Repeated-Measures ANOVA

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FSS MU,

14. 11. 2016

Repeated Measures

Repeated Measures ANOVA

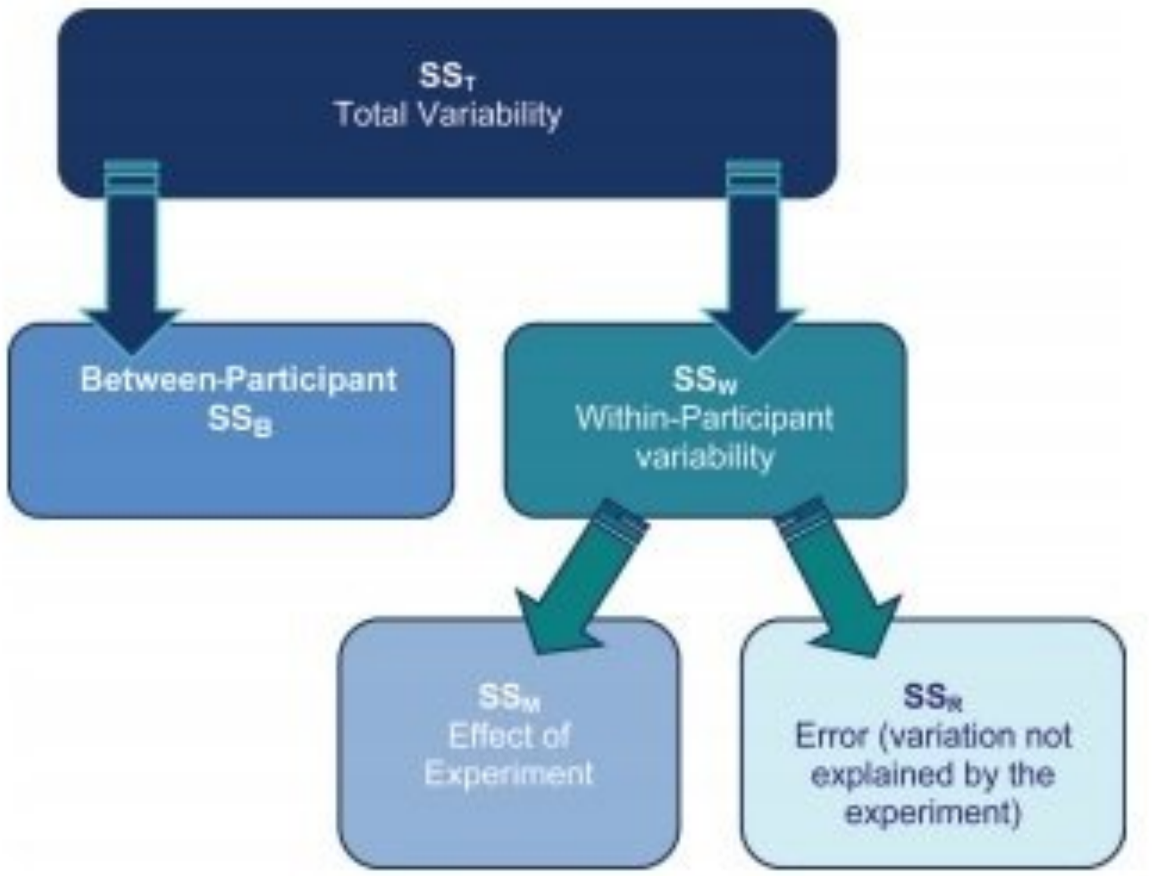
Úvod

- Slouží pro **srovnání skupinových průměrů** napříč **3 a více podmínkami**
- Within-subject a longitudinální design:
  - Sledujeme **vývoj** nějaké proměnné **v čase**
  - Vystavujeme **stejné jedince několika experimentálním podmínkám** a hledáme rozdíl ve změně

Představuje řešení situace, kdy je při opakovaných měřeních **porušen předpoklad** *ANOVA* či *lineární regrese* o **nezávislosti pozorování**

Repeated Measures ANOVA

<u>Rozptyl v rámci Repeated Measures ANOVA</u> (dle Field, 2012)



Repeated Measures ANOVA

<u>Rozptyl v rámci Repeated Measures ANOVA</u> **(dle Field, 2012)**

Celkový rozptyl (SS_T) $$SS_T = s^2_{grand}(N-1)$$

- The grand variance in the equation is simply the **variance of all scores when we ignore the group to which they belong**

Variabilita mezi subjekty (SS_W) $$SS_W = s^2_{Person\ 1}(n_1-1) + s^2_{Person\ 2}(n_2-1) + s^2_{Person\ 3}(n_3-1) + \ldots + s^2_{Person\ n}(n_n-1)$$

This equation simply means that we are looking at the **variation in an individual's scores and then adding these variances for all the people in the study**. The **n_s** simply represent *the number of scores on which the variances are based*

- Variance created by individuals' performances under different conditions
  - <u>Some of this variation is the result of our experimental manipulation and some of this variation is simply random fluctuation</u>.

Variabilita mezi měřeními (SS_M) $$SS_M = \sum_{i=1}^{k} n_i(\bar{x}_i - \bar{x}_{grand})^2$$

We worked out **how much variation could be explained by our experiment (the model SS) by looking at the means for each group and comparing these to the overall mean**. *So, we measured the variance resulting from the differences between group means and the overall mean*

1. Calculate the difference between the mean of each group and the grand mean.
2. Square each of these differences.
3. Multiply each result by the number of participants that contribute to that mean (ni).
4. Add the values for each group together:

Repeated Measures ANOVA

<u>Rozptyl v rámci Repeated Measures ANOVA</u> (dle Field, 2012)

Chybový rozptyl (SSR) $SS_R = SS_W - SS_M$ $SS_R = \sum_{i=1}^{n}(x_i - \overline{x}_i)^2$

- The final sum of squares is the **residual sum of squares (SSR)**, which tells us **how much of the variation cannot be explained by the model**.
  - This value is the <u>*amount of variation caused by extraneous factors outside of experimental control*</u>

The between-participant sum of squares (SSB) $SS_B = SS_T - SS_W$

- This term represents **individual differences between cases**
  - Its value is the <u>*amount of variation caused by differences between individuals*</u>

Repeated Measures ANOVA

Rozptyl v rámci Repeated Measures ANOVA (dle Field, 2012)

The mean squares

SS_M tells us **how much variation the model** (*e.g. the experimental manipulation*) **explains** and **SS_R** tells us **how much variation** is due to **extraneous factors**.

- *However, because both of these values are summed values the number of scores that were summed influences them. We eliminate this bias by calculating the average sum of squares.*

$$MS_M = \frac{SS_M}{df_M}$$

$$MS_R = \frac{SS_R}{df_R}$$

F-ratio

- The F-ratio is a measure of the **ratio of the variation explained by the model and the variation explained by unsystematic factors**.
  - *It can be calculated by dividing the model mean squares by the residual mean squares*

$$F = \frac{MS_M}{MS_R}$$

Repeated Measures ANOVA

Rozptyl v rámci Repeated Measures ANOVA

Source	SS	df	MS	F
Conditions	$SS_{conditions}$	$(k-1)$	$MS_{conditions}$	$\frac{MS_{conditions}}{MS_{error}}$
Subjects	$SS_{subjects}$	$(n-1)$	$MS_{subjects}$	$\frac{MS_{subjects}}{MS_{error}}$
Error	SS_{error}	$(k-1)(n-1)$	MS_{error}	
Total	SS_T	$(N-1)$		

k - *počet podmínek (treatments)*

n - *počet participantů (pozorování)*

df_Total - *number of observations (across all levels of the within-subjects factor, n) – 1*

Repeated Measures ANOVA

Výhody

Vyšší statistická síla

- The repeated measures design is statistically more powerful because of the **change in the nature of the error term** or the **unsystematic variance**. Subjects may consistently reveal individual differences.
    - For example: *participants which are not taking the experiment seriously, do not gain anything in the working memory training. So, they will not show an effect*.
- Repeated measures design will take this variability across subjects into account and will see whether there is **systematic variability that can be contributed to the individual** people.

Menší počet potřebných participantů

- **Nižší náklady** (je třeba méně zkoumaných osob)

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Nevýhody

Order effect

- *Jak ošetřit vliv pořadí experimentálních podmínek na sledovanou proměnnou?*
    - Counterbalancing
        - Randomised - A2,A1,A1,A2
            - Randomizace pořadí podmínek
        - Blocked - a) A1,A2 b) A2,A1
            - Náhodné zařazení do skupiny reprezentující pořadí podmínek
    - Latin Square - Každý respondent je vystaven každému pořadí experimentálních podmínek právě jednou

Missing values

- Vzhledem k obvykle menšímu počtu participantů představuje *riziko* - změnou variability či výpočetní komplikací (kupř. fit modelu)
    - Otázka nakládání s chybějícími daty - **listwise**, **pairwise**, **vážení** atd.
    - Souvisí s povahou chybějících dat - MCAR, MAR, MNAR

Repeated Measures ANOVA

Data: Cognitive training

- Four independent groups (8, 12, 17, 19 sessions)
- Measured IQ of 20 subjects after 4 sessions of training
- **Dependent variable** is **IQ gain after a particular session**
- **Null hypothesis**: *There are no differences among the treatment groups*
- **Alternative hypothesis**: *There is one (or more) mean differences among the treatment groups*

Repeated Measures ANOVA

Data: Cognitive training

```
setwd()
dir()

install.packages("readxl")
library("readxl")

excel_sheets("RMANOVA.xlsx")

RMANOVA = read_excel("RMANOVA.xlsx", sheet = 1)
View(RMANOVA)

RMANOVA$condition2 = factor(RMANOVA$condition, order = TRUE, levels =
c("8 days", "12 days", "17 days", "19 days"))
RMANOVA$subject2 = factor(RMANOVA$subject, order = FALSE)
```

Repeated Measures ANOVA

Statistický popis dat

 # Summary statistics by group

```
library(psych)
describeBy(RMANOVA, group = RMANOVA$condition2)
```

 # Boxplot

```
library(ggplot2)
bp1 = ggplot(RMANOVA, aes(condition2, iq))
bp1 + geom_boxplot(aes(fill=condition2), alpha=I(0.5)) +
 geom_point(position="jitter", alpha=0.5) +
 geom_boxplot(outlier.size=0, alpha=0.5) +
 theme(
   axis.title.x = element_text(face="bold", color="black", size=12),
   axis.title.y = element_text(face="bold", color="black", size=12),
   plot.title = element_text(face="bold", color = "black", size=12)) +
 labs(x="Condition",
      y = "IQ gain",
      title= "IQ gain by the days of training") + theme(legend.position='none')
```

Repeated Measures ANOVA

F-test a F-Ratio

Funkce aov

- Because you're using repeated measures, you have to add a term Error(wm$subject / wm$condition).
  - This term tells R that you need a special error term since you are working in a repeated measures design and the error term differs.
- So you add this term, saying that the subjects are measured repeatedly across conditions.

```
# Apply the aov function
model <- aov(iq ~ condition2 + Error(subject2 / condition2), data = RMANOVA)

# Look at the summary table of the result
summary(model)
```

Repeated Measures ANOVA

F-test a F-Ratio

Funkce ezANOVA

 # ez package

```
install.packages("ez")
library("ez")
```

```
newModel <- ezANOVA(data = dataFrame, dv = .(outcome variable), wid = .(variable that identifies participants), within =  .(repeated measures predictors), between = .(between-group predictors), detailed = TRUE, type = 2)
```

 # Apply the ezANOVA function

```
model2 <- ezANOVA(data = RMANOVA, dv = .(iq), wid = .(subject2), within =  .(condition2), detailed = TRUE, type = 3)
model2
```

Repeated Measures ANOVA

F-test a F-Ratio

Multilevel approach

```
install.packages("nlme")
library("nlme")

model3 = lme(iq ~ condition2, random = ~1|subject2/condition2, data =
RMANOVA, method = "ML")

summary(model3)
```

Repeated Measures ANOVA

Velikost účinku

$$\eta^2 = \frac{SS_b}{SS_t}$$

$$\omega^2 = \frac{df_{\text{effect}} \times (MS_{\text{effect}} - MS_{\text{error}})}{SS_{\text{total}} + MS_{\text{subjects}}}$$

ss_cond = 196.1
ss_total = 196.1 + 297.8
eta_sq <- ss_cond / ss_total

dfeffect = 3
MSeffect = 65.36
MSerror = 5.22
SStotal = 196.1 + 297.8
MSsubjects = 9.242

Omega2 = (dfeffect * (MSeffect - MSerror)) / (SStotal + MSsubjects)

Repeated Measures ANOVA

Předpoklady použití

Povaha proměnných

- "Závislá" proměnná kardinální úrovně měření

Normalita rozložení závislé proměnné

- V rámci každé sledované skupiny
- **Neparametrická** alternativa – Friedmanův test

Sféricita

- Sphericity is the condition where the **variances of the differences between all combinations of related groups (levels) are equal**
  - *Violation of sphericity is when the variances of the differences between all combinations of related groups are not equal*.
- The violation of sphericity is serious for the **repeated measures ANOVA**, with **violation causing the test to become too liberal** (i.e., *an increase in the Type I error rate*)

Repeated Measures ANOVA

Předpoklady použití

Mauchy's test

```
# Define the iq data frame
iq <- cbind(RMANOVA$iq[RMANOVA$condition == "8 days"],
        RMANOVA$iq[RMANOVA$condition == "12 days"],
        RMANOVA$iq[RMANOVA$condition == "17 days"],
        RMANOVA$iq[RMANOVA$condition == "19 days"])


# Make an mlm object
mlm <- lm(iq ~ 1)


# Mauchly's test
mauchly.test(mlm, x = ~ 1)
```

Repeated Measures ANOVA

Předpoklady použití

 # Normalita rozložení

```
ggplot(data=RMANOVA, aes(RMANOVA$iq)) +
  geom_histogram(breaks=seq(0, 20, by = 2),
                 col="red",
                 aes(fill=..count..)) +
  scale_fill_gradient("Count", low = "green", high = "red")+
  labs(title="Histogram for IQ Gain") +
  labs(x="IQ Gain", y="Count") + theme(legend.position='none')


ggplot(RMANOVA, aes(x=iq, fill=condition2)) +
  geom_histogram(position="identity", binwidth=1, alpha=0.5)
```

Repeated Measures ANOVA

Friedmanův test

```
friedman.test(iq ~ condition2 | subject2, data = RMANOVA)
```

Post-Hoc testy

<u>Úvod</u>

Allow for multiple pairwise comparisons without an increase in the probability of a Type I error

<u>Používáme, pokud nemáme dopředu jasné hypotézy</u>

- Srovnávají **vše se vším** – každou skupinu s každou (ale **neumí slučovat skupiny jako kontrasty**)

Z principu jsou <u>oboustranné</u>

Je jich mnoho – <u>liší se v několika parametrech</u>:

- **Konzervativní** (Ch. II. typu) versus **Liberální** (Ch. I. typu)
  - *Most liberal = no adjustment*
  - *Most conservative = adjust for every possible comparison that could be made*
- Ne/vhodné pro **porušení předpokladu sféricity**

Post-Hoc testy

Doporučení podle Fielda (2012)

Not only does **sphericity** create problems for the F in repeated-measures ANOVA, but also it causes some amusing **complications for post hoc tests**

- The default is to have no adjustment and simply perform a **Tukey LSD post hoc test** (this is not recommended).
- The second option is a **Bonferroni correction** (recommended for the reasons mentioned above), and the final option is a **Sidak correction**, which should be selected if you are concerned about the loss of power associated with Bonferroni corrected values.

Post-Hoc testy

Tukey

 # funkce glht

```
library(multcomp)
posthoc = glht(model3, linfct = mcp(condition2 = "Tukey"))
summary(posthoc)
```

Post-Hoc testy

Bonferroni

```
# Pairwise t-test
with(RMANOVA, pairwise.t.test(iq, condition, p.adjust.method =
"bonferroni", paired = T))
```

```
# glht
summary(glht(model3, linfct=mcp(condition2 = "Tukey")), test =
adjusted(type = "bonferroni"))
```

Post-Hoc testy

Šidák

 # dunn.test

```
install.packages("dunn.test")
library("dunn.test")
dunn.test (x = RMANOVA$iq, g=RMANOVA$condition2,
method="sidak", kw=TRUE, label=TRUE,
        wrap=FALSE, table=TRUE, list=FALSE, rmc=FALSE, alpha=0.05)
```

Kontrasty

Úvod

Umožňují porovnat jednotlivé skupiny v jednom kroku bez nutnosti korigovat hladinu významnosti (**bez snížení síly testu**)

- Jen když máme dopředu hypotézy
- Kontrastů lze provést tolik, kolik je počet skupin – 1

Každý kontrast **srovnává 2 podmínky**

- Průměr skupiny nebo průměr více skupin dohromady
- *Např. "19 dnů" vs. "8 dnů" nebo "17 dnů" vs. "12 dnů"*

Kontrasty

Name	Definition	Contrast	Three Groups			Four Groups		
Deviation (first)	Compares the effect of each category (except first) to the overall experimental effect	1	2	vs.	(1,2,3)	2	vs.	(1,2,3,4)
		2	3	vs.	(1,2,3)	3	vs.	(1,2,3,4)
		3				4	vs.	(1,2,3,4)
Deviation (last)	Compares the effect of each category (except last) to the overall experimental effect	1	1	vs.	(1,2,3)	1	vs.	(1,2,3,4)
		2	2	vs.	(1,2,3)	2	vs.	(1,2,3,4)
		3				3	vs.	(1,2,3,4)
Simple (first)	Each category is compared to the first category	1	1	vs.	2	1	vs.	2
		2	1	vs.	3	1	vs.	3
		3				1	vs.	4
Simple (last)	Each category is compared to the last category	1	1	vs.	3	1	vs.	4
		2	2	vs.	3	2	vs.	4
		3				3	vs.	4
Repeated	Each category (except the first) is compared to the previous category	1	1	vs.	2	1	vs.	2
		2	2	vs.	3	2	vs.	3
		3				3	vs.	4
Helmert	Each category (except the last) is compared to the mean effect of all subsequent categories	1	1	vs.	(2, 3)	1	vs.	(2, 3, 4)
		2	2	vs.	3	2	vs.	(3, 4)
		3				3	vs.	4
Difference (reverse Helmert)	Each category (except the first) is compared to the mean effect of all previous categories	1	3	vs.	(2, 1)	4	vs.	(3, 2, 1)
		2	2	vs.	1	3	vs.	(2, 1)
		3				2	vs.	1

Kontrasty

Příklad

 # Helmertův kontrast

```
options(contrasts = c("contr.helmert", "contr.poly"))
contrasts(RMANOVA$condition2) <- "contr.helmert"
model3 = lme(iq ~ condition2, random = ~1|subject2/condition2, data =
RMANOVA, method = "ML")
summary(model3)
```

Základní literatura

Field, A., Miles, J., & Field, Z. (2012). Discovering Statistics Using R. Sage: UK.

Navarro, D. J. (2014). Learning statistics with R: A tutorial for psychology students and other beginners. Available online: http://health.adelaide.edu.au/psychology/ccs/teaching/lsr/