T_2 - T_1)}{R'} \tag{5.24}$$

where R' is called the thermal resistance and is defined by

$$R' = \frac{x_2 - x_1}{kA} = \frac{\Delta x}{kA} \tag{5.25}$$

In Eq. (5.24), $(T_2 - T_1)$ may be assumed as the thermal potential and is responsible for heat flow.

Thus, one can define heat flow by:

$$Heat\ Flow = \frac{Thermal\ Potential}{Thermal\ Resistance} \tag{5.26}$$

Thermal resistance (R') is analogous to electrical resistance, and q and $(T_2 - T_1)$ may be viewed as the current and potential difference in Ohm's law, respectively. Like an electrical circuit, the thermal resistance may be in series and thus provides a very useful method of analyzing heat transfer through a composite wall or slab made up of layers of dissimilar material. In Fig. 5.4 is shown a wall constructed of three different materials. The heat transferred by conduction through this wall is given by Eq. (5.27):

$$R' = R_A + R_B + R_C = \frac{\Delta x_1}{k_1 A} + \frac{\Delta x_2}{k_2 A} + \frac{\Delta x_3}{k_3 A} \tag{5.27}$$

The thermal resistance model works much like the electrical resistance model. In Fig. 5.4, the thermal resistances are in series. We can also apply the concept of thermal resistances in parallel as shown in Fig. 5.5. The application of parallel network is also analogous to the electrical model.

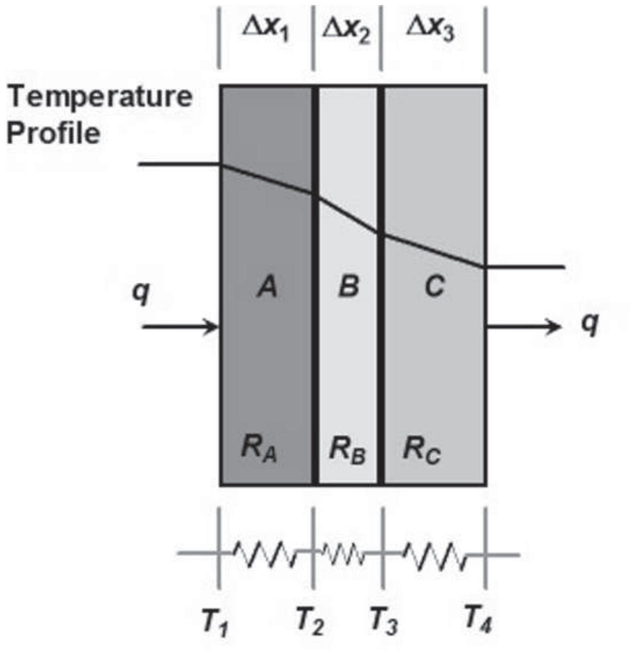

Fig. 5.4. One dimensional heat flow through a composite wall with the corresponding electrical analog.

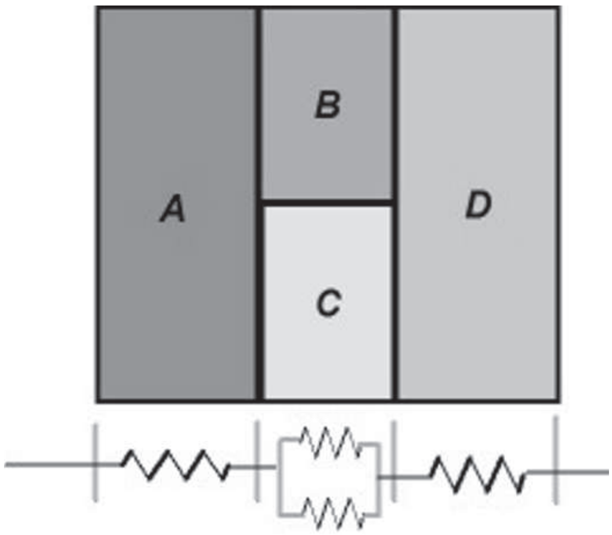

Fig. 5.5. The thermal equivalent of parallel resistance.

Equation (5.2) is developed for Cartesian coordinates (x-y-z). The application of Eq. (5.2) is restricted to a plane wall where the cross-sectional area is a constant. However, Eq. (5.2) can be extended to both polar and spherical coordinate system. Consider a long, hollow cylinder whose cross section is shown in Fig. 5.6. In this case, area A is a function of radius r. Therefore, Eq. (5.1) can be written as

$$\dot{q}_r = -k(2\pi rL)\frac{dT}{dr} \tag{5.28}$$

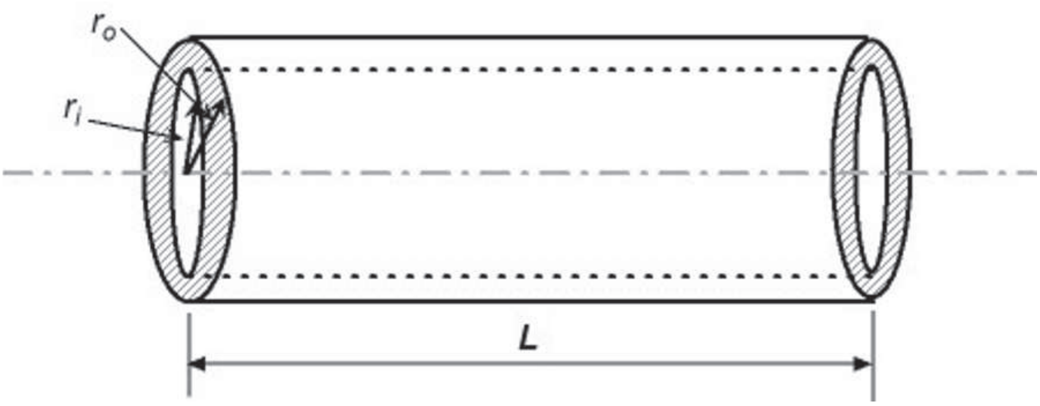

Fig. 5.6. A cylinder considered for heat transfer.

where,

r = Radius of the cylinder
L = Length of the cylinder

Integration of Eq. (5.28) yields

$$q_r = \frac{2\pi kL}{\ell n\left(\frac{r_0}{r_i}\right)}(T_i - T_0) \tag{5.29}$$

Equation (5.29) can be written in terms of thermal resistance

$$q_r = \frac{(T_i - T_0)}{R'} \tag{5.30}$$

where,

$$R' = \frac{\ell n\left(\frac{r_0}{r_i}\right)}{2\pi kL}$$

5.7 R Values

Insulation industry uses the concept of R-values to describe the thermal resistance of insulations [8–10]. It should be noted that R-values are not defined in the same manner as the thermal resistance. An R-value is defined as:

$$R = \frac{\Delta T}{q/A} \tag{5.31}$$

The heat flow on a per unit area basis is used in this equation.

Example 5.5

What is the difference in heat flow per unit area for insulation with an *R* value of 13 versus 19? Assume that the temperature indoors is 70°F and outdoors is 50°F.

Solution

$$q/A = \frac{\Delta T}{R} \text{ Btu/h-ft}^2$$

$$q/A = \frac{20}{13} = 1.538 \text{ Btu/h-ft}^2$$

$$q/A = \frac{20}{19} = 1.053 \text{ Btu/h-ft}^2$$

$$\Delta \frac{q}{A} = 1.538 - 1.053 = 0.485 \text{ Btu/h-ft}^2$$

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Problems

1. Aluminum has a specific heat of 0.902 J/g x °C. How much heat is lost when a piece of aluminum with a mass of 23.984 g cools from a temperature of 415.0°C to a temperature of 22.0°C?
2. The temperature of a sample of water increases by 69.5°C when 24,500 J are applied. The specific heat of liquid water is 4.18 J/g x °C. What is the mass of the sample of water?
3. 850 calories of heat are applied to a 250 g sample of liquid water with an initial temperature of 13.0°C. Find (a) the change in temperature and (b) the final temperature. (Remember, the specific heat of liquid water, in calories, is 1.00 cal/g x °C.)
4. When 34,700 J of heat are applied to a 350 g sample of an unknown material the temperature rises from 22.0°C to 173.0°C. What is the specific heat of this material?
5. The wall of a furnace is comprised of three layers as shown in the figure. The first layer is refractory (whose maximum allowable temperature is 1,400°C) while the second layer is insulation (whose maximum allowable temperature is 1,093°C). The third layer is a plate of 6.35 mm thick steel [thermal conductivity = 45 W/(m K)]. Assume the layers to be in very good thermal contact.

The temperature T_0 on the inside of the refractory is 1370°C, while the temperature T_3 on the outside of the steel plate is 37.8°C. The heat loss through the furnace wall is expected to be 15,800 W/m^2. Determine the thickness of refractory and insulation that results in the minimum total thickness of the wall.

Given thermal conductivities in W/(m K):

Layer	**k at 37.8°C**	**k at 1,093°C**
Refractory	3.12	6.23
Insulation	1.56	3.12