Game theory 1

Lukáš Lehotský
llehotsky@mail.muni.cz

Agent vs. structure

- On the **individualist extreme** of agent-structure spectrum
- Regards the social phenomena as interactions of individuals – **structure is a product** of individual behavior
- Individual behavior is **unconstrained**

Criticism against rational choice theory

- Common criticism of rational choice – people behave irrationally
- Rationality ≠ Sensibility
- Ordering preferences
- I can mostly prefer taking over the world and least painful death, but equally prefer most painful death and least taking over the world

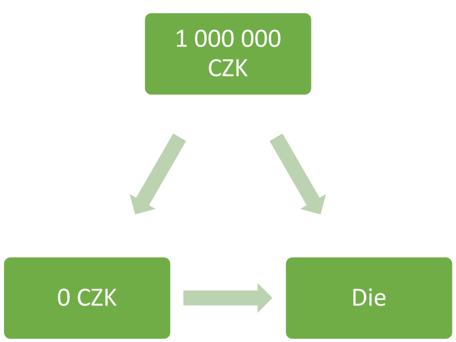
Rationality

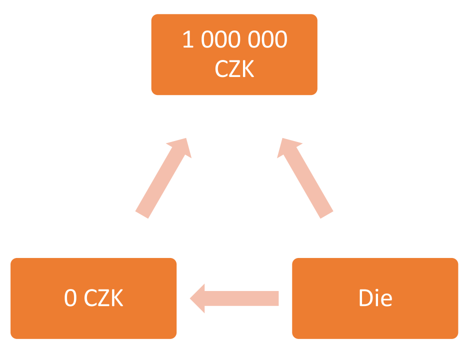
- Defined by two key premises
  - Completeness
  - Transitivity

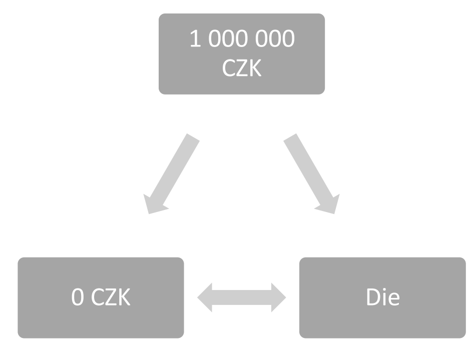
- **Indifferent to normative assessment** of preferences and choices

Completeness

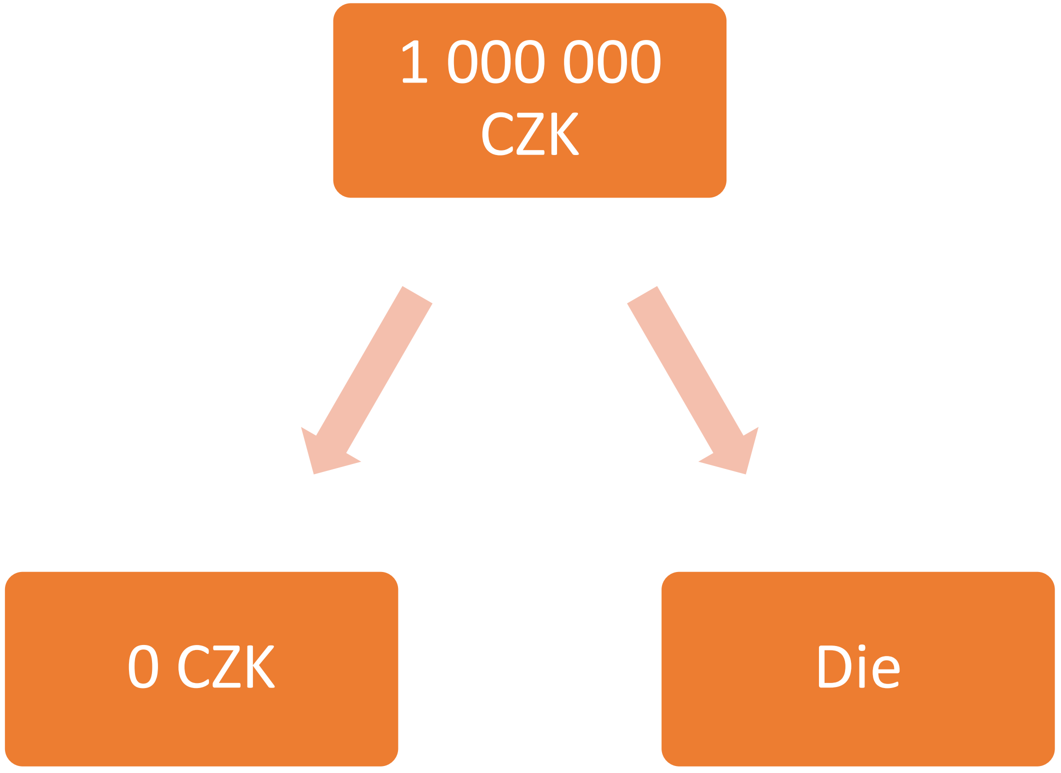
- Preference ordering complete if and only if for any two outcomes X and Y individual:
  - A) Prefers X to Y – strong preference relation
  - B) Prefers Y to X – strong preference relation
  - C) Is indifferent – weak preference relation





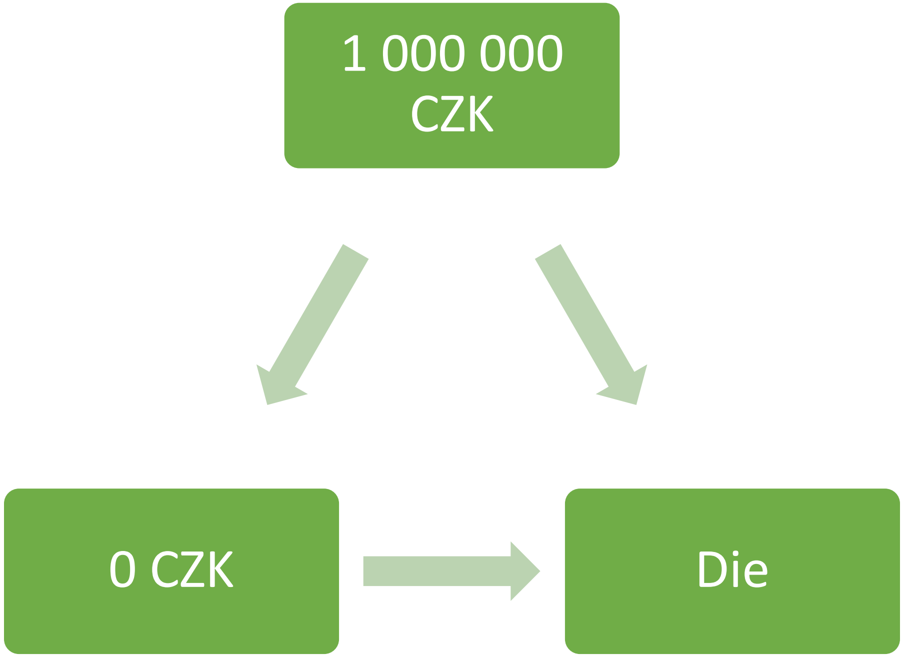


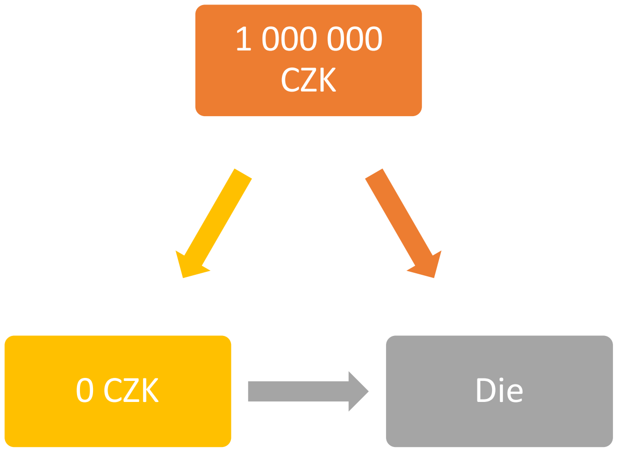
Incomplete preferences

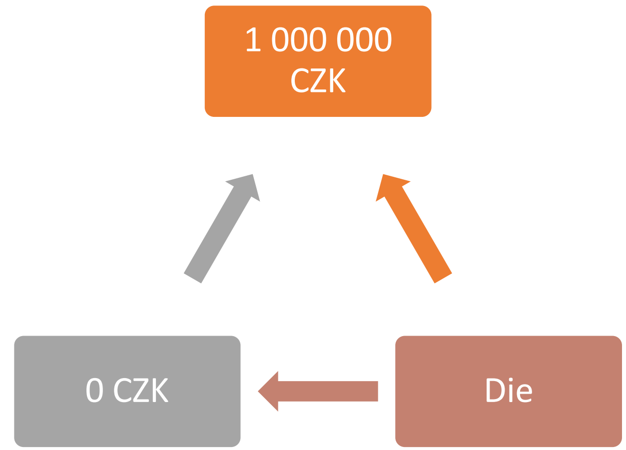


Transitivity

- For any three outcomes X, Y and Z, if X is preferred to Y and Y is preferred to Z, X must be preferred to Z




1 000 000 CZK
0 CZK
Die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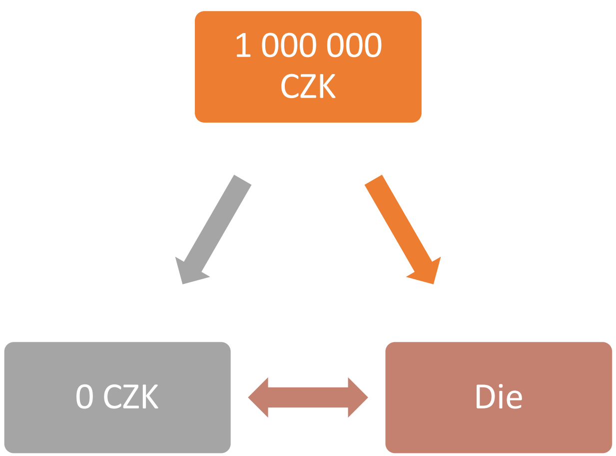
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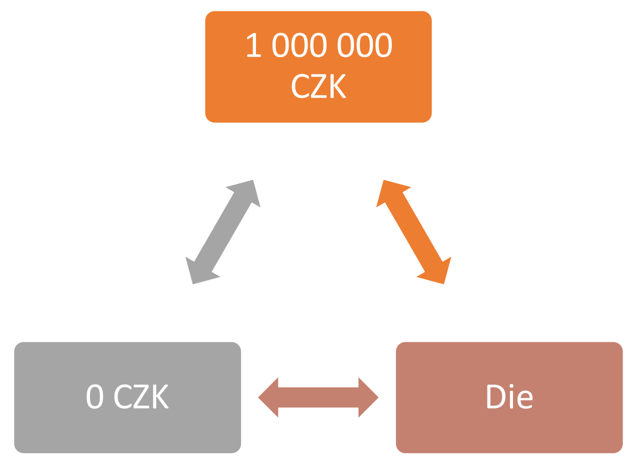
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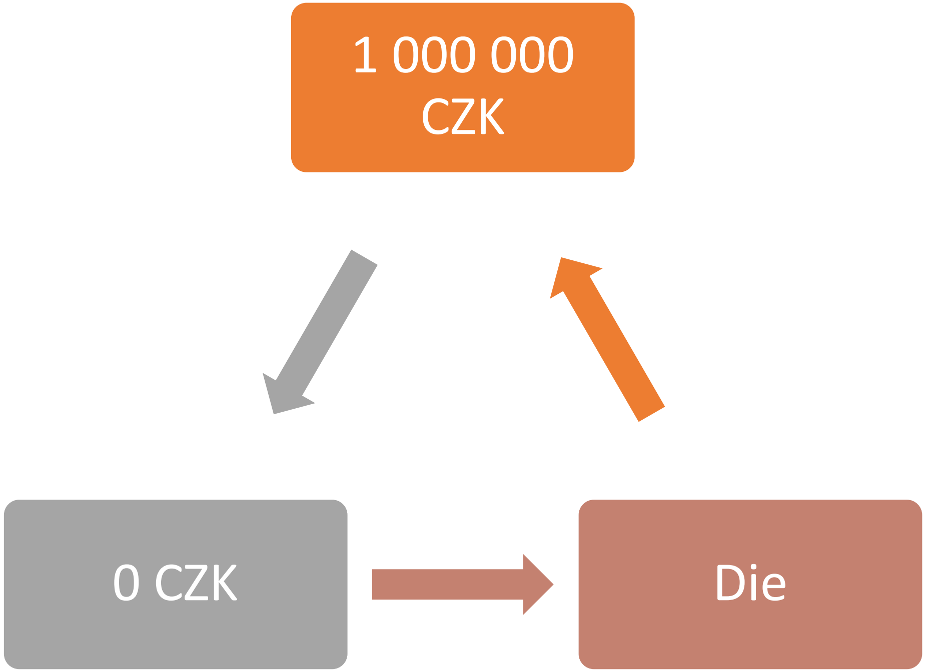
0 CZK
Die

Intransitive preferences

- Prefer X to Y, Y to Z and Z to X
- Doesn't make sense



Other notions about preferences

- Preferences over outcomes are **stable** and do **not change in the time of making decision** – are fixed

- Preferences are **ordinal** – they order actions but the difference between the two values has no meaning unless they state utility

- Compare two situations
  - $u(C_1) = 1$, $u(C_2) = 2$, $u(C_3) = 0$
  - $u(C_1) = 1$, $u(C_2) = 200$, $u(C_3) = -50$

- Both situations have same preference ordering
  - C_2 p C_1 p C_3

Other notions about rationality

- Rational choice theory is not attempting to explain cognitive processes happening in individuals

- Rationality tells **nothing about preferences over outcomes**

- Rational actors may **differ in choices in same situation**

- Rational actors can **err**

Types of games

Types of games

- Games of perfect information
- Games of imperfect information

- Cooperative games
- Non-cooperative games

- Constant-sum game
- Positive-sum game

Games of perfect/imperfect information

Perfect information games

- All players know other players' strategies available to them
- All players know payoffs over actions
- All players know other players know

Imperfect information games

- Some information about other players' actions is not know to the player

Cooperative/non-cooperative games

Cooperative games

- Actors are allowed to make enforceable contracts
- Players do not need to cooperate, but cooperation is enfoceable by an outside party

Non-cooperative games

- Actors unable to make enforceable contracts outside of those specifically modeled in the game
- Players might cooperate, but any cooperation must be self-enforcing

Constant-sum/Positive-sum games

Constant sum games

- Sum of all players' payoffs is the same for any outcome
- Gain for one participant is always at the expense of another
- Special case of zero-sum game where all outcomes involve a sum of all player's payoffs of 0

Positive-sum games

- Combined payoffs of all players are not the same in every outcome of the game
- Positive-sum game implies that players may have interests in common, to achieve an outcome that maximizes total payoffs.

Introducing a game

What makes a game the game

- Players
- Actions
- Strategies
- Outcomes
- Payoffs of player

Game of grades

- Each pair can choose 2 actions: α or β
- If both choose α, both will receive **C**
- If both choose β, both will receive **B**
- If one chooses α and other β, one will receive **A** and other **D**

Game of grades – my grades

My opponent

Me		α	β
	α	C	**A**
	β	**D**	B

Game of grades – my opponent's grades

My opponent

		α	β
Me	α	C	**D**
	β	**A**	B

Game of grades – normal form

My opponent

Me	α	β
α	C , C	A , D
β	D , A	B , B

Games in normal form

Normal form representation of a game

- Called also “strategic form” or “matrix form”
- Visualized as a **matrix**
- Represents a game as if agents were acting **simultaneously**

Utilities (Payoffs)

- Grades are not **utilites**

- Utilities for game:
  - EU(A) = 3
  - EU(B) = 2
  - EU(C) = 1
  - EU(D) = 0

- Preference over outcomes: A > B > C > D -> APBPCPD

Game of grades with payoffs

My opponent

Me		α	β
	α	1 , 1	3 , 0
	β	0 , 3	2 , 2

Solution concepts

- Nash Equilibrium
  - Dominant Strategy Equilibrium
  - Pure Strategy Equilibrium
  - Mixed Strategy Equilibrium
- Subgame Perfect Equilibrium
- Bayesian Equilibrium
- Weak Perfect Bayesian Equilibrium

My opponent

Me		α	β
	α	1 , 1	3 , 0
	β	0 , 3	2 , 2

My opponent

		α	β
Me	α	1 , 1	3 , 0
	β	0 , 3	2 , 2

My opponent

		α	β
Me	α	1 , 1	3 , 0
	β	0 , 3	2 , 2

My opponent

Me	α	β
α	C , C	A , D
β	D , A	B , B

Prisoner’s dilemma

- Both players are tempted to **defect**, since cooperate is **strictly dominated** by defect

- The outcome of the game is that both players betray the other one and end up choosing α

- Both will end up with outcome that is less preferred than the optimal outcome β, β by seeking maximal gain from own action

- Though β, β is Pareto Efficient outcome – brings best outcomes for all players, while no one could be better-off without making someone worse-off

Dominance

Dominant Strategy Equilibrium

- Strategy might be dominant

Two types of dominance

- Strict (strong) dominance
- Weak dominance

Strict dominance

- Player i
- Payoff u_i
- Dominant strategy s_i
- Dominated strategy s_i'
- Strategy of all other players s_{-i}

- Player i‘s strategy s_i' is strictly dominated by player i‘s strategy s_i if and only if
- $u_i(s_i , s_{-i}) > u_i(s_i' , s_{-i})$ for **all** s_{-i}

- utility of playing s_i against others’ strategies s_{-i} is **greater** than utility of playing s_i' against others’s strategies s_{-i} for all others’ strategies s_{-i}

Game of grades – strict dominance

My pair

Me		α	β
	α	1 , 1	3 , 0
	β	0 , 3	2 , 2

Weak dominance

- Player i
- Payoff u_i
- Dominant strategy s_i
- Dominated strategy s_i'
- Strategy of all other players s_{-i}

- Player i's strategy s_i' is weakly dominated by player i's strategy s_i if
- $u_i(s_i , s_{-i}) \geq u_i(s_i' , s_{-i})$ for **all** s_{-i} and
- $u_i(s_i , s_{-i}) > u_i(s_i' , s_{-i})$ for **some** s_{-i}

- utility of playing s_i against others' strategies s_{-i} is **greater or equal to** utility of playing s_i' against others's strategies s_{-i} for all others' strategies s_{-i} and **greater for some** others' strategies s_{-i}

Game of grades – weak dominance

My pair

Me	α	β
α	1 , 1	3 , 0
β	0 , 3	3 , 2

Never play dominated strategies

- Dominated strategy **brings lesser payoffs** than dominant strategy
- Dominated strategy brings lesser payoffs **no matter what strategy is selected by other player**
- Can’t control minds of others to force them not to play dominant strategy
- Event if **could** control minds of others and be sure they’ll play dominated strategy, than **rational to play dominant strategy anyway**

Choosing numbers

- Choose integer between 1 – 100 incl.
- All numbers will be averaged
- Winner is the one who will be closest to the 2/3 of the group's average

Choosing numbers

- Average = 100
- 2/3 of average = ~ 66.66
- X > 67 is strictly dominated strategy
  - Even if everyone else selected 100
  - One selected 67
  - I selected 68
  - Outcome – 68 is dominated by 67

- What is the rational choice for this game?

If all players were strictly rational, result is 1

I know you know

- I know
  - Numbers above 67 are never rational
- You know that I know
  - You'll never select number above 67, therefore numbers above 46 are never rational either
- I know You know that I know
  - I know that You'll never select above 46, hence I should never select number higher than 30
- You know that I know that You know that I know
  - You know that I won't select above 30, therefore I should never select number above 20

Get into opponent’s shoes

Real life results

- 2012 Game theory online course

- 10 000 + players

- Mean 34
- Mode 50
- Median 33
- Winner 23

- Spikes: 50, 33, 20, 1

Iterated deletion of dominated strategies

Iterated deletion of dominated strategies

- Can delete dominated strategies as if they were not present in the game
- Game becomes simpler than the original one
- Can find equilibriums quickly – games are dominance-solvable

Game of grades

Me \ My pair	α	β
α	1 , 1	~~3 , 0~~
β	0 , 3	~~2 , 2~~

My pair

		α
Me	α	1 , 1
	β	~~0 , 3~~

My pair

		α
Me	α	1 , 1

This game is dominance-solvable

Opponent

Me		s_1	s_2	s_3
	S_1	0 , 1	-2 , 3	4 , -1
Me	S_2	0 , 3	3 , 1	6 , 4
	S_3	1 , 5	4 , 2	5 , 2

S_1 vs S_2

Opponent

Me		s_1	s_2	s_3
	S_1	0 , 1	-2 , 3	4 , -1
Me	S_2	0 , 3	3 , 1	6 , 4
	S_3	1 , 5	4 , 2	5 , 2

S_1 vs S_3

Me \ Opponent	s_1	s_2	s_3
S_1	0 , 1	-2 , 3	4 , -1
S_2	0 , 3	3 , 1	6 , 4
S_3	1 , 5	4 , 2	5 , 2

S_2 vs S_3

Opponent

		s_1	s_2	s_3
	S_1	0 , 1	-2 , 3	4 , -1
Me	S_2	0 , 3	3 , 1	6 , 4
	S_3	1 , 5	4 , 2	5 , 2

S_1 VS S_3

Opponent

Me		s_1	s_2	s_3
	S_1	0 , 1	-2 , 3	4 , -1
Me	S_2	0 , 3	3 , 1	6 , 4
	S_3	1 , 5	4 , 2	5 , 2

S_1 VS S_2

		Opponent s_1	Opponent s_2	Opponent s_3
Me	S_1	0 , 1	-2 , 3	4 , -1
Me	S_2	0 , 3	3 , 1	6 , 4
Me	S_3	1 , 5	4 , 2	5 , 2

S_2 VS S_3

		Opponent s_1	Opponent s_2	Opponent s_3
Me	S_1	0 , 1	-2 , 3	4 , -1
Me	S_2	0 , 3	3 , 1	6 , 4
Me	S_3	1 , 5	4 , 2	5 , 2

Opponent

		s_1	s_2	s_3
	S_1	~~0 , 1~~	~~-2 , 3~~	~~4 , -1~~
Me	S_2	0 , 3	3 , 1	6 , 4
	S_3	1 , 5	4 , 2	5 , 2

Opponent

Me		s_1	s_2	s_3
	S_2	0 , 3	3 , 1	6, 4
	S_3	1, 5	4 , 2	5 , 2

s_1 vs s_3 after deletion

Opponent

Me	s_1	s_2	s_3
S_2	0 , 3	3 , 1	6 , 4
S_3	1 , 5	4 , 2	5 , 2

s_1 vs s_2 after deletion

Opponent

Me		s_1	s_2	s_3
	S_2	0 , 3	3 , 1	6 , 4
	S_3	1 , 5	4 , 2	5 , 2

s_2 vs s_3 after deletion

		Opponent s_1	Opponent s_2	Opponent s_3
Me	S_2	0 , 3	3 , 1	6 , 4
	S_3	1 , 5	4 , 2	5 , 2

Opponent

Me		s_1	s_2	s_3
	S_2	0 , 3	~~3 , 1~~	6 , 4
	S_3	1 , 5	~~4 , 2~~	5 , 2

Opponent

		s_1	s_3
Me	S_2	0 , 3	6 , 4
	S_3	1 , 5	5 , 2

Opponent

Me

	s_1	s_3
S_2	0 , 3	6 , 4
S_3	1 , 5	5 , 2

Opponent

		s_1	s_3
Me	S_2	0 , 3	6 , 4
	S_3	1 , 5	5 , 2

Games sometimes not dominance solvable, but simplified

Limits of iterated deletion of dominated strategies

- **Strictly** dominated strategies may be deleted in a **random** order
- Deleting **weakly** dominated strategies in some order **might delete** equilibriums
- This solution concept is not always applicable – **sometimes game simply don’t have dominance**

How to solve the game without dominance?

Opponent

Me		s_1	s_3
	S_2	0 , 3	6 , 4
	S_3	1 , 5	5 , 2

Nash Equilibrium

Nash Blonde Game

- 2 or more lusty males
- Several interested females
- At least one more female than male
- Just one female blonde
- Every male prefers blonde to brunette and brunette to no companion

Nash Blonde Game – normal form

		M2	
		Bl	Br
M1	Bl	0 , 0	2 , 1
	Br	1 , 2	1 , 1

Nash Equilibrium

- Set of strategies, one for each player, such that **no player has incentive to unilaterally change** her action
- Players are in equilibrium if a change in strategies by any one of them would lead player to earn less (considering strategies of others') than if she remained with her current strategy
- **Mutual best response** to others' choices

A

B		L	C	R
	T	1 , 1	0 , 0	0 , 0
B	M	0 , 2	1 , 1	2 , -1
	B	0 , 0	1 , 2	2 , 1

A

B		L	C	R
	T	1 , 1	0 , 0	0 , 0
	M	0 , 2	1 , 1	2 , -1
	B	0 , 0	1 , 2	2 , 1

A

B		L	C	R
	T	1 , 1	0 , 0	0 , 0
B	M	0 , 2	1 , 1	2 , -1
	B	0 , 0	1 , 2	2 , 1

		A		
		L	C	R
B	T	1 , 1	0 , 0	0 , 0
	M	0 , 2	1 , 1	2 , -1
	B	0 , 0	1 , 2	2 , 1

Games might have more NE

Pure strategy equilibrium

- Two equilibriums in this game
- (T , L)
  - u(A) = 1
  - u(B) = 1
- (C , B)
  - u(A) = 1
  - u(B) = 2
- These are **pure strategy equilibriums**

Other basic games

Chicken

		B: s	B: h
A	S	5 , 5	0 , 10
A	H	10 , 0	-10 , -10

Chicken NE

- Pure strategies NE
  - (H , s)
    - EU(A) = 10
    - EU(B) = 0
  - (S , h)
    - EU(A) = 0
    - EU(B) = 10
- Mixed strategies NE
  - (1/2 S , 1/2 s)
    - EU(A) = 5/2
    - EU(B) = 5/2

B

A	s	h
S	5 , 5	0 , 10
H	10 , 0	-10 , -10

Stag hunt

		B: s	B: r
A	S	5 , 5	0 , 3
A	R	3 , 0	3 , 3

Stag hunt NE

- Pure strategies NE
  - (S , s)
    - EU(A) = 5
    - EU(B) = 5
  - (R , r)
    - EU(A) = 3
    - EU(B) = 3
- Mixed strategies NE
  - (3/5 S , 3/5 s)
    - EU(A) = 3
    - EU(B) = 3

B

A	s	R
S	5 , 5	0 , 3
R	3 , 0	3 , 3

B

A		s	r
	S	5 , 5	0 , 7
	R	7 , 0	3 , 3

Stag hunt

		B	
		s	r
A	S	5 , 5	0 , 3
	R	3 , 0	3 , 3

Problems with games in normal form

- Treat decision-making as **simultaneous**
- Some games have **more NE** and **not easy to find out which is more probable** than the other
- Introduction of **time** and ability to observe actions of other players changes the decision-making of an actor