Theory of Consumer Choice

V. Hajko

2014

The principal interests of this lecture

- How can we represent the consumer's preferences?
- How can we represent the choices a consumer can afford?
- What determines how a consumer divides his or her resources between two goods?

Rational behavior

- The rational consumer behavior is driven by the effort to achieve the highest level of satisfaction
  - In the economics, the consumers (sic!) are being satisfied by the consumption of goods and services
  - The highest level of satisfaction is seen according to the consumer's *preferences*
  - With limited resources, people face *tradeoffs*
    - if you go swimming, you will have less time for jogging
    - if you buy more of one good, you will have less money to buy something else
    - if you work more, you will earn more, but you will have less of the leisure time
- The consumers seek to consume the "best" bundle of goods they can afford:
  - they maximize their utility functions subject to their budgetary constrains

The preference relations

- In order to represent the what is “better” or "worse", the consumer has to be able to compare the choices.
  - This comparison can be represented by the preference relations.
  - This logic of comparison is used in the construction of the so-called "utility function" - a representation of the consumer's preferences.
- The preference relation allows for the comparison of pairs of alternatives, x and y
  - $x \succeq y$ means x is at least as good as y; ($x \succ y$ means x is strictly preffered to y)
  - $x \preceq y$ means y is at least as good as x; ($x \prec y$ means y is strictly preffered to x)
  - $x \sim y$ means x is indifferent to y

The rational preference relations

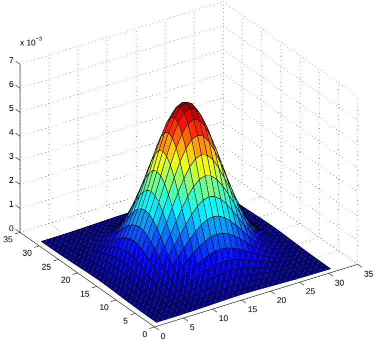
- The preference relation is rational, if it fulfills two basic assumptions: *completeness* and *transitivity*
  - Completeness: for any pair of $x, y \in X$ we have $x \succeq y$, $x \preceq y$ or both (i.e. $x \sim y$).
    - consumer is able to compare any two pairs and decide whether x is preferred to y, y is preferred to x, or whether he or she is indifferent between x and y
  - Transitivity: for all alternatives $x, y.z \in X$, if $x \succeq y$ and $y \succeq z$ then $x \succeq z$.
    - rational behavior rules out a preference cycle

Utility functions

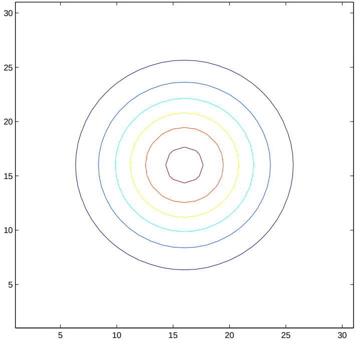
- A function $u: X \to R$ is a utility function representing preference relation $\succeq$, if for all $x, y \in X$ we have $x \succeq y \Leftrightarrow u(x) \geq u(y)$
- The utility function is used to represent the preferences (the logical relationships) with a numerical representation
  - The utility function is strictly *individual*
- A preference relation $\succeq$ can be represented by a utility function only if it is rational

Figure : The visual representations, function $f(x, y) = z$

(a) We need 3-D graphs to visually capture the value of the function with two inputs



(b) Contour plot for $f(x, y) = z$.

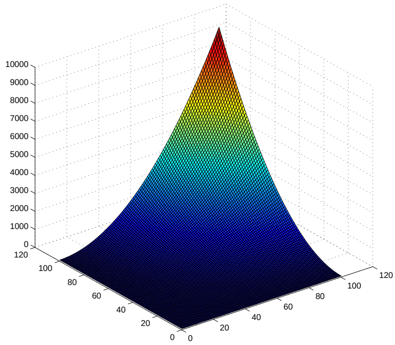


Utility function

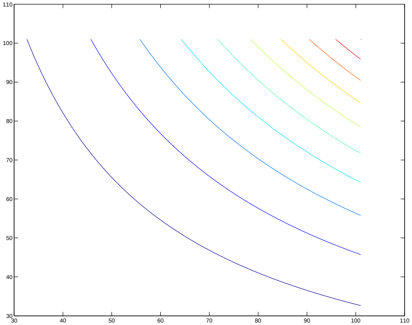
- The contour lines of the utility function represent all points, where the various combinations of amounts of x and y lead to the same value of the utility function
  - Assume the utility function $U = X^2Y^2$; for the given value of utility function, e.g. $U = 100$, we can write down the specific contour line (which is called the *indifference curve*) as $100 = X^2Y^2$, or $Y = \sqrt{\frac{100}{X^2}}$
  - The *indifference curve* shows all combination of bundles of x and y that bring the same utility. Therefore the consumer is *indifferent* between all bundles lying on the indifference curve.
- Properties of the *well behaved* utility function
  - monotonicity (or its weaker version, local nonsatiation)
    - in essence, the larger amount of consumed goods or services will not decrease the utility (the goods are desirable)
  - convexity
    - a formal expression of a basic inclination of economic agents for diversification (the assumption that consumers prefer variety)

Figure : The visual representation of the convex utility function with nonsation

(a) The utility function:



(b) The indifference map is the contour plot of the utility function:

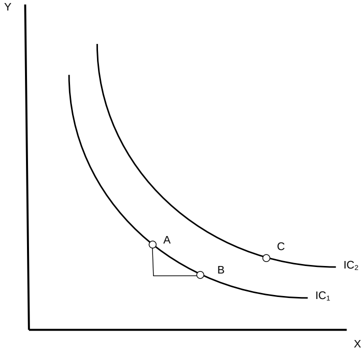


Indifference curves

- For well behaved utility functions, the higher indifference curves show the contours of higher values of the utility function
  - The higher indifference curves are preferred to lower ones
- Indifference curves do not cross (this would violate the transitivity of the preference relation)
- Indifference curves are bowed inward (becase of the convexity property)

Marginal rate of substitution in consumption

- Marginal rate of substitution in consumption (MRS_C) is the rate at which a consumer is willing to trade one good for another
- It is represented by the slope of the indifference curve
- $MRS_C = \frac{MU_X}{MU_Y}$, also $MRS_C = -\frac{\Delta Y}{\Delta X}$

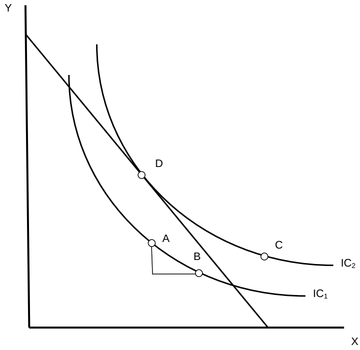


Budget constraint

- Due to the limited resources, some of the consumption bundles are unavailable to the consumer.
- The limitation of the consumption is called *the budget constraint*: $I \geq p_x X + p_y Y$, where I is consumers disposable income, p_x is the price of one unit of X and X is the number of units of X consumed.
- The slope of the budget line represents the so called Marginal rate of substitution in exchange ($MRS_E = -\frac{p_X}{p_Y}$) - this represents the ratio at which the goods are valued by the market (e.g. "1 apple = 2 bananas" - this is called the relative price)

Budget constraint

- An increase in income shifts the budget constraint outward
- A change in the prices causes the budget constraint to change its slope ("rotate")



Consumer's optimization problem

- The consumer wants to attain the maximal utility
    - The higher level of utility is equivalent with higher value of the utility function
    - Therefore, the consumer seeks to find such a combination that will *maximize* the value of the utility function AND satisfy the budget constraint
- $max\, U(X, Y)$ subject to: $I \geq p_x X + p_y Y$
    - since we are assuming monotonicity, we can simplify the budget constraint to $I = p_x X + p_y Y$
    - we can write the Lagrangean for this optimization problem as: $L(X, Y, \lambda) = U(X, Y) + \lambda(I - p_x X - p_y Y)$
    - By differentiating L with respect to x,y and λ and setting the derivatives equal to zero, we get the first order conditions:
        - $\frac{\partial U}{\partial X} - \lambda p_x = 0$
        - $\frac{\partial U}{\partial Y} - \lambda p_y = 0$
        - $I - p_x X - p_y Y = 0$

The optimal consumption

- The first order conditions imply the consumer will seek to consume the goods in the ratio that fulfills $MRS_C = MRS_E$, i.e. $\frac{MU_X}{MU_Y} = \frac{p_X}{p_Y}$ or $\frac{MU_X}{p_X} = \frac{MU_Y}{p_Y}$
  - The marginal utility MU_X is the change in the total utility caused by infinetisimaly small change in the consumption of X, i.e. $MU_X = \frac{\partial U}{\partial X}$
  - This implies the optimum will occur at the point where the indifference curve and the budget constraint are tangent.
- The the attainable consumption, hence also the attainable utility, is limited by the budget constraint.
  - Combined with the previous condition, this implies the optimum will occur at the point where the *highest* indifference curve and the budget constraint are tangent.

Income and substitution effects

- The income effect is the change in consumption that results when a price change moves the consumer to a higher or lower indifference curve.
  - the price change increases (decreases) the purchasing power of the consumer (his relative wealth), which encourages (discourages) the consumer to buy the goods in question
- The substitution effect is the change in consumption that results when a price change moves the consumer along an indifference curve to a point with a different marginal rate of substitution.
  - the price change encourages greater (lower) consumption of the good, since it has become relatively cheaper (more expensive)

Normal goods vs. inferior goods

- Based on the *income elasticity* of demand ($\varepsilon_{ID} = \frac{\frac{\partial Q^D}{Q}}{\frac{\partial I}{I}}$) we can distinguish normal and inferior goods:
    - With the increase in income, the consumer buys more of a *normal* good.
    - With the increase in income, the consumer buys less of an *inferior* good.

Individual demand

- The demand function $Q^D = f(p, \cdot)$ represents the quantity of the given goods demanded, dependent on the price (and possibly other factors, e.g. $Q_X^D = f(p_x, p_y, I)$).
- Consumer's demand curve summarizes the solutions of the optimization problems for the various levels of price (given no changes in the other factors, e.g. for the given level of income and prices of other goods).
- Typically a demand curve slopes downward (with a decrease in price, the consumer would buy more of the goods).
    - There is an exception (but very rare): the so-called Giffen goods - an *increase* in the price raises the quantity demanded (the income effect overwhelmes the substitution effect).