Theory of Consumer Choice and Theory of Firms

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Key questions

- How can we represent the consumer's preferences?
- How can we represent the choices a consumer can afford?
- What determines how a consumer divides his or her resources between two goods?
  - Consumer Optimization Problem

Rational behavior

- The rational consumer behavior is driven by the effort to achieve the highest level of satisfaction
  - In the economics, the consumers (sic!) are being satisfied by the consumption of goods and services
  - The highest level of satisfaction is seen according to the consumer's *preferences*
  - With limited resources, people face *tradeoffs*
    - if you go swimming, you will have less time for jogging
    - if you buy more of one good, you will have less money to buy something else
    - if you work more, you will earn more, but you will have less of the leisure time
- The consumers seek to consume the "best" bundle of goods they can afford:
  - they maximize their utility functions subject to their budgetary constrains

The preference relations

- In order to represent the what is “better” or "worse", the consumer has to be able to compare the choices.
  - This comparison can be represented by the preference relations.
  - This logic of comparison is used in the construction of the so-called "utility function" - a representation of the consumer's preferences.
- The preference relation allows for the comparison of pairs of alternatives, x and y
  - $x \succeq y$ means x is at least as good as y; ($x \succ y$ means x is strictly preffered to y)
  - $x \preceq y$ means y is at least as good as x; ($x \prec y$ means y is strictly preffered to x)
  - $x \sim y$ means x is indifferent to y

The rational preference relations

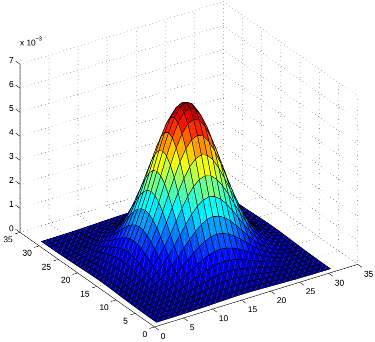
- The preference relation is rational, if it fulfills two basic assumptions: *completeness* and *transitivity*
  - Completeness: for any pair of $x, y \in X$ we have $x \succeq y$, $x \preceq y$ or both (i.e. $x \sim y$).
    - consumer is able to compare any two pairs and decide whether x is preferred to y, y is preferred to x, or whether he or she is indifferent between x and y
  - Transitivity: for all alternatives $x, y.z \in X$, if $x \succeq y$ and $y \succeq z$ then $x \succeq z$.
    - rational behavior rules out a preference cycle

Utility functions

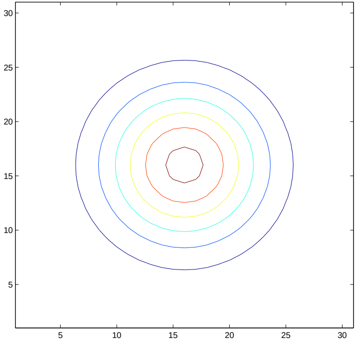
- A function $u: X \to R$ is a utility function representing preference relation $\succeq$, if for all $x, y \in X$ we have $x \succeq y \Leftrightarrow u(x) \geq u(y)$
- The utility function is used to represent the preferences (the logical relationships) with a numerical representation
  - The utility function is strictly *individual*
- A preference relation $\succeq$ can be represented by a utility function only if it is rational

Figure: The visual representations, function $f(x, y) = z$

(a) We need 3-D graphs to visually capture the value of the function with two inputs



(b) 2-D contour plot for $f(x, y) = z$

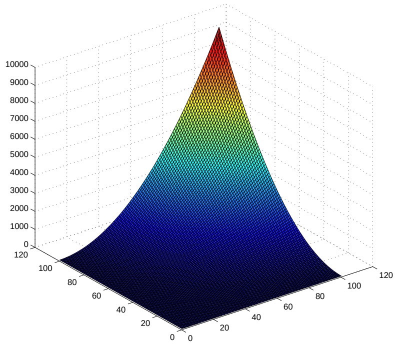


Utility function

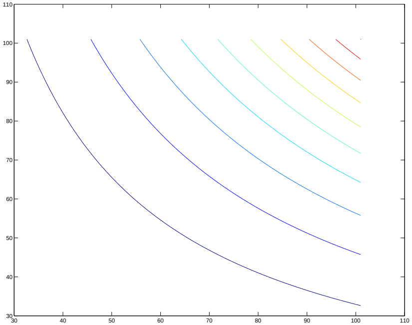
- The contour lines of the utility function represent all points, where the various combinations of amounts of x and y lead to the same value of the utility function
  - Assume the utility function $U = X^2Y^2$; for the given value of utility function, e.g. $U = 100$, we can write down the specific contour line (which is called the *indifference curve*) as $100 = X^2Y^2$, or $Y = \sqrt{\frac{100}{X^2}}$
  - The *indifference curve* shows all combination of bundles of x and y that bring the same utility. Therefore the consumer is *indifferent* between all bundles lying on the indifference curve.
- Properties of the *well behaved* utility function
  - monotonicity (or its weaker version, local nonsatiation)
    - in essence, the larger amount of consumed goods or services will not decrease the utility (the goods are desirable)
  - convexity
    - a formal expression of a basic inclination of economic agents for diversification (the assumption that consumers prefer variety)

Figure: The visual representation of the convex utility function with nonsation

(a) The utility function:



(b) The indifference map is the contour plot of the utility function:

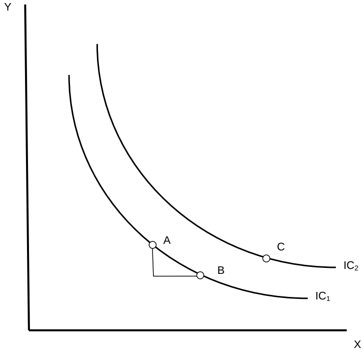


Indifference curves

- For well behaved utility functions, the higher indifference curves show the contours of higher values of the utility function
    - The higher indifference curves are preferred to lower ones
- Indifference curves do not cross (this would violate the transitivity of the preference relation)
- Indifference curves are bowed inward (becase of the convexity property)

Marginal rate of substitution in consumption

- Marginal rate of substitution in consumption (MRS_C) is the rate at which a consumer is willing to trade one good for another
- It is represented by the slope of the indifference curve
- $MRS_C = \frac{MU_X}{MU_Y}$, also $MRS_C = -\frac{\Delta Y}{\Delta X}$



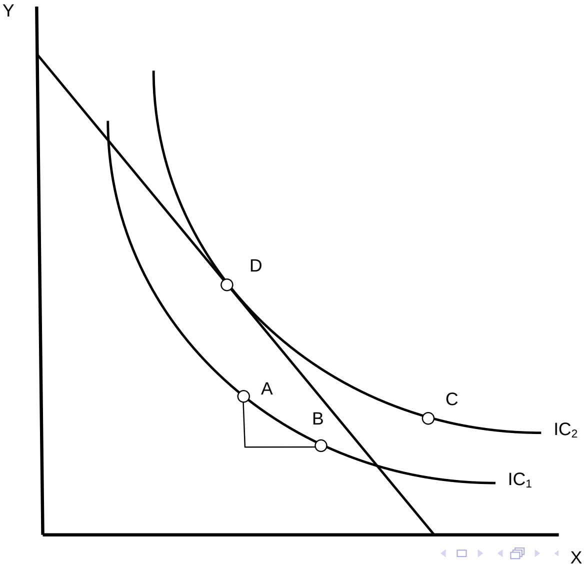
1. Consumer choice
   - Consumer behavior
   - Consumer Preferences
   - Utility functions and indifference curves
   - Consumer's optimization problem

2. Theory of Firms
   - Firm behavior
   - Production function
   - Costs
   - Revenues and demand
   - Profit maximization problem

Budget constraint

- Due to the limited resources, some of the consumption bundles are unavailable to the consumer.
- The limitation of the consumption is called *the budget constraint*: $I \geq p_x X + p_y Y$, where I is consumers disposable income, p_x is the price of one unit of X and X is the number of units of X consumed.
- The slope of the budget line represents the so called Marginal rate of substitution in exchange ($MRS_E = -\frac{p_X}{p_Y}$)
  - this represents the ratio at which the goods are valued by the market (e.g. "1 apple = 2 bananas" - this is called the relative price)
- An increase in income shifts the budget constraint outward
- A change in the prices causes the budget constraint to change its slope ("rotate")

Budget constraint



Consumer's optimization problem

- The consumer wants to attain maximum utility
  - The higher level of utility is equivalent with higher value of the utility function
  - Therefore, the consumer seeks to find such a combination that will *maximize* the value of the utility function AND satisfy the budget constraint
- $max\, U(X, Y)$ subject to: $I \geq p_x X + p_y Y$
  - since we are assuming monotonicity, we can simplify the budget constraint to $I = p_x X + p_y Y$
  - we can write the Lagrangian for this optimization problem as: $L(X, Y, \lambda) = U(X, Y) + \lambda (I - p_x X - p_y Y)$
  - By differentiating L with respect to x,y and λ and setting the derivatives equal to zero, we get the first order conditions:
    - $\frac{\partial U}{\partial X} - \lambda p_x = 0$
    - $\frac{\partial U}{\partial Y} - \lambda p_y = 0$
    - $I - p_x X - p_y Y = 0$

The optimal consumption

- The first order conditions imply the consumer will seek to consume the goods in the ratio that fulfills $MRS_C = MRS_E$, i.e. $\frac{MU_X}{MU_Y} = \frac{p_X}{p_Y}$ or $\frac{MU_X}{p_X} = \frac{MU_Y}{p_Y}$
  - The marginal utility MU_X is the change in the total utility caused by infinetisimaly small change in the consumption of X, i.e. $MU_X = \frac{\partial U}{\partial X}$
  - This implies the optimum will occur at the point where the indifference curve and the budget constraint are tangent
- The the attainable consumption, hence also the attainable utility, is limited by the budget constraint
  - Combined with the previous condition, this implies the optimum will occur at the point where the *highest* indifference curve and the budget constraint are tangent

Income and substitution effects

- The income effect is the change in consumption that results when a price change moves the consumer to a higher or lower indifference curve.
  - the price change increases (decreases) the purchasing power of the consumer (his relative wealth), which encourages (discourages) the consumer to buy the goods in question
- The substitution effect is the change in consumption that results when a price change moves the consumer along an indifference curve to a point with a different marginal rate of substitution.
  - the price change encourages greater (lower) consumption of the good, since it has become relatively cheaper (more expensive)

Normal goods vs. inferior goods

- Based on the *income elasticity* of demand ($\varepsilon_{ID} = \frac{\frac{\partial Q^D}{Q}}{\frac{\partial I}{I}}$) we can distinguish normal and inferior goods:
  - With the increase in income, the consumer buys more of a *normal* good.
  - With the increase in income, the consumer buys less of an *inferior* good.

Individual demand

- The demand function $Q^D = f(p, \cdot)$ represents the quantity of the given goods demanded, dependent on the price (and possibly other factors, e.g. $Q_X^D = f(p_x, p_y, I)$).
- Consumer's demand curve summarizes the solutions of the optimization problems for the various levels of price (given no changes in the other factors, e.g. for the given level of income and prices of other goods).
- Typically a demand curve slopes downward (with a decrease in price, the consumer would buy more of the goods).
    - There is an exception (but very rare): the so-called Giffen goods - an *increase* in the price raises the quantity demanded (the income effect overwhelmes the substitution effect).

Key questions

- What is the goal of the firm?
- How do firms make decisions?
  - Production choices maximizing the firm's profit
- What is a production function, marginal product, fixed and variable costs, marginal costs and marginal revenue?
- What is a perfectly competitive market and what is imperfet competition?
- What is the link to the demand function?
- How the firm determines its supply function?

The goal of the firm

- The producers of the goods and services are called firms (to distinguish the specific production units and their behavior)
- The business operation of the firm is the transformation of (typically multiple) *inputs* into (one or more) *outputs*.
  - The formal description of this transformation is called *the production function*
  - This transformation faces economic constraints - there are *costs* of production
  - The firm sells its production - it has to be aware of the demand in order to calculate its *revenues*
- The goal of the firm is to *maximize profits* (profit = total revenues - total costs, $\Pi = TR - TC$)
  - Can you provide the reason why?

Firm's profit maximization problem

- In general, the firm aims to maximize profits $\Pi = TR - TC$
  - the solution to the problem is found by setting the optimal Q (since $\Pi(Q) = TR(Q) - TC(Q)$
  - the optimal Q can be produced using various combinations of inputs (typically, K and L) (i.e. $Q(K, L)$), hence the firm can find the solution by setting optimal K and L (i.e. $\Pi(Q(K, L)) = TR(Q(K, L)) - TC(Q(K, L)))$
  - alternative method is to formulate *the cost minimization problem* to produce a given quantity $\bar{Q}$

The production function

- The production function ($Q = f(F_1, F_2, \dots F_N)$) describes the technological process of the transformation of inputs to outputs.
- The typical production factors (F) considered in the production of the firm are: capital (K) and labor (L).
  - $Q = f(K, L)$

Short run and long run

- Due to the technological limitations, it is usually time consuming to change the amount of capital used in production.
- This assumption is reflected in the distinction between the the short run and the long run
    - in the short run, we consider some of the production factors (typically capital (K)) as fixed and only some as variable (typically labor (L))
        - i.e. $Q = f\left(\bar{K}, L\right)$, or for the sake of simple notation $Q = f\left(L\right)$
    - in the long run, we consider *all* the production factors as *variable*

Marginal product

- The marginal product of factor F_i ($MP_{F_i} = \frac{\partial Q}{\partial F}$; note it also shows the slope of the production function!) is the increase in output caused by the additional unit of the input (while other inputs remain constant)
- The marginal product is not constant, but is (typically) *diminishing*
  - In contrast, if you hire additional worker, or buy additional machine, the costs of the aditional unit are often the same as with the previous unit
  - You can compare the marginal benefit (the increase in production) with the marginal costs (the increase in costs) to determine whether or not you want to add the additional unit of the input.
- Try to provide an example

Figure: Production function in the long run

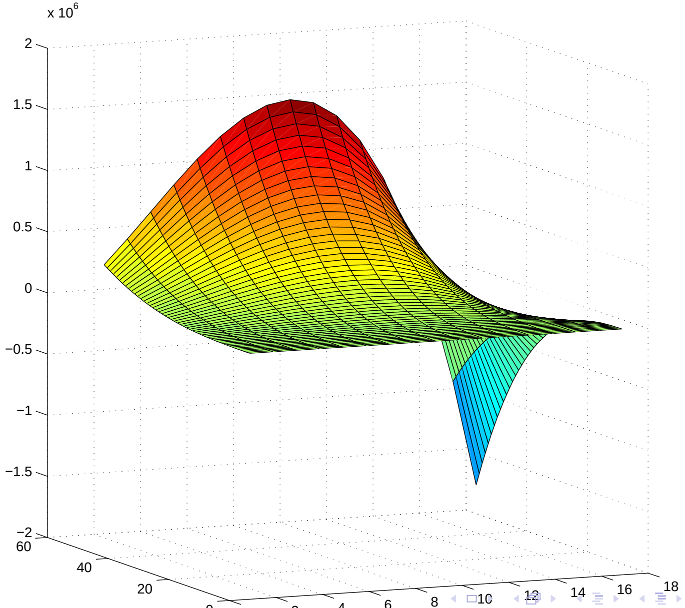


Figure: Production function in the short run

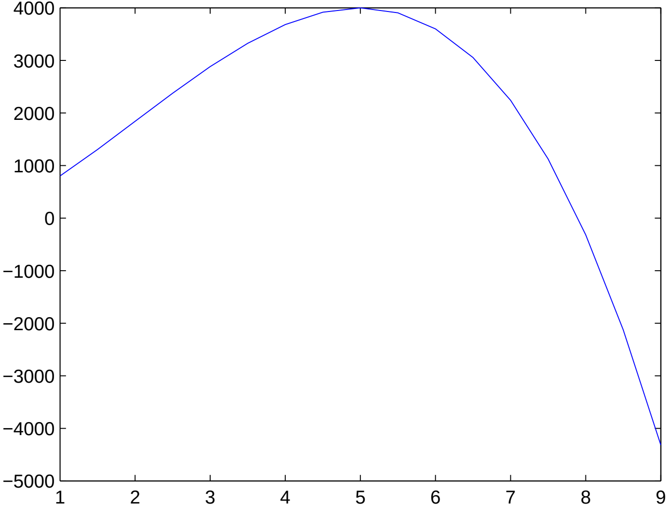
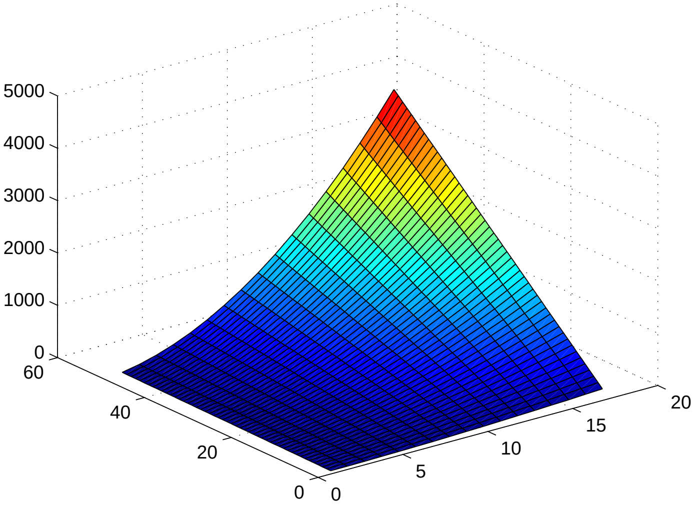


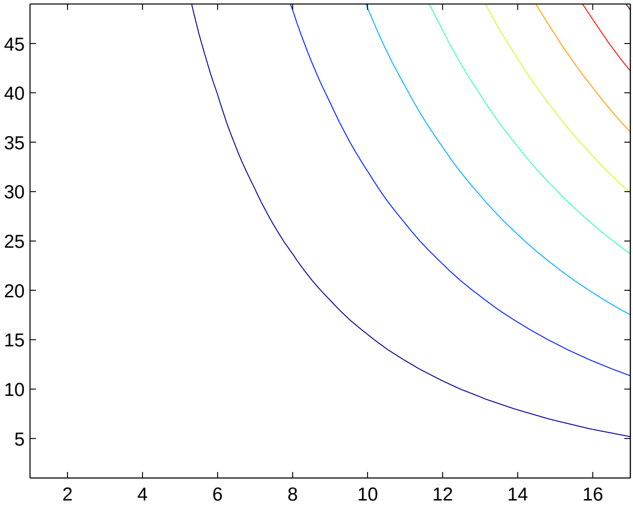
Figure: Production function in the long run, no diminishing returns (non-decreasing function)



Isoquants

- *Isoquants* are the contour lines of the production function.
- Isoquants represent all points, where the various combinations of amounts of inputs (K and L) lead to the same values of the production function.
  - they show the one given level of output that can be produced using various combinations of inputs
  - In a similar fashion as *the indifference curves*, the *isoquants* are the 2-D representation of the 3-D figure
  - the slope of an isoquant is called *the marginal rate of technical substitution* - it shows the ratio in which we can substitute the production factors while keeping the output constant

Figure: Isoquants of the production function in the long run, no diminishing returns (non-decreasing function)



Costs

- In order to produce something, the firm has to pay for the inputs.
- We need to consider both *implicit* and *explicit* costs
- Explicit costs: everything that will show up in the accounting (e.g. the electricity bill, the wages of hired workers)
- Implicit costs: the opportunity costs (the value of foregone alternative use of resources)

Illustration: you want to buy a machine, the price of the machine is $10,000, you can both lend or borrow money, with the fixed interest rate 3%.

- you can take a loan for $10,000 and pay 3% interest to the bank → explicit cost = $300
- you can take your own $10,000 and pay 0% interest to the bank → explicit cost = $0
  - But you could also lend $10,000 to someone else and collect the 3% interest!
  - By using your own money your foregone interest is $300 = implicit costs

Accounting profit vs. economic profit

- Accounting can only record actual events, not the alternate foregone opportunities.
- BUT economic costs are not meaningless $\rightarrow$ they influence the decisions! (it would not be rational / economical to pursue a certain activity, if there is a more profitable alternative)
- Accounting profit = total revenue minus total *explicit* costs
- Economic profit = total revenue minus (total *explicit and* total *implicit* costs)

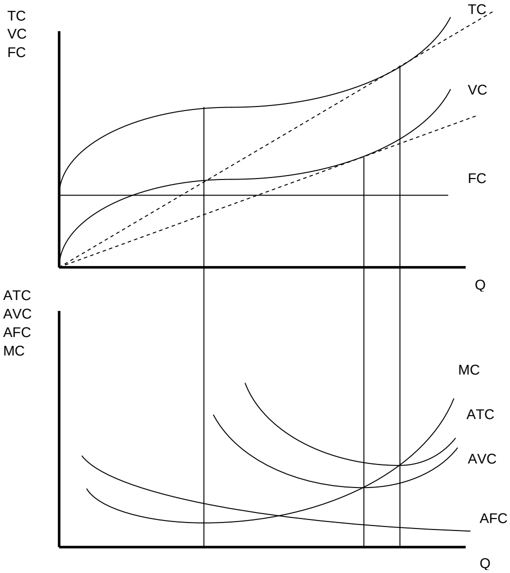
Fixed costs vs. variable costs

- We have already differentiated between the short run and the long run
  - In the short run, we can not change the quantity of some production factors
    - BUT we still have to pay for them
    - These costs are *fixed* by the inability to change the amount of certain production factors in the short run
- If we can change the quantity of the inputs we include into the production, we are dealing with the **variable** costs
- In the **short run**, there are **fixed costs** (typically the cost of capital) **and variable costs** (cost of labor)
- In the **long run**, all inputs and hence **all costs** are ***variable***

Marginal costs

- Marginal costs ($MC = \frac{\partial TC}{\partial Q}$; note it also shows the slope of the function of total costs!):
    - the change in total costs caused by producing the additional unit of the output
- The marginal costs are not constant, but are (typically) *increasing* (due to the **diminishing returns** of the production function!).

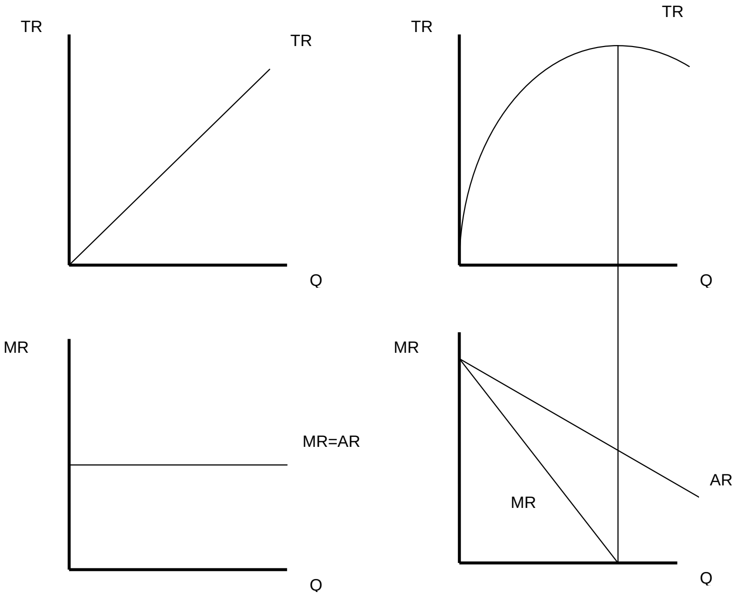
Figure: Total and marginal costs



Revenues

- Firms generate revenues by selling their output, i.e. $TR = p.Q\left(K, L\right)$
  - we have seen the firm is able to influence the quantity of the product it produces by using various amounts of inputs
- Depending on the market structure (influencing the demand for their products), firms either take the price from the market (p is constant), or they are able to influence the market price by changing the quantity produced (p is a function of Q)
  - perfect competition: $TR = p.Q\left(K, L\right)$
  - imperfect competition: $TR = p\left(Q\left(K, L\right)\right).Q\left(K, L\right)$

Figure: Total, marginal and average revenue



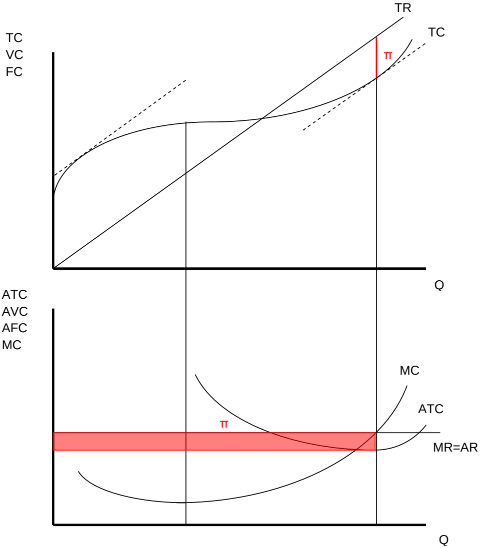
Marginal revenue

- The marginal revenue ($MR = \frac{\partial TR}{\partial Q}$; note it also shows the slope of the function of total revenues)
  - the change in the total revenue caused by producing the additional unit of the output
- The demand function is different in perfect competition and in imperfect competition: the demand function shows the quantity demanded for the given price
- The perfectly competetive firm can sell as many units as it can produce for the market price
  - The demand function is a horizontal line, AR$=MR$
- The imperfect competitive firm: in order to sell more units of Q, it has to lower the price on *all* units $\rightarrow$ it faces downward sloping demand
  - It has downward sloping marginal revenues

Profit maximization problem

- To maximize the profits $\Pi = TR - TC$, the firm will set the quantity produced so that
$\frac{\partial \Pi}{\partial Q} = \underbrace{\frac{\partial TR}{\partial Q}}_{MR} - \underbrace{\frac{\partial TC}{\partial Q}}_{MC} = 0 \rightarrow MR - MC = 0 \rightarrow MR = MC$
- In other words, the firm will produce additional unit of production, as long as the marginal revenue of this additional unit is larger or equal to what this additional unit of production costs
  - Holds true for both perfectly and imperfectly competitive firms

Figure: Total and marginal revenues



Supply function

- The supply function of the firm is the upward sloping part of the *MC* curve above the *AVC* (or *ATC* in the long run)
  - The profit maximization problem explains the **supply of the firm** - a firm will supply as long as the last unit of the product that is still profitable
  - In the short run (with existence of fixed costs), a firm might choose to produce, even if the production ends in red numbers (in loss)
    - The alternative option is to produce nothing, yet still pay fixed costs
    - If a firm covers at least variable costs for producing given amount of product, it will engage in production