

# Additional Topics

PSYb5440 – Introduction to Factor Analysis

Week 13

# Additional Topics

- Today's lesson will be a bit of an amorphous cross-over
- We'll talk about some topics that exceed the basics of FA that we have learned during the semester
- If we had more time, the topics presented today would be presented in a more thorough way over the course of multiple days, but...

# Model comparison

- In many cases, you will have multiple models that are all plausible candidates. Your goal might be to select one of them – the one which is superior to the rest.
- You should compare interpretability
- You should compare model-data fit
- Ideally, you should compare both

# Direct model comparison

- In order to compare models directly, the compared models must be *nested*.
- Model A is nested within Model B if Model A is a *special case* of Model B, or Model B is a *general case* of Model A.
- More specifically, Model A is nested within Model B if Model A can be obtained by *imposing additional restrictions* on Model B.
- The free parameters of Model A are a subset of those of Model B.

# Direct model comparison

- Sounds arcane?
- Some examples of nested models:
- Any (usual) restricted model with  $m$  factors is nested within an unrestricted model with  $m$  factors
- An orthogonal model with  $m$  factor is nested within an oblique model with  $m$  factors (if restricted, they must have the same loading structure)
- A model where two parameters are constrained to be equal is nested within the model without this restriction.

# Direct model comparison

- Nested models:
  - Will have more degrees of freedom
  - Will have fewer free parameters (that's the same thing)
  - Will have equal or greater value of the same discrepancy function (the model have the same or greater discrepancy from data)

# Direct model comparison

- You can test how two nested models differ in fit.
- Basically, you can test whether the additional constraints have a statistically significant negative impact on model fit:

$$H_0: F_{0A} - F_{0B} = 0$$

$$H_A: F_{0A} - F_{0B} > 0$$

- The test statistic is a  $\chi^2$  difference,  $\Delta\chi^2 = \chi_A^2 - \chi_B^2 = (N - 1)(\hat{F}_A - \hat{F}_B)$

# Direct model comparison

- Under the null hypothesis, the  $\Delta\chi^2$  is chi-square distributed with degrees of freedom equal to the difference of degrees of freedom of the two models (or the difference in the number of free parameters)
- $df = df_A - df_B$
- If the test statistic exceeds a critical value (based on the  $\alpha$ -level), then the null hypothesis is rejected. Model A fits worse than Model B.
- However, this approach suffers from the same issues that affect the test of perfect fit.

# Indirect model comparison

- For non-nested models, the  $\Delta\chi^2$  test cannot be conducted.
- If one wants to compare two non-nested models, other comparisons can be employed, though:
  - 1) Compare fit indices – however, no test can be performed (among other things, we don't know the distribution of fit indices)
  - 2) Compare information criteria – let's take a look at that

# Information criteria

- Remember the log-likelihood from way back when we talked about maximum likelihood estimation?
- The log-likelihood is usually a relatively large, negative number. The smaller it gets (the more negative it gets), the smaller the likelihood. In case data is the same, worse models will result in smaller likelihood.
- Sometimes, a *deviance* is calculated:  $-2 * \text{log-likelihood}$  (so, a relatively large, positive number). The larger the deviance, the smaller the likelihood (the more the model *deviates* from data)
- *Deviance* is used to calculate the so-called information criteria.

# Information criteria

- Information criteria are **relative** measures that combine information on model's goodness of fit and its complexity, and are supposed to capture the model's **relative quality**
- Akaike's Information Criterion ( $k$  = number of parameters):  
$$AIC = 2k - 2\log\text{likelihood}$$
$$AIC = 2k + \text{Deviance}$$
- AIC takes into account the model fit (deviance) and model complexity ( $k$ )
- If two different factor models are fit on the same data, the model with larger deviance fits worse, but AIC also takes into account model complexity.

# Information criteria

- Information criteria are **relative** measures that combine information on model's goodness of fit and its complexity, and are supposed to capture the model's **relative quality**
- Bayesian (Schwarz) Information Criterion ( $k$  = number of parameters):  
$$BIC = \ln(n) k - 2 \log \text{likelihood}$$
$$BIC = \ln(n) k + \text{Deviance}$$
- BIC takes into account the model fit (deviance) and model complexity ( $k$ ), as well as sample size ( $n$ ). It penalizes the model for complexity relatively more than AIC.

# Information criteria

- You can only compare models on their information criteria if the models were fit to the **same data**
- Moreover, even if the information criteria values differ for two models, we don't know how much is too much – there is no “effect size” for information criteria.
- So treat the AIC and BIC as sources of information, but keep the above in mind.

# Bi-factor model

- What is the bi-factor model?
  - 1) All items load on a single “general” factor
  - 2) All items also load on one, and only one, additional “specific” factor
  - 3) All factors are uncorrelated
- So, the  $\Lambda$  matrix has  $m$  columns, where one of these columns is full of free parameters and the remaining  $m-1$  columns contain free parameters each for a set of MVs, these sets do not overlap. The  $\Phi$  matrix is diagonal.

# Bi-factor model

- Why can the bi-factor model be useful?
- It's a “multidimensional unidimensional model” 😊
- It might have interesting interpretations
- It usually fits better than a 1-factor model