

# Social network analysis 3 + 4

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# Outline

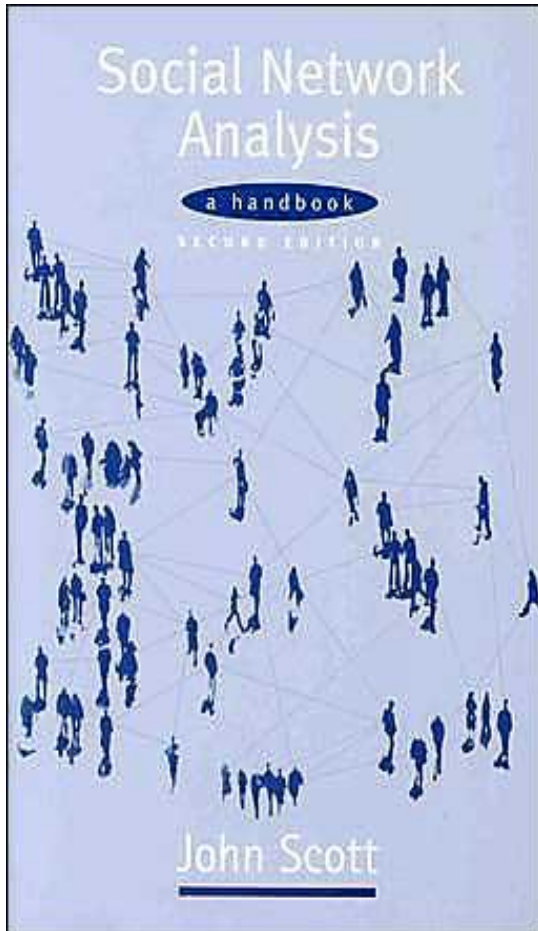
- Centrality and centralization
- Dyads and reciprocity
- Triads and transitivity
- Segments
- R: SNA mini-case

# “Red bible”

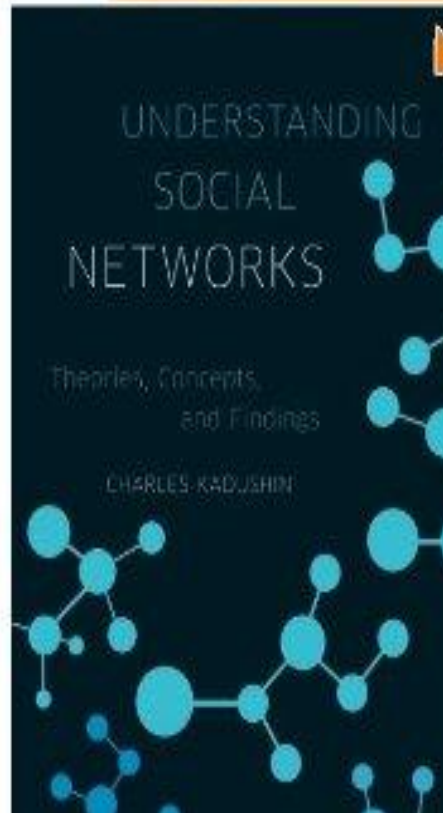
- Stanley Wasserman and Katherine Faust (1994): *Social Network Analysis: Methods and Applications*. Cambridge University Press.



# Introductory sources



LOOK INSIDE!



LOOK INSIDE!



# Graph theory

- **Network topology** is defined by two main concepts: connectivity and centrality.
- **Connectivity** describes interconnectedness of nodes in network (focus on **flows**).
- **Centrality** describes location of nodes in network (focus on **positions**).

# Graph theory: notation

- $G$  = graph/network
- $N$  = # of nodes in network,  $n$  = individual node
- $e$  = edge,  $g$  = geodesic
- $i, j, \dots$  = indices (labels for selected elements)
- $g_{ij}$  = geodesic between nodes  $i$  and  $j$ ,  $n_i$  = node  $i$
- $k$  = # of selected elements (typically nodes)
- $Cd'(G)$  = ' indicates that the measured value is standardized
- Upper case: global measures
- Lower case: local measures
- $cd(n_i)$  = node  $i$  degree centrality
- $Cd(G)$  = graph  $G$  degree centralization

# Centrality

- **Local measure:** characterizes position of a particular node within a network.
- Different measures for different network classes! (e.g. bipartite networks, directed or weighted ties).
- Simplest case: undirected binary network.
- Different types of centralities:
  - Degree centrality
  - Closeness centrality
  - Betweenness centrality

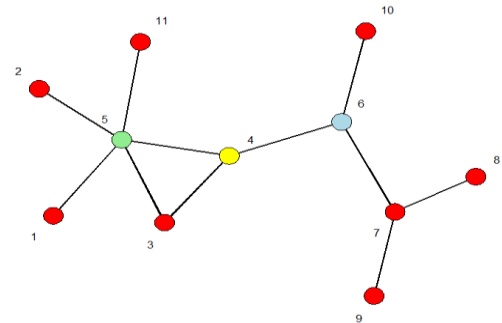
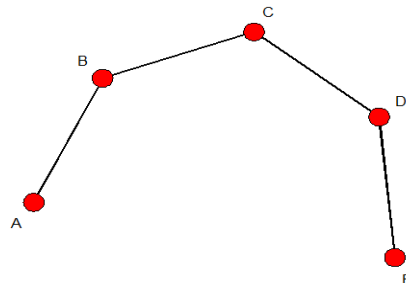
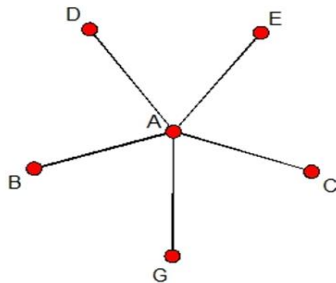
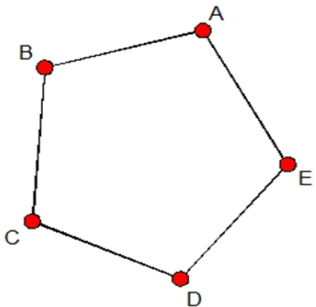
# Centralization

- **Global measure:** characterizes extent to which a certain local feature prevails in a network.
- **Centralization** is measured as a difference of all centrality values from the highest centrality value.
- Again, we need to consider the network class.
- Analogically, there are different types of centralizations.
- **centralization  $\neq$  centrality**



# Degree centrality

- Nodes with most connections.
- **Theoretical importance:**
  - Most powerful actors in network.
  - Indication of prestige (in-degree centrality).
  - Indication of influence (out-degree centrality).
  - *Depends on tie conceptualization.*
  - *Limited only to adjacent nodes (neighborhood)!*



# Degree centrality

- **Undirected graph:** # of connections of a node.

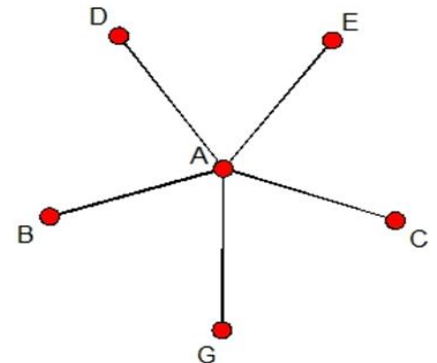
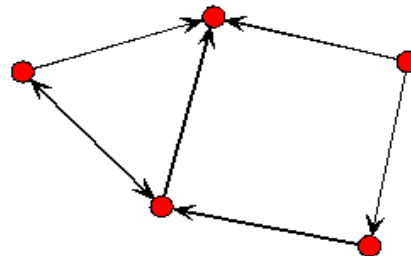
$$c_D(n_i) = d(n_i)$$

- **Standardization:** division by # of all possible connections (# of all nodes - 1).

$$c_D'(n_i) = d(n_i) / (N - 1)$$

- **Directed graph:** # of connections from/to a node.

- In-degree centrality
- Out-degree centrality



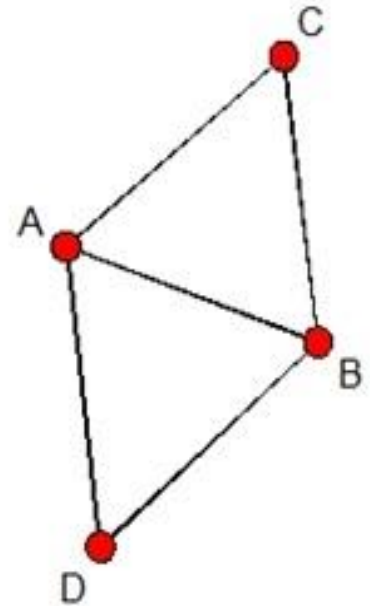
# Degree centrality

$$c_D(n_i) = d(n_i)$$

$$c_D(n_A) = d(n_A) = 3$$

$$c_D'(n_i) = d(n_i) / (N - 1)$$

$$c_D'(n_A) = 3 / 3 = 1$$



# Degree centralization

1. # of connections of each node (degree centrality).
2. Differences of centralities from the highest centrality value.
3. Summation of differences.

$$C_D(G) = \sum (c_D(n_{MAX}) - c_D(n_i))$$

4. Standardization: division by  $(N - 1) * (N - 2)$

$$C_D'(G) = \sum (c_D(n_{MAX}) - c_D(n_i)) / (N - 1) * (N - 2)$$

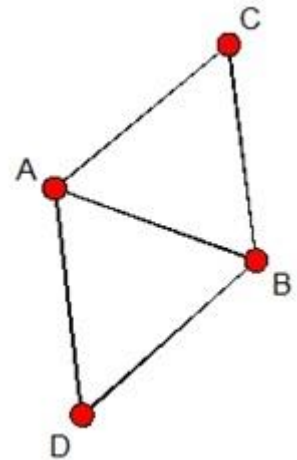
# Degree centralization

$$C_D(G) = \sum (c_D(n_{MAX}) - c_D(n_i))$$

$$C_D(G) = (0 + 0 + 1 + 1) = 2$$

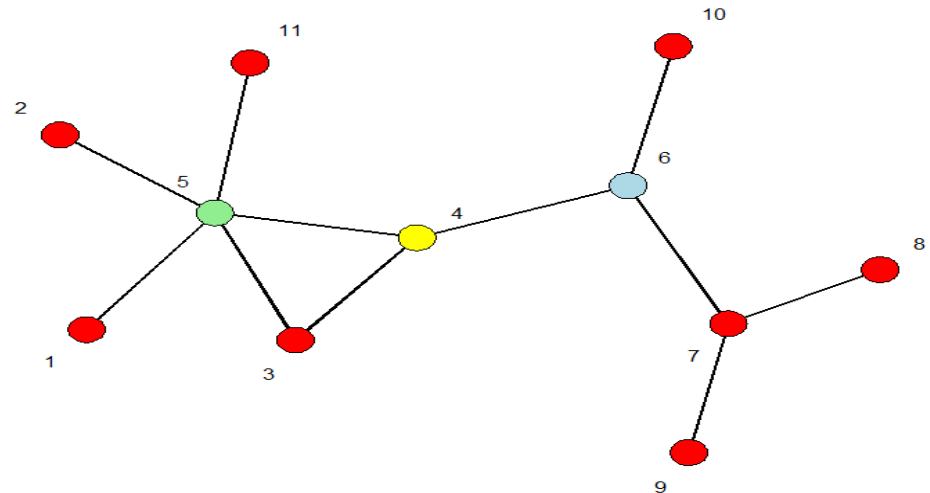
$$C_D'(G) = \sum (c_D(n_{MAX}) - c_D(n_i)) / (N - 1) * (N - 2)$$

$$C_D'(G) = 2 / (4 - 1) * (4 - 2) = 2 / 6 = 0.33$$



# Closeness centrality

- Nodes with shortest average path length.
- Node is closer to more actors than any other node.
- **Theoretical importance:**
  - Shorter path lengths = quicker and more efficient access to sources.



# Closeness centrality

- **Distance:**  $\sum$  geodesics of a node to all other nodes.
- **Closeness:** inverse concept  $\rightarrow 1 / \text{distance}$ :

$$c_C(n_i) = 1 / \sum g(n_i, n_j) = c_C(n_i) = [ \sum g(n_i, n_j) ]^{-1}$$

- Standardization: multiplication by # of nodes - 1.

$$c_C'(n_i) = [ \sum g(n_i, n_j) * (N - 1) ]^{-1}$$

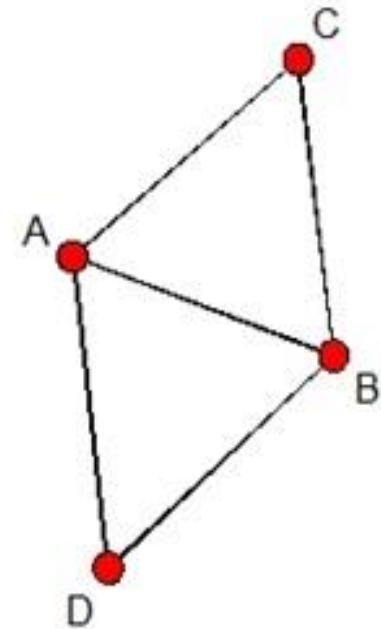
# Closeness centrality

$$c_C(n_i) = [ \sum g(n_i, n_j) ]^{-1}$$

$$c_C(n_A) = [ (1 + 1 + 1) ]^{-1} = 0.33$$

$$c_C'(n_i) = [ \sum g(n_i, n_j) * (N - 1) ]^{-1}$$

$$c_C'(n_A) = 0.33 * (4 - 1) = 1$$

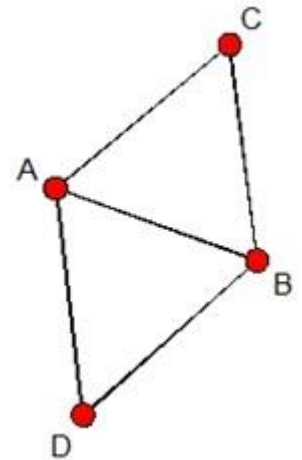




# Closeness centralization

1. **Standardized** closeness centrality values for each node.
2. Differences of centralities from the highest centrality value.
3. Summation of differences.

$$C_C'(G) = \sum (c_C'(n_{MAX}) - c_C'(n_i))$$



4. Standardization:  $(N - 1) * (N - 2) / (2N - 3)$

$$C_C'(G) = \sum (c_C'(n_{MAX}) - c_C'(n_i)) / [(N - 1) * (N - 2) / (2N - 3)]$$

# Closeness centralization

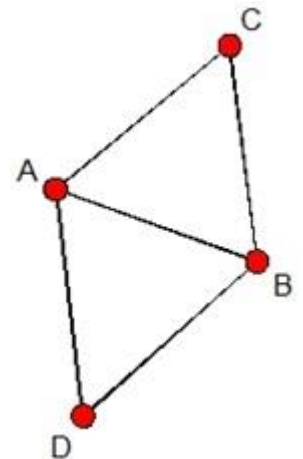
$$C_C'(G) = \sum (c_C'(n_{MAX}) - c_C'(n_i))$$

$$C_C'(G) = \sum (1 - 1) + (1 - 1) + (1 - 0.75) + (1 - 0.75)$$

$$C_C'(G) = \sum (c_C'(n_{MAX}) - c_C'(n_i)) / [ (N - 1) * (N - 2) / (2N - 3) ]$$

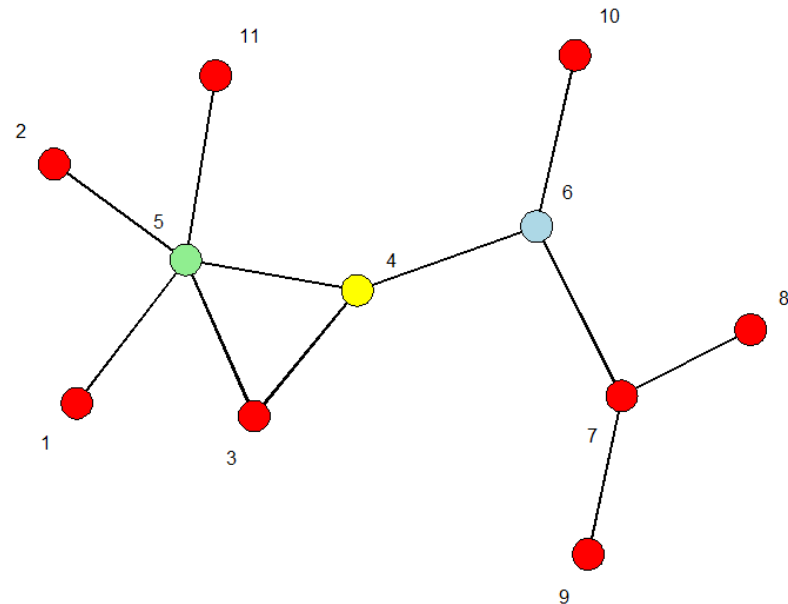
$$C_C'(G) = \sum (1 - 1) + (1 - 1) + (1 - 0.75) + (1 - 0.75) / [ (4 - 1) * (4 - 2) / (8 - 3) ]$$

$$C_C'(G) = 0.5 / (12 / 5) = 0.5 / 1.2 = 0.42$$



# Betweenness centrality

- Nodes which are most in-between other nodes.
- **Theoretical importance:**
  - Crucial for flow control (gatekeepers, brokers).
  - Bridges to otherwise weakly connected parts of network (access to sources).



# Betweenness centrality

- **Betweenness:** ratio of geodesics upon which a node lies to all geodesics in network.

$$c_B(n_i) = \sum g_{jk}(n_i) / g_{jk}$$

- Standardization: division by # of all possible geodesics upon which node can lie.

$$c_B'(n_i) = [ \sum g_{jk}(n_i) / g_{jk} ] / [ (N - 1) * (N - 2) / 2 ]$$

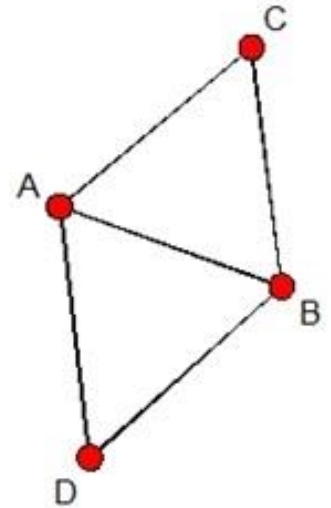
# Betweenness centrality

$$c_B(n_i) = \sum g_{jk}(n_i) / g_{jk}$$

$$c_B(n_A) = 0.5 / 3 = 0.17$$

$$c_B'(n_i) = [ \sum g_{jk}(n_i) / g_{jk} ] / [ (N - 1) * (N - 2) / 2 ]$$

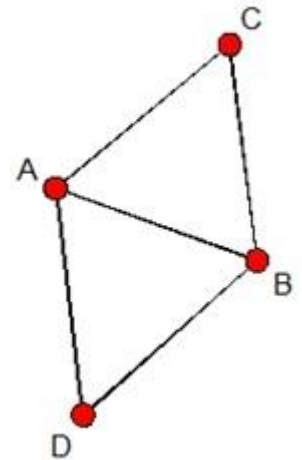
$$c_B'(n_A) = 0.17 / [ ((4 - 1) * (4 - 2)) / 2 ] = 0.17 / 3 = 0.06$$



# Betweenness centralization

1. Betweenness centrality values for each node.
2. Differences of centralities from the highest centrality value.
3. Summation of differences.

$$C_B'(G) = \sum (c_B(n_{MAX}) - c_B(n_i))$$



4. Standardization:  $(N - 1)^2 * (N - 2) / 2$

$$C_B'(G) = \sum (c_B(n_{MAX}) - c_B(n_i)) / [ (N - 1)^2 * (N - 2) / 2 ]$$

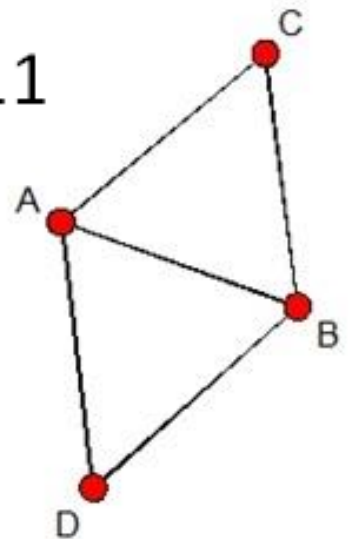
# Betweenness centralization

$$C_B'(G) = \sum (c_B(n_{MAX}) - c_B(n_i))$$

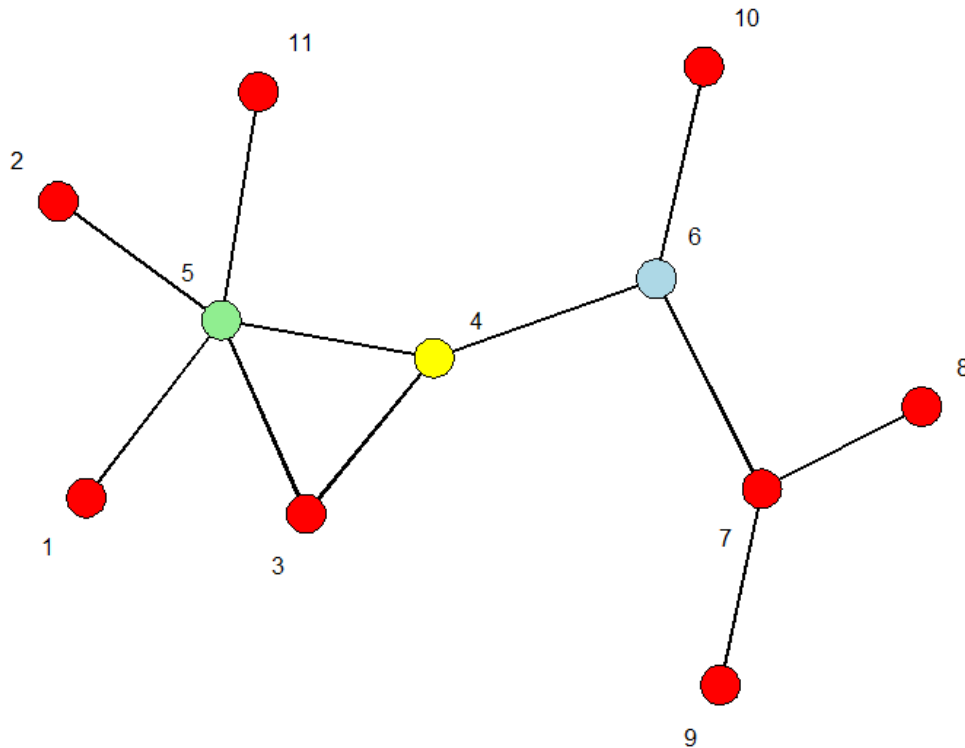
$$C_B(G) = \sum (0.5 - 0.5) + (0.5 - 0.5) + (0.5 - 0) + (0.5 - 0) = 1$$

$$C_B'(G) = \sum (c_B(n_{MAX}) - c_B(n_i)) / [ (N - 1)^2 * (N - 2) / 2 ]$$

$$C_B(G)' = 1 / [ (4 - 1)^2 * (4 - 2) / 2 ] = 1 / 9 = 0.11$$



# Degree vs. closeness vs. betweenness



	1	2	3	4	5	6	7	8	9	10	11
degree	1	1	2	3	5	3	3	1	1	1	1
closeness	0.33	0.33	0.42	0.53	0.48	0.5	0.4	0.29	0.29	0.34	0.33
betweenness	0	0	0	50	48	54	34	0	0	0	0

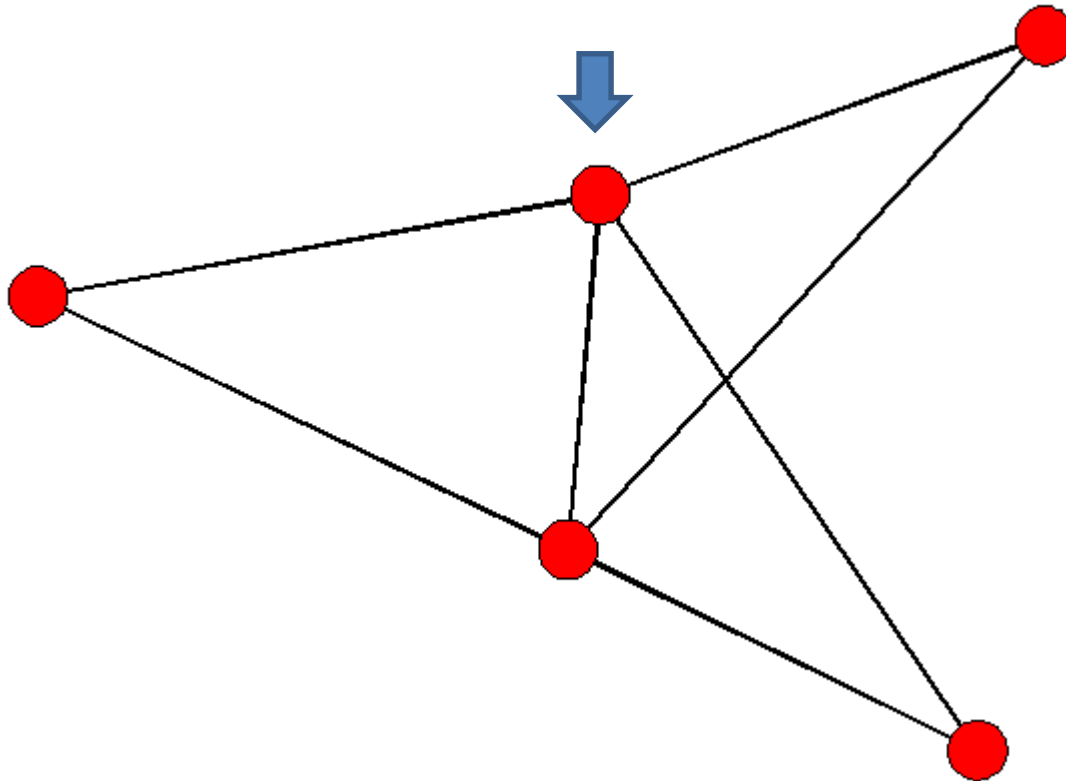


# Centrality classification (Moody)

	Low Degree	Low Closeness	Low Betweenness
High Degree		Embedded in cluster that is far from the rest of the network	Ego's connections are redundant - communication bypasses him/her
High Closeness	Key player tied to important important/active alters		Probably multiple paths in the network, ego is near many people, but so are many others
High Betweenness	Ego's few ties are crucial for network flow	Very rare cell. Would mean that ego monopolizes the ties from a small number of people to many others.	

# Exercise

- Calculate degree and closeness centrality of a given node. What is degree centralization?

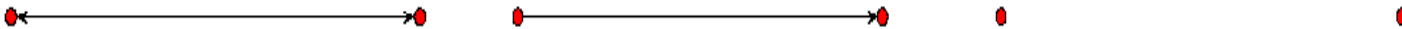


# Dyads

- Dyad: a most basic relational unit.
- 2 dyads for undirected graphs:



- **Isomorphism:** structural interchangeability of nodes, edges or their configurations.
- 3 isomorphic dyads for directed graphs.
- **MAN:** **M:** mutual, **A:** asymmetric, **N:** null:



# Reciprocity (Wasserman & Faust 2009)

- **Reciprocity:** there is a bidirectional (mutual) tie between two nodes.

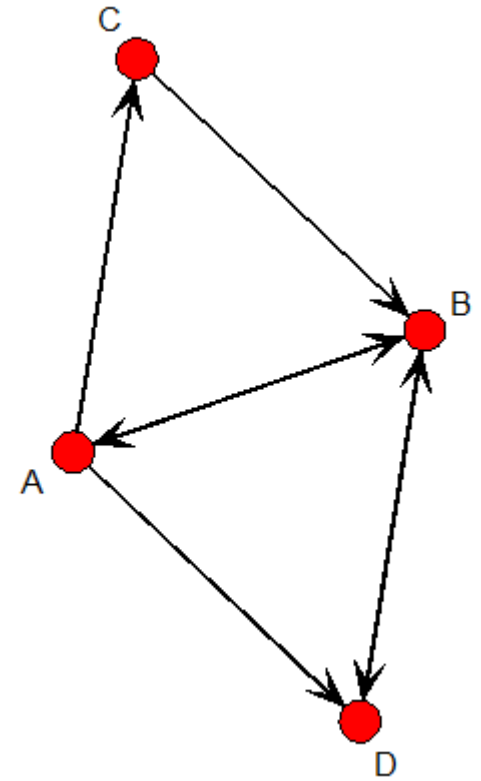


- Reciprocity in graph is given as a ratio of reciprocal (**mutual**) dyads to total number of connections (mutual + **asymmetrical** dyads).

$$R(G) = 2M / (2M + A)$$

# Reciprocity

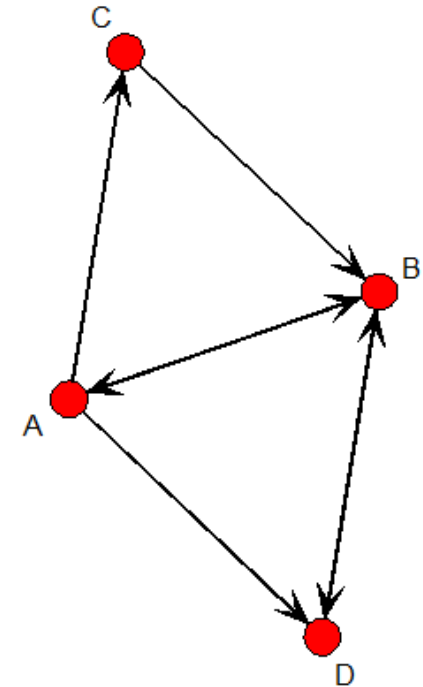
$$R(G) = 2M / (2M + A)$$



# Reciprocity

$$R(G) = 2M / (2M + A)$$

$$R(G) = 2 * 2 / (2 * 2 + 3) = 4 / 7 = 0.57$$



- R default

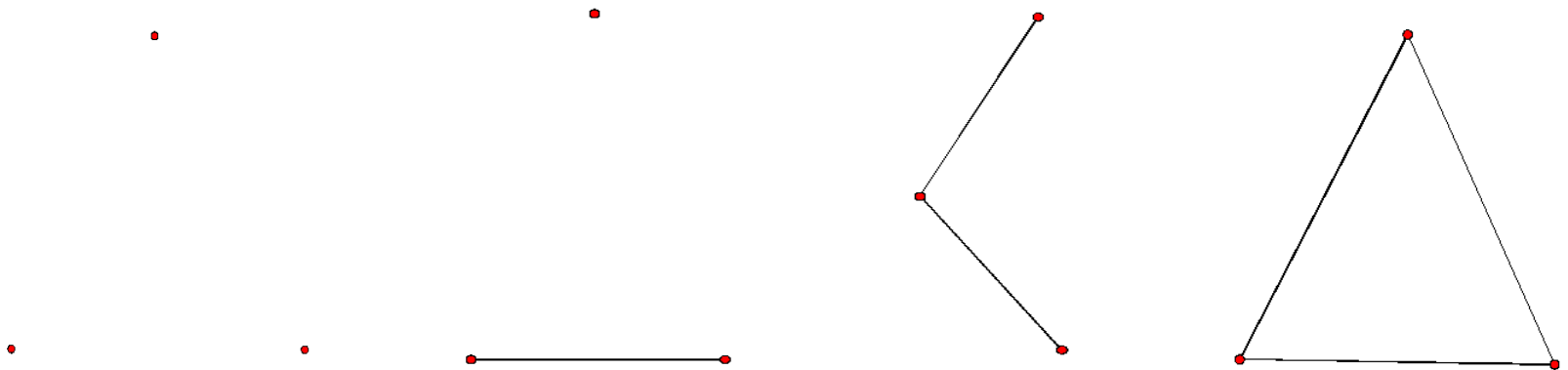
$$R(G) = M / (M + A + N) = (1 + 1) / ((1 + 1) + (1 + 1 + 1) + 1) = 2 / 6 = 0.33$$

- R dyadic.nonnul

$$R(G) = M / (M + A) = (1 + 1) / ((1 + 1) + (1 + 1 + 1)) = 2 / 5 = 0.4$$

# Triplets and triads in undirected graph

- Triad as a **basic unit of social organization**.
- triplet = empty triad
- In undirected graph:  $2^3 = 8$  combinations of triads with preserved identity (anisomorphic).
- 4 isomorphic triads:



# Triplets and triads in directed graph

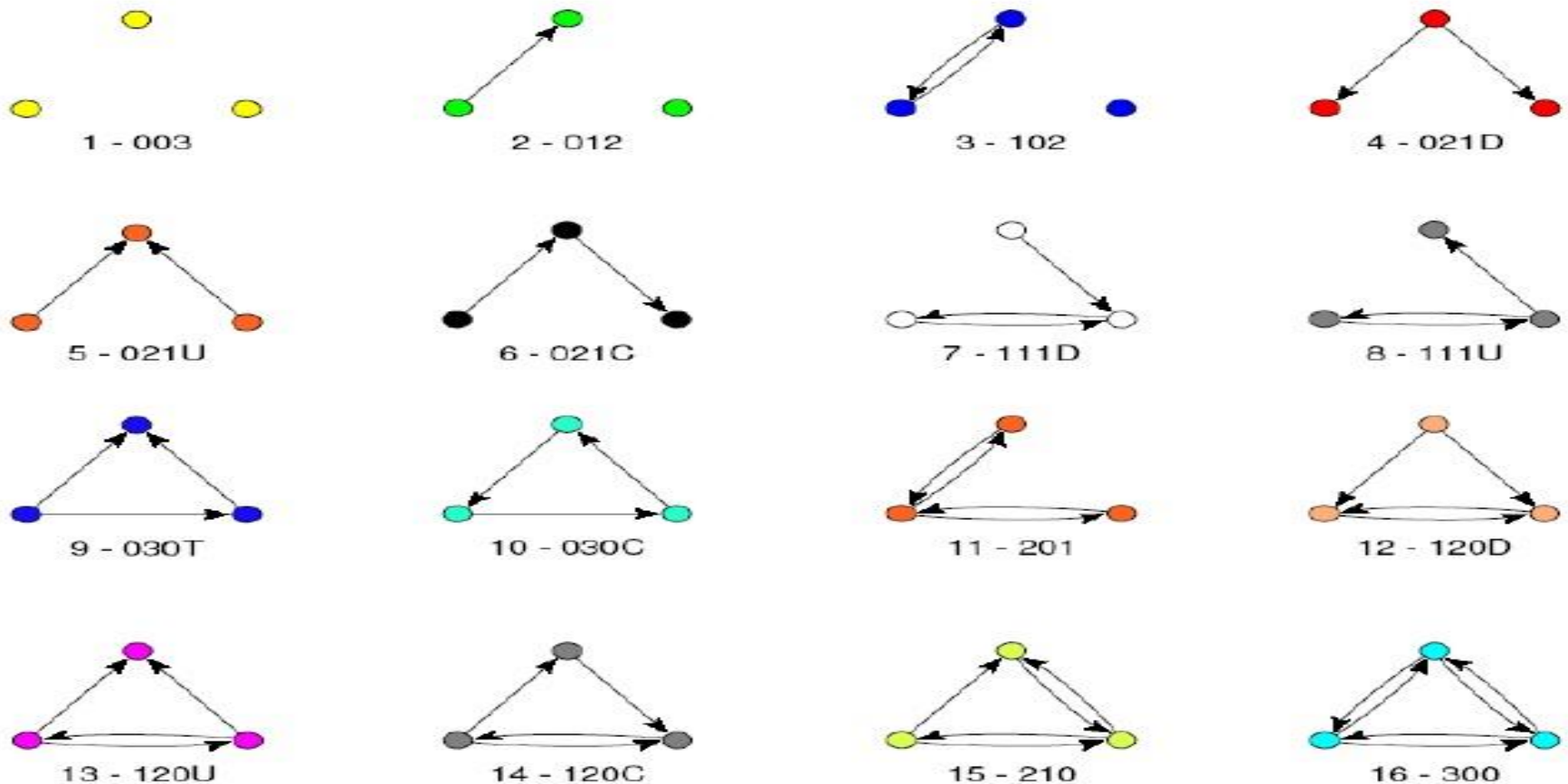
- There is a 6 combinations of triplets: (a, b, c); (a, c, b); (b, a, c); (b, c, a); (c, a, b); (c, b, a).
- Thus:  $2^6 = 64$  combinations of anisomorphic triads.
- If we neglect node identities, we get  $2^4 =$  **16 isomorphic triads = triadic census.**
- # of isomorphic triads in directed graph is given by:

$$\sum T_u = N! / (k! * (N - k)!)$$



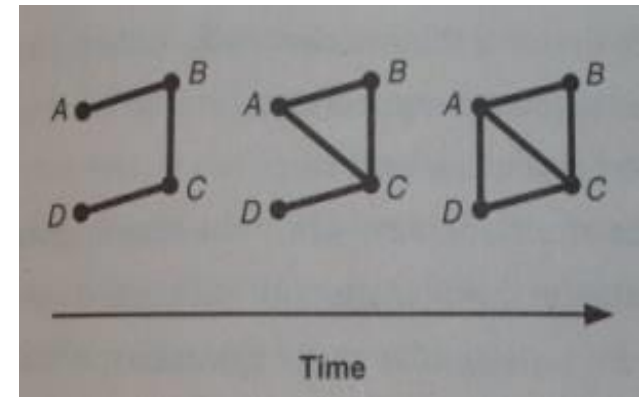
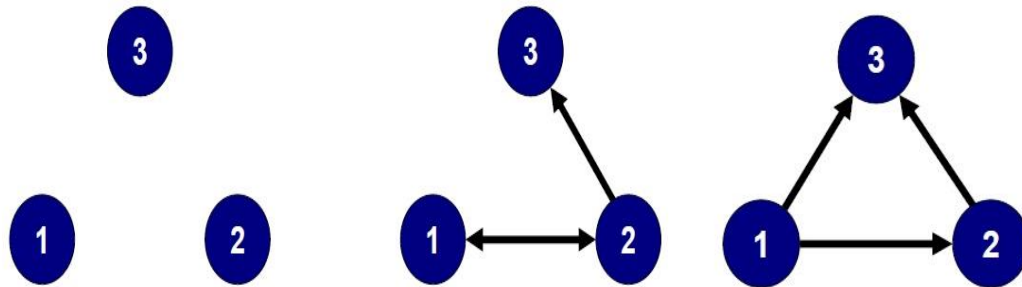
# Triadic census

- MAN (**M**utual-**A**symmetric-**N**ull), tie direction (**U**p/**D**own), transitivity (**T**) vs. cycle (**C**)



# Transitivity

- Transitivity: *a friend of my friend is my friend.*
- Triadic closure:

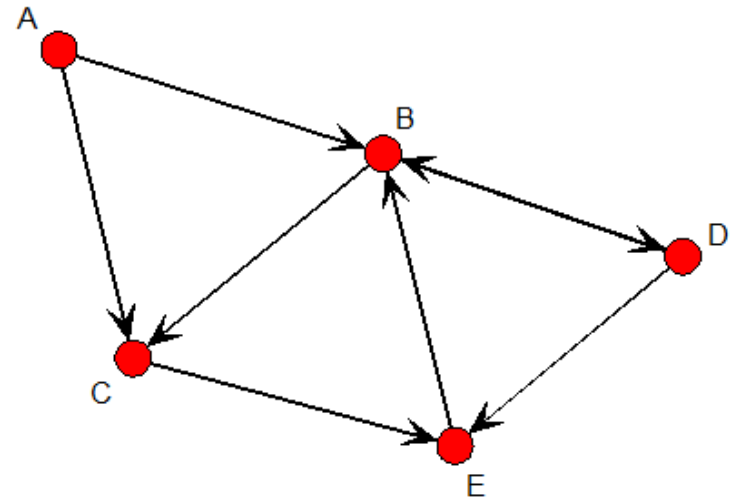


Thurner 2012

- **Measure:** # of transitive triads over # of all triads.  
$$T(G) = \sum_{ijk} e_{ij}, e_{jk}, e_{ik} / (N! / (k! * (N - k)!))$$

# Transitivity

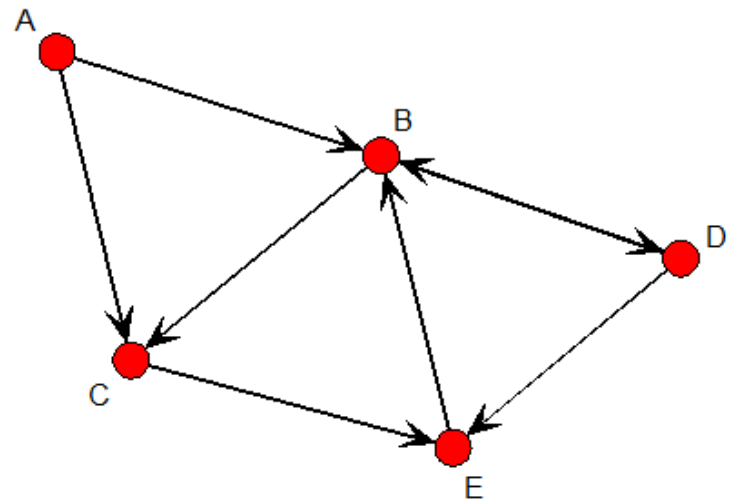
$$T(G) = \sum_{ijk} \mathbf{e}_{ij}, \mathbf{e}_{jk}, \mathbf{e}_{ik} / (N! / (k! * (N - k)!))$$



# Transitivity

$$T(G) = \sum_{ijk} e_{ij}, e_{jk}, e_{ik} / (N! / (k! * (N - k)!))$$

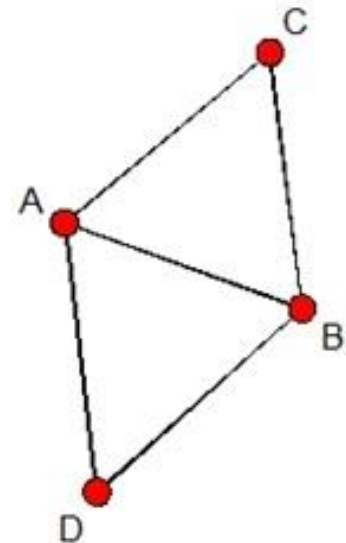
$$T(G) = 1 / (120 / 6 * 2) = 1 / 10 = 0.1$$



# Local clustering coefficient

- Measures level of a given node's neighborhood's interconnectedness.
- Measure: # of interconnections of adjacent nodes over # of all possible interconnections of adjacent nodes.

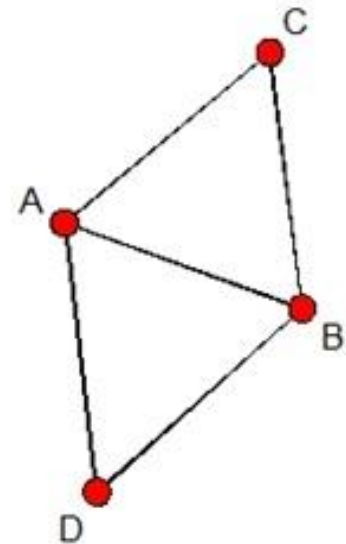
$$cc_i = e_{jk} / (n_{jk} * (n_{jk} - 1) / 2)$$



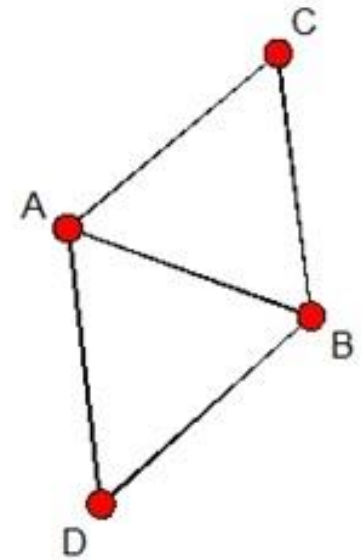
# Local clustering coefficient

$$cc_i = e_{jk} / (n_{jk} * (n_{jk} - 1) / 2)$$

$$cc_A = 2 / ((3 * 2) / 2) = 2 / 3 = 0.67$$



# Clustering coefficient



- Global measures:
- **(1) global cluster coefficient (GCC):**
  - # of closed triads / # all connected triads
- **(2) average cluster coefficient (ACC):**
  - Arithmetic mean of local cluster coefficient values.

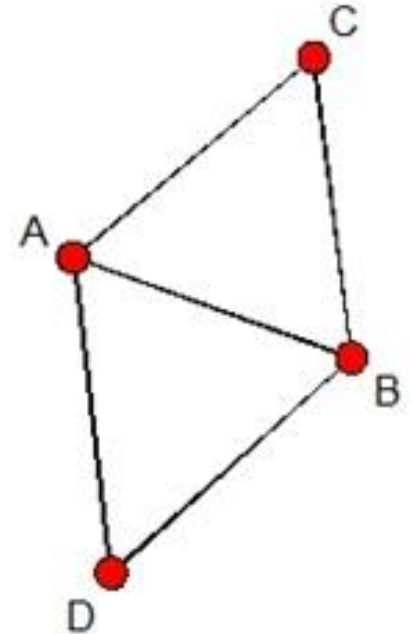
$$ACC(G) = \sum c_i / N$$

# Clustering coefficient

$$ACC(G) = \sum cc_i / N$$

$$ACC(G) = \sum(0.67 + 0.67 + 1 + 1) / 4 = 3.34 / 4 = 0.835$$

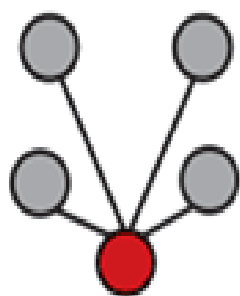
$$GCC(G) = 2 / 4 = 0.5$$



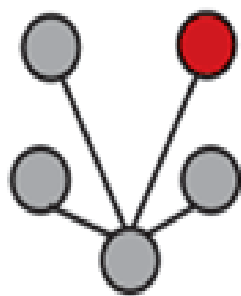


**a**

Degree



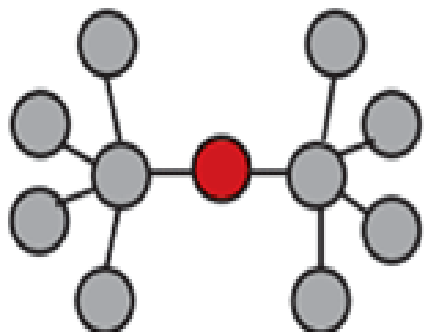
High-degree node



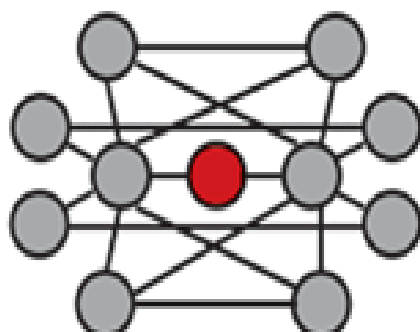
Low-degree node

**b**

Bridging centrality



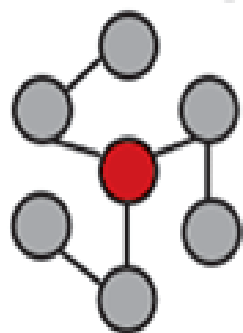
High bridging centrality



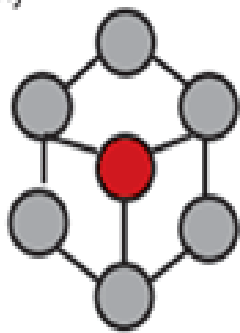
Low bridging centrality

**c**

Betweenness centrality



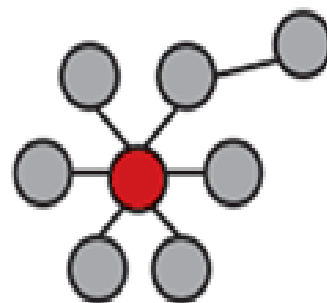
High betweenness centrality



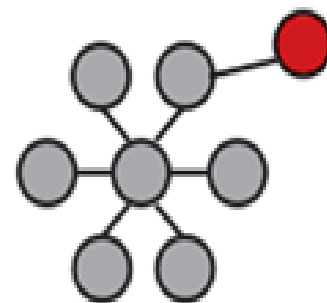
Low betweenness centrality

**d**

Closeness centrality



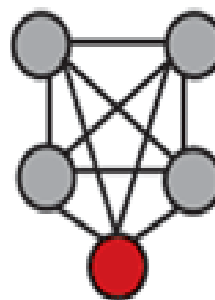
High closeness centrality



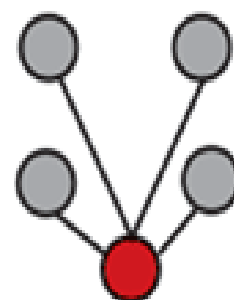
Low closeness centrality

**e**

Clustering coefficient



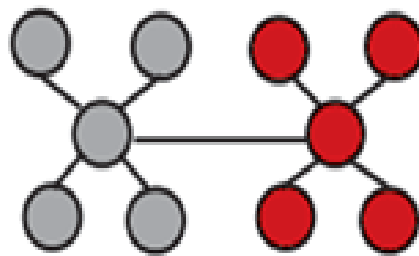
High clustering coefficient



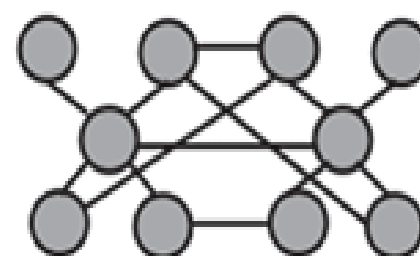
Low clustering coefficient

**f**

Modularity



Highly modular network



Nonmodular network

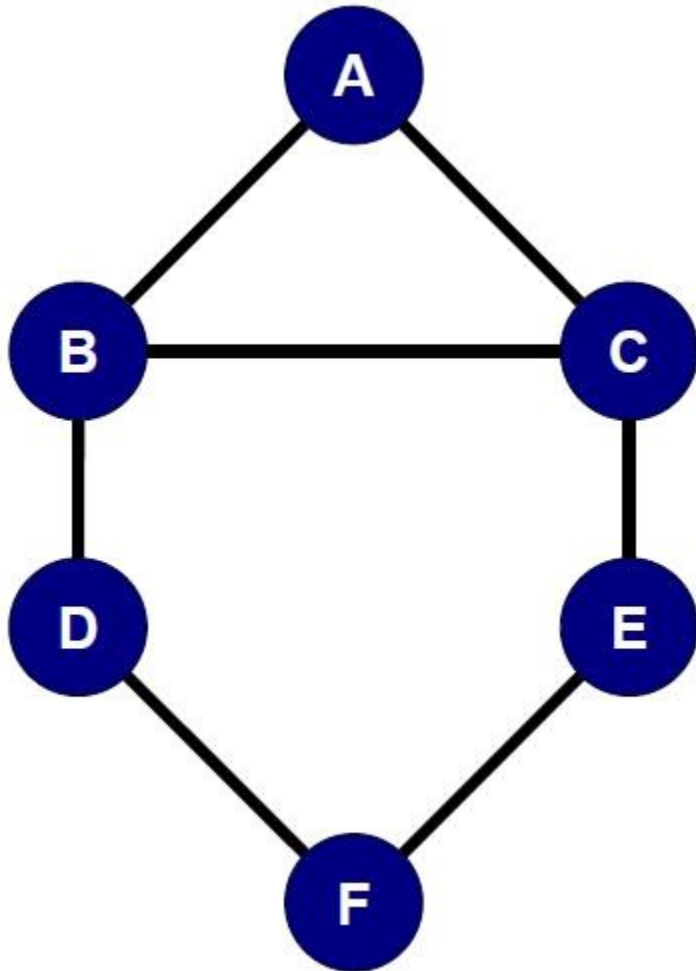
# Segments

- SNA toolkit allows to describe larger parts of the graph than triads.
- The most used concepts for network segmentation measurement:
  - cliques / n-cliques
  - k-cores
  - ... and many others

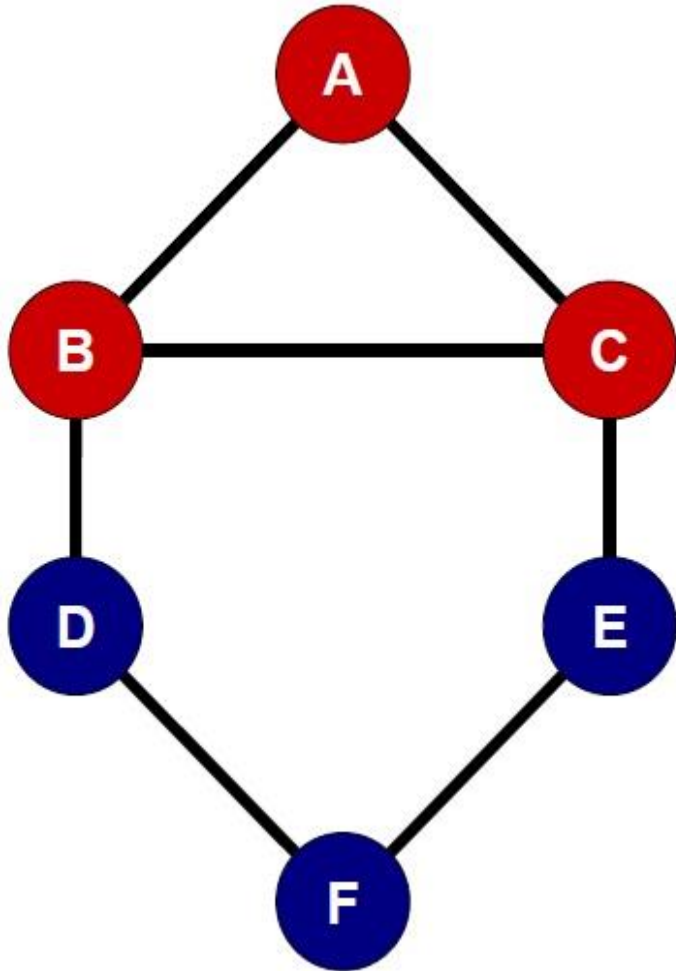
# Cliques and n-cliques

- **Clique** is a **maximal complete subgraph** that consists from three and more nodes.
- → each member of the clique has to be connected to all other members of the clique.
- Strong assumption → **n-clique**.
- N-clique is a maximal subgraph where **longest geodesic** between any two members is not greater than **n**.
- Thus in 2-clique every members is connected to all other members in 1 or 2 steps.

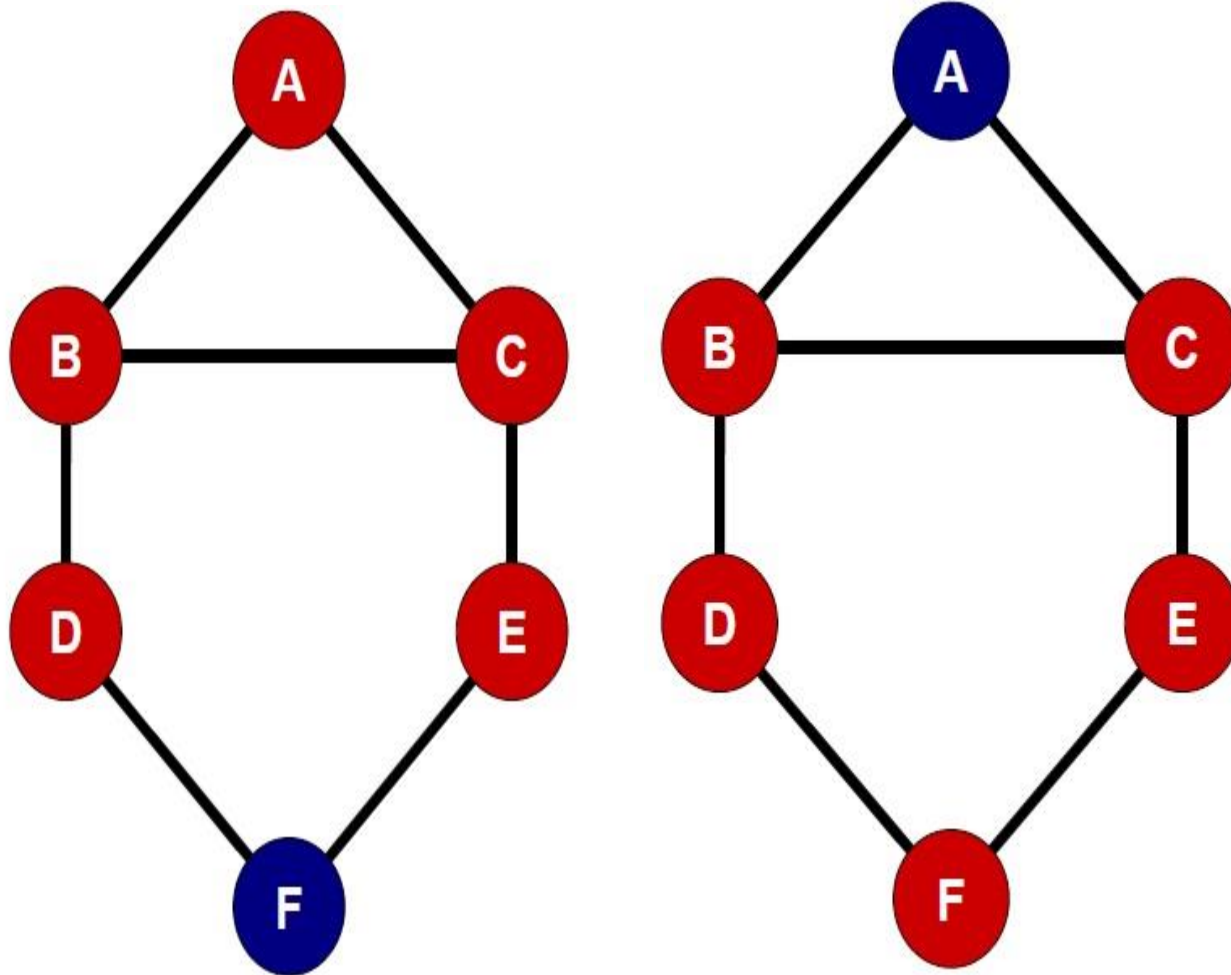
# Cliques and n-cliques



# Cliques and n-cliques



# Cliques and n-cliques

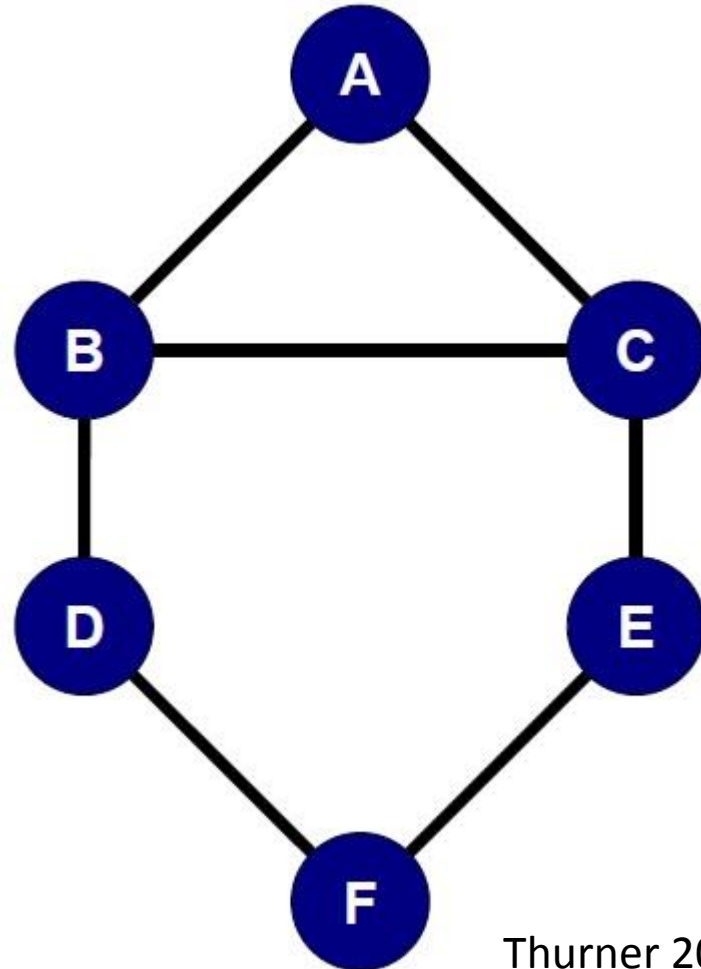


# K-core

- **K-core** is maximal subgraph where all nodes are connected with specific ( $k$ ) minimal # of nodes in the subgraph.
- Thus:  **$k$  indicates how many connections each member of the subgraph has to have.**
- Therefore: it is not important how many connections to other members is missing.

# K-core

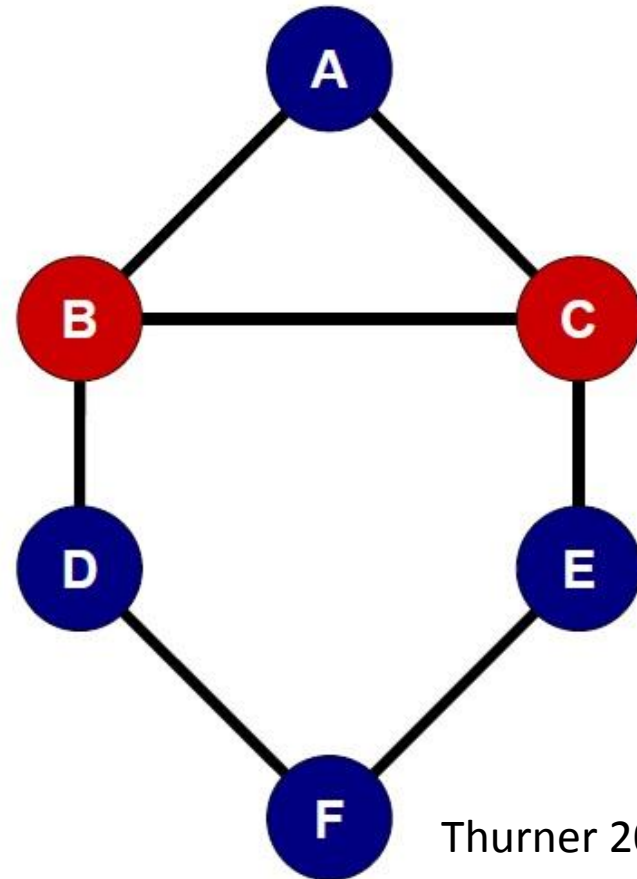
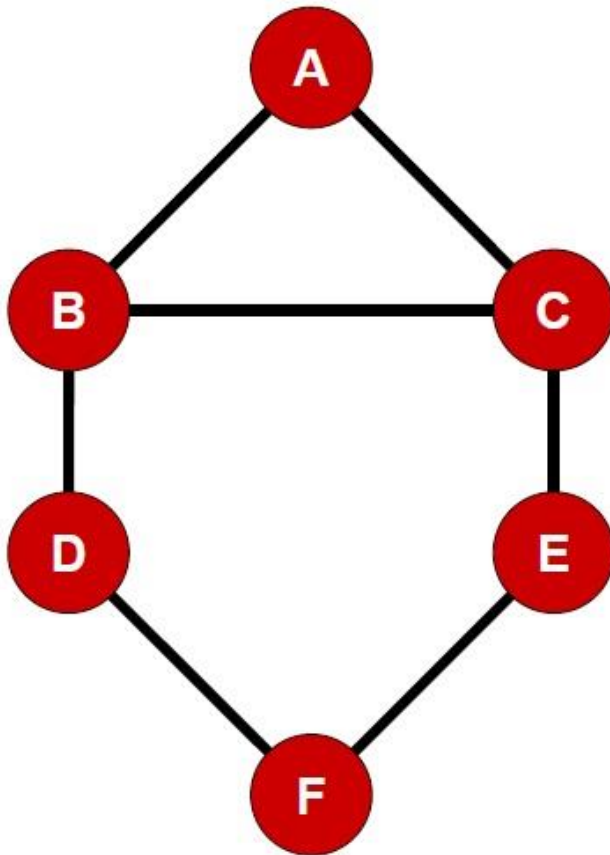
- Find 2-core and 3-core



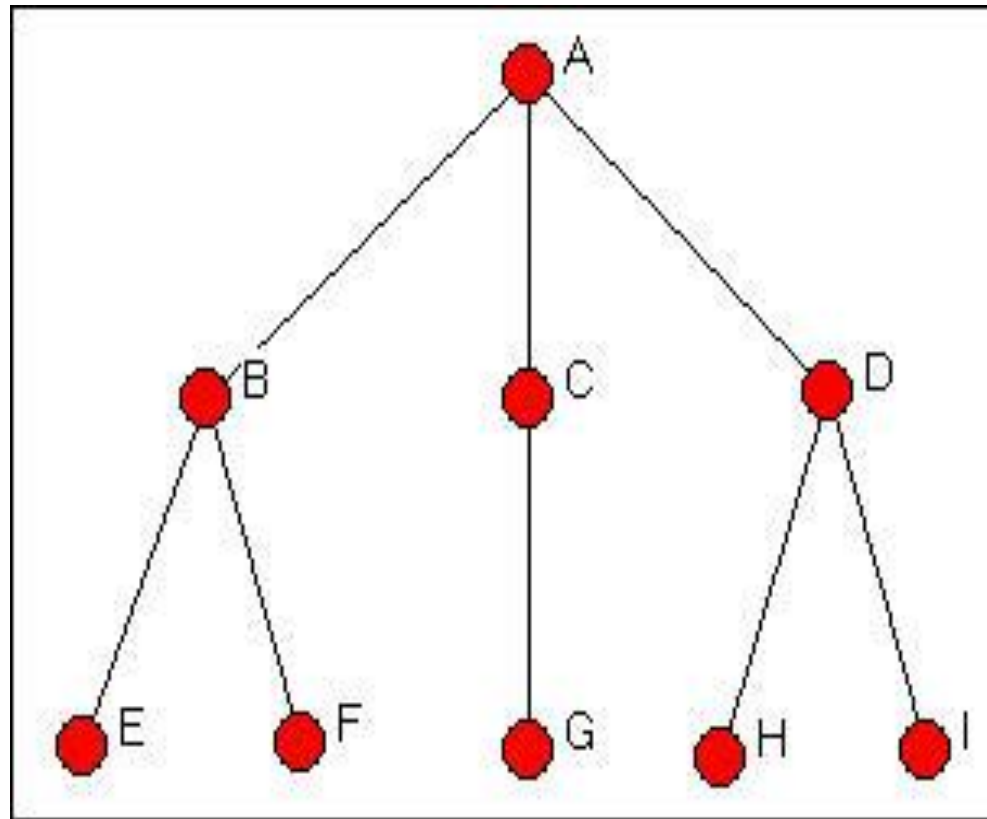


# K-core

- Find 2-core and 3-core:



# Structural equivalence



Hanneman & Riddle 2005

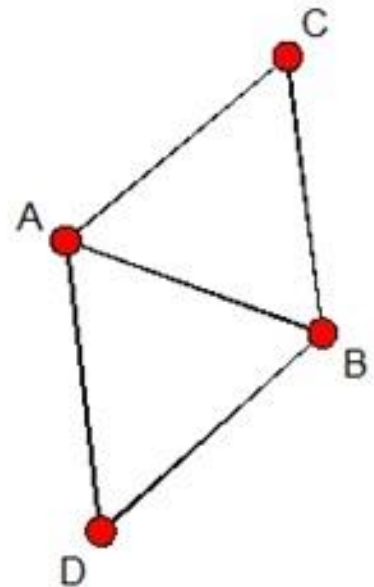
# Structural equivalence

- **Social position:** similar ties to other nodes.
  - E.g. Ph.D. students at our department have similar ties to others (under/graduates, department members, supervisors, etc.) as Ph.D. students at other departments.
- **Social role:** pattern of ties to other positions.
  - E.g. professional ties with department members, competitive ties with other Ph.D., friendship ties with other students, etc.

# Euclidean distance

- **Euclidean distance (ED):** is a distance of nodes  $i$  and  $j$  in relation to all other nodes in graph.
- ED of structurally equivalent nodes = 0.
- It is possible to classify nodes based on their ED.

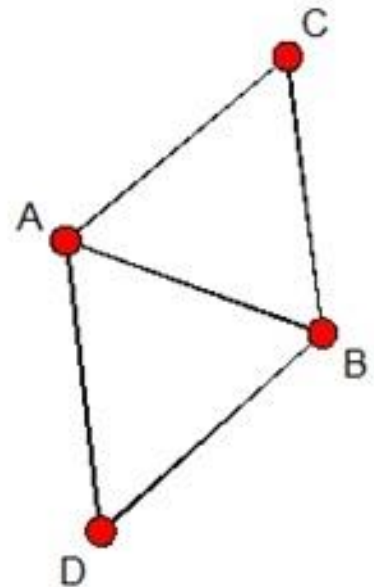
$$d_{ij} = \text{sqrt} [ \sum \sum [ (n_{ik} - n_{jk})^2 + (n_{ki} - n_{kj})^2 ] ]$$



# Euclidean distance

- **(1)**: differences of distances of  $i$  and  $j$  to all other nodes.
- **(2)**: sum of squares of the differences (SSD).
- **(3)**: square root of the result (SSD).

$$d_{ij} = \text{sqrt} [ \sum \sum [ (n_{ik} - n_{jk})^2 + (n_{ki} - n_{kj})^2 ] ]$$



# Euclidean distance

$$d_{ij} = \text{sqrt}[ \sum \sum [ (n_{ik} - n_{jk})^2 + (n_{ki} - n_{kj})^2 ] ]$$

$$d_{ac} = \text{sqrt}[ \sum \sum [ (n_{ak} - n_{ck})^2 + (n_{ka} - n_{kc})^2 ] ] ; k = \{b, d\}$$

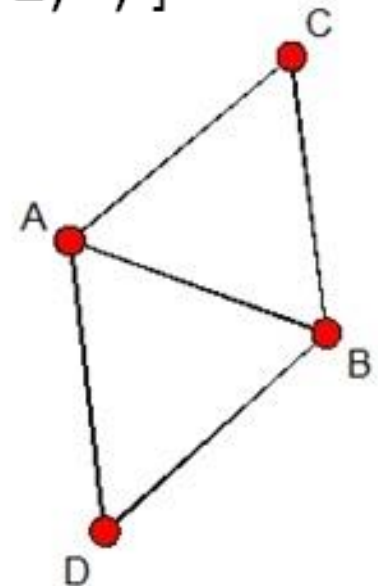
$$d_{ac} = \text{sqrt}[ ( (ab - cb)^2 + (ba - bc)^2 ) + ( (ad - cd)^2 + (da - dc)^2 ) ]$$

$$d_{ac} = \text{sqrt}[ ( (1 - 1)^2 + (1 - 1)^2 ) + ( (1 - 2)^2 + (1 - 2)^2 ) ]$$

$$d_{ac} = \text{sqrt}(0 + 0 + 1 + 1) = \text{sqrt}(2) = 1.41$$

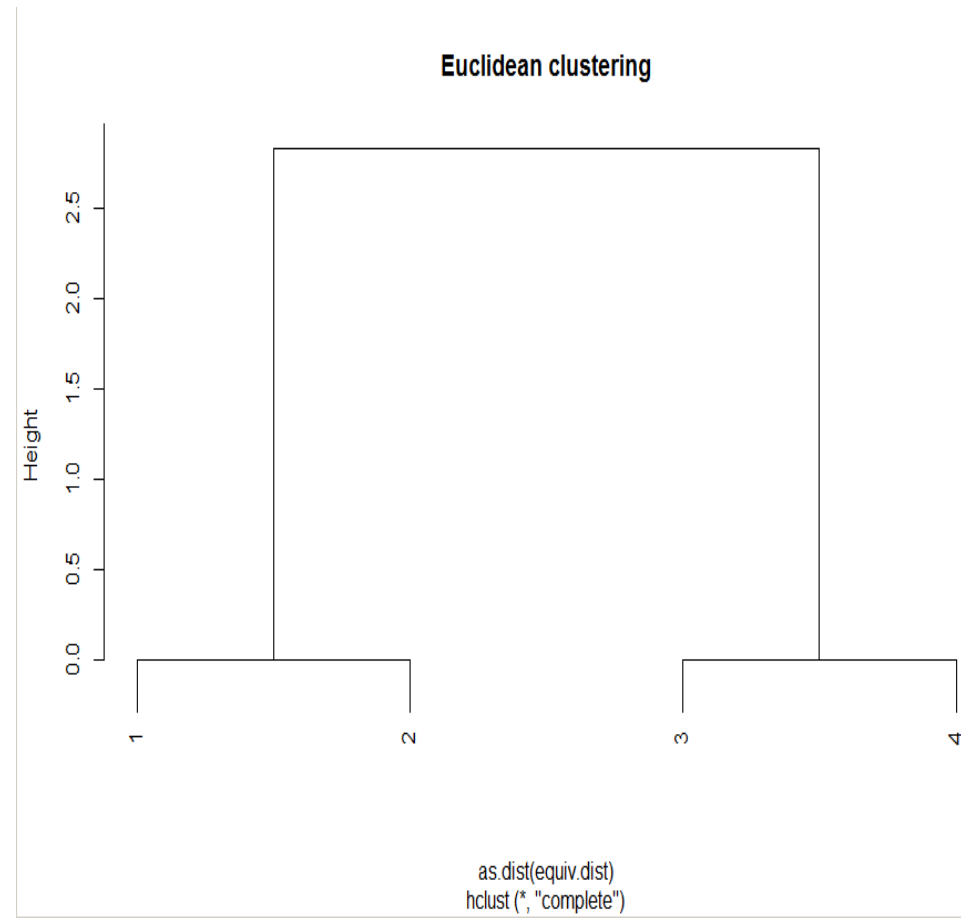
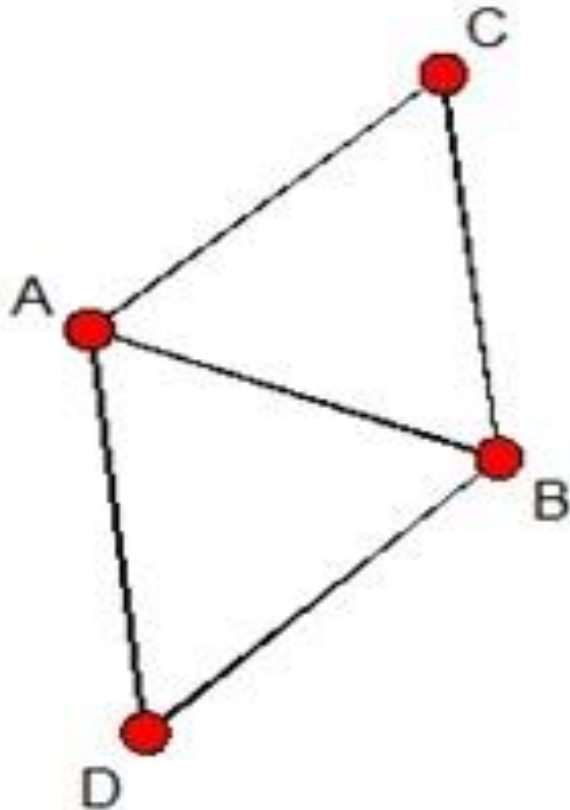
0	1	1	1
1	0	1	1
1	1	0	0
1	1	0	0

0	0	1.41	1.41
0	0	1.41	1.41
1.41	1.41	0	0
1.41	1.41	0	0



0	1	1	1
1	0	1	1
1	1	0	0
1	1	0	0

0	0	1.41	1.41
0	0	1.41	1.41
1.41	1.41	0	0
1.41	1.41	0	0



# Illustration: energy interdependence

- **Research objective:**
  - Mapping energy interdependence relations at the European natural gas market (exploratory objective).
- **Research question:**
  - What is the level of energy interdependence at the European market?
- **Research importance:**
  - Collection of data and exploration.
  - Necessary step for explanatory research (liberal peace hypotheses).



# Network border delineation

- positional strategy of border delineation:
  - European NG consumers and their suppliers.
- sample ~ population :
  - EU28 + Norway, FYROM, Ukraine, Belarus, Turkey and its suppliers.

# Conceptualization / operationalization

- Operationalization (Barbieri 1996):
  - Interdependence has two dimensions: saliency and symmetry.
  - **Trade share (TS)**: bilateral trade flow over total trade flow
    - $TS = \text{trade}_{AB}/\text{trade}_{AW} ; \text{trade}_{BA}/\text{trade}_{BW}$
  - **Saliency (S)**: trade shares product of A and B
    - $S = \sqrt{\text{trade}_{AB}/\text{trade}_{AW} * \text{trade}_{BA}/\text{trade}_{BW}}$
  - **Symmetry (Y)**: difference of trade shares of A and B
    - $Y = 1 - \text{abs}(\text{trade}_{AB}/\text{trade}_{AW} - \text{trade}_{BA}/\text{trade}_{BW})$
  - **Interdependence (I)**:
    - $I = \sqrt{V * S}$
- Attribute variables:
  - exporter / importer
  - Composite Index of National Capability (CINC)

# Interdependence: calculation

<b>CZE-RUS</b>	
share (bcm/y)	CZE: $7/9 = 0.77$ (77 %); RUS: $7/150 = 0.05$ (5 %)
saliency	$\sqrt{0.77 \cdot 0.05} = 0.20$ (20 %)
symmetry	$1 - \text{abs}(0.05 - 0.77) = 1 - 0.72 = 0.28$ (28 %)
interdependence	$\sqrt{0.20 \cdot 0.28} = 0.24$ (24 %)
weighted interdependence	$\sqrt{0.24 \cdot \sqrt{0.20 \cdot 0.04}} = 0.15$ (15 %)

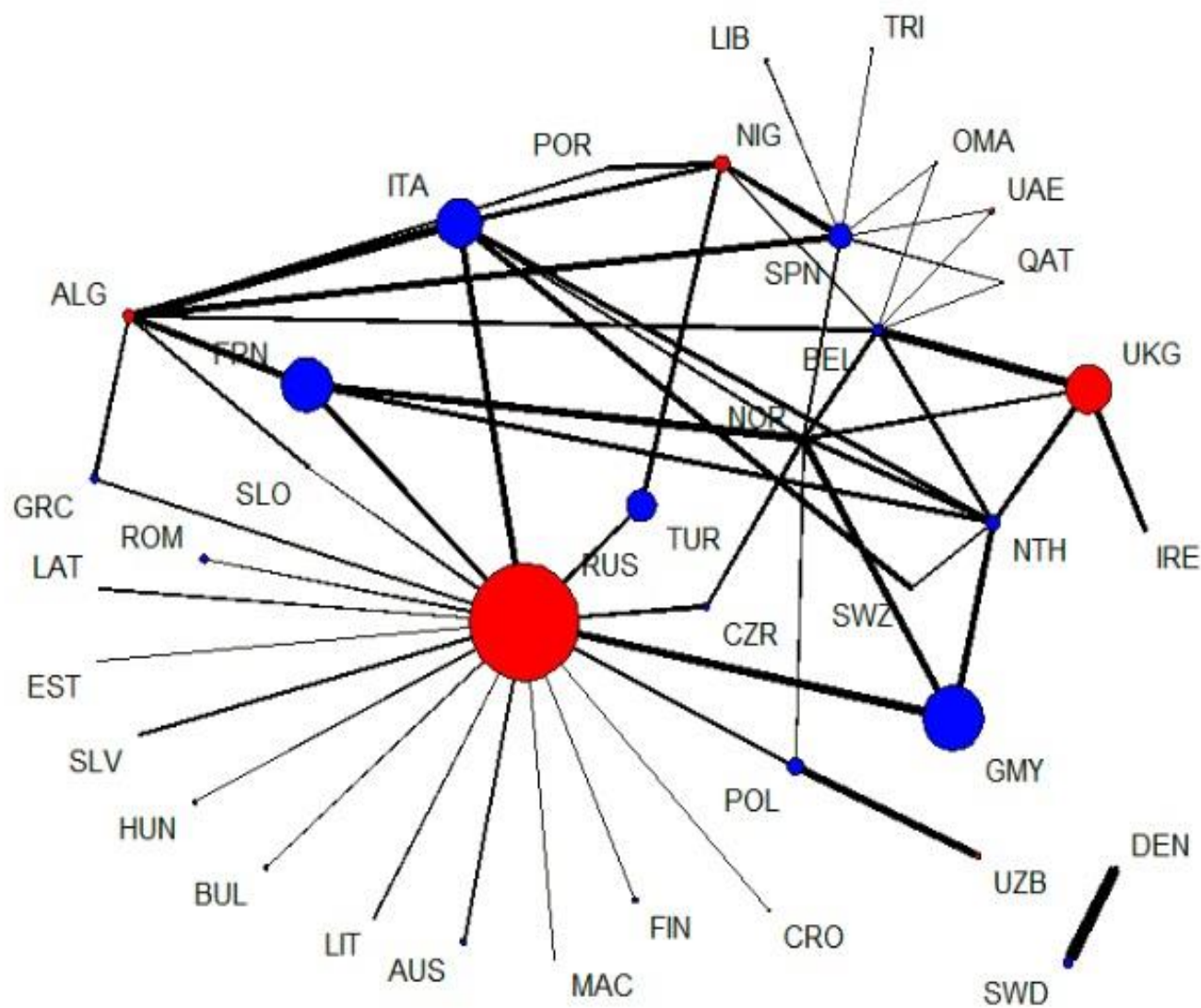
<b>GER-RUS</b>	
share (bcm/y)	GER: $40/85 = 0.47$ (47 %); RUS: $40/150 = 0.27$ (27 %)
saliency	$\sqrt{0.47 \cdot 0.27} = 0.36$ (36 %)
symmetry	$1 - \text{abs}(0.47 - 0.27) = 1 - 0.2 = 0.8$ (80 %)
interdependence	$\sqrt{0.36 \cdot 0.80} = 0.54$ (54 %)
weighted interdependence	$\sqrt{0.54 \cdot \sqrt{0.24 \cdot 0.04}} = 0.23$ (23 %)

# Attribute variables

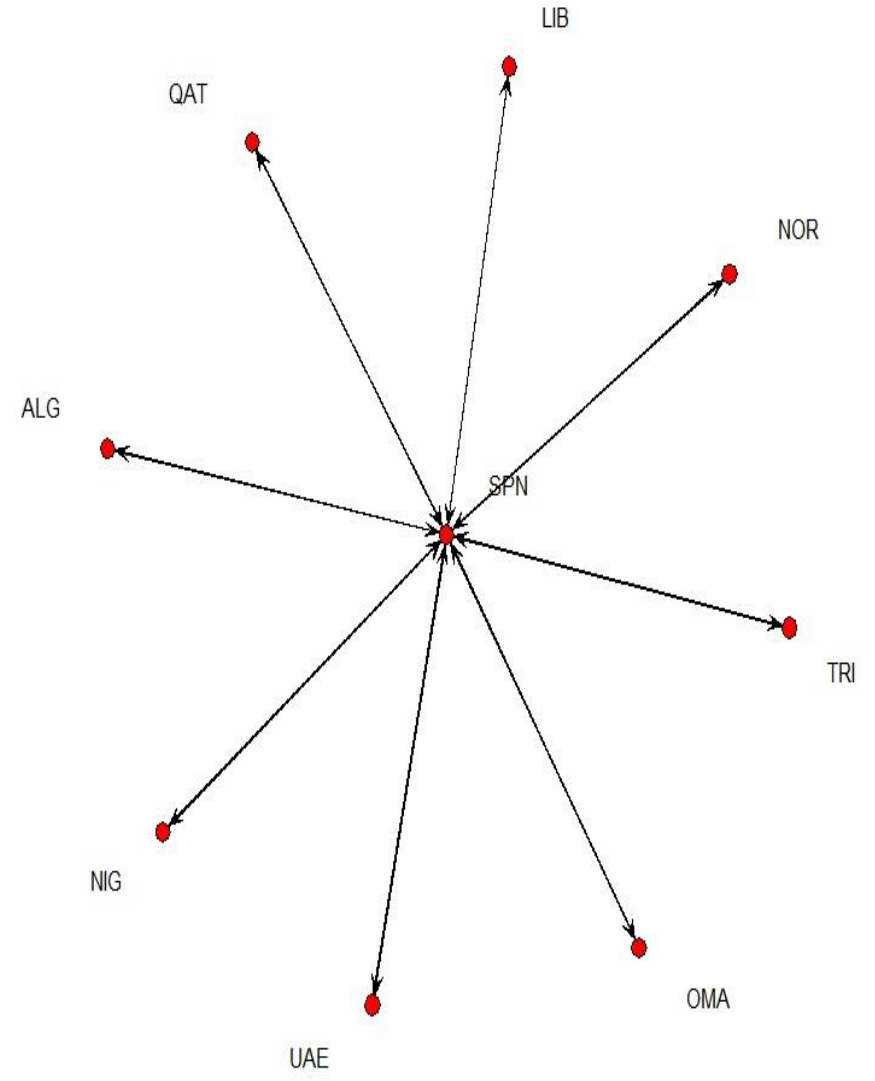
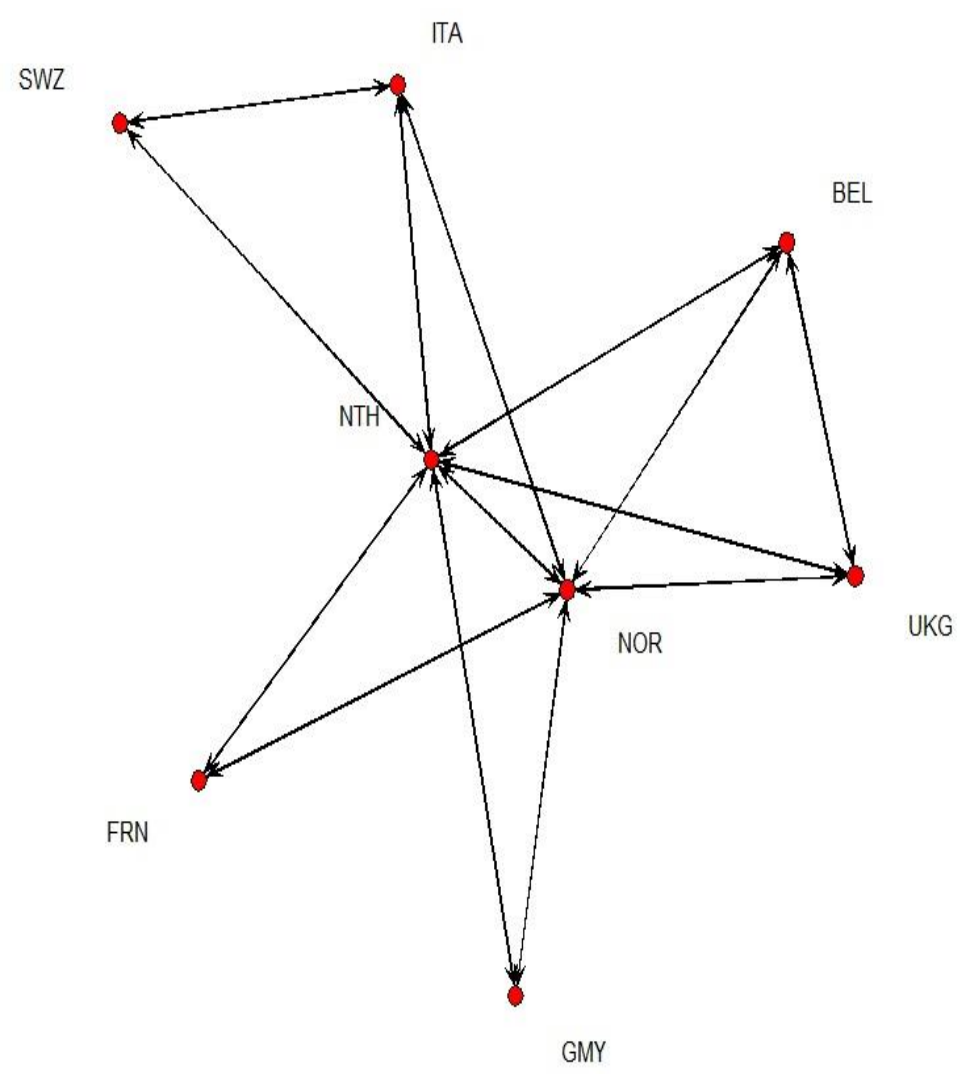
variable	operationalization (bcm/y)	CZE	GER
NG consumption / TPES	NG consumption / TPES	0.20 (20 %)	0.24 (24 %)
import / consumption	import NG / consumption NG	98 %	90/95 = 0.95 (95 %)
diversification (concentration)	$HH = \sum \text{shares}^2$	$0.7^2 + 0.3^2 = 0.58$ (58 %)	$0.23^2 + 0.33^2 + 0.40^2 = 0.32$ (32 %)
substitutability	N - 1	0.2 (20 %)	0.6 (60 %)
storage capacity / total consumption	storage capacity / consumption NG	$3/9 = 0.3$ (30 %)	$24/95 = 0.25$ (25 %)
political regime	Freedom House Index	0.82 (82 %)	0.84 (84 %)
...			



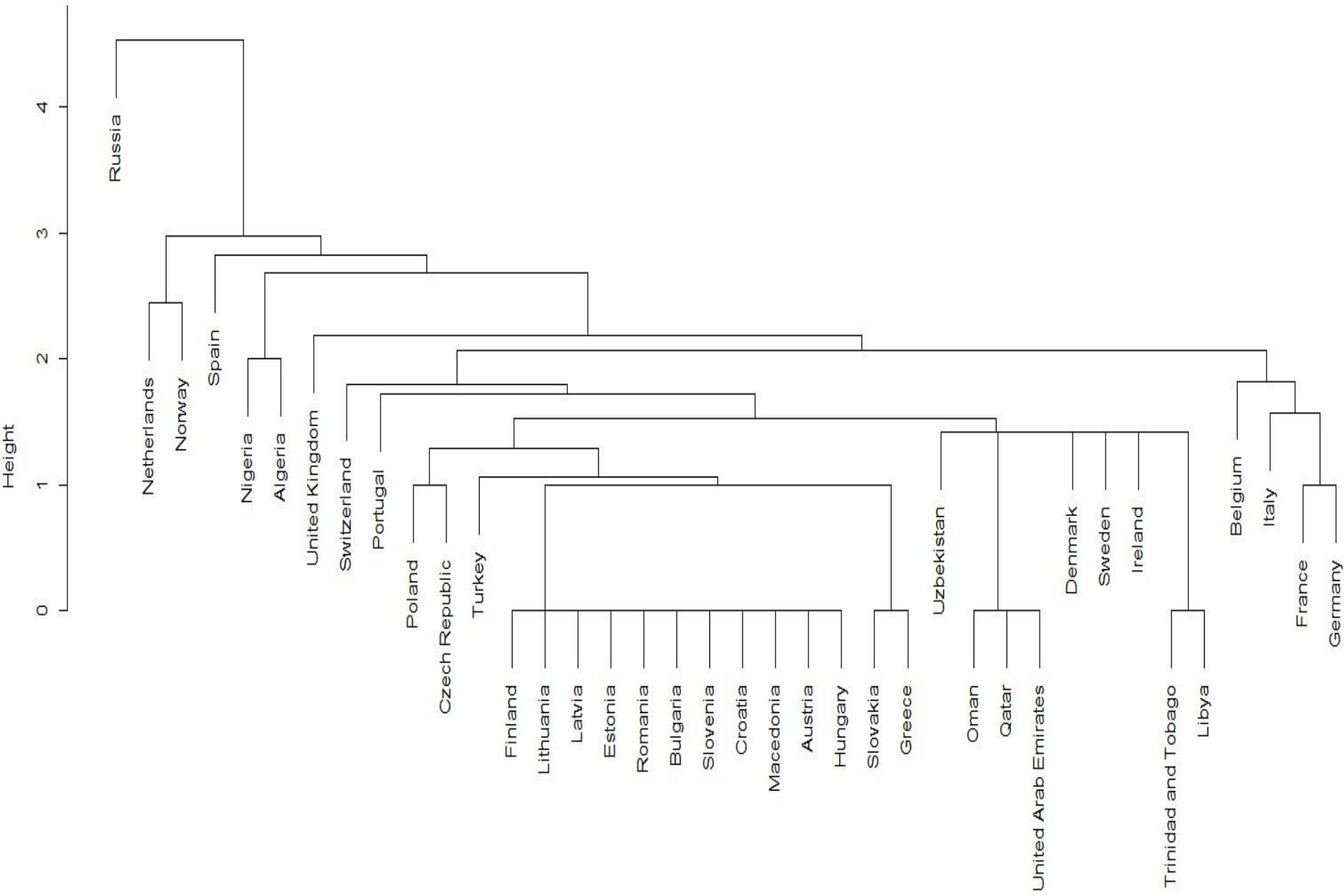
Interdependence network (European natural gas market (2001))



edge width: interdependence  
node size: national capabilities  
exporter (red), importer (blue)



# Cluster Dendrogram





# Assignment 2

- Create a new script.
- Generate a random graph with 5 nodes.
- Calculate degree, closeness, and betweenness centrality.
- Calculate degree centralization.
- Visualize graph and report the centrality / centralization results.
- Bonus: display node size as a function of its degree centrality.