CEFA: Comprehensive Exploratory Factor Analysis

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The accompanying program is CEFA 3.04. The associated Graphical User Interface is CEFAtool 3.04.

Mathematical design and specification: Michael W. Browne, Robert Cudeck, Krishna Tateneni and Gerhard Mels.
Program architecture and programming, CEFA: Krishna Tateneni, Gerhard Mels, Robert Cudeck and Michael W. Browne.
Program architecture and programming, CEFAtool: Krishna Tateneni.

Acknowledgments

CEFA is written in Lahey FORTRAN 95. CEFAtool is written in Sun Java 6.

Portions of CEFA are derived from routines given in

Press, W.H., Teukolsky, S.A., Vetterling, W.T. and Flannery, B.P. (1992). *Numerical Recipes in Fortran:The Art of Scientific Programming, Second Edition*. Cambridge: Cambridge University Press

and from algorithms published in *Applied Statistics*.

The code for automatic derivatives in CEFA was derived from FORTRAN 77 code generated by ADIFOR:

Bischoff, C., Carle, A., Khademi, P., Mauer, A. and Hovland, P. (1995). *ADIFOR 2.0 User's Guide, Revision C*. Technical Report ANL/MCS-TM-192. Mathematics and Computer Science Division, Argonne National Laboratory, IL.

The algorithm for the generation of random normal deviates in CEFA is from

Leva, J. (1992). Algorithm 712; A Normal Random Number Generator, *ACM Transactions on Mathematical Software (TOMS)*, **18**, 454-455.

A subroutine for evaluating the cumulative distribution function of the bivariate normal distribution is based on

Yihong Ge & Alan Genz. BVN/BVNU: *A function for computing bivariate normal probabilities.* [Computer software]. Retrieved from http://www.math.wsu.edu/math/faculty/genz/homepage

The rotation routines in CEFA were substantially influenced by OBORMIN, a FORTRAN77 program for factor rotation developed by Michael W. Browne and Carina Tornow, and by TARROT, a program for target rotation developed by Michael W. Browne.

Exploratory factor analysis routines in CEFA were substantially influenced by FCAP and PACE, FORTRAN 77 programs developed by Robert Cudeck. A routine for standard errors using the delta method was influenced by FAS, a FORTRAN 77 program for standard errors of rotated factor loadings developed by Robert Cudeck. Routines for computing polychoric correlation coefficients were substantially influenced by Fortran 95 subroutines written by

Guangjian Zhang. A printing routine that overlays the target and the rotated matrix for target rotation was substantially influenced by a subroutine written by Longjuan Liang.

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CEFA and CEFAtool are distributed free of charge, without support and with no warranty. Please send bug reports to Michael W. Browne, e-mail: browne.4@OSU.edu.

Persons who would like to include a reference to CEFA in a journal article may use the following format (Publication Manual of the American Psychological Association, Fifth Edition, Example 83, page 280):

Browne, M. W., Cudeck, R., Tateneni, K. & Mels G. (2008). CEFA: Comprehensive Exploratory Factor Analysis, Version 3.03 [Computer software and manual]. Retrieved from http://faculty.psy.ohio-state.edu/browne/

CEFA: Comprehensive Exploratory Factor Analysis

1. Introduction

Exploratory factor analysis is a widely used method of analysis, particularly in the behavioral sciences. Although there have been some substantial developments in the last twenty five years, many have not been included in readily available factor analysis programs and consequently cannot be tried out by practitioners.

Most factor analysis programs have facilities for obtaining maximum likelihood and ordinary least squares estimates of factor loadings which suffice in most practical situations. There are, however, alternatives to maximum likelihood (Swain, 1975a) that have not been tried much because they are not generally available. Frequently, noniterative solutions with squared multiple correlation coefficients as appproximations for communalities are provided. Noniterative solutions may, however, use consistent estimates of communalities (Albert, 1946; Ihara and Kano, 1986) rather than approximations. Again, this alternative has not been available to the practitioner.

The choices of rotation procedure are often limited to Varimax (Kaiser, 1958) for orthogonal rotation and Direct Quartimin (Jennrich and Sampson, 1966) for oblique rotation. These rotation criteria are well suited to situations where a perfect cluster solution with only one substantial loading per row of the factor matrix exists. They are not well suited for more complex solutions with several nonzero loadings per row. The originator of multiple factor analysis, Thurstone (1935, 1947) originally intended the concept of simple structure to incorporate complex solutions and provided his “Box Data” (Thurstone, 1947, pp. 369–376) as an example of a complex simple-structure. Yates (1987) has discussed this matter carefully. Currently, available programs are not able to recover simple structure from the Box Data. Some rotation criteria that can recover the simple structure from these data have been proposed (e.g. McCammon, 1966; McKeon, 1968; Yates, 1987) but are not well-known. An alternative approach was to use Varimax (or Direct Quartimin) in conjunction with a weighting scheme (Cureton & Mulaik, 1975) that enabled the recovery of simple structure from the Box Data. Rotation approaches that are worth trying, but are not readily available, do therefore exist.

One of the reasons for the current emphasis on confirmatory factor analysis is that standard errors for factor loading and factor correlation estimates are available in a number of programs. This is not the case for exploratory factor analysis. Methods for estimating standard errors of rotated factor loadings have been given by Jennrich and coauthors in a sequence of papers starting with Archer & Jennrich (1973) and culminating with Jennrich and Clarkson (1980). The algebra involved is quite complicated and the results have not been implemented in readily accessible computer programs.

We have been engaged in research intended to make improved rotation techniques readily available. The main aims were:

- to adapt the approaches of Kaiser (1958) and of Jennrich and Sampson (1966) to obtain a general rotation method that can easily be adapted to individual rotation criteria.
- to develop general methods for obtaining asymptotic standard errors of factor loadings and inter-factor correlations that can easily be adapted to individual rotation criteria.
- to implement these results in a generally available computer program that will enable a user to carry out an exploratory factor analysis including the initial estimation of the factor matrix, the assessment of fit, the rotation of the factor matrix and the estimation of standard errors of rotated factor loadings and inter-factor correlations.

In this work, the generality of the method has been given precedence over its speed, although speed of algorithms has not been disregarded. A number of potentially interesting rotation criteria have been found and incorporated in the program. The methods developed, however, will make it possible to incorporate new, as yet undiscovered, rotation criteria with little difficulty. Because the evaluation of standard errors is rather complicated, and implementation errors can easily be made, two methods of evaluating standard errors have been incorporated. One, the bordered information matrix method is intended for general use. The other, the delta method with numerical derivatives, is slower and is intended primarily for checking purposes. All asymptotic theory developed is based on the asymptotic distribution of the correlation matrix, not the covariance matrix. Standard deviations are treated as nuisance parameters and partialled out.

This research has culminated in a computer program, CEFA (Comprehensive Exploratory Factor Analysis), that is described in subsequent sections of this report. Version 1 of CEFA was made available in 1998. The current Version 3 of CEFA has some additional facilities made available in the meantime. CEFA, Version 3 may be downloaded free of charge from

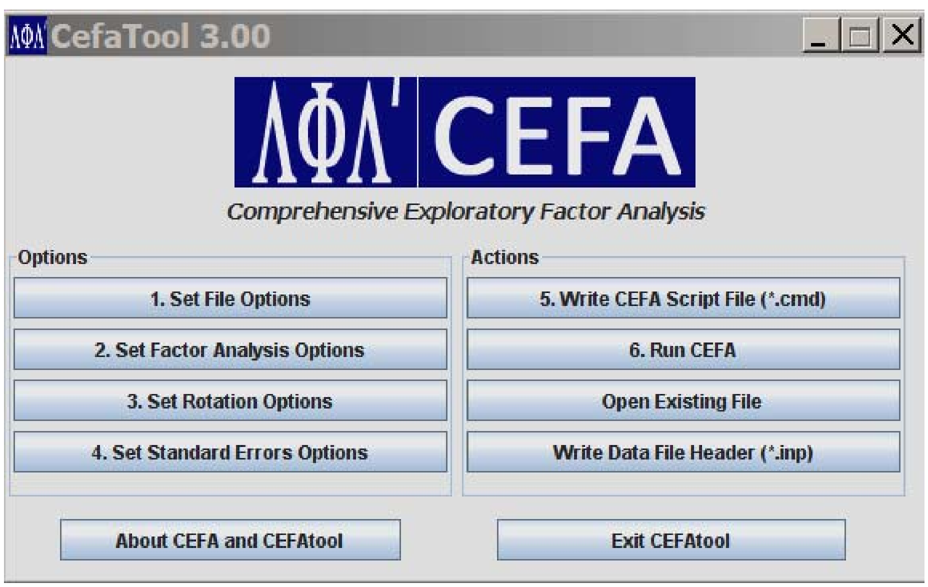
http://faculty.psy.ohio-state.edu/browne/

CEFA is able (i) to input raw data and calculate a product moment correlation matrix (ii) to input ordinal raw data and calculate a polychoric correlation matrix (iii) to input a correlation matrix (or covariance matrix which is immediately scaled to a correlation matrix) or (iv) to input a factor matrix directly for rotation. At the factor extraction stage CEFA is able to use maximum Wishart likelihood (MWL), any other member of the Swain (1975) family, ordinary least squares (OLS) and the non-iterative partitioned correlation matrix (PACE) method described by Cudeck (1991). CEFA is able to use all rotation criteria reviewed in Browne (2001), provide asymptotic standard errors for rotated factor loadings for MWL and other members of the Swain family of discrepancy functions as well as for OLS. Test statistics and associated fit measures for MWL, the Swain family, OLS and PACE are also provided (cf. Section 5).

2. CEFA: Getting Started

CEFA should be installed according to the specifications in the Readme.txt file downloaded at the same time as the present documentation.

CEFA may be run directly under the Command or DOS prompt of Windows (See Section 3). It is more convenient, however, to run CEFA via the more user-friendly Graphical User Interface, CEFAtool (see Section 3). To bring up this interface, double-click on the CEFAtool icon ΛΦΛ'. You will then see the main window



Main Window

The six numbered steps in the Main Window follow the sequence of steps involved in carrying out a typical exploratory factor analysis using CEFA. However, depending on the specific situation, not all six steps may be necessary. Typically the Main Window will remain open until further CEFA runs are no longer required. It may be closed by clicking on Exit CEFAtool.

CEFA requires two files, a data file, *.inp (*.dat in earlier versions), that contains the data to be analyzed, and a script or command file, *.cmd, that gives instructions to CEFA on how to carry out the analysis. The first four steps in the Main Window, labeled Options, allow the user to specify the instructions to be written to the script or command file, while the fifth step actually writes those instructions into a file. After executing the sixth step, Run CEFA, an output file, *.out, containing the results of the analysis, is produced.

The data file may be prepared using any text editor, for example Notepad. If you use a word processor, you should ensure that the file produced is a plain text file.

Here we shall give an example of how CEFA may be used to carry out a factor analysis of unpublished data due to Holzinger reported in Harman (1960, Table 11.3).

2.1 The Data File

A data file is given in the file Example.inp that is included with CEFA (see Section 3g). We shall show how the reader may construct this data file which has been named Holzinger.inp here, to avoid overwriting the example data file that is included with CEFA.

2.1.1 Data file: Holzinger.inp

`696 9`	*(Number of cases, variables)*
`1`	*(Datatype)*
`0`	*(Random seed used when random starts in rotation are required)*
`1`	*(Variable names)*
`WrdMean SntComp OddWrds`	
`MxdArit Remndrs MissNum`	
`Gloves  Boots   Hatchts`	
`1`	(*Factor names)*
`Verb Arith Spat`	
`1`	*(Order/Target matrix)*

PTO

```
? 0 0
? 0 0
? 0 0
0 + 0
0 ? 0
0 ? 0
0 0 ?
0 0 ?
0 0 ?
    1
  .75  1
  .78 .72  1
  .44 .52 .47  1
  .45 .53 .48 .82  11
  .51 .58 .54 .82 .74
  .21 .23 .28 .33 .37 .35  1
  .30 .32 .37 .33 .36 .38 .45  1
  .31 .30 .37 .31 .36 .38 .52 .67  1
```

(This matrix is used as a Target matrix in the Target rotation method only. In all other rotation methods it is used as an Order matrix. It prespecifies the ordering and direction of columns of a rotated matrix. The symbols + and ? refer to unspecified elements. Any element not equal to + or ? represents a specified value to be matched as closely as possible. See Section 2.1.3)

2.1.2 Data File: Header information:

Datatype: 1 for correlations/covariances, 2 for raw data, 3 for a factor loading matrix, 4 for raw data intended for polychoric correlations. If type 1 is specified a correlation or covariance matrix should be given at the end of the file as shown in this example. If type 2 is specified the data matrix should be given at the end of the file. If type 3 is specified a factor matrix (unrotated or orthogonally rotated) should be given at the end of the file. If type 4 is specified the data matrix should be given at the end of the file; note that the data should consist of discrete integer values with a maximum of 10 categories per variable.

Random seed: Used for random starts in rotation (see Section 6). 0 means choose one based on the system clock; otherwise type in a value given in a previously obtained output file[1].

Variable names: 1 if provided, 0 if not. If $=1$, type names on following lines in free-format separated by blanks.

Factor names: 1 if provided, 0 if not. If $=1$, type names on following lines in free-format separated by blanks.

Order/Target matrix: 1 if provided, 0 if not. If $=1$, type the matrix on the following lines. (Section 2.1.3).

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[1]Note that the random number generator used in CEFA 3.00 is different from that used in previous versions, and uses a single seed value rather than two.

Alternatively, the lines preceding the Order/Target matrix may be prepared using option `Write Data File Header` of the CEFAtool Main Window. (Section 2.1.4). Note that you will still need to append your order matrix (if required) and data to the file created by CEFAtool.

2.1.3 Data File: Order/Target Matrix

An order matrix and a target matrix have the same appearance but have different functions. An order/target matrix should be of the same order as the factor matrix input directly or extracted from input data. Any element that is ? will be regarded as unspecified, while any element that is + will be required to be positive, but is otherwise unspecified. Specified values (usually, but not necessarily $= 0$) should be assigned to all remaining elements of the order/target matrix. In any given column, there should be at most one + element. No column with some specified elements not equal to zero may contain a + element. (A 9 may be used instead of a ? for compatibility with previous versions of CEFA.)

An order matrix may be employed after a blind rotation (i.e. any rotation in Section 6 except for Target Rotation e.g. CF-Varimax) has been carried out. The order matrix is only employed after a blind rotation has been completed and has no effect on the magnitudes of the elements. Columns of the previously rotated factor matrix are reordered and reflected (all elements of the column multiplied by -1) so as to minimize the sum of squares of differences of rotated elements and corresponding specified elements of the order matrix. If a column of the order matrix contains a + element the corresponding column of the rotated matrix is reflected, if necessary, to ensure a positive element in the position corresponding to the +. This reordering and reflection is not essential but can be convenient for comparative purposes, particularly when several different rotation criteria are applied to the same data.

A target matrix is required for the Target Rotation option (see Section 6.6). It is defined in exactly the same way as an order matrix but it influences the values of the rotated loadings, not only the order and signs of columns. The Target Rotation procedure (see Section 6) performs an oblique or orthogonal rotation so as to minimize the sum of squares of differences of rotated elements and corresponding *specified* target elements. The elements of the target therefore influence the actual rotation performed and the target matrix is used simultaneously with the rotation procedure, not thereafter.

If all specified elements of a column of an order/target matrix are zero and it contains no (unspecified) + element, columns of the rotated factor matrix are reflected so as to ensure positive column sums.

An example of a order/target matrix follows. All specified elements of the first column of the order/target matrix are zero so that the sum of all elements of the first column of the rotated matrix must be positive. All specified elements of the second column of the order/target matrix are zero but the 4th element (unspecified) is a + so that the 4th element of the rotated factor matrix must be positive. The third column of the order/target matrix has a non-zero (specified) element (.3). Hopefully the corresponding element of the rotated matrix will be close to .3 but it could be negative.

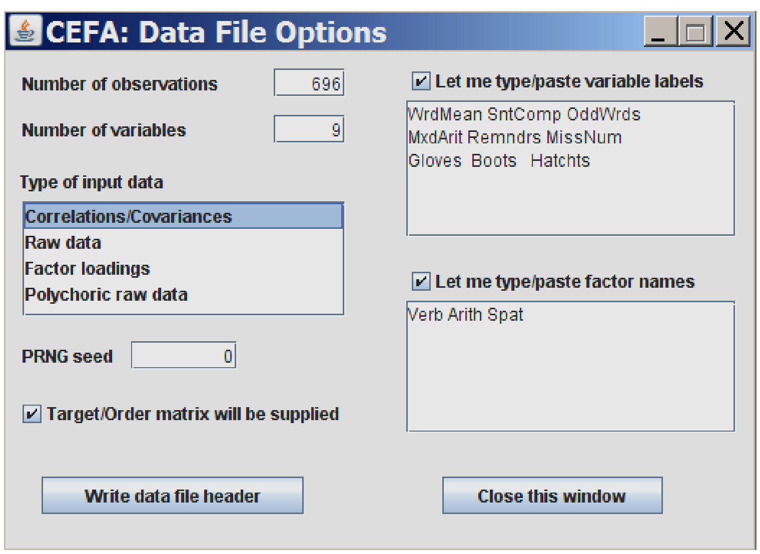
```
? 0 0
? 0 0
? 0 0
0 + 0
0 ? 0
0 ? 0
0 0 .3
0 0 ?
0 0 ?
```

2.1.4 Using CEFAtool for the Data File

To prepare the initial lines of the Data file, click on

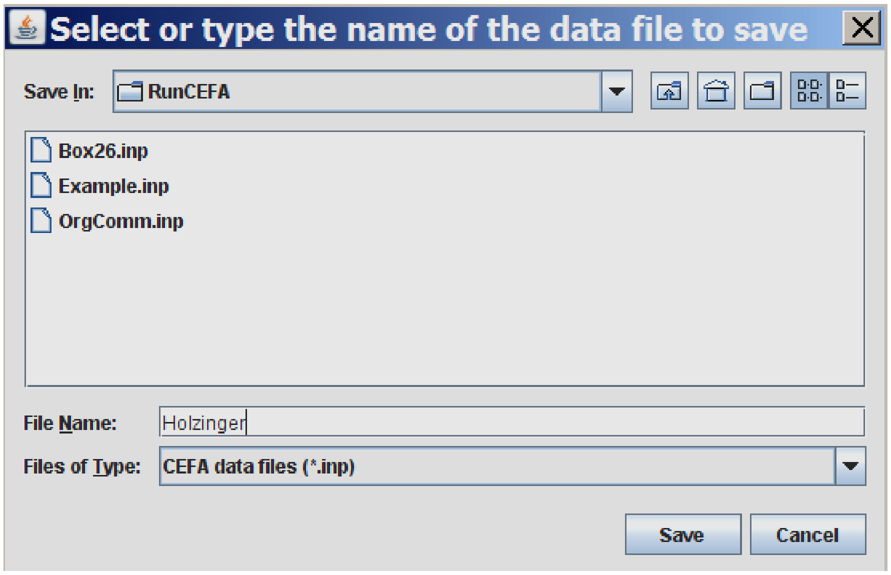
`Write Data File Header (*.inp)`

from the Main Window. The window shown below appears.

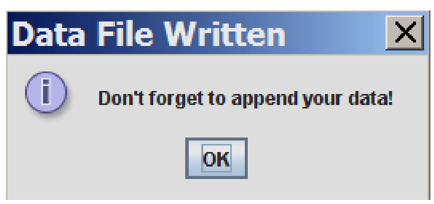


Data File Window

Fill in the required information and click on Write data file header. The following window appears



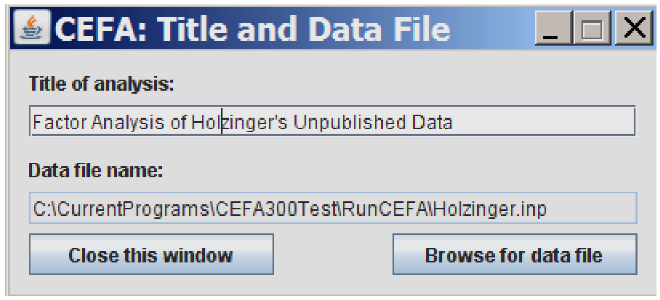
Type in a file name (e.g. `Holzinger` or `Holzinger.inp`). If omitted, the extension `.inp` is added automatically. The lines of the data file (`Holzinger.inp`) preceding the Order/Target matrix and data are then written to disk. Click on Save and the following window will appear.



Click on OK. The lines of the data file (`Holzinger.inp`) preceding the Order/Target matrix and data are then written to disk. Open the data file with Notepad and type in the Order/Target matrix (if appropriate) and data. Alternatively copy the Order/Target matrix and data from another file and paste it into the data file.

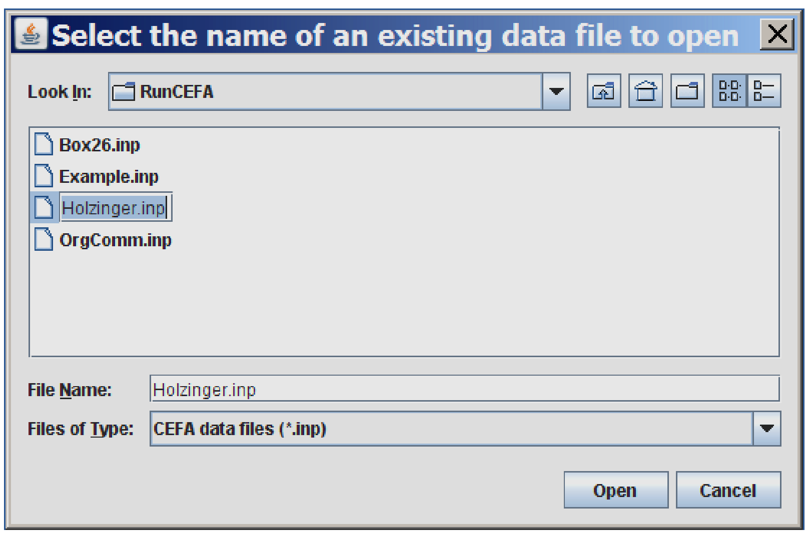
2.2 The Script File

The data file may be used for several runs. A new script file is usually required for each run. Although the script file may be prepared using a text editor (e.g. Notepad) alone it is more convenient to use options 1-5 of the Main Window. First click on 1: Set File Options in the Main window. This will bring up the following window. In the figure shown, an appropriate title has been typed and the name and path of the Data file has been entered.



Window 1: Title and Data Options

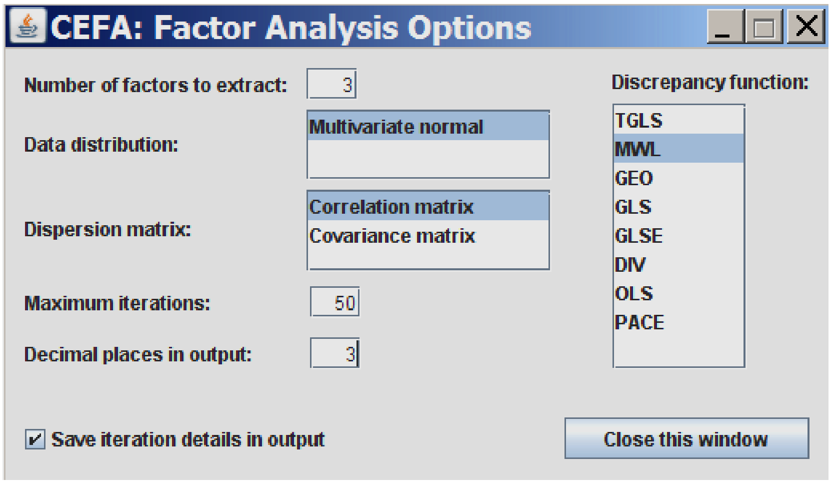
This is done by clicking on Browse for data file, choosing the appropriate file from the list that appears (see Window 1a)



Window 1a: Data File Options

and clicking on the Open button below.

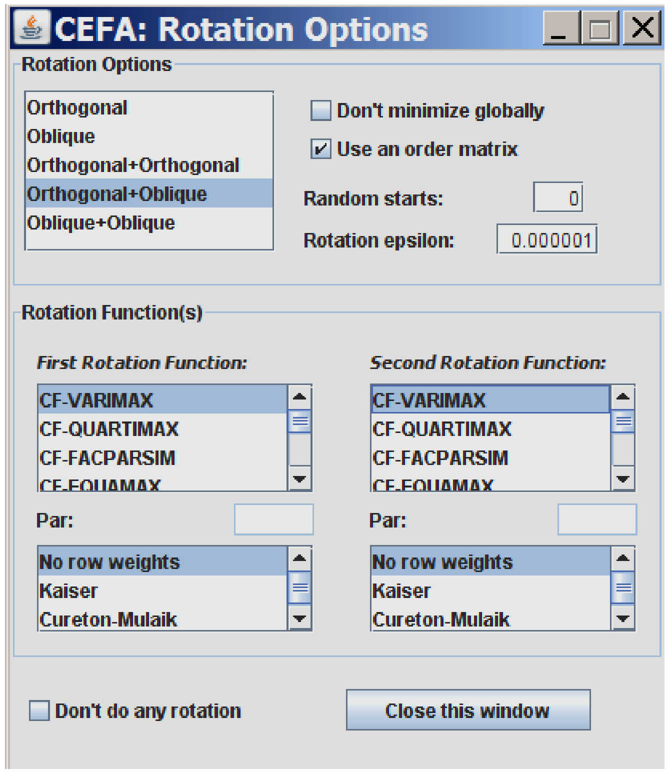
Now click on 2: Set Factor Analysis Options in the Main Window and the following window will appear (see Section 5).



Window 2: Factor Analysis Options

In this Window the Number of factors extracted has been set to 3 and the defaults of Correlation Matrix for Dispersion Matrix, 50 for Maximum Number of Iterations, 2 for Decimal places in output and MWL for Discrepancy Function have been kept. The only option available in the current version of CEFA for Data Distribution is Multivariate Normal. Click on Save Iteration Details to obtain iteration details for both the Factor Analysis and Rotation procedures. Close Window 2 by clicking on the Finished button.

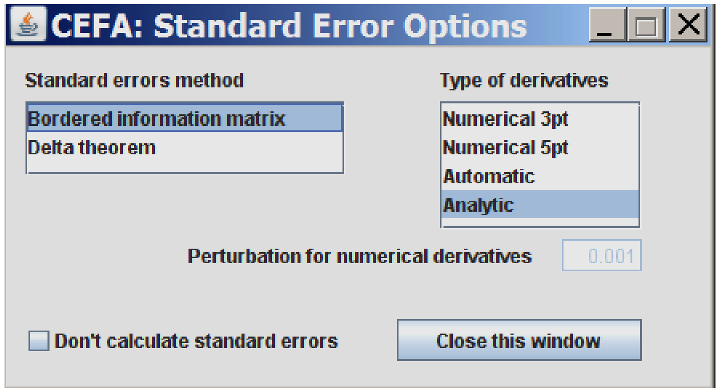
Now click on 3: Set Rotation Options in the Main Window to Obtain Window 3 (see Section 6). In the window shown,



Window 3: Rotation Options

Orthogonal+Oblique (orthogonal rotation followed by oblique rotation) has been chosen, the order matrix given in the Holzinger.inp file will be used, and the Crawford-Ferguson equivalent of Varimax rotation (CF-VARIMAX) has been chosen for both orthogonal and oblique rotation. Several other rotation functions are not visible in the picture but may be found on the screen by moving the highlighted area with the down arrow of the keyboard past CF-EQUAMAX. The default value, 0.000001, of the rotation convergence criterion (Rotation Epsilon) yields an accurate solution. In those situations where one wishes to investigate possible local minima of the rotation criterion, Random Starts should be assigned a positive value (e.g. 10). The initial random number required is then obtained from the Data File. When a previous run with random starts is to be duplicated, the previously obtained initial random numbers should be given in the Data File. If no rotation is required, click on Don't do any rotation.

After clicking on Close this window, go back to the Main window, click on 4: Set Standard Errors Options and obtain Window 4 (see Section 7).



Window 4: Standard Error Options

If Cureton-Mulaik weighting (see Window 3) is *not used*, the alternatives chosen, Bordered Information Matrix and Analytic Derivatives, are recommended for general use in conjunction with all rotation criteria, except Minent and Infomax[2]. If the requirements for correct standard errors are not met (for example, if the PACE option has been used in Window 2 or if a principal component matrix has been input with Datatype 3 in the Data file), ensure that a ✓ occurs next to Don't claculate standard errors by clicking if necessary. Note that if two consecutive rotations have been chosen in Window 3, standard errors are only provided for the second option.

After closing Window 4 with Close this window, go to the Main Window and click on 5: Write CEFA Script File. This gives Window 5.

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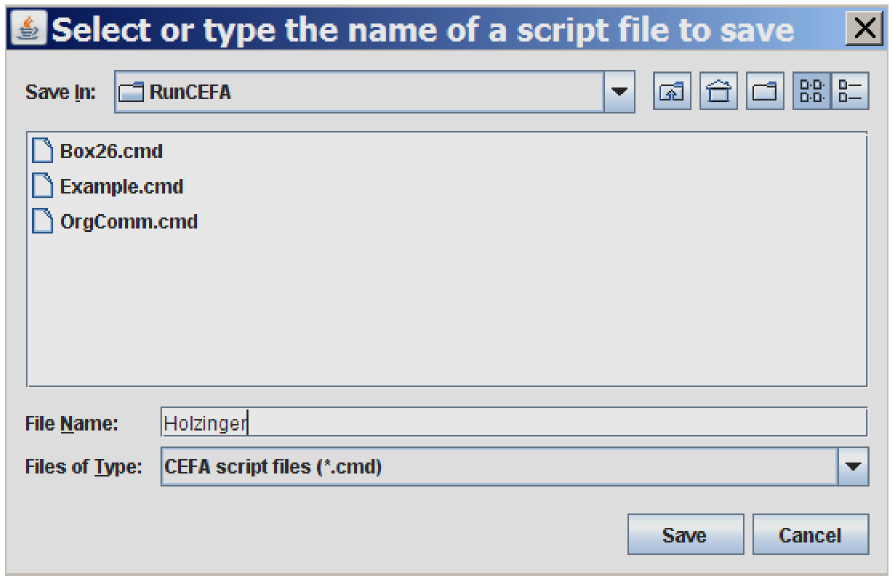
[2]For Minent and Infomax the alternatives Bordered Information Matrix and Automatic Derivatives are recommended. If Cureton-Mulaik weighting is used, Bordered Information Matrix and Numerical3 are recommended in conjunction with all rotation criteria.

If Delta Theorem is used instead of Bordered Information Matrix, then Numerical3 is recommended in conjunction with all rotation criteria, unless accuracy his important. Numerical5 then is appropriate.



Window 5: Write CEFA Script File

The file name for the script file must be specified either by clicking on a name in the list or by typing it. If omitted, the extension, .cmd, will automatically be added.

The following script file is produced by CEFAtool when Windows 1-5 have been completed as shown earlier. The instructions produced appear in a typewriter font. Annotations have been inserted in italics.

2.2.1 File: Holzinger.cmd

```
TITLE     Factor Analysis of Holzinger's Unpublished Data
```

Up to 80 characters may be used for a descriptive title.

```
DATAFILE  C:\CurrentPrograms\CEFA302Test\RunCEFA\Holzinger.inp
```

The name of the data file need not include the full path, if CEFA.exe is invoked from the same directory. Note that this path may vary from user to user.

```
NUMFACT   3
```

The number of factors extracted. This should match the number of columns of supplied factor or order/target matrices in the data file!

```
DISCFUN   2
```

The discrepancy function can be: 1 (TGLS), 2 (MWL), 3 (GEO), 4 (GLS), 5 (GLSE), 6 (DIV), 7 (OLS), or 8 (PACE).

`DISPMAT 1`

The dispersion matrix used in the exploratory factor analysis can be 1 (correlations) or 2 (covariances).

`MAXITER 50`

The maximum number of iterations for the Newton algorithm in the exploratory factor analysis step of CEFA. (50 is the default)

`DATADIST 1`

At present, the data distribution can only be 1 (multivariate normal)

`ITERINFO 1`

This can be 1 (save iteration details) or 0. ITERINFO applies simultaneously to iterations for the factor extraction procedure and for the rotation procedure.

`NUMDP 3`

Number of decimal places required in the output.

`TYPROT 3`

The type of rotation can be 1 (orthogonal), 2 (oblique), 3 (orthogonal followed by another orthogonal), or 4 (orthogonal followed by oblique).

`FNAME CF-VARIMAX`

The rotation criterion name for the first rotation can be: CF-VARIMAX, CF-QUARTIMAX, CF-FACPARSIM, CF-EQUAMAX, CF-PARSIMAX, MINENT1, INFOMAX, GEOMIN, CRAWFER, or TARGET. The first five are special cases of CRAWFER.

`ROWWT1 1`

Row weights for the first rotation may be 1 (no row weights), 2 (Kaiser weights), or 3 (Cureton-Mulaik weights).

`FNAME2 CF-VARIMAX`

If TYPROT is 3 or 4, a second rotation criterion is required

`ROWWT2 1`

Row weights for the second rotation may be 1 (no row weights), 2 (Kaiser weights), or 3 (Cureton-Mulaik weights).

`ROTEPS 0.000001`

The default value, 0.000001, of the rotation convergence criterion (Rotation Epsilon) yields a very accurate solution. With large problems, it could be made a little larger if time is a consideration.

`RNDSTARTS 0`

In those situations where one wishes to investigate possible local minima of the rotation criterion, rndstarts should be assigned a positive value (e.g. 10).

`USEORDMAT 1`

This can be 1 (sort columns of the rotated factor loading matrix using the specified order matrix), or 0 (sort columns in descending order of sums of squares). The order matrix has been provided in the data file.

`NOMIN      0`

This can be 1 (find the closest local minimum instead of optimizing the rotation criterion), or 0.

`SERMETH    1`

This can be 1 (use a bordered information matrix), or 2 (use the Delta theorem). The latter option is available for the Swain family of discrepancy functions but not for OLS. Standard errors are not available for PACE.

`TYPDER     4`

The derivatives used for computation of standard errors can be 1 (3-point numerical), 2 (5-point numerical), 3 (automatic), or 4 (analytic).

`DEREPS     0.001`

The perturbation size for numerical derivatives. Not used because analytic derivatives are required.

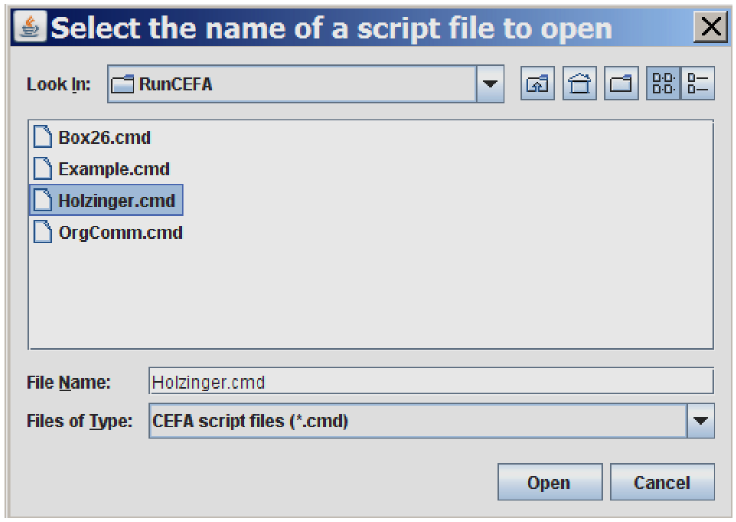
`STOP`

The last keyword in the script file must be STOP.

Note that it is often not necessary to look at the script file when CEFAtool is being used. However, saved script files may be used in the future to repeat the analysis. In addition, small changes to the analysis may be made by directly editing a saved script file using a text editor (e.g., Notepad).

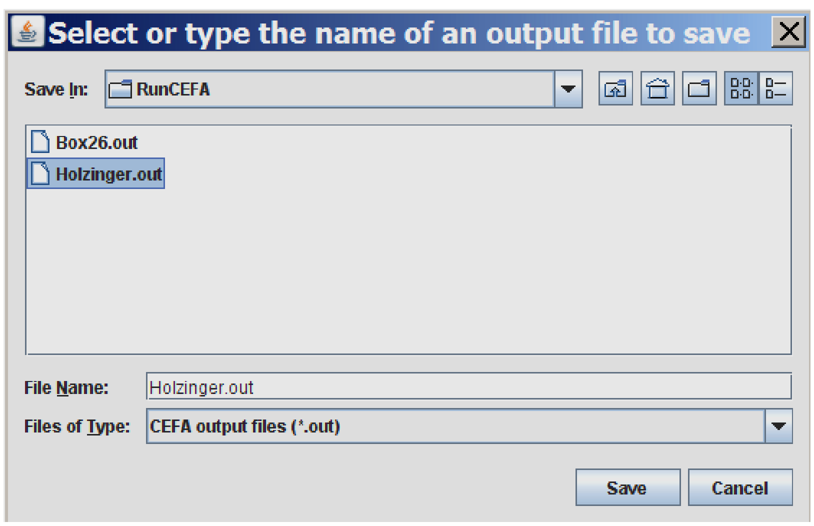
2.3 Running CEFA

Once you have prepared the data and script files you may run CEFA. Click on 6: Run CEFA in the Main Window. First the following window appears.



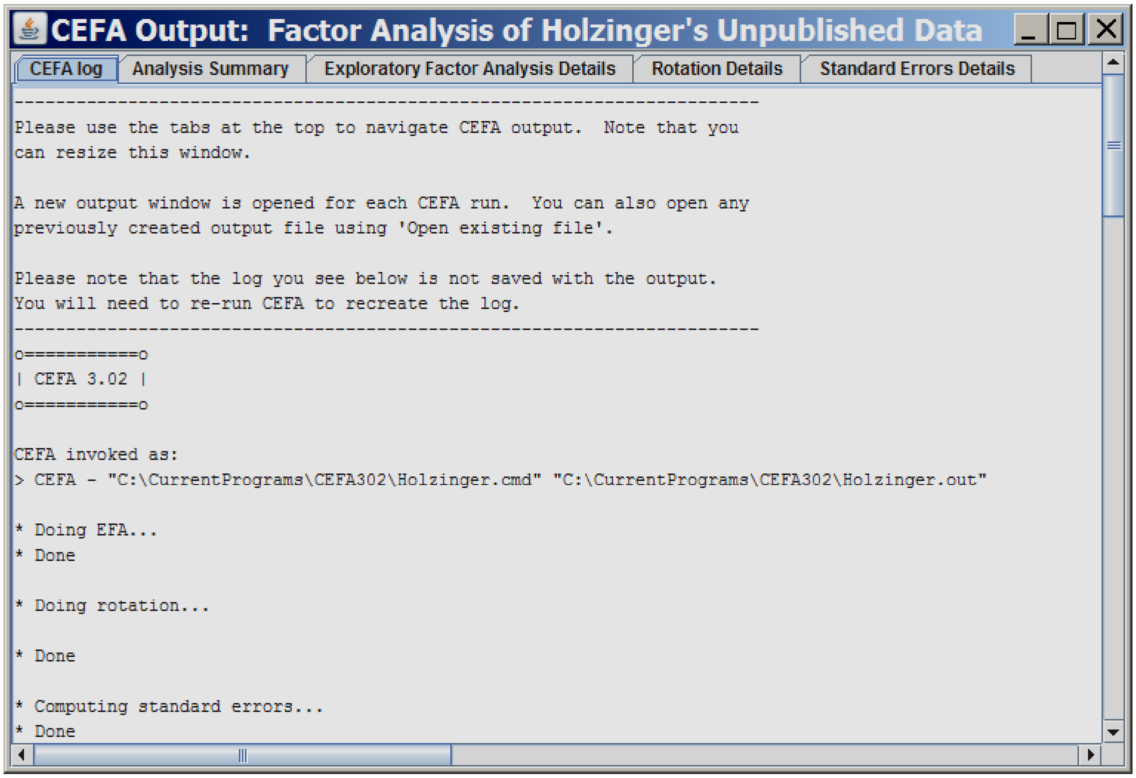
Window 6a: Specify Script File to be Used in Run

Either type in the Script File name (the .cmd may be omitted) or click on an existing file. Click on Open and the following window appears.



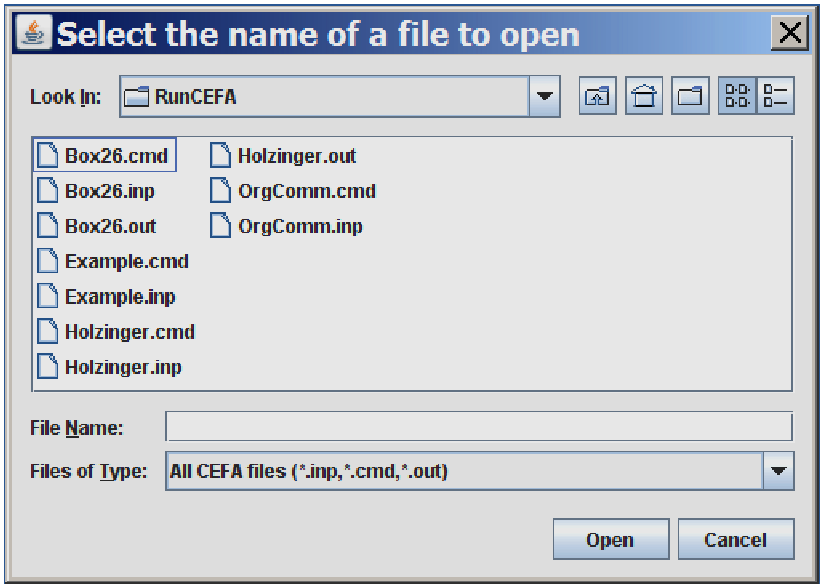
Window 6b: Name File to be Used for Output and Run CEFA

Type in an output file name (the .out may be omitted) or click on an existing file name to be overwritten and click on Save. CEFA starts running and the following window appears after the run is complete.



Window 6c: CEFA Output

The CEFA Output window has a series of tabs, each of which contains a different section from the output file. You may browse the output by clicking on the separate tabs. You may also open previously created output files using Open Existing File in the Main Window. The following window appears



Window: Open an Existing File

Click on the appropriate file name and then click on Open. To show only files of a specified type in the list, click on the list labeled "Files of Type:" and select the type of files you want to show.

2.4 The Output file

The CEFA run specified earlier produces the following output file. (The font has been changed to a smaller font for lines that would not fit on the page using the standard font.)

2.4.1File: Holzinger.out

```
o-----------------------------------------------o
| CEFA: Comprehensive Exploratory Factor Analysis |
|                                                 |
|              Release Version 3.03               |
|                  December 2008                  |
|                                                 |
|           Mathematical Specification:           |
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|                                                 |
|                   Programming:                  |
|         Krishna Tateneni, Gerhard Mels,         |
|       Robert Cudeck, and Michael W. Browne.      |
o-----------------------------------------------o
```

```
   Date: 2008-12-09
   Time: 15:35:15

 ***************************************************
 * Factor Analysis of Holzinger's Unpublished Data *
 ***************************************************

 o======================o
 | Details of Analysis |
 o======================o

   Data    file                                                             :
C:\CurrentPrograms\CEFA300Test\RunCEFA\Holzinger.inp

 Number of observations :   696
 Number of variables    :     9
 Number of factors      :     3

 Data assumed to follow multivariate normal distribution

 Discrepancy function   : MWL
 Dispersion matrix      : Correlations
 Max EFA iterations     :  50

 Rotation type          : Orthogonal+Orthogonal
 Sort columns using     : Order matrix

 Rotation Criterion     : CF-VARIMAX
 Row weights            : None

 Rotation Criterion 2   : CF-VARIMAX
 Row weights            : None

 Rotation convergence   : 0.100E-05

 Std Errors using       : Bordered information matrix
 Derivatives            : Analytic
```

```
o===========================o
| Sample Correlation Matrix |
o===========================o
```

(The original font size has been reduced to fit the matrix onto the page)

	WrdMean	SntComp	OddWrds	MxdArit	Remndrs	MissNum	Gloves	Boots	Hatchts
WrdMean	1.000								
SntComp	0.750	1.000							
OddWrds	0.780	0.720	1.000						
MxdArit	0.440	0.520	0.470	1.000					
Remndrs	0.450	0.530	0.480	0.820	1.000				
MissNum	0.510	0.580	0.540	0.820	0.740	1.000			
Gloves	0.210	0.230	0.280	0.330	0.370	0.350	1.000		
Boots	0.300	0.320	0.370	0.330	0.360	0.380	0.450	1.000	
Hatchts	0.310	0.300	0.370	0.310	0.360	0.380	0.520	0.670	1.000

```
Eigenvalues of Sample Correlation Matrix:
0.4771E+01  0.1405E+01  0.1055E+01  0.5627E+00  0.3247E+00  0.2732E+00  0.2552E+00  0.2087E+00
0.1447E+00


 ***************************************
 * Exploratory Factor Analysis Details *
 ***************************************

o=========================================================o
| Non iterative Unique Variances, Communalities, and SMCs |
o=========================================================o
```

(Always provided, even if PACE is not requested. It is interesting to compare how well the noniterative communalities and usual SMC's approximate the communalities obtained later)

Variable	Unique Variance	Communality	SMC
WrdMean	0.177	0.823	0.684
SntComp	0.294	0.706	0.648
OddWrds	0.248	0.752	0.672
MxdArit	0.063	0.937	0.775
Remndrs	0.268	0.732	0.704
MissNum	0.246	0.754	0.724
Gloves	0.631	0.369	0.325
Boots	0.427	0.573	0.484
Hatchts	0.207	0.793	0.525

```
o==================================o
| Iteration Details for  3 Factors |
o==================================o
```

(Of little interest to users. Intended for program developers)

```
-------------------------------------------------------
               MWL  Iteration Details
-------------------------------------------------------
Iter          Disc Fun   Max Grad    Step NSH NZP POB
-------------------------------------------------------
   0      0.132764741017  0.8371908
   1      0.089219229955  1.4320760  8.6E-1   0   0   1
   2      0.027357386318  0.3266576  1.0E+0   0   0   0
   3      0.016690327543  0.0530229  1.0E+0   0   0   0
   4      0.016359700002  0.0022382  1.0E+0   0   0   0
   5      0.016359082741  0.0000046  1.0E+0   0   0   0
   6      0.016359082738  0.0000000  1.0E+0   0   0   0
-------------------------------------------------------
```

```
o================================o
| MWL  Unrotated Factor Loadings |
o================================o

                Fac1        Fac2        Fac3
  WrdMean      0.678       0.533      -0.273
  SntComp      0.717       0.377      -0.231
  OddWrds      0.699       0.486      -0.164
  MxdArit      0.900      -0.340      -0.033
  Remndrs      0.836      -0.200       0.028
  MissNum      0.862      -0.128       0.002
  Gloves       0.421       0.089       0.431
  Boots        0.478       0.246       0.540
  Hatchts      0.483       0.302       0.673
```

```
o==========================================o
| MWL  Unique Variances and Communalities |
o==========================================o
```

Variable	Unique Variance	Communality
WrdMean	0.181	0.819
SntComp	0.290	0.710
OddWrds	0.248	0.752
MxdArit	0.073	0.927
Remndrs	0.260	0.740
MissNum	0.241	0.759
Gloves	0.629	0.371
Boots	0.420	0.580
Hatchts	0.222	0.778

```
o==================================o
| MWL  Scaled Standard Deviations |
o==================================o

  1.000   1.000   1.000   1.000   1.000   1.000   1.000   1.000   1.000
```

```
o==========================o
| MWL  Matrix of Residuals |
o==========================o
```

(The original font size has been reduced to fit the matrix onto the page)

	WrdMean	SntComp	OddWrds	MxdArit	Remndrs	MissNum	Gloves	Boots	Hatchts
WrdMean	0.000								
SntComp	-0.001	0.000							
OddWrds	0.002	-0.003	0.000						
MxdArit	0.002	-0.005	0.000	0.000					
Remndrs	-0.003	0.012	-0.003	0.000	0.000				
MissNum	-0.005	0.011	0.000	0.001	-0.006	0.000			
Gloves	-0.006	-0.006	0.012	-0.005	0.024	-0.002	0.000		
Boots	-0.008	0.009	0.005	0.001	-0.005	-0.001	-0.006	0.000	
Hatchts	0.005	-0.005	-0.005	0.000	-0.002	0.002	-0.001	0.001	0.000

Value of the maximum absolute residual = 0.0237

```
o=====================o
| MWL  Measures of Fit |
o=====================o
```

(See Browne & Cudeck, 1993)

```
Sample discrepancy function value            : 0.01635908

Population discrepancy function value, Fo
        Bias adjusted point estimate        : 0.000
      90 percent confidence interval        : ( 0.000; 0.016)

Root mean square error of approximation, RMSEA = SQRT(Fo/DF)
                      Point estimate        : 0.000
      90 percent confidence interval        : ( 0.000; 0.037)

Expected cross-validation index
       Point estimate (modified AIC)        : 0.111
      90 percent confidence interval        : ( 0.112; 0.129)
 ECVI (modified AIC) for the saturated model : 0.129

Chi-square test statistic                    :      11.370

Exceedance Probabilities
 Perfect fit (Ho: RMSEA = 0.0)              : 0.498
 Close fit (Ho: RMSEA <= 0.05)              : 0.995

Multiplier for obtaining test statistic      :   695.0
Degrees of freedom                           :    12
Effective number of parameters               :    33
```

```
*******************
* Rotation Details *
*******************

o====================================o
| Row Weights (Not Necessarily Used!) |
o====================================o
 Kaiser weights, Cureton-Mulaik weights, and
 their products (final Cureton-Mulaik weights)
```

(Always provided, even if the Cureton-Mulaik weighting is not requested: small values of v_i forecast complexity after rotation and large values forecast simplicity.

	$h_i^{-\frac{1}{2}}$	v_i	$v_i h_i^{-\frac{1}{2}}$
WrdMean	1.105	0.712	0.787
SntComp	1.187	0.479	0.568
OddWrds	1.153	0.531	0.612
MxdArit	1.039	0.649	0.674
Remndrs	1.163	0.427	0.496
MissNum	1.148	0.302	0.346
Gloves	1.641	0.833	1.366
Boots	1.313	0.882	1.158
Hatchts	1.134	0.965	1.094

See Section 6.7)

```
o======================o
| Iteration Information |
o======================o
```

(Of little interest to users. Intended for program developers)

```
-----------------------------------------------------------------
Cycle          FunVal        MinDelta         MaxDelta  BMin  BMax
-----------------------------------------------------------------
    1  2.228537095532  0.064833747028  0.293206944581     6     6
    2  2.193854560846  0.007428544016  0.044418170543     5     6
    3  2.193730635887  0.000507054780  0.002552850867     5     6
-----------------------------------------------------------------
      Switched to Minimum Bracketing Procedure...
-----------------------------------------------------------------
    4  2.193729942348  0.000039650581  0.000186675505     5     5
    5  2.193729938095  0.000003037996  0.000014713292     5     5
    6  2.193729938070  0.000000233149  0.000001130631     5     5
    7  2.193729938070  0.000000017921  0.000000086611     2     5
-----------------------------------------------------------------
```

```
o==================================o
| CF-VARIMAX Rotated Factor Matrix |
o==================================o
```

	Verb	Arith	Spa
WrdMean	0.863	0.241	0.126
SntComp	0.750	0.363	0.125
OddWrds	0.792	0.280	0.216
MxdArit	0.231	0.926	0.126
Remndrs	0.275	0.788	0.207
MissNum	0.352	0.768	0.214
Gloves	0.097	0.264	0.541
Boots	0.194	0.211	0.706
Hatchts	0.181	0.175	0.845

CF-VARIMAX Criterion: 2.193729938069513

```
o=====================o
| Factor Correlations |
o=====================o
```

	Verb	Arith	Spa
Verb	1.000		
Arith	0.000	1.000	
Spa	0.000	0.000	1.000

(Iteration information. for the second rotation)

```
-----------------------------------------------------------------
Cycle          FunVal        MinDelta        MaxDelta  BMin  BMax
-----------------------------------------------------------------
    1  0.992110257727  0.124490924523  0.336577931571     6     6
    2  0.947011891974  0.002896285859  0.065779439250     6     6
    3  0.946875963661  0.001130338462  0.004298060448     6     6
-----------------------------------------------------------------
      Switched to Minimum Bracketing Procedure...
-----------------------------------------------------------------
    4  0.946868971358  0.000040981549  0.000841438562     5     6
    5  0.946868669449  0.000021390404  0.000213382311     5     6
    6  0.946868662493  0.000005167144  0.000030758996     5     5
    7  0.946868662359  0.000000821069  0.000003978921     5     5
    8  0.946868662356  0.000000125737  0.000000578009     4     5
-----------------------------------------------------------------


o==================================o
| CF-VARIMAX Rotated Factor Matrix |
o==================================o

                  Verb        Arith          Spa
  WrdMean        0.894        0.006        0.022
  SntComp        0.737        0.175        0.018
  OddWrds        0.790        0.052        0.122
  MxdArit        0.014        0.954        0.006
  Remndrs        0.084        0.770        0.106
  MissNum        0.175        0.723        0.108
  Gloves        -0.042        0.160        0.549
  Boots          0.054        0.040        0.725
  Hatchts        0.025       -0.025        0.883

CF-VARIMAX Criterion: 0.9468686623561622

o=====================o
| Factor Correlations |
o=====================o

                  Verb        Arith          Spa
  Verb           1.000
  Arith          0.481        1.000
  Spa            0.345        0.372        1.000
```

```
**************************
* Standard Errors Details *
**************************
```

(Provided only for the last rotation. (Second rotation: Oblique here))

```
o=========================================o
| Bordered Information Matrix Diagnostics |
o=========================================o

 No errors trapped in calculations.
Maximum constraint violation:       0.757E-06

Elapsed Seconds:      0

o================================o
| Standard Errors after Rotation |
o================================o

                 Verb       Arith         Spa
  WrdMean       0.017       0.016       0.016
  SntComp       0.022       0.024       0.022
  OddWrds       0.021       0.021       0.021
  MxdArit       0.013       0.014       0.012
  Remndrs       0.021       0.020       0.021
  MissNum       0.022       0.021       0.021
  Gloves        0.035       0.036       0.035
  Boots         0.027       0.027       0.032
  Hatchts       0.018       0.016       0.030

                 Verb       Arith         Spa
  Verb          0.000
  Arith         0.025       0.000
  Spa           0.030       0.029       0.000

o========================================o
| One-At-A-Time 90% Confidence Intervals |
o========================================o

                    Verb                Arith                Spa
   WrdMean ( 0.866; 0.922)   (-0.021; 0.032)   (-0.004; 0.048)
   SntComp ( 0.701; 0.774)   ( 0.136; 0.215)   (-0.019; 0.054)
   OddWrds ( 0.756; 0.824)   ( 0.018; 0.086)   ( 0.088; 0.156)
   MxdArit (-0.007; 0.035)   ( 0.931; 0.977)   (-0.014; 0.026)
   Remndrs ( 0.049; 0.119)   ( 0.736; 0.803)   ( 0.071; 0.140)
```

```
   MissNum ( 0.139; 0.211)   ( 0.689; 0.758)   ( 0.074; 0.142)
   Gloves  (-0.100; 0.016)   ( 0.101; 0.219)   ( 0.492; 0.605)
   Boots   ( 0.010; 0.098)   (-0.005; 0.085)   ( 0.672; 0.778)
   Hatchts (-0.005; 0.055)   (-0.052; 0.002)   ( 0.833; 0.932)

   Arith   ( 0.439; 0.521)
   Spa     ( 0.294; 0.394)   ( 0.323; 0.419)

o================o
| CEFA Completed |
o================o
```

3. CEFA: An Overview

(a) Brief Description

CEFA is a program for carrying out exploratory factor analysis with rotation and standard errors of estimates. A variety of estimation methods and rotation criteria are available, and two independent methods for standard errors are included. CEFA is compatible with the Microsoft Windows family of operating systems.

(b) CEFA Files

There are three types of files used by CEFA: (1) Data files, usually named with the extension inp (or dat), (2) Script or command files, usually named with the extension cmd, and (3) Output files, usually named with the extension out. All of these files contain plain text, and may be edited using any text editor, such as Notepad, a standard component in Microsoft Windows.

The data file contains data for analysis. CEFA accepts the following types of data: (1) Raw data, (2) Correlation or covariance matrices, (3) Factor loading matrices where the factors are mutually uncorrelated, and (4) Discrete data for polychoric correlations.

The script file consists of several pairs of keywords and their associated values. These keywords specify options for the analysis. Multiple script files may be associated with the same data file.

CEFA is invoked with the names of two files, a script file, and an output file; after processing the script file, CEFA reads data from the associated data file. All output is directed to the output file.

(c) Running CEFA

To run CEFA from the command line, just type the name of the executable, and respond to the prompts for input and output files (the input file is the script file containing commands for the analysis):

```
> cefa
* Input file: "data\my script.cmd"
* Output file: data\cefa.out
```

Notice that quotes are used when there are spaces in the path.

Alternatively, the names of the input and output files may be specified on the command line, and the prompts may be avoided:

```
> cefa "data\my script.cmd" data\cefa.out
```

The latter form is likely to be convenient for invocation from batch files, or for recalling commands when command-line editing is enabled e.g., by using the DOSKEY program.

After completing the analysis, CEFA pauses for a key to be pressed; to run CEFA in a completely non-interactive manner, include a hyphen after the name of the program, as shown below:

```
> cefa -
* Input file: "data\my script.cmd"
* Output file: data\cefa.out
```

or:

```
> cefa - "data\my script.cmd" data\cefa.out
```

(d) Console Messages

While CEFA is running, various messages are displayed on the screen. In particular, when standard errors are computed using the method based on the Delta theorem, a countdown is displayed of the number of rotations. When 3-point numerical derivatives are used, the total number of rotations will be $1 + 2pm$ where p is the number of manifest variables and m is the number of factors. For 5-point numerical derivatives, the number is $1 + 4pm$.

When random starts are carried out, the number of random starts are counted down; because each starting point is different, it is not unusual for the counter to go faster or slower on specific rotations as the rate of change depends upon the number of iterations that are required to attain convergence.

If CEFA is run from the graphical user interface CEFAtool, the console messages are shown on the first tab of the output window after the run is complete.

(e) CEFAtool

CEFAtool is a Graphical User Interface to CEFA. With the exception of the case where batch file processing is used, CEFAtool is intended to eliminate the need to use CEFA directly.

CEFAtool can be used to: (1) Generate script files conveniently by providing information via user-friendly dialog boxes, (2) Generate header information for data files, so that only the data matrix need be appended, (3) Invoke CEFA transparently, and (4) Edit any text file using one-click access to Notepad.

One convenient feature of CEFAtool is that once options have been set and a script file written, one or two changes can be made to the dialogs, and a new script can be generated easily.

Please see Section 3 for more details.

(f) Data and Script Files

For a detailed description of the contents of a data file and a script file, please see the sample files:

```
Example.inp
Example.cmd
```

Users are strongly encouraged to minimize the possibility of errors by using CEFAtool to generate files.

(g) Sample Files

The sample files included with CEFA are:

- Example.inp Holzinger's unpublished data with nine psychological tests. This is an annotated data file intended to act as a reference; you should not delete it. A three factor solution fits very well. In this example a correlation matrix is provided in the data file.

- Example.cmd A sample script file which uses Example.dat. This is an annotated script file intended to act as a reference; do not delete it.

- Box26.inp Thurstone's box data set. The 26 variables are various functions of length, width, and height in a random collection of 30 boxes. This data set consists of a factor loading matrix obtained by a "principal axes" factorization of the original correlation matrix given by Cureton and Mulaik (1975). Note: formulae for standard errors (Section 7) are not valid for this data set since efficient estimates were not used for factor loadings. In this example a factor matrix is provided in the data file.

- Box26.cmd, A sample script file which uses Box26.dat. Since the factor loading estimates provided in the data file are not efficient it would be inappropriate to obtain standard errors with CEFA.

- OrgComm.inp, Organizational commitment data analyzed by Boshoff & Mels. In this example the raw data are provided in the data file.

- OrgComm.cmd, A sample script file which uses OrgComm.dat.

4. CEFAtool: An Overview

(a) Brief Description

CEFAtool is a Graphical User Interface to CEFA, a console-based application for exploratory factor analysis with rotation and standard errors. CEFA and CEFAtool are compatible with the Microsoft Windows family of operating systems.

CEFAtool can be used to: (1) Generate script files conveniently by providing information via user-friendly dialog boxes, (2) Generate header information for data files, so that only the data matrix need be appended, (3) Invoke CEFA transparently, and (4) Edit any text file using one-click access to Notepad.

(b) Starting CEFAtool

You may run CEFAtool.exe directly by double-clicking it. You may also create shortcuts to CEFAtool.exe using the various methods provided for this by Microsoft Windows.

CEFAtool looks for CEFA.exe when it is asked to run a script file. Usually, CEFA.exe is installed in the same directory as CEFAtool.exe.

The main window of CEFAtool consists of two sets of four command buttons each; the first set is labeled "Options" and the second set is labeled "Actions". These are discussed in Sections 3c and 3d.

(c) Options Group

The options group of buttons in CEFAtool allow you to specify various options which can then be written to a CEFA script file. Some of the options have default values, but others must be specified before a script file can be generated. These are highlighted below.

File Options.

A data file name must be specified. Click on the button next to the text field to select a file name. By convention, data files end in the extension inp or dat, although this is not strictly necessary.

A default title is generated by using the current date and time when CEFAtool was started, but this title can be modified.

Factor Analysis Options.

The number of factors must be specified. Note that if an order/target matrix is provided in the data file, the number of factors must match the number of columns in the Order/Target matrix.

By default, the dispersion matrix for the factor analysis (if applicable) is a correlation matrix, and the discrepancy function used is MWL. The data distribution is assumed to be multivariate normal. These details may be changed if necessary. In addition, the maximum number of iterations for the Newton algorithm used in the factor analysis is capped at 50.

The number of decimal places may be changed if desired. In addition, technical details of the iterative process may be saved. Note that if this option is selected, iteration details from the rotation algorithm will also be saved.

Rotation Options.

A method of rotation, as well as a rotation criterion, must be specified. If the method of rotation is a chained rotation, a second rotation criterion must be specified. A parameter value is necessary when using the generalized Crawford-Ferguson family, or the Geomin criterion.

A row weighting scheme must be selected. If no row weights are to be used, that option may be selected from the list.

If an order matrix is supplied in the data file, it may be used to order columns of the rotated factor matrix by checking the "use order matrix" option. By default, columns are sorted in decreasing order of sum of squared loadings in the column.

If global minimization of the rotation criterion is to be avoided, and the closest local minimum is to be used, check the option. In most cases, this option should probably be left as it is. Similarly, the default convergence criterion of 0.000001 for the rotation should be appropriate for most cases.

If random starts are desired, type the appropriate number.

Finally, it is possible to use CEFA without carrying out any rotation; check the box if you do not want any rotations.

Standard Errors Options.

The defaults should cover most cases; for the Swain family of discrepancy functions, an alternative estimation method based on the Delta theorem may be used as a means of checking results.

The type of derivative used and the perturbation size for numerical derivatives should only be changed by experienced users.

Computation of standard errors can be skipped if so desired.

(d) Actions Group

Write CEFA Script File.

When all required options have been set, a CEFA script file can be generated. By default, script files are named with the extension cmd. It is not necessary to specify the extension when typing the filename.

Run CEFA.

CEFA can be invoked from CEFAtool. An existing script file must be selected, as well as an output file. By default, output files are named with the extension out.

Write Data File Header.

The header information for CEFA data files can also be generated using CEFAtool. Type the number of variables and the number of observations in the appropriate fields, and select the type of data you will be supplying as well as the type of distribution.

The random number seed may be left at its default values of 0,0; in this case, when random starts are carried out, the system clock is used to generate a seed. If a specific random start is to be reproduced, you will need to look up the seed information from the previous output file, and copy the numbers.

Default labels are generated within CEFA for variables and factors. If you would like to provide your own, check the appropriate box, and type the names, separated by spaces, in the text fields that appear.

Finally, if an order/target matrix will be provided in the data file, select the option for that.

After you click "Finished", CEFAtool will allow you to save the header information you typed in a "inp" (or "dat") file. To complete the data file, you should open the file using Notepad, and append the order/target matrix (if applicable), and your data at the end of the header information.

For raw data, the variables should index the columns, and the individual cases should index the rows. Numbers may be typed free-format. For a covariance or correlation matrix, enter the lower triangle, including the diagonal elements. For a factor loading matrix, enter the data so that factors index columns and variables index rows.

See the sample data files for more information.

Open Existing File.

Using this option, any text file can be examined and edited using Notepad. By default, all files with the extensions inp, dat, or out are listed. Select "Files of Type:" to see files with only the specified extension. Files with the extension "out" are assumed to be CEFA output files, and opened in a special tabbed window (for viewing only, not for editing.)

5. Window 2: Factor Analysis Options

Number of factors extracted. The number of factors extracted should match the number of columns of the Order/Target matrix (Section 2.1.3) and of the factor matrix if provided in the data file (Section 2.1.1)

Data Distribution. The only permissible option in the current version of CEFA is Multivariate Normal.

Dispersion Matrix. The default option is Correlation Matrix since factor analysis is intended primarily for the analysis of correlation matrices. If Covariance Matrix is chosen, a factor matrix is obtained using the correlation matrix and rows are rescaled by standard errors. With all options, except OLS and PACE, this is equivalent to obtaining the factor matrix from the covariance matrix. Rotation and standard errors of factor loadings are not possible when Covariance Matrix is chosen.

Max Iterations. This option sets the maximum number of iterations allowed in the process for estimating the factor matrix. The default is 50. It has no effect with PACE.

Decimal places in output. This option sets the number of decimal places for matrix output throughout the CEFA run, in the estimation, rotation and standard error stages. The default is 3.

Save iteration details. When this option is checked, iteration details are written to the output file *both* in the *estimation stage* and in the *rotation stage*. This option is intended primarily for the program developers and will be of less interest to users.

Discrepancy Function. This option specifies the discrepancy function to be minimized to yield estimates of factor loadings and unique variances.

The following notation will be employed here: $\boldsymbol{R}$ represents a $p \times p$ sample correlation matrix, $\boldsymbol{P}^* = \boldsymbol{A}^*\boldsymbol{A}^{*\prime} + \boldsymbol{D}_\psi^*$ represents a covariance matrix reproduced according to the factor analysis model, $\boldsymbol{A}^*$ is a factor matrix, $\boldsymbol{D}_\psi^*$ is a diagonal unique covariance matrix and $F(\boldsymbol{R}, \boldsymbol{P}^*)$ is a discrepancy function to be minimized. The eigenvalues of $\boldsymbol{R}^{-\frac{1}{2}}\boldsymbol{P}^*\boldsymbol{R}^{-\frac{1}{2}}$ are represented by $\theta_1 \leq \theta_2 \leq \cdots \leq \theta_p$.

Most of the estimators obtained (i.e. excluding MWL and OLS) do not yield a $\boldsymbol{P}^*$with unit diagonal elements. Therefore $\boldsymbol{A}^*$ and $\boldsymbol{D}_\psi^*$ are standardized to yield $\boldsymbol{A}$ and $\boldsymbol{D}_\psi$ such that

$\boldsymbol{P} = \boldsymbol{A}\boldsymbol{A}' + \boldsymbol{D}_\psi$ has unit diagonals. The matrices output by CEFA are $\boldsymbol{A}$, $\boldsymbol{D}_\psi$ and the residual matrix, $\boldsymbol{R} - \boldsymbol{P}$. Asymptotic theory appropriate for this standardization is employed throughout.

Swain (1975a, b) has provided a family of discrepancy functions that yield asymptotically equivalent and efficient estimators. CEFA can provide minimum discrepancy estimates using the following members of the Swain family (cf. Swain, 1975b, Table 1) of discrepancy functions.

TGLS True Generalized Least Squares:

$$F_1(\boldsymbol{R}, \boldsymbol{P}^*) = \tfrac{1}{2} tr[\boldsymbol{P}^{*-1}(\boldsymbol{R} - \boldsymbol{P}^*)]^2$$

MWL Maximum Wishart Likelihood:

$$F_2(\boldsymbol{R}, \boldsymbol{P}^*) = ln|\boldsymbol{P}^*| - ln|\boldsymbol{R}| + tr[\boldsymbol{R}\boldsymbol{P}^{*-1}] - p$$

GEO Geodesic Distance:

$$F_3(\boldsymbol{R}, \boldsymbol{P}^*) = \tfrac{1}{2}\sum_{i=1}^{p}(ln\,\theta_i)^2$$

GLS Generalized Least Squares:

$$F_4(\boldsymbol{R}, \boldsymbol{P}^*) = \tfrac{1}{2} tr[\boldsymbol{R}^{-1}(\boldsymbol{R} - \boldsymbol{P}^*)]^2$$

GLSE Generalized Least Squares Extended: $F_5(\boldsymbol{R}, \boldsymbol{P}^*) = \frac{1}{2}\sum_{i=1}^{p}(\theta_i - 1)^2 e^{(\theta_i - 1)}$

DIV Divergence: $F_6(\boldsymbol{R}, \boldsymbol{P}^*) = \frac{1}{2}\left\{tr[\boldsymbol{R}\boldsymbol{P}^{*-1}] + tr[\boldsymbol{R}^{-1}\boldsymbol{P}^*] - 2p\right\}$

The algorithm used in CEFA is adapted from Swain (1975a, 1975b), Jennrich and Robinson (1969) and Jennrich and Sampson (1968).

Formulae for asymptotic standard errors are the same for all members of the Swain family. The maximum likelihood estimator, MWL, is the default.

In addition CEFA can provide the OLS estimator (Jöreskog, 1977)

OLS Ordinary Least Squares: $F_7(\boldsymbol{R}, \boldsymbol{P}^*) = \frac{1}{2} tr[(\boldsymbol{R} - \boldsymbol{P}^*)]^2$

Finally, CEFA will provide the non-iterative estimator:

PACE Partitioned Covariance Estimator.

The algorithm described by Cudeck (1991) is used in CEFA. Similar results would be given by the algorithm given by Kano (1990). PACE is noniterative and fast and may be used for large correlation matrices. It also has the advantage of seldom giving Heywood cases.

Standard errors of rotated factor loadings for the OLS option are based on Proposition 4 of Browne (1984). Standard errors are not produced by CEFA if the PACE option is used.

The test statistic provided in conjunction with members of the Swain family is $(N-1)F(\boldsymbol{R}, \boldsymbol{P}^{*})$. An adaptation of the test statistic given in Browne (1984, Proposition 4) is employed in conjunction with OLS and PACE. Measures of model fit (Browne & Cudeck, 1993) associated with the test statistics are also provided.

6. Window 3: Rotation Options

Rotation Info This option specifies whether an orthogonal rotation, an oblique rotation or a sequence of two rotations will be carried out. The alternatives are:

Orthogonal	Orthogonal rotation only
Oblique	Oblique rotation only
Orth + Orth	An orthogonal rotation is carried out and is used as initial solution for another orthogonal rotation
Orth + Oblq	An orthogonal rotation is carried out and is used as initial solution for an oblique rotation

In general a single rotation will be chosen. A sequence of two rotations is carried out when the first rotation is intended to be used an initial solution for the second. This may be convenient when the second rotation criterion has multiple minima and the first does not. Also, it is sometimes convenient to obtain both an orthogonal rotation and an oblique rotation simultaneously using the same rotation criterion. In the second rotation no attempt is made to find a global optimum and the local optimum closest to the initial solution is sought. If a sequence of two rotations is chosen, standard errors of rotated factor loadings (see Section 7) are *only provided for the second rotation*.

Don't Minimize Globally If this option is checked, no attempt is made to search for a global minimum of the rotation criterion. The local minimum closest to the starting point is sought.

Use an Order Matrix This option may be checked only if an order matrix has been provided in the data file (see Section 2.1.1). The columns of the rotated factor matrix will then be reordered and reflected so as to match the specified elements of the order matrix as closely as possible (see Section 2.1.3). On the other hand, it is not essential to check this option whenever an order matrix has been provided in the data file. When this option is not checked, columns are ordered with column sums of squares in descending order of magnitude and columns are reflected to ensure nonnegative column sums.

Random Starts This option specifies the number of times the rotation procedure is to be repeated from new starting points obtained by postmultiplying the initial factor matrix by random orthogonal matrices. This procedure is useful for investigating the existence of local minima in a rotation criterion (Gebhardt, 1968; Kiers, 1994). CEFA will output each different solution and give the number of times it occurred. It will also output a pair of seeds for the random number generator corresponding to each solution. This pair of seeds can be included in the data file (see Section 2.1) if the same starting point is required in a subsequent run. If two zeroes are input as seeds as shown in the data file of Section 2.1.1 then a pair of seeds for the starting point is generated using the computer time clock. When two or more random starts are employed, standard errors of rotated loadings are not computed. The default for the random start option is 0 (the random start procedure is not used).

Rotation Epsilon This provides a convergence criterion for the iterative rotation process. It corresponds roughly to the number of decimal places accuracy of the solution. The default value is .000001.

Turn Rotation Off If this option is checked, all entries in the window vanish and no rotation of the factor matrix is carried out.

Function Info For each of the rotations (1 or 2) requested at Rotation Info the rotation criterion and row standardization procedure to be used are required.

The following rotation criteria are available.

CF-Varimax	Crawford-Ferguson equivalent of Varimax (see Section 6.2)
CF-Quartimax	Crawford-Ferguson equivalent of Quartimax (see Section 6.2)

CF-FacParsim	Factor Parsimony (see Section 6.2)
CF-Equamax	Crawford-Ferguson equivalent of Equamax (see Section 6.2)
CF-Parsimax	Parsimax (see Section 6.2)
Minent	McCammon Minimum Entropy (see Section 6.3)
Infomax	McKeon Infomax (see Section 6.4)
Geomin	Yates Geomin. ϵ is required. (see Section 6.5)
Crawfer	Crawford Ferguson Family. κ is required. (see Section 6.2)
Target	Target rotation (see Section 6.6)

The row standardization procedure may be one of the following (see Section 6.7)

No row weights	No row standardization.
Kaiser row weights	Kaiser's row standardization by inverse communality square roots.
Cureton-Mulaik	Cureton-Mulaik standardization to reduce the influence of rows that are anticipated to be complex

6.1 Notation

Further information will be given on individual rotation criteria. Some notation will be required for this purpose. The original $p \times m$ unrotated matrix will be denoted by $\boldsymbol{A}$ and the corresponding diagonal matrix of communalities will be represented by

$$\boldsymbol{D}_h = Diag[\boldsymbol{A}\boldsymbol{A}'].$$

The rows of $\boldsymbol{A}$ may be weighted before rotation using a diagonal weight matrix $\boldsymbol{D}_w$ and the rotation applied to $\boldsymbol{D}_w\boldsymbol{A}$. A transformation matrix, $\boldsymbol{T}$, is found so as to minimize a rotation criterion $f(\boldsymbol{\Gamma})$ of the transformed matrix

$$\boldsymbol{\Gamma} = \boldsymbol{D}_w\boldsymbol{A}\boldsymbol{T}$$

with typical element, γ_{ir}. In orthogonal rotation, $\boldsymbol{T}$ is required to be orthogonal, $\boldsymbol{T}\boldsymbol{T}' = \boldsymbol{I}$, and in oblique rotation the diagonal elements of the factor correlation matrix

$$\boldsymbol{\Phi} = \boldsymbol{T}^{-1}\boldsymbol{T}^{-1'}$$

are required to be equal to 1, $Diag[\boldsymbol{\Phi}] = \boldsymbol{I}$. After rotation, the rotated factor matrix is given by

$$\boldsymbol{\Lambda} = \boldsymbol{D}_w^{-1}\boldsymbol{\Gamma}.$$

Typically, a rotation criterion may be expressed in terms of squared elements of $\boldsymbol{\Gamma}$. The following notation is employed in the descriptions that follow.

$$s_{ir} = \gamma_{ir}^2 \quad S_{i.} = \sum_{r=1}^{m} s_{ir} \quad S_{.r} = \sum_{i=1}^{p} s_{ir} \quad S = \sum_{i=1}^{p}\sum_{r=1}^{m} s_{ir}$$

Some of the rotation criteria to be considered make use of the entropy function. Suppose that $x_i \geq 0$, $i = 1, \ldots, p$ and that $\sum_{i=1}^{p} x_i = 0$ and let $\boldsymbol{x} = (x_1, x_2, \ldots, x_p)'$. The entropy function is

$$Ent(\boldsymbol{x}) = \sum_{i=1}^{p} e(x_i)$$

where

$$e(x_i) = x_i \; ln(x_i), \quad x_i > 0, \qquad e(0) = 0$$

The greatest lower bound of $Ent(\boldsymbol{x})$ is 0, which is attained if a single element of $\boldsymbol{x}$ is $= 1$ and the rests are $= 0$. The least upper bound is $ln\, p$, which is attained if all elements of $\boldsymbol{x}$ are equal to $\frac{1}{p}$.

All rotation criteria to be considered are bounded below by zero and are minimized to yield the rotated solution. CEFA only makes use of the rotation criteria, and not the optimization algorithms, suggested by the original authors.

6.2 The Crawford-Ferguson Family of Rotation Criteria

The Crawford-Ferguson (Crawford & Ferguson, 1970) family of rotation criteria may be employed both in orthogonal rotation and in oblique rotation (Crawford, 1975). In orthogonal rotation members of the family are equivalent to the well known Orthomax family of criteria, giving the same results as Quartimax, Varimax and Equamax. In oblique rotation members of the family cannot result in collapse due to factor intercorrelations that tend towards one during the iterative process (Crawford, 1975).

The Crawford-Ferguson family may be expressed (e.g. Browne, 1974) in a form indexed by a single parameter, $0 \leq \kappa \leq 1$:

$$f_\kappa(\Gamma) = (1-\kappa)\sum_{i=1}^{p}\sum_{r=1}^{m}\sum_{s\neq r}^{m}\gamma_{ir}^2\gamma_{is}^2 + \kappa\sum_{r=1}^{m}\sum_{i=1}^{p}\sum_{j\neq i}^{p}\gamma_{ir}^2\gamma_{jr}^2$$

The first term (if $\kappa < 1$) is a measure of row parsimony and attains its greatest lower bound of zero when no row of $\boldsymbol{\Gamma}$ has more than one nonzero element. The second, (if $\kappa > 0$) is a measure of column parsimony and reaches zero if no column has more than one nonzero element. A Crawford-Ferguson criterion is therefore a weighted sum of a measure of row simplicity and a measure of column simplicity of $\boldsymbol{\Gamma}$.

Individual members of the Crawford-Ferguson family are shown in the following table.

$\kappa = 0$	CF-Quartimax (Quartimin)
$\kappa = \frac{1}{p}$	CF-Varimax
$\kappa = \frac{m}{2p}$	CF-Equamax
$\kappa = \frac{m-1}{p+m-2}$	CF-Parsimax
$\kappa = 1$	CF-Factor Parsimony

The members of the Crawford-Ferguson family shown in this table may be chosen directly in Window 3. Other members may be used by choosing the Crawfer alternative and entering the required value of κ ($0 \leq \kappa \leq 1$) in the box that appears.

6.3 McCammon's Minimum Entropy Criterion

McCammon (1966) proposed the following minimum entropy criterion for orthogonal rotation:

$$f(\Gamma) = \frac{\sum_{r=1}^{m}\sum_{i=1}^{p} e\left(\frac{s_{ir}}{S_{.r}}\right)}{\sum_{r=1}^{m} e\left(\frac{S_{.r}}{S}\right)}$$

The greatest lower bound of the numerator is zero and is attained if all columns of $\boldsymbol{\Gamma}$ have not more than one nonzero element and the least upper bound of the denominator is attained if all column sums of squares, $S_{.r}$, are equal. Consequently the criterion aims at within column simplicity and equal column sums of squares in $\boldsymbol{\Gamma}$.

The minimum entropy criterion was intended for orthogonal rotation (McCammon, 1966) and does not give good results in oblique rotation. It is one of the few rotation criteria that can yield a good orthogonal solution (without the Cureton-Mulaik row standardization) for Thurstone's 26 variable Box data (when the Random Starts option is used).

6.4 McKeon's Infomax Criterion

McKeon (1968) proposed a rotation criterion that is derived from on the likelihood ratio test of no association in a contingency table. He regarded the squared factor loadings as if they were entries in a contingency table and suggested that the test statistic be maximized. In order to obtain a rotation criterion that is to be minimized, McKeon's infomax criterion has been subtracted from its least upper bound, $\mathit{ln}\, m$, to give

$$f(\Gamma) = \mathit{ln}\, m - \left\{ \sum_{r=1}^{m} \sum_{i=1}^{p} e\left(\frac{s_{ir}}{S}\right) - \sum_{i=1}^{p} e\left(\frac{S_{i.}}{S}\right) - \sum_{r=1}^{m} e\left(\frac{S_{.r}}{S}\right) \right\}$$

This rotation criterion is suitable for both orthogonal and oblique rotation. It can yield good orthogonal or oblique solutions (without the Cureton-Mulaik row standardization) for Thurstone's 26 variable Box data (when the Random Starts option is used).

6.5 Yates Geomin Criterion

Yates (1987, formula (38)) suggested that the sum of within row geometric means of squared factor loadings be used as a rotation criterion. The following modification of Yates' criterion is used in CEFA:

$$f(\Gamma) = \sum_{i=1}^{p} \left\{ \prod_{r=1}^{m} (s_{ir} + \epsilon) \right\}^{\frac{1}{m}} \quad \text{where} \quad \epsilon \geq 0$$

Yates' original criterion is the special case of the criterion given here where $\epsilon = 0$. This results in indeterminacy when a loading in a row of $\boldsymbol{\Gamma}$ is zero since other loadings in the row then have no effect on the criterion. The value of ϵ should be sufficiently large to avoid near indeterminacy while remaining close to zero. A value of $\epsilon = 0.01$ has been tried and found satisfactory in a few trials with factor matrices where $m = 3$. In particular, this value of ϵ can yield good orthogonal or oblique solutions (without the Cureton-Mulaik row standardization) for Thurstone's 26 variable Box data (when the Random Starts option is used). It may be necessary to use larger values of ϵ if the number of factors is large.

The (original) Yates Geomin criterion differs from the other exploratory rotation criteria considered here, in that it will attain zero when Thurstone's first criterion for simple structure is met; that is if each row of $\boldsymbol{\Gamma}$ has one zero element (see Yates, 1987, Chapter 2).

As pointed out by Yates (1987, formulae (32), (37)) his criterion is a modification of a rotation criterion suggested by Thurstone (1935).

6.6 Rotation to a Partially Specified Target

Let $\boldsymbol{B}$ represent the target matrix described in Section 2.1.3. An orthogonal (Browne, 1972a) or oblique (Browne, 1972b) rotation to a least squares fit to the specified elements of $\boldsymbol{B}$ is carried out by minimizing the criterion

$$f(\mathit{\Gamma}) = \sum_{r=1}^{m} \sum_{i \in I_r} (\gamma_{ir} - b_{ir})^2$$

where I_r is the set of subscripts for specified target loadings, b_{ir} ($b_{ir} \neq 9$), in column r of B. This criterion differs from the other rotation criteria in that it is used in confirmatory, rather than purely exploratory, rotation.

6.7 Row Standardization

Row standardization was originally developed with Varimax in mind but has been used successfully with some other rotation criteria. It is inadvisable to use row standardization in target rotation.

Kaiser (1958) noticed that in orthogonal rotation, Varimax tended to give complex solutions for rows with low communalities and suggested the weights (see Section 6.1)

$$\boldsymbol{D}_w = \boldsymbol{D}_h^{-\frac{1}{2}}$$

to overcome this difficulty. This standardization has become widely used in conjunction with other criteria of the Crawford-Ferguson family, both in orthogonal and oblique rotation.

Cureton and Mulaik (1975) found that Varimax, with or without Kaiser standardization, does not recover a good approximation to the known structure of the Thurstone 26 variable box data and developed an alternative weighting scheme. This makes use of the first principal axis of $\boldsymbol{A}$ to forecast how complex each row of the rotated factor matrix will be and assigns lower weights to complex rows.

Let $\boldsymbol{b}$ be the first principal component of $\boldsymbol{AA}'$ and let $\boldsymbol{a} = \boldsymbol{D}_h^{-\frac{1}{2}}\boldsymbol{b}$. The i-th Cureton-Mulaik weight, v_i, may be expressed as

$$v_i = \cos^2(x(a_i)\,) + .001$$

where

$$x(a_i) = \frac{\cos^{-1}\left(m^{-\frac{1}{2}}\right) - \cos^{-1}(|a_i|)}{\cos^{-1}\left(m^{-\frac{1}{2}}\right) - \delta(a_i)} \times \frac{\pi}{2}$$

and

$$\delta(a_i) = 0 \qquad \text{if } |a_i| \geq m^{-\frac{1}{2}}$$

$$\delta(a_i) = \frac{\pi}{2} \qquad \text{if } |a_i| < m^{-\frac{1}{2}}$$

The final diagonal weight matrix, $\boldsymbol{D}_w$, (Section 6.1) is then given by

$$\boldsymbol{D}_w = \boldsymbol{D}_v \boldsymbol{D}_h^{-\frac{1}{2}}.$$

Cureton and Mulaik (1975) showed that their weighting system enabled the weighted Varimax rotation to produce a good solution for the Thurstone's 26 variable data and some subsets of these data. Some preliminary applications of this weighting scheme with other criteria in applying both orthogonal and oblique rotations to the Thurstone data have been encouraging.

7. Window 4: Standard Errors Options

Standard Errors Method There are two alternative methods of obtaining standard errors available in CEFA:

Bordered Information Matrix This method is an extension of an approach employed by Jennrich (1974) for obtaining standard errors of rotated factor loadings in orthogonal rotation. Allowance is made for oblique rotation, for row standardization, and for arbitrary rotation criteria. This approach requires first and second derivatives of the rotation criterion with respect to the factor loadings. Exact derivatives may be used, if available, for speed and accuracy. The Bordered Information Matrix alternative is the default and the recommended approach in general.

Delta Theorem This method is a faster and more general version (Tateneni, 1998) of a simplification by Cudeck and O'Dell (1994) of an approach employed by Archer and Jennrich (1973) and by Jennrich (1973). First order numerical derivatives of rotated loadings with respect to unrotated loadings are employed. The Delta

Method alternative in CEFA is slower than the Bordered Information Matrix alternative and is intended primarily for checking purposes. The two methods should give the same results for the same problem.

Type of Derivative First and second order derivatives of the simplicity function with respect to the factor loadings are required for the Bordered Information Matrix method. First order derivatives of rotated factor loadings with respect to unrotated factor loadings are required for the Delta Method. Alternatives are:

Numerical3 The double sided difference (three point) formula (e.g. Burden, Faires & Reynolds, 1981, formula 4.13) is used to give a numerical approximation to the derivative required. When this alternative is employed a box requiring the perturbation to be employed appears. The default is 0.001. Tateneni (1998) has examined the accuracy of this approximation and concluded that it is sufficiently accurate for practical purposes when obtaining standard errors. This approximation procedure is very easily implemented in conjunction with new criteria but is slower than the analytic and automatic derivatives. Double sided difference approximations for derivatives may be employed with any of the rotation criteria available in CEFA.

Numerical5 The Richardson extrapolation (five point) formula (e.g. (e.g. Burden, Faires & Reynolds, 1981, formula 4.22) is less well known than the double sided difference formula but has been found effective for obtaining a numerical approximation to a Hessian by Jamshidian and Jennrich (1998). It involves substantially more computation than the double sided difference formula but, with a careful choice of perturbation, can give more accurate results. A comparison of the two numerical approximation methods for obtaining standard errors of rotated loadings is given by Tateneni (1998). Again, a box for the perturbation to be used appears when this option is chosen. Richardson extrapolation approximations for derivatives may be employed with any of the rotation criteria available in CEFA.

Automatic Automatic differentiation (Bischoff, Carle, Khademi & Mauer, 1995) involves the repeated application of the chain rule in computer program code. The programmer provides a subroutine for evaluating the function and the automatic differentiation program supplements it by providing code to evaluate the derivative. Derivatives evaluated are exact. Tateneni (1998) discusses the use of automatic derivatives in the evaluation of standard errors of rotated loadings. Automatic derivatives are available for use with all the rotation criteria available in CEFA in conjunction with no standardization or Kaiser standardization. They cannot be used when Cureton-Mulaik standardization is employed.

Analytic Exact analytic derivatives available for use with all the rotation alternatives available in CEFA except Minent and Infomax in conjunction with no standardization or Kaiser standardization. They cannot be used when Cureton-Mulaik standardization is employed.

When Cureton-Mulaik standardization is employed and when the Delta method is employed numerical derivatives must be employed.

Turn Std Errs Off This box may be checked when no standard errors are required. If a factor matrix that has not been obtained using an asymptotically efficient estimation method is input in the Data matrix (for example, if the Box26.dat file is used, Section 3g) this box should be checked. It is automatically checked if OLS or PACE estimates are obtained.

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