

PSY544 – Introduction to Factor Analysis

Homework assignment 1, Fall 2018

Due midnight, October 17, 2018

Suppose we have the following matrices:

$$A = \begin{bmatrix} 8 & 4 \\ 3 & 12 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 3 & 9 \\ 11 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & -8 & 0 \\ 0 & 0 & 3 \\ 5 & -7 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 13 & 5 \\ 1 & 0 \\ 7 & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} 8 \\ 1 \\ 2 \\ 5 \\ 4 \end{bmatrix}$$

Compute the following. Show your work.

1. $X'X$
2. DA
3. CB
4. $(DB)C$
5. $BD + A$
6. $C + D$
7. Is $B'A$ equal to AB ?
8. Compute $|A|$. We haven't covered this in class, so look up online how to do it by hand.

Now, suppose we have a simple, ordinary linear regression model based on $N = 5$ observations. Using scalar notation, we would write the model as follows:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

for $i = 1, \dots, N$. The x_i s are the predictor scores, and the ε_i s are error terms that have zero means (by definition). Therefore, the expected value of y_i is a linear function of the regression coefficients and the predictor scores:

$$E(y_i) = \mu_i = \beta_0 + \beta_1 x_i$$

Let the $N \times 1$ vector of means be denoted as $\boldsymbol{\mu}$. Write down each element of this vector in terms of the regression coefficients and predictor scores:

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} = \begin{bmatrix} \beta_0 + \beta_1 x_1 \\ \beta_0 + \beta_1 x_2 \\ \beta_0 + \beta_1 x_3 \\ \beta_0 + \beta_1 x_4 \\ \beta_0 + \beta_1 x_5 \end{bmatrix}$$

Given what you've just written down, re-express the mean vector $\boldsymbol{\mu}$ as a matrix product between a $N \times 2$ matrix \mathbf{X} and a 2×1 vector $\boldsymbol{\beta}$:

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} = \mathbf{X}\boldsymbol{\beta} = \begin{bmatrix} 1 & x_1 \\ & \\ & \\ & \\ & \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$$

Now, let's call the $N \times 1$ vector from the previous equation \mathbf{y} . Also, let's aggregate all the ε_i s into an $N \times 1$ vector of error terms. How would you formulate the original regression equation using matrix notation?

$$\mathbf{y} = \begin{bmatrix} 1 & x_1 \\ & \\ & \\ & \\ & \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \end{bmatrix} = \text{---} + \boldsymbol{\varepsilon}$$