Additional Topics

PSY544 – Introduction to Factor Analysis

Week 13

Additional Topics

- Today’s lesson will be a bit of an amorphic cross-over
- We’ll talk about some topics that exceed the basics of FA that we have learned during the semester
- If we had more time, the topics presented today would be presented in a more thorough way over the course of multiple days, but…

Model comparison

- In many cases, you will have multiple models that are all plausible candidates. Your goal might be to select one of them – the one which is superior to the rest.

- You should compare interpretability
- You should compare model-data fit
- Ideally, you should compare both

Direct model comparison

- In order to compare models directly, the compared models must be *nested*.
- Model A is nested within Model B if Model A is a *special case* of Model B, or Model B is a *general case* of Model A.
- More specifically, Model A is nested within Model B if Model A can be obtained by *imposing additional restrictions* on Model B.
- The free parameters of Model A are a subset of those of Model B.

Direct model comparison

- Sounds arcane?

- Some examples of nested models:
- Any (usual) restricted model with *m* factors is nested within an unrestricted model with *m* factors
- An orthogonal model with *m* factor is nested within an oblique model with *m* factors (if restricted, they must have the same loading structure)
- A model where two parameters are constrained to be equal is nested within the model without this restriction.

Direct model comparison

- Nested models:

- Will have more degrees of freedom
- Will have fewer free parameters (that’s the same thing)
- Will have equal or greater value of the same discrepancy function (the model have the same or greater discrepancy from data)

Direct model comparison

- You can test how two nested models differ in fit.

- Basically, you can test whether the additional constraints have a statistically significant negative impact on model fit:

$$H_0: F_{0A} - F_{0B} = 0$$
$$H_A: F_{0A} - F_{0B} > 0$$

- The test statistic is a χ^2 difference, $\Delta\chi^2 = \chi^2_A - \chi^2_B = (N-1)(\hat{F}_A - \hat{F}_B)$

Direct model comparison

- Under the null hypothesis, the $\Delta\chi^2$ is chi-square distributed with degrees of freedom equal to the difference of degrees of freedom of the two models (or the difference in the number of free parameters)

- $df = df_A - df_B$

- If the test statistic exceeds a critical value (based on the α-level), then the null hypothesis is rejected. Model A fits worse than Model B.
- However, this approach suffers from the same issues that affect the test of perfect fit.

Indirect model comparison

- For non-nested models, the $\Delta\chi^2$ test cannot be conducted.

- If one wants to compare two non-nested models, other comparisons can be employed, though:

- 1) Compare fit indices – however, no test can be performed (among other things, we don't know the distribution of fit indices)

- 2) Compare information criteria – let's take a look at that

Information criteria

- Remember the log-likelihood from way back when we talked about maximum likelihood estimation?

- The log-likelihood is usually a relatively large, negative number. The smaller it gets (the more negative it gets), the smaller the likelihood. In case data is the same, worse models will result in smaller likelihood.
- Sometimes, a *deviance* is calculated: -2*log-likelihood (so, a relatively large, positive number). The larger the deviance, the smaller the likelihood (the more the model *deviates* from data)
- *Deviance* is used to calculate the so-called information criteria.

Information criteria

- Information criteria are **relative** measures that combine information on model's goodness of fit and its complexity, and are supposed to capture the model's **relative quality**

- Akaike's Information Criterion *(k* = number of parameters)*:*

$$AIC = 2k \; - 2loglikelihood$$
$$AIC = 2k + Deviance$$

- AIC takes into account the model fit (deviance) and model complexity (*k*)
- If two different factor models are fit on the same data, the model with larger deviance fits worse, but AIC also takes into account model complexity.

Information criteria

- Information criteria are **relative** measures that combine information on model's goodness of fit and its complexity, and are supposed to capture the model's **relative quality**

- Bayesian (Schwarz) Information Criterion *(k* = number of parameters)*:*

$$BIC = ln(n)\, k\ - 2loglikelihood$$
$$BIC = ln(n)\, k + Deviance$$

- BIC takes into account the model fit (deviance) and model complexity (*k*), as well as sample size (*n*). It penalizes the model for complexity relatively more than AIC.

Information criteria

- You can only compare models on their information criteria if the models were fit to the **same data**

- Moreover, even if the information criteria values differ for two models, we don't know how much is too much – there is no "effect size" for information criteria.

- So treat the AIC and BIC as sources of information, but keep the above in mind.

Bi-factor model

- What is the bi-factor model?

- 1) All items load on a single “general” factor
- 2) All items also load on one, and only one, additional “specific” factor
- 3) All factors are uncorrelated

- So, the $\mathbf{\Lambda}$ matrix has *m* columns, where one of these columns is full of free parameters and the remaining *m*-1 columns contain free parameters each for a set of MVs, these sets do not overlap. The $\mathbf{\Phi}$ matrix is diagonal.

Bi-factor model

- Why can the bi-factor model be useful?

- It’s a “multidimensional unidimensional model” ☺
- It might have interesting interpretations
- It usually fits better than a 1-factor model