PSY544 – Introduction to Factor Analysis

Homework assignment 1, Fall 2019

Due midnight, October 21, 2019

Suppose we have the following matrices:

$$\boldsymbol{A} = \begin{bmatrix} 8 & 4 \\ 3 & 12 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} 6 & 3 & 9 \\ 11 & 0 & 1 \end{bmatrix} \qquad \boldsymbol{C} = \begin{bmatrix} 0 & -8 & 0 \\ 0 & 0 & 3 \\ 5 & -7 & 0 \end{bmatrix}$$

$$\boldsymbol{D} = \begin{bmatrix} 13 & 5 \\ 1 & 0 \\ 7 & 3 \end{bmatrix} \qquad \boldsymbol{X} = \begin{bmatrix} 8 \\ 1 \\ 2 \\ 5 \\ 4 \end{bmatrix}$$

Compute the following, if possible. Show your work.

1. ***X'X***
2. ***DA***
3. ***CB***
4. ***(DB)C***
5. ***BD + A***
6. ***C + D***
7. Is ***B'A*** equal to ***AB?***
8. Compute |***A***|. We haven't covered this in class, so look up online how to do it by hand.

Now, suppose we have a simple, ordinary linear regression model based on N = 5 observations. Using scalar notation, we would write the model as follows:

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

for *i* = 1, 2, ..., N. The x_is are the predictor (independent variable) scores, and the ε_is are error terms (residuals) that have zero means (by definition). Therefore, the expected value of y_i is a linear function of the regression coefficients and the predictor scores:

$$E(y_i) = \mu_i = \beta_0 + \beta_1 x_i$$

Let the N x 1 vector of means be denoted as $\boldsymbol{\mu}$. Write down each element of this vector in terms of the regression coefficients and predictor scores:

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} = \begin{bmatrix} \beta_0 + \beta_1 x_1 \\ . \\ . \\ . \\ . \end{bmatrix}$$

Given what you've just written down, re-express the mean vector $\boldsymbol{\mu}$ as a matrix product between a N x 2 matrix $\boldsymbol{X}$ and a 2 x 1 vector $\boldsymbol{\beta}$:

$$\boldsymbol{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \\ \mu_5 \end{bmatrix} = \boldsymbol{X\beta} = \begin{bmatrix} 1 & x_1 \\ . & . \\ . & . \\ . & . \\ . & . \end{bmatrix} \begin{bmatrix} . \\ . \end{bmatrix}$$

Now, let's call the N x 1 vector resulting from the previous equation $\boldsymbol{y}$. Also, let's aggregate all the ε_is into an N x 1 vector of error terms. How would you formulate the original regression equation using matrix notation?

$$\boldsymbol{y} = \begin{bmatrix} 1 & x_1 \\ . & . \\ . & . \\ . & . \\ . & . \end{bmatrix} \begin{bmatrix} . \\ . \end{bmatrix} + \begin{bmatrix} . \\ . \\ . \\ . \\ . \end{bmatrix} = \_\_\_\_ + \boldsymbol{\varepsilon}$$