Analýza závislosti: intervalová a poměrová data (Pearsonovo *r*)

Petr Ocelík

MVZn4003 Úvod do kvantitativních metod

Opakování

- K čemu používáme **chí-kvadrát test nezávislosti**?

Opakování

- K čemu používáme **Kendallovo tau**?

Opakování

- Co nám říká **p-hodnota** (statistická významnost)?

Míry závislosti

- Existuje **mnoho měr závislosti** (MZ), korelační koeficienty jsou pouze podmnožinou MZ
- MZ měří **přítomnost** nebo/a **sílu závislosti** mezi proměnnými
- MZ typicky nabývají hodnot v intervalech <0,1> nebo <-1,1>

→ **neomezujme se na korelaci**, či dokonce Pearsonovo r

- Korelace neimplikuje kauzalitu

- Kauzalita může být založena na různých typech závislosti

Míry závislosti

úrovně měření	míry
nominální ordinální	chí-kvadrát test nezávislosti
ordinální	Kendallův korelační koeficient tau
metrická	Pearsonův korelační koeficient r

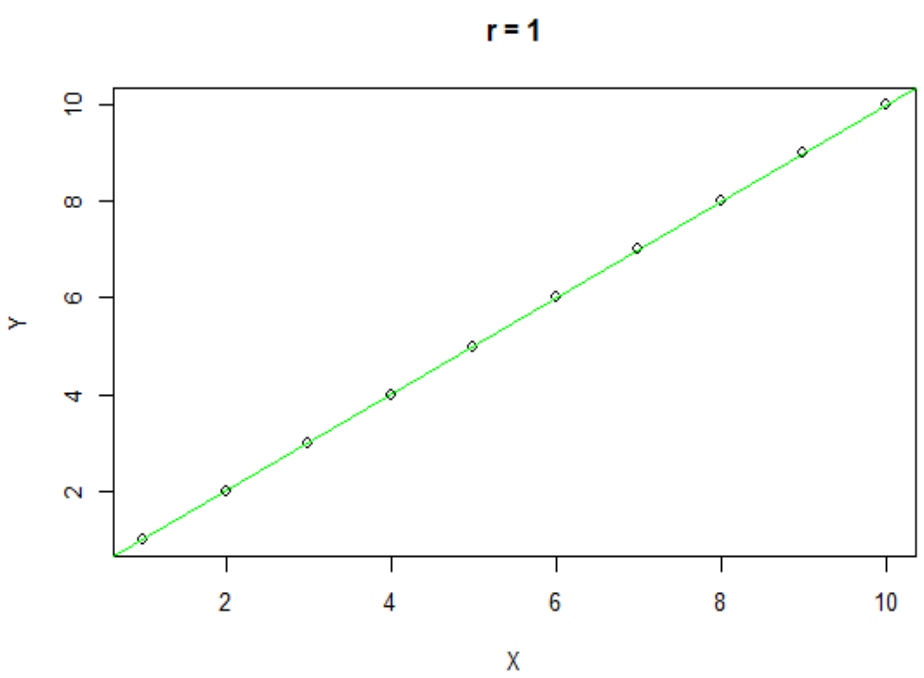
Pearsonův korelační koeficient r

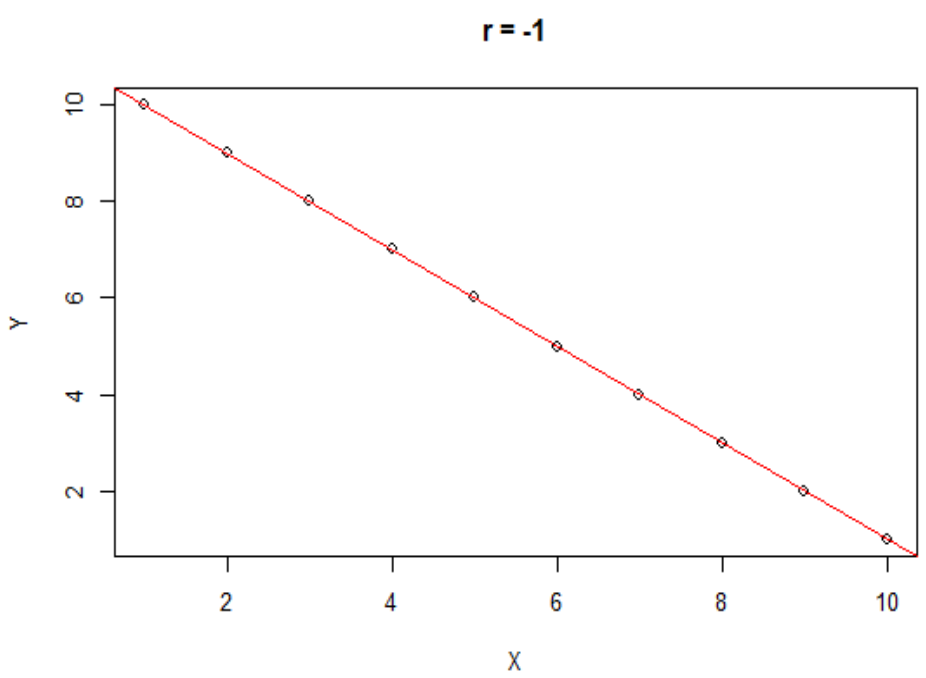
- Pearsonův korelační koeficient (***r***)
- Pearsonovo *r* měří **sílu a směr lineární závislosti mezi dvěma spojitými proměnnými**

wikimedia

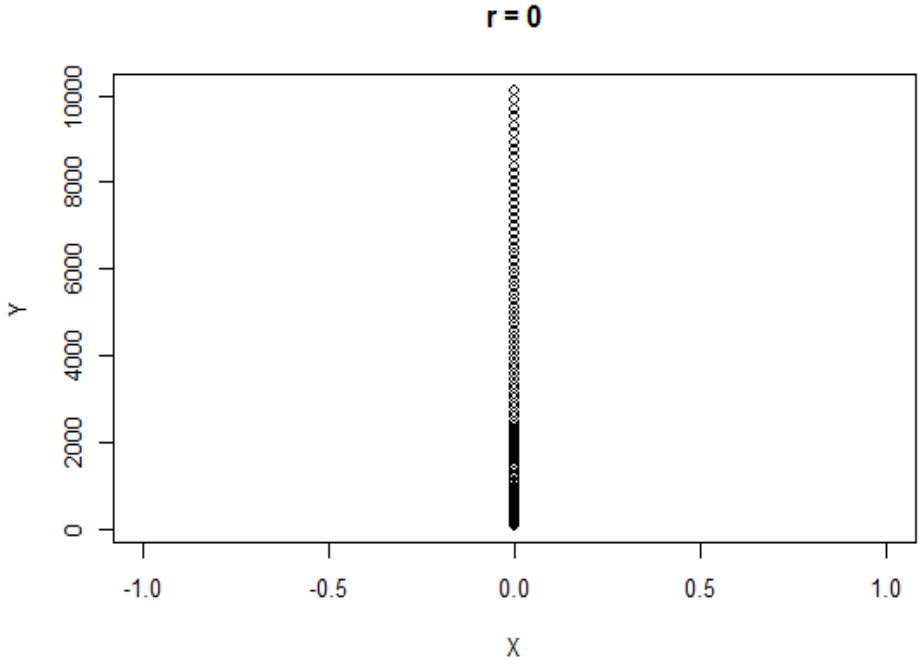
- Pearsonovo *r* nabývá hodnot <-1,1>
  - Dokonale pozitivní lineární vztah = 1
  - Dokonale negativní lineární vztah = -1
  - Žádný lineární vztah = 0

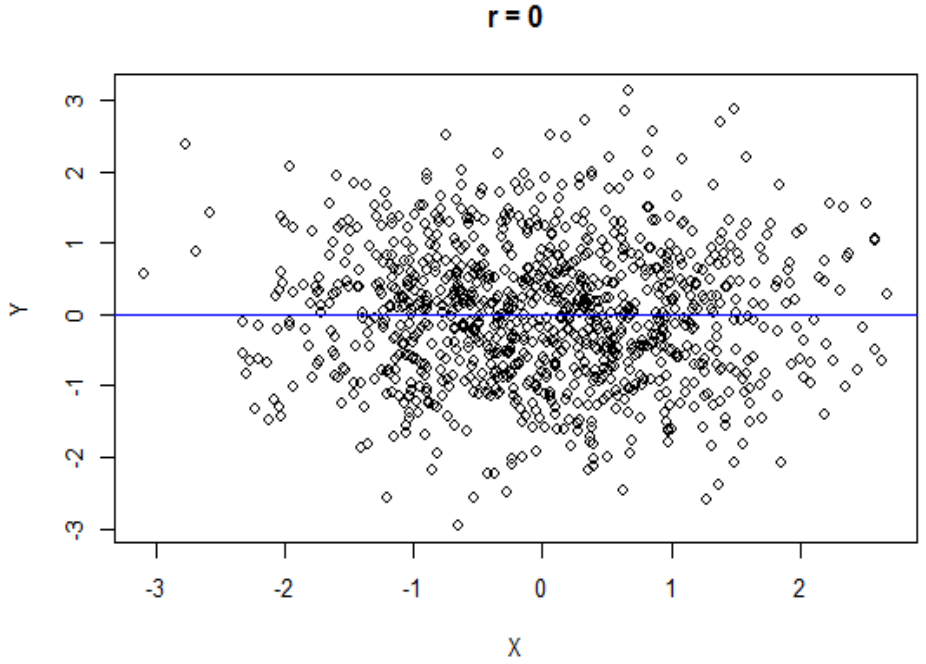
- Symetrický vztah: r(X, Y) = r(Y, X)
- Hodnota *r* není závislá na jednotkách proměnných





http://guessthecorrelation.com/





- Kellstedt & Whitten (2013) příklad korelace volebního zisku inkumbenta (vládní politické strany) a změny HDP během předcházejícího volebního období.
- Statisticky významná korelace = 0.574

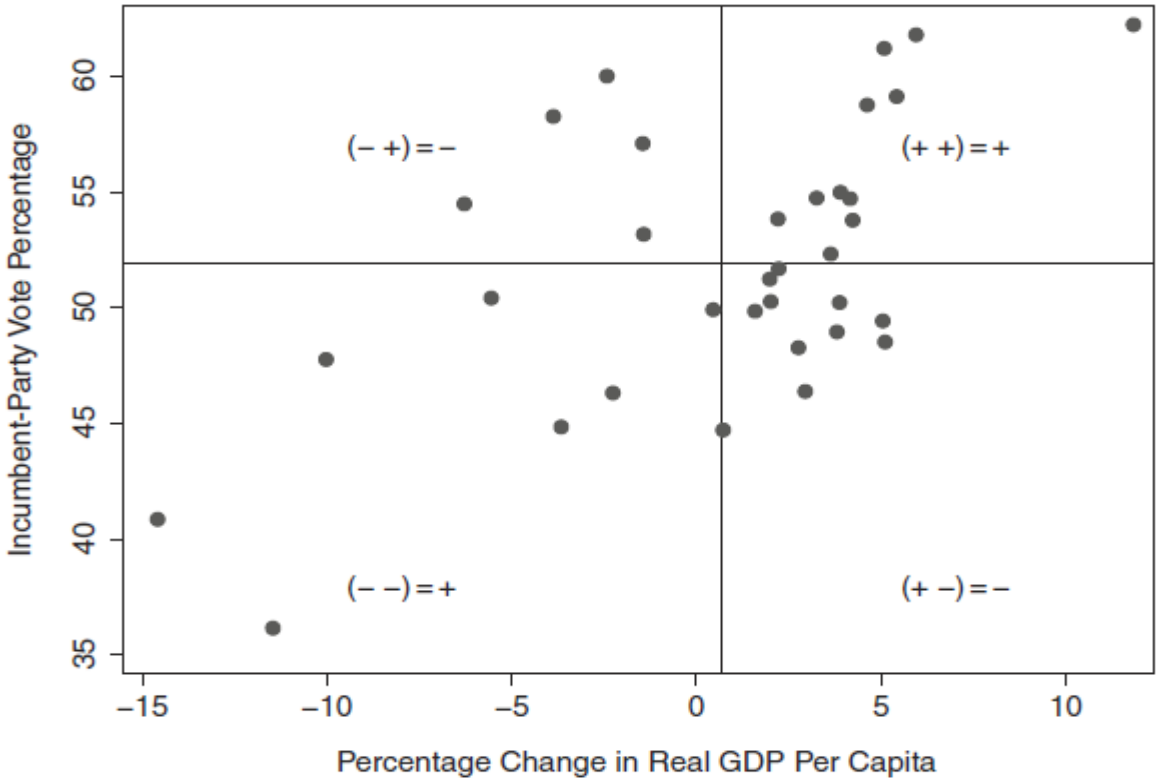
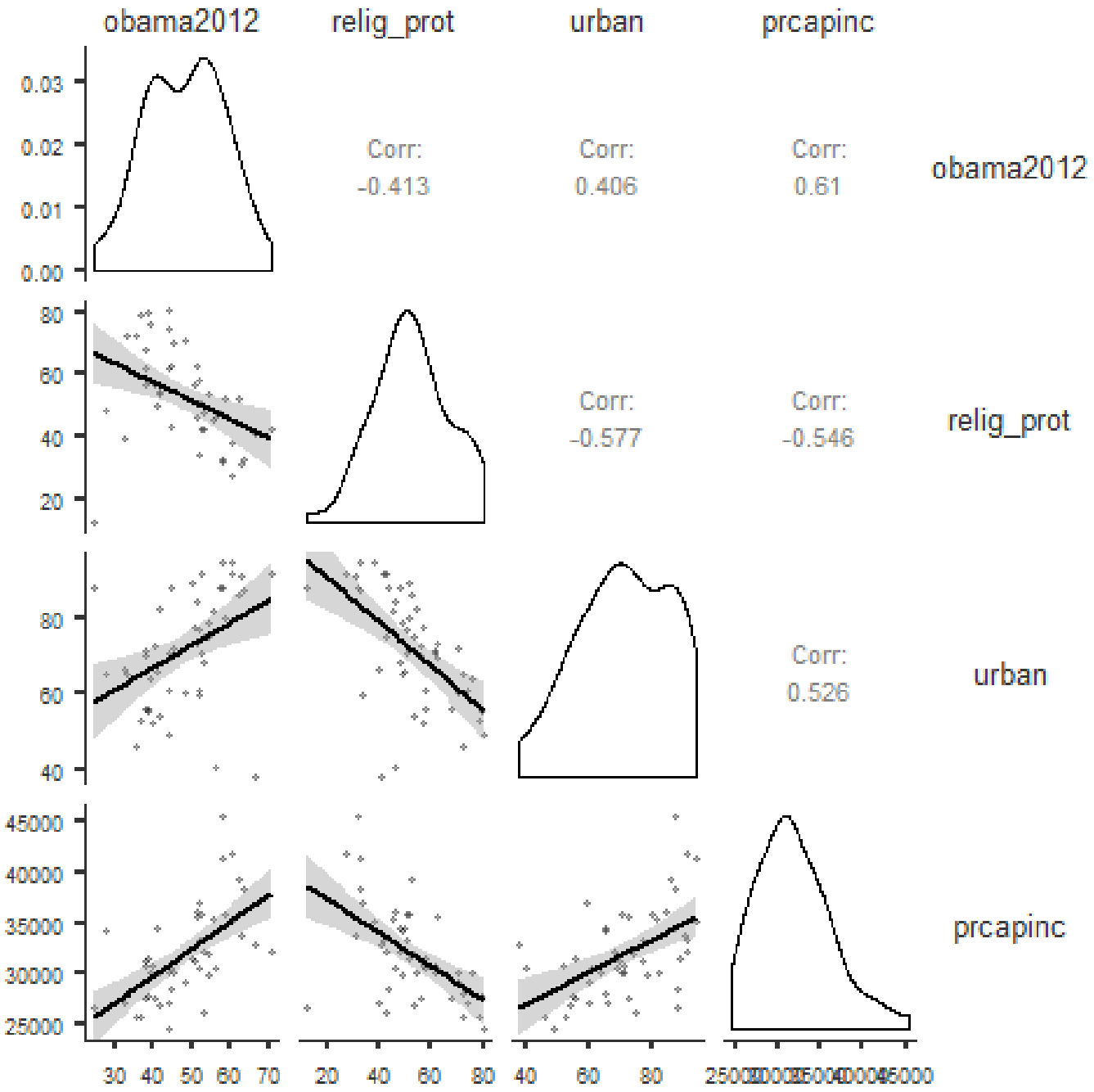


Figure 7.4. Scatter plot of change in GDP and incumbent-party vote share with mean-delimited quadrants.


obama2012
relig_prot
urban
prcapinc
Corr:
-0.413
Corr:
0.406
Corr:
0.61
obama2012
Corr:
-0.577
Corr:
-0.546
relig_prot
Corr:
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prcapinc
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45000
40000
35000
30000
25000
30 40 50 60 70
20 40 60 80
40 60 80

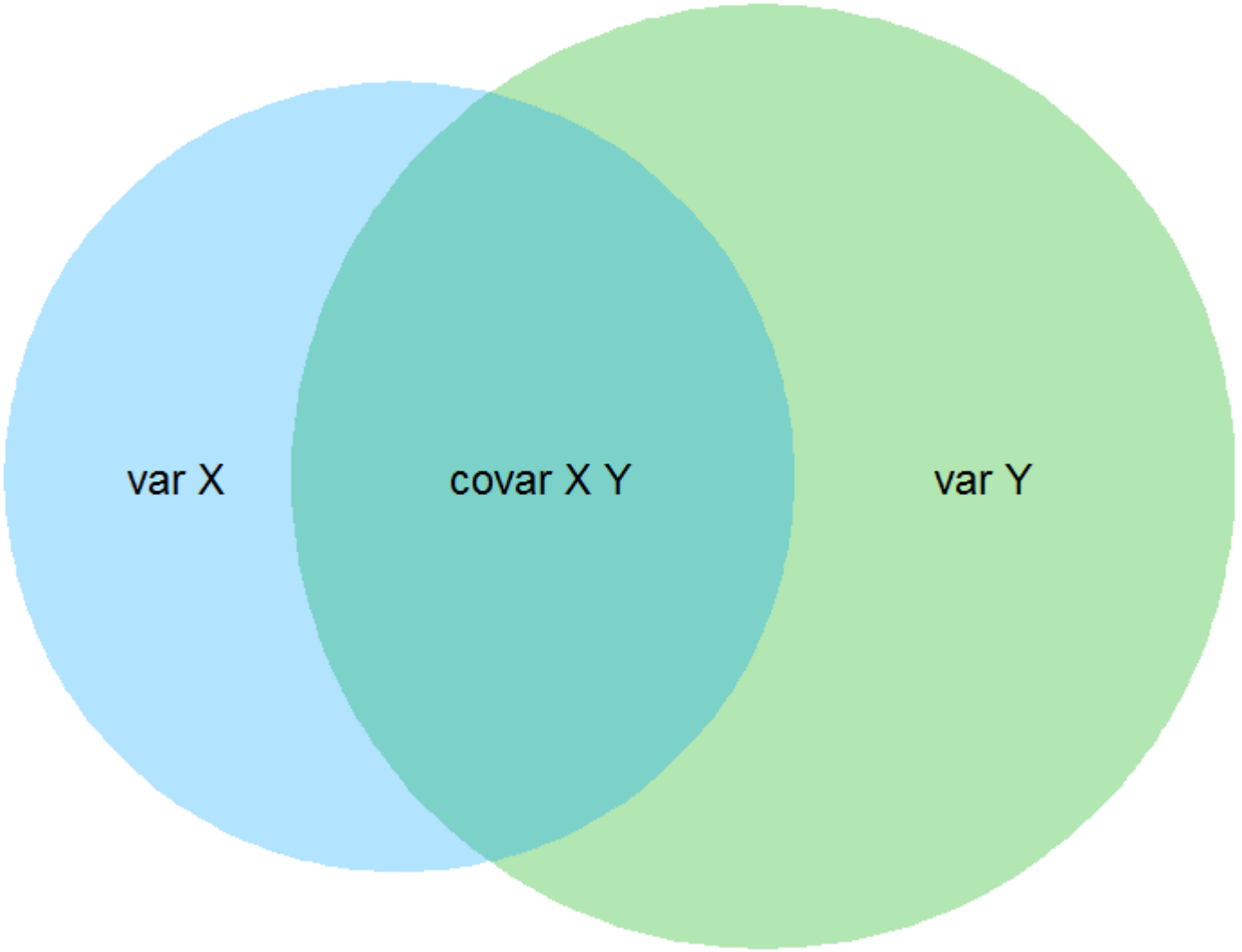
Pearsonův korelační koeficient r

hodnota Pearsonova *r*	interpretace
0.00–0.19	velmi slabá
0.20–0.39	slabá
0.40–0.59	mírná
0.60–0.79	silná
0.80–1.00	velmi silná

Ovšem vždy závislé na kontextu!

Evans 1996

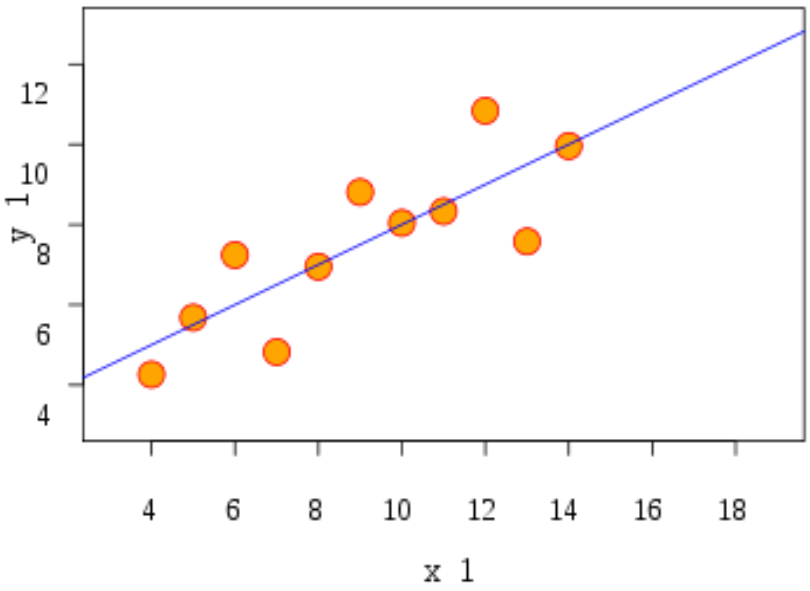
- r = kovariance X, Y / kombinovaný rozptyl X, Y

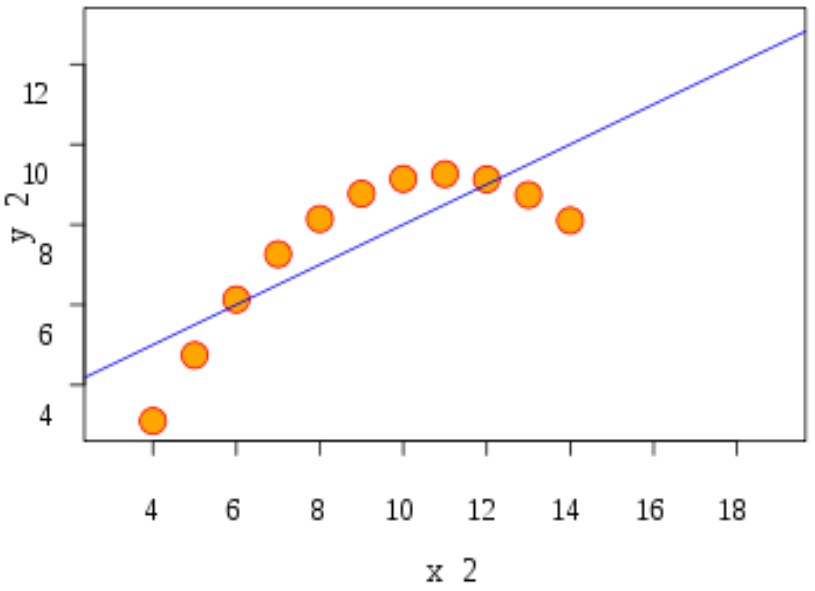


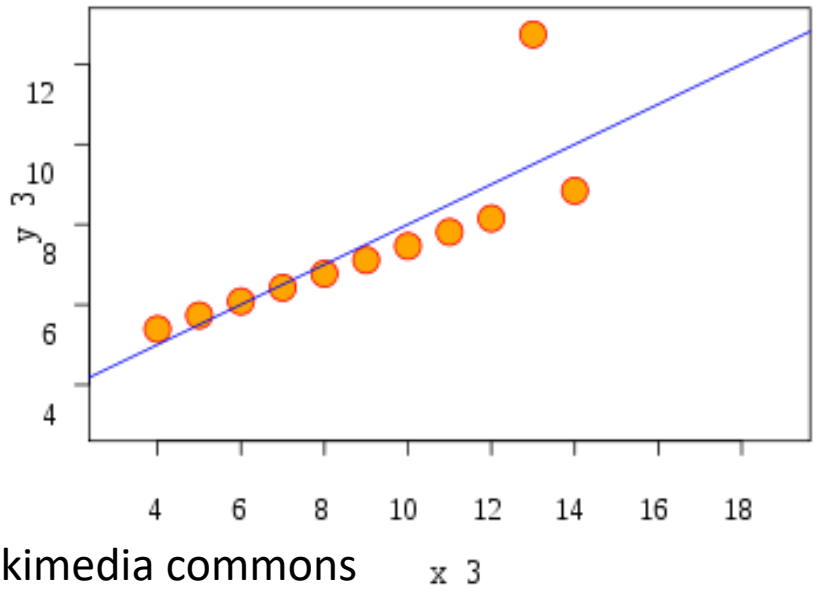
Pearsonovo r: předpoklady a omezení

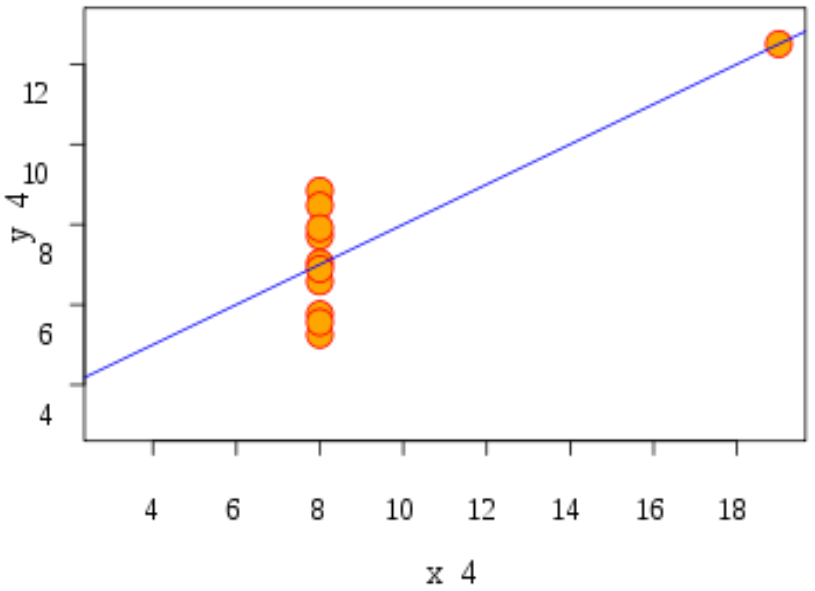
- Metrická úroveň (intervalová nebo poměrová) měření
- **Lineární vztah mezi proměnnými X a Y**
- Homoskedasticita (nezávislost rozptylu)
- Citlivost vůči odlehlým hodnotám (outliers)

Anscombe's quartet









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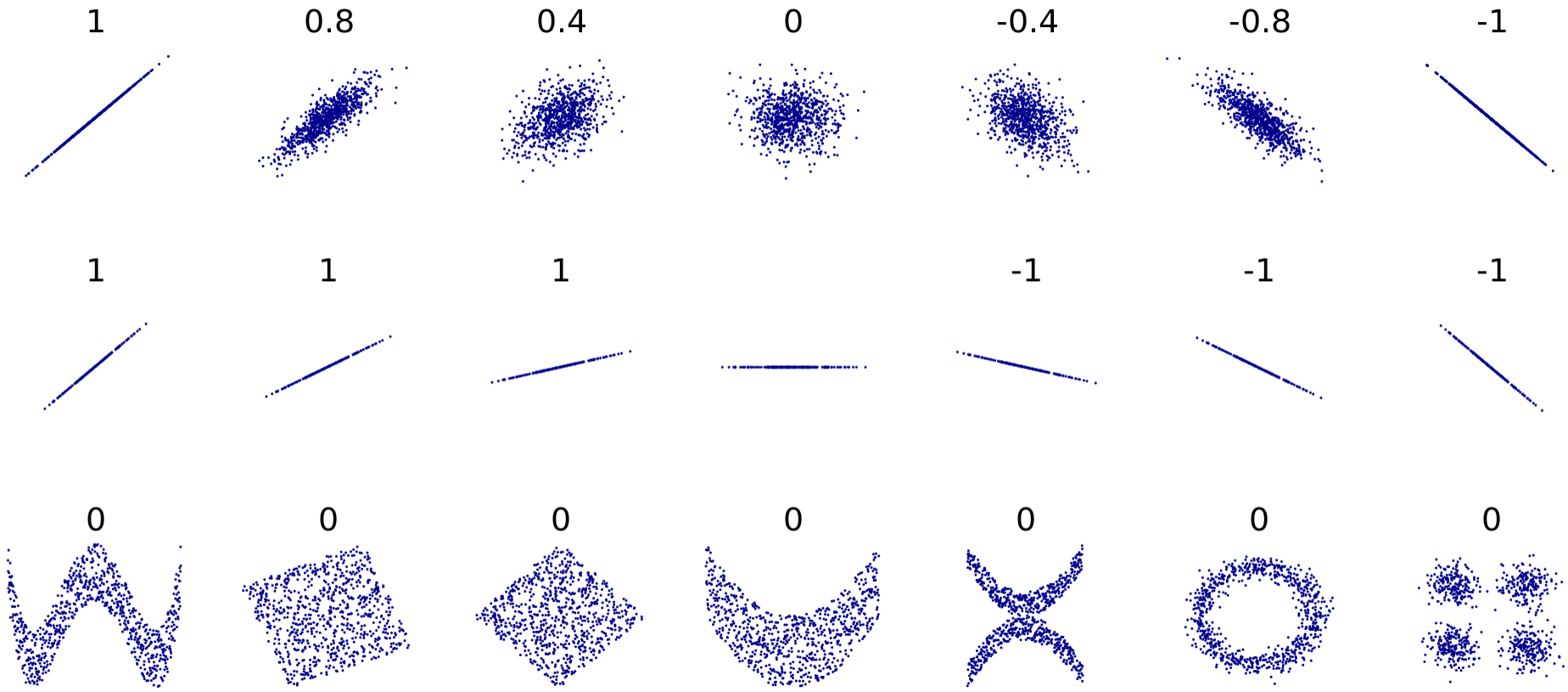
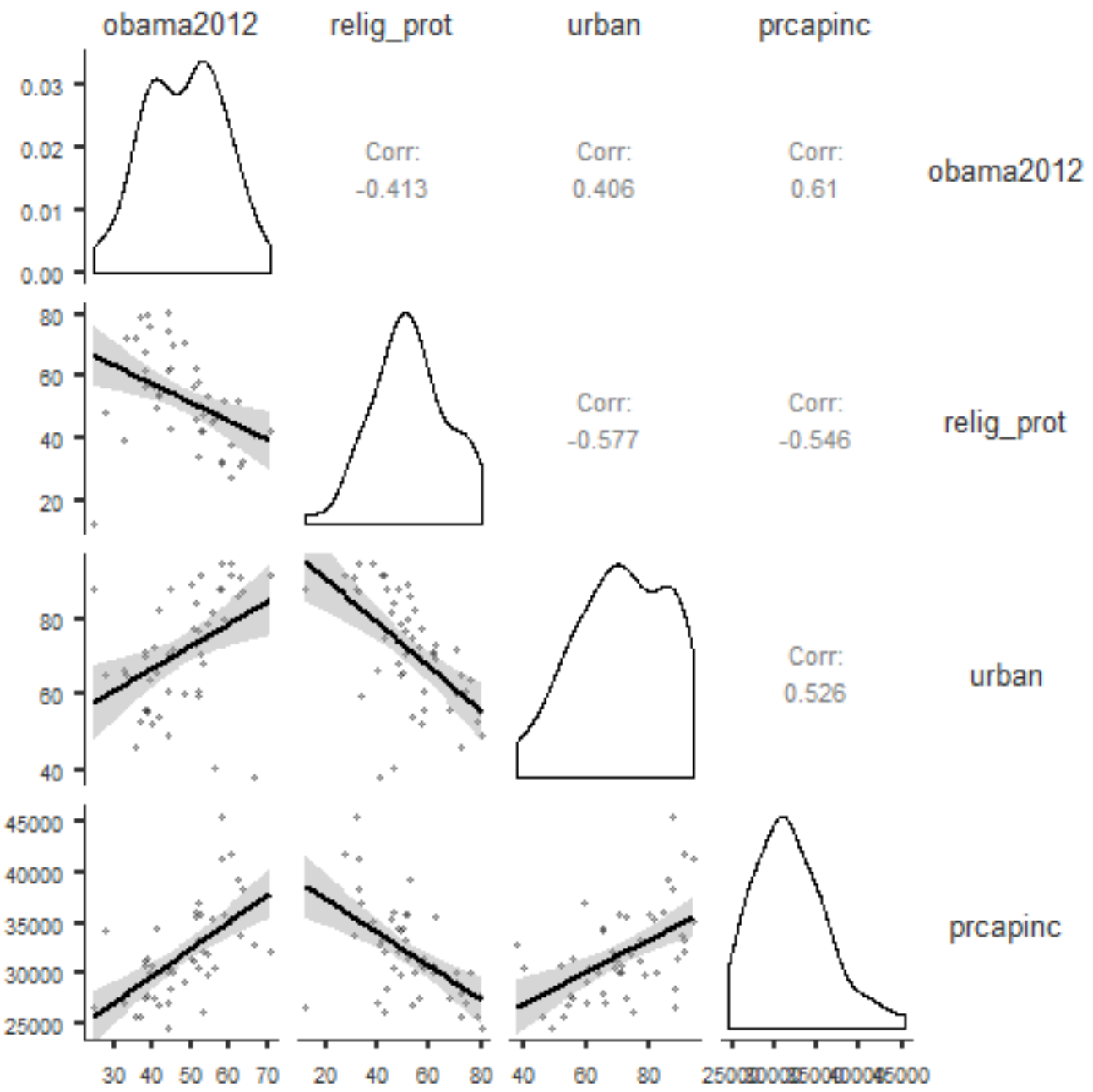


wikimedia commons


obama2012
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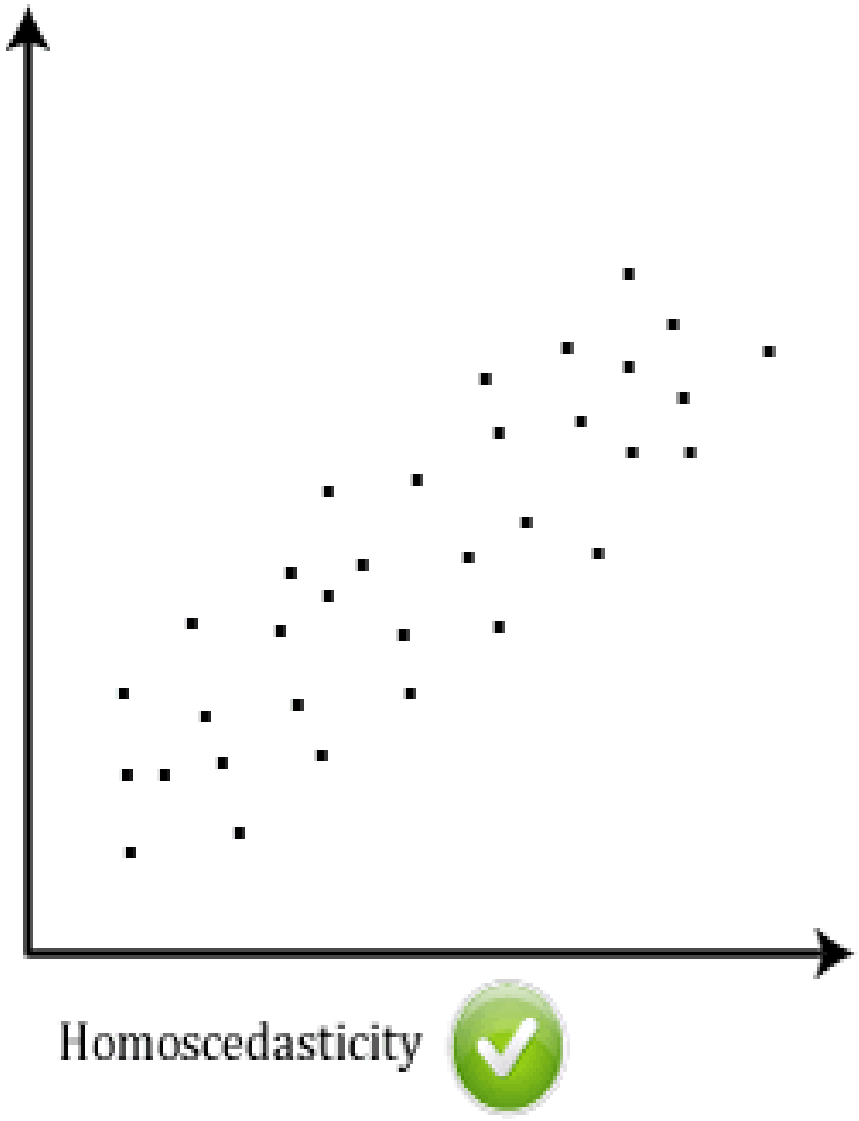
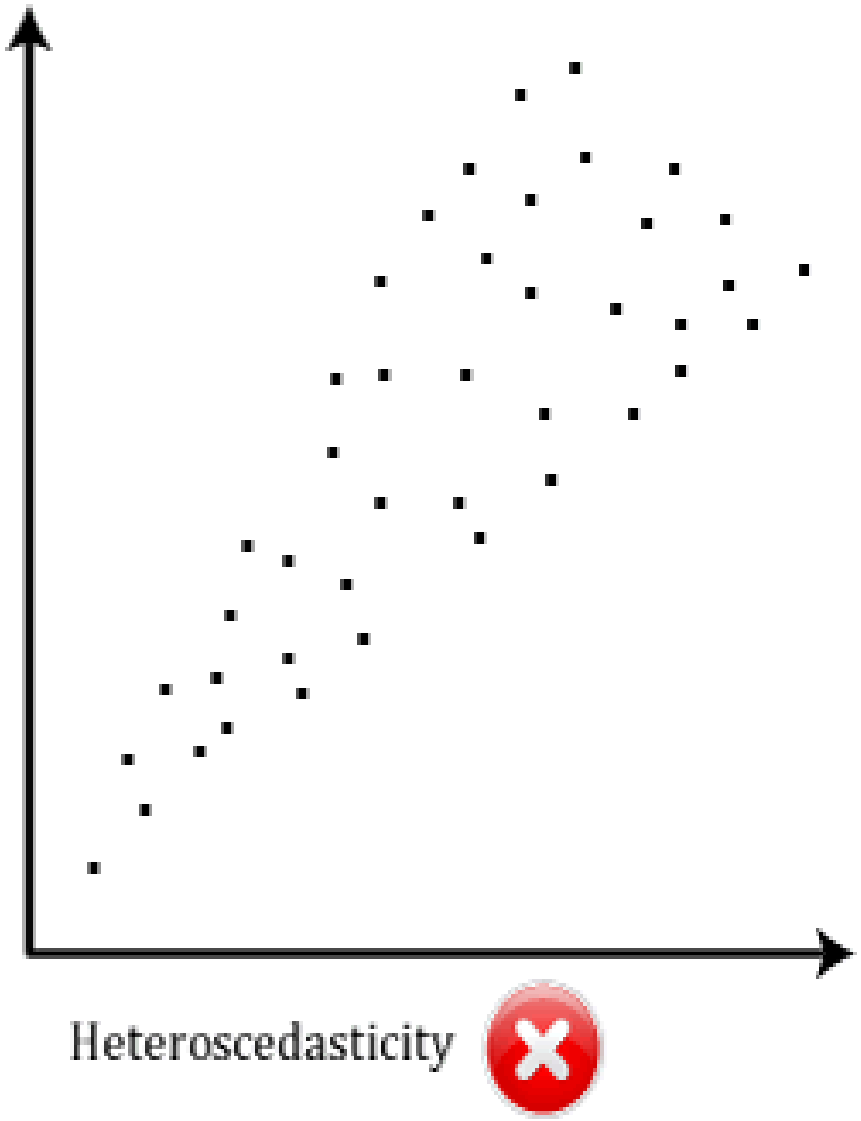
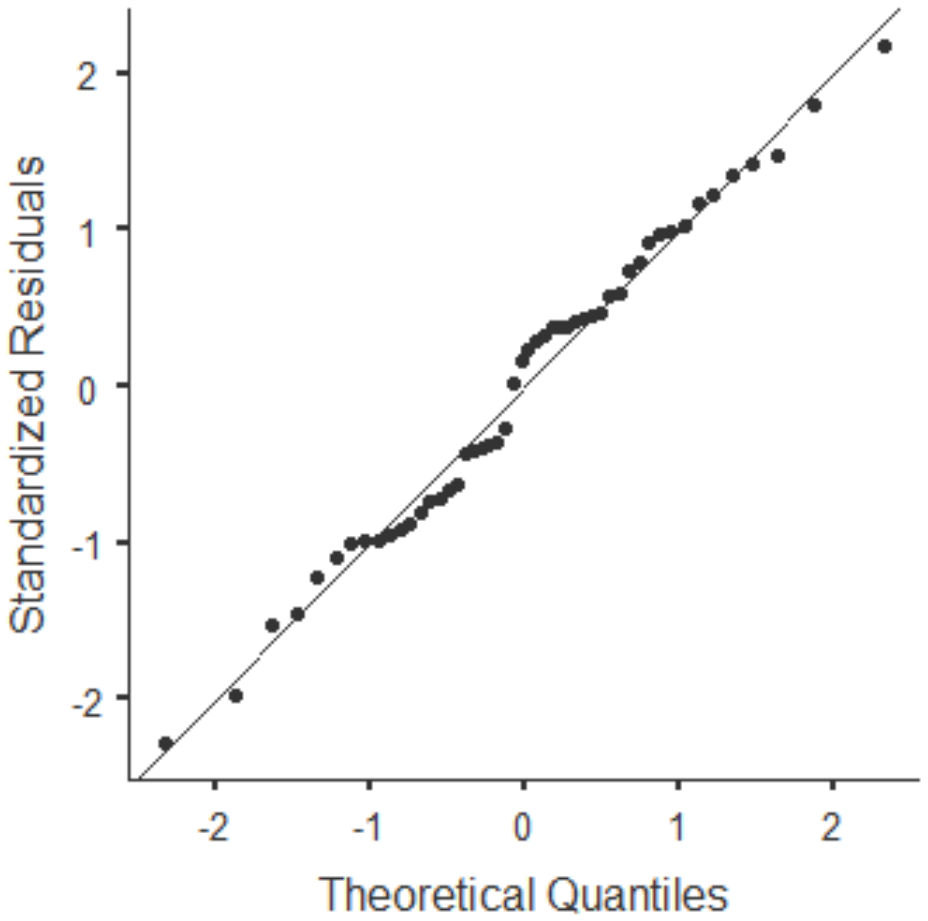
- Citlivost vůči odlehlým hodnotám (outliers)

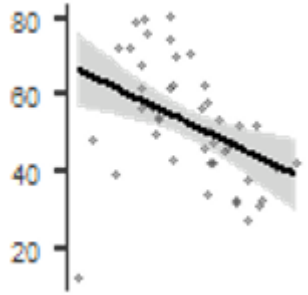




stats.stackexchange.com

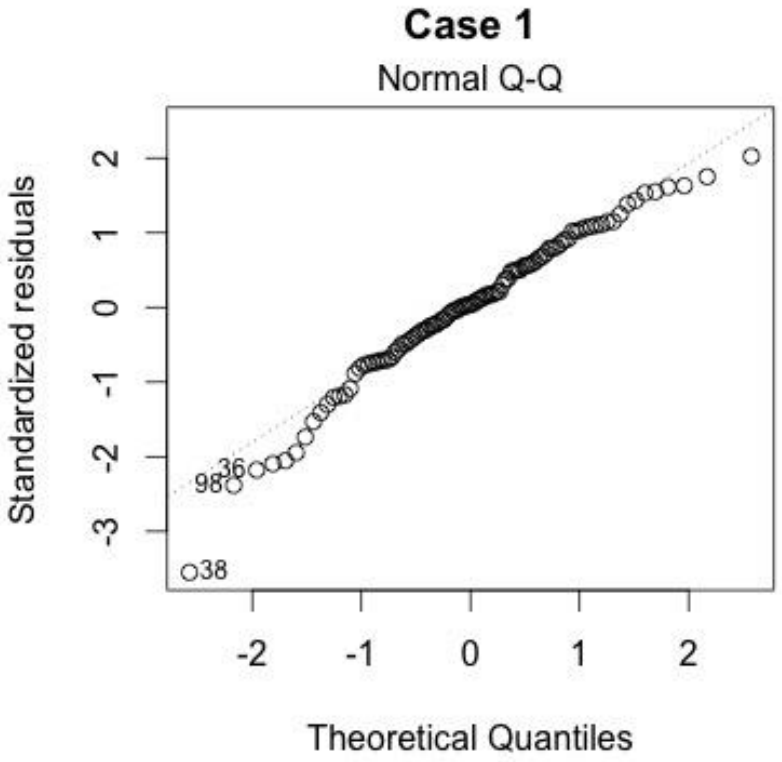
Kvantilový graf (Q-Q plot)

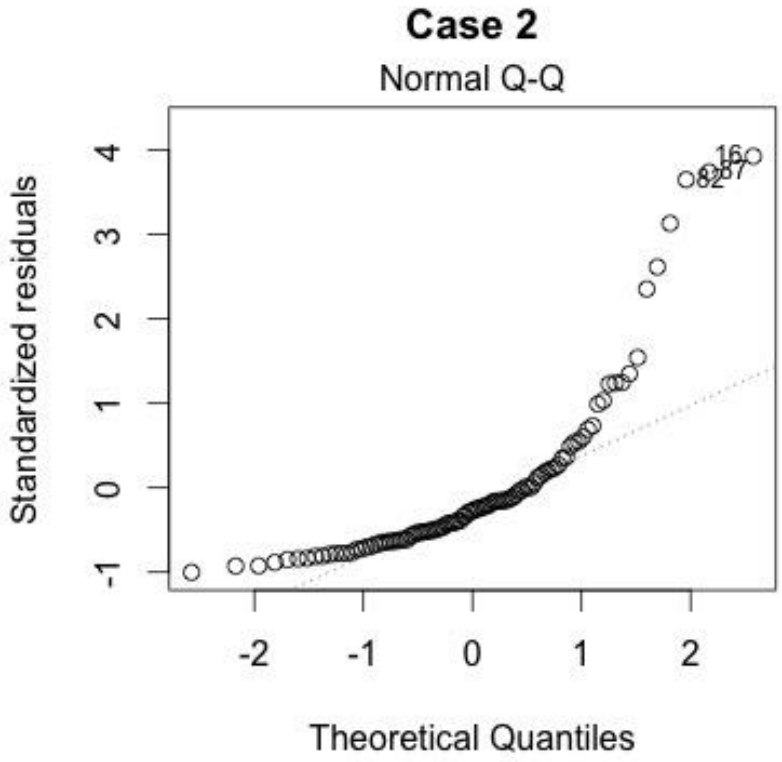




relig_prot x
obama2012

Kvantilový graf





Kim 2019

Pearsonovo r: předpoklady a omezení

- Metrická úroveň (intervalová nebo poměrová) měření
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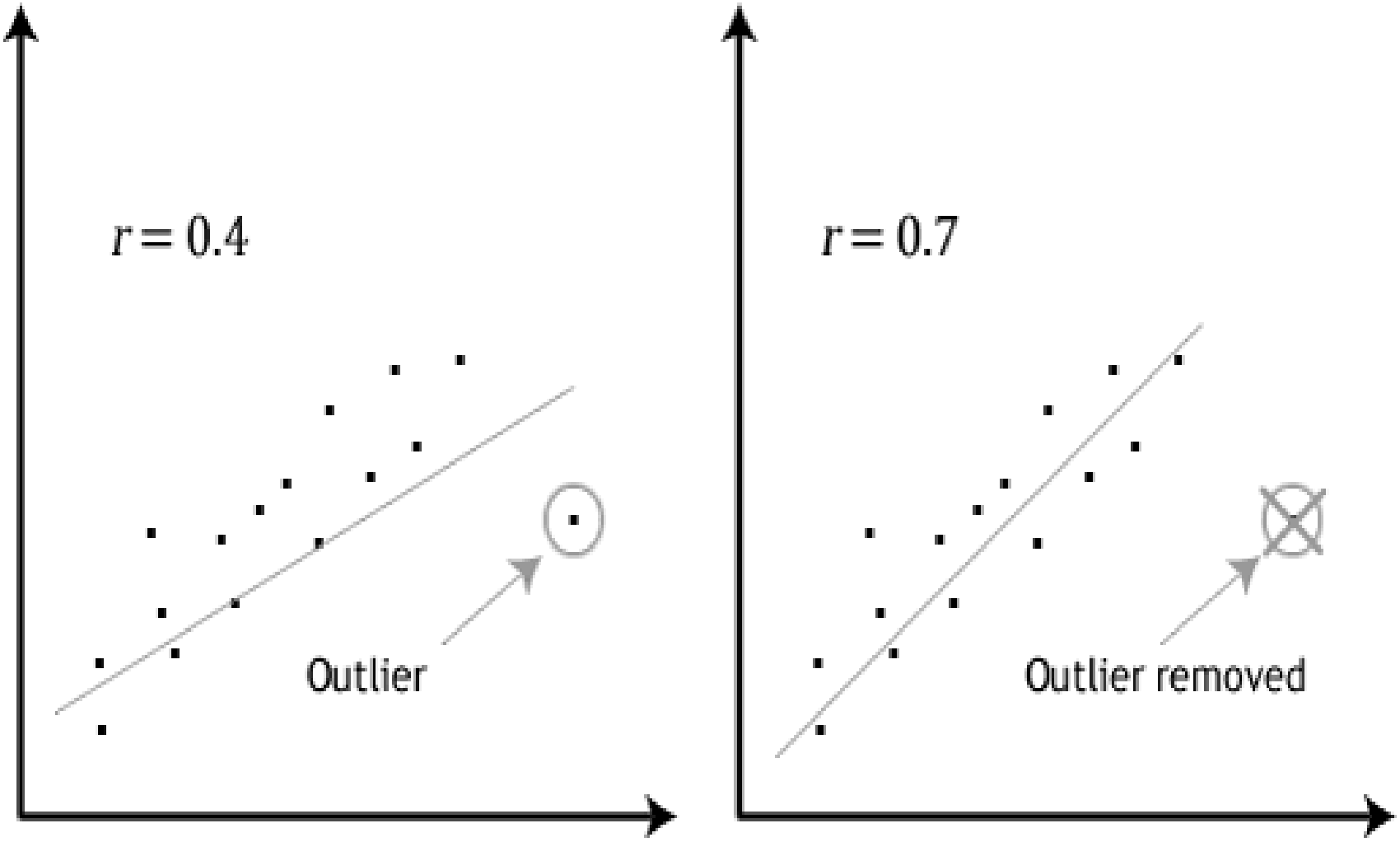
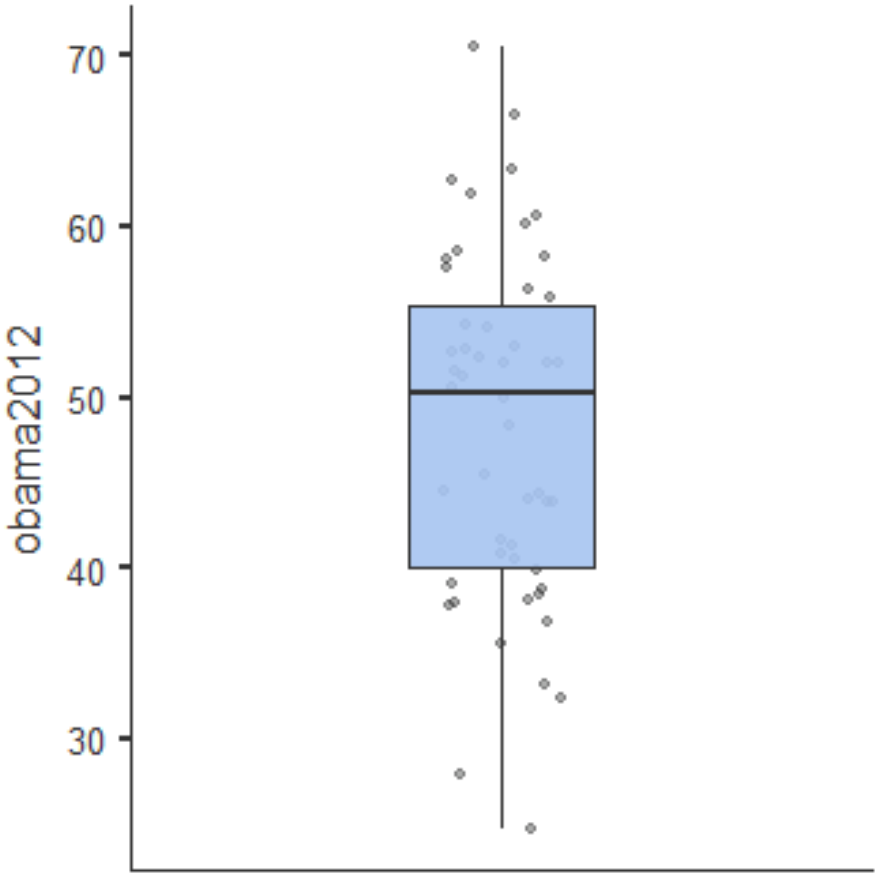
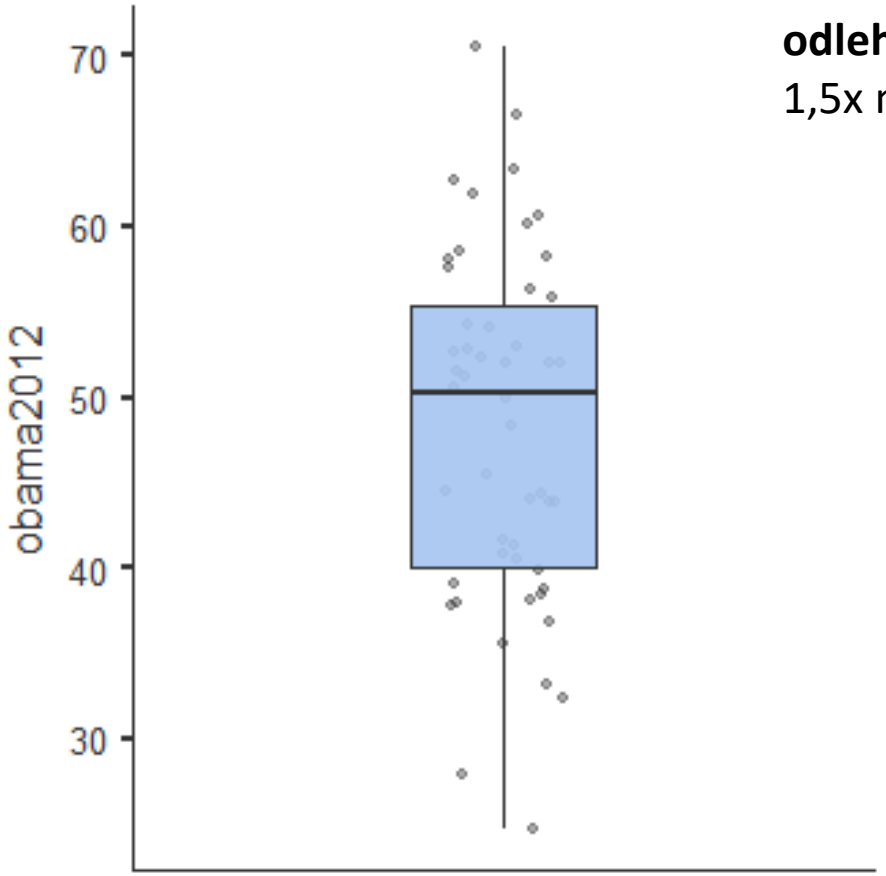

r = 0.4
Outlier
r = 0.7
Outlier removed

Krabicový graf (box plot)

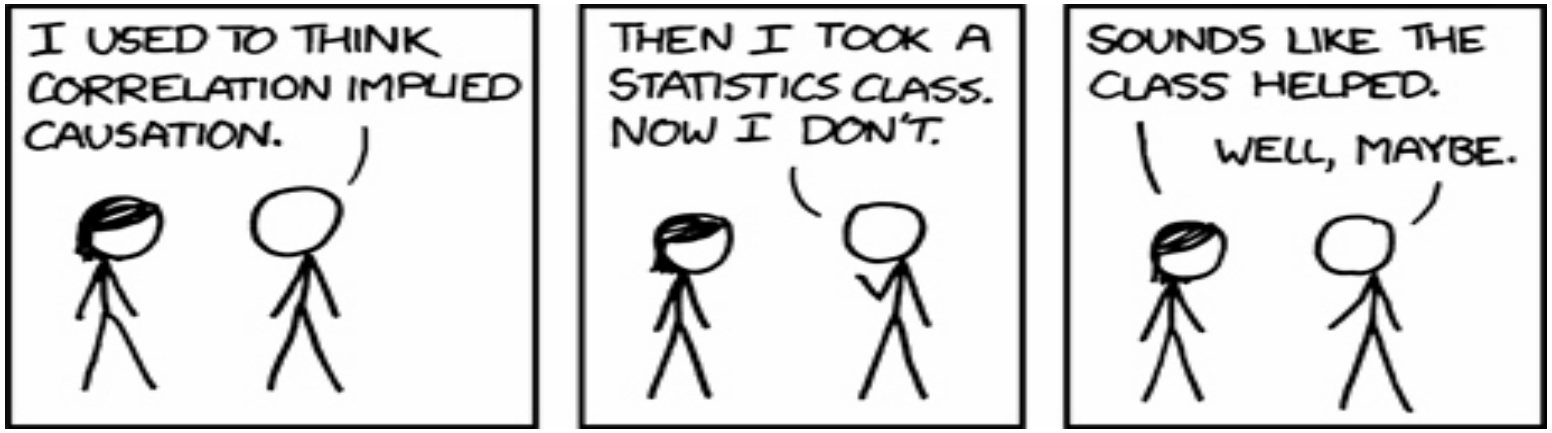


Krabicový graf (box plot)



odlehlá hodnota =
1,5x mezi-kvartilového rozpětí

Korelace a kauzalita



- Korelace je nezbytnou (v SAD) nikoli však postačující podmínkou kauzality (viz Kelldstedt and Whitten 2013)

Obecné vzorce

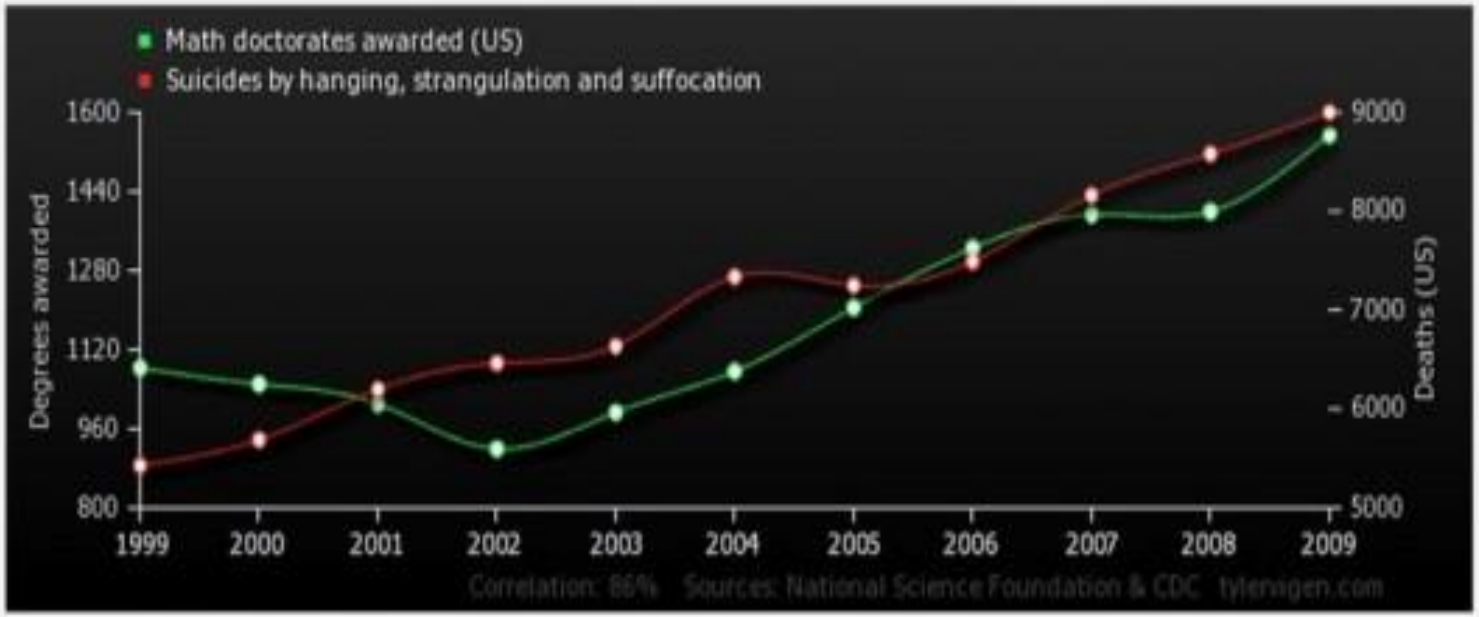
1. X způsobuje Y a Y způsobuje X (**obousměrný vztah**):
   - Vyšší úroveň demokracie (X) zvyšuje ekonomický růst (Y) a vyšší ekonomický růst (Y) zvyšuje úroveň demokracie (X)
2. Y způsobuje X (**obrácený vztah**):
   - Vyšší ekonomický růst (Y) zvyšuje úroveň demokracie (X)
3. Vztah X a Y jako důsledek **společné příčiny**:
   - Korelace mezi vyšším ekonomický růstem (X) a nižší úrovní konfliktu (Y) je způsobena vyšší úrovní demokracie (Z)
4. Mezi X a Y je náhodná závislost (**koincidence**):
   - „Nesmyslné korelace”

- **Korelace sama o sobě nedokáže kontrolovat žádný z těchto efektů**!

Math doctorates awarded (US)

correlates with

Suicides by hanging, strangulation and suffocation



	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Math doctorates awarded (US) *Degrees awarded (National Science Foundation)*	1,083	1,050	1,010	919	993	1,076	1,205	1,325	1,393	1,399	1,554
Suicides by hanging, strangulation and suffocation *Deaths (US) (CDC)*	5,427	5,688	6,198	6,462	6,635	7,336	7,248	7,491	8,161	8,578	9,000

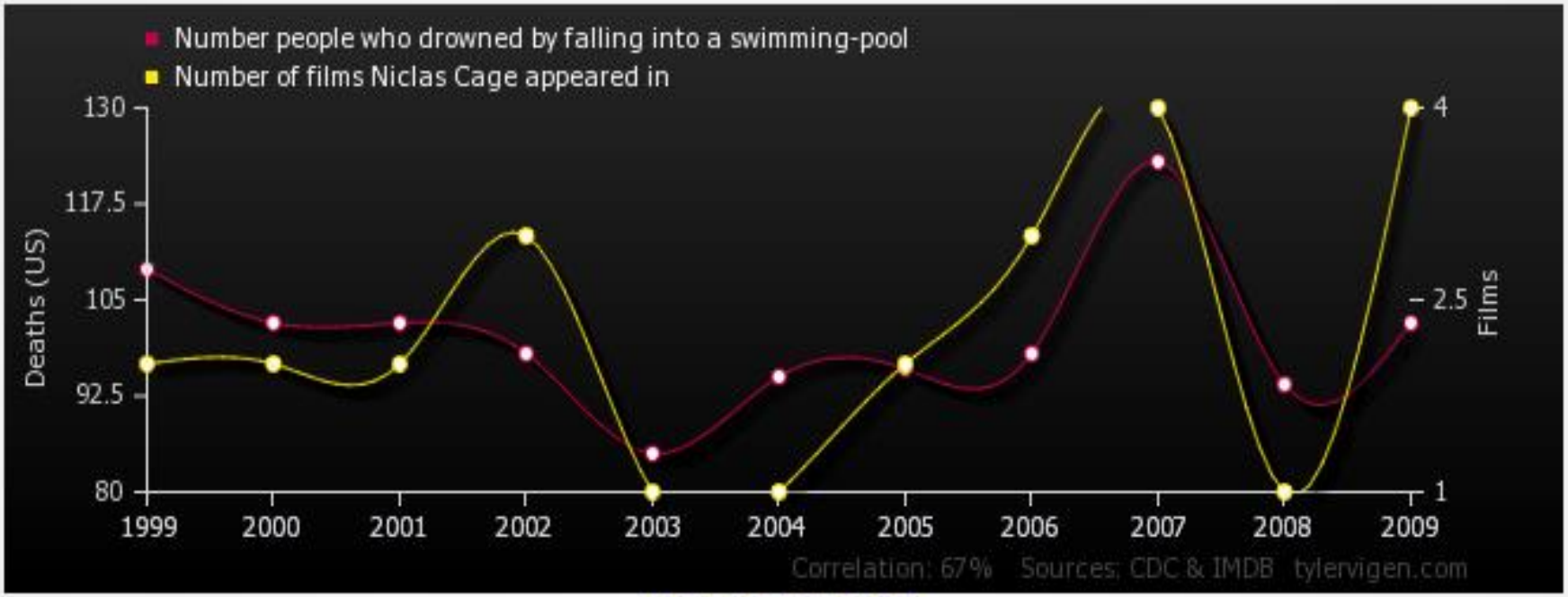
Correlation: 0.860176

Permalink - Not interesting

Number people who drowned by falling into a swimming-pool

correlates with

Number of films Nicolas Cage appeared in

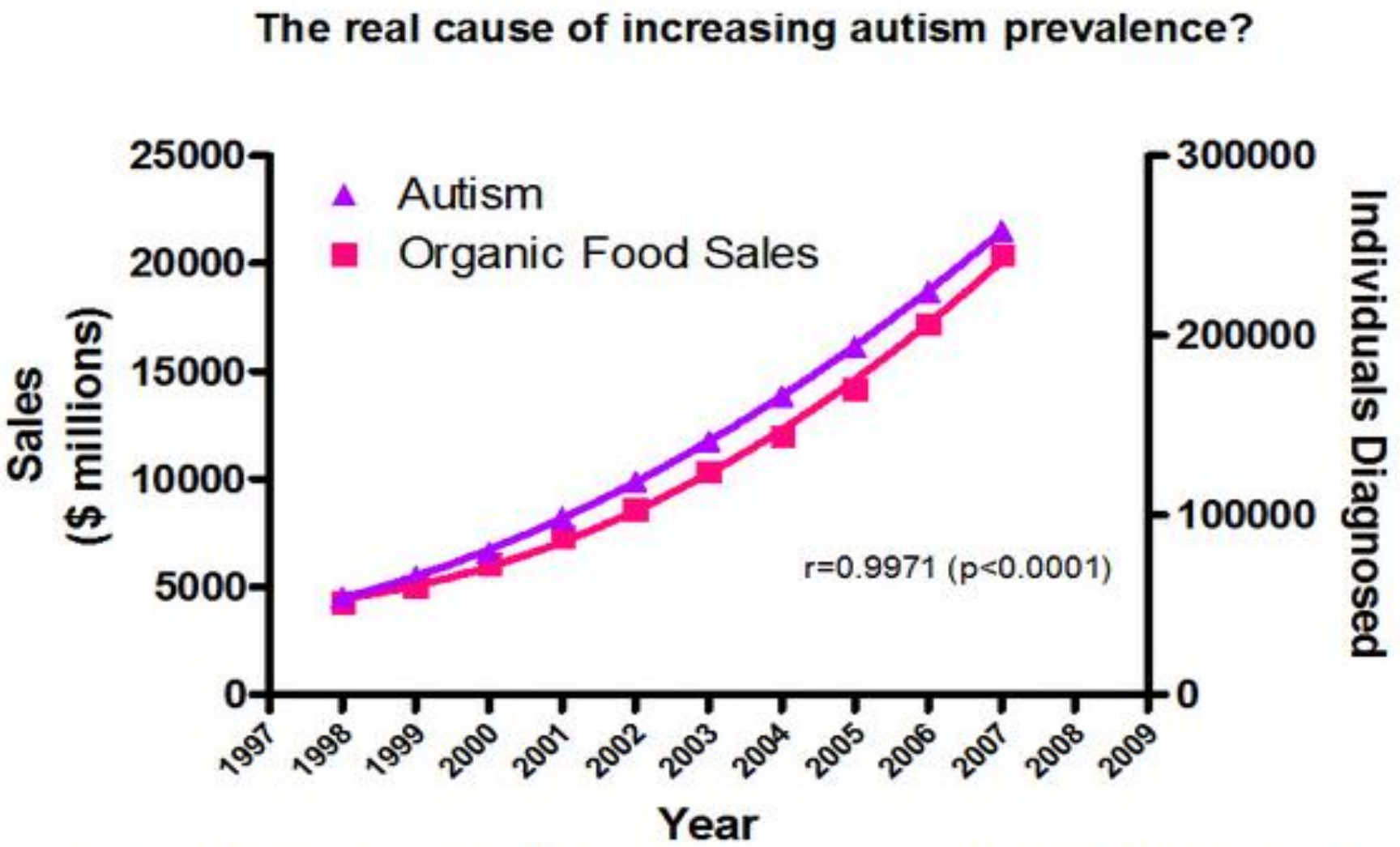


Upload this image to imgur

	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Number people who drowned by falling into a swimming-pool Deaths (US) (CDC)	109	102	102	98	85	95	96	98	123	94	102
Number of films Nicolas Cage appeared in Films (IMDB)	2	2	2	3	1	1	2	3	4	1	4

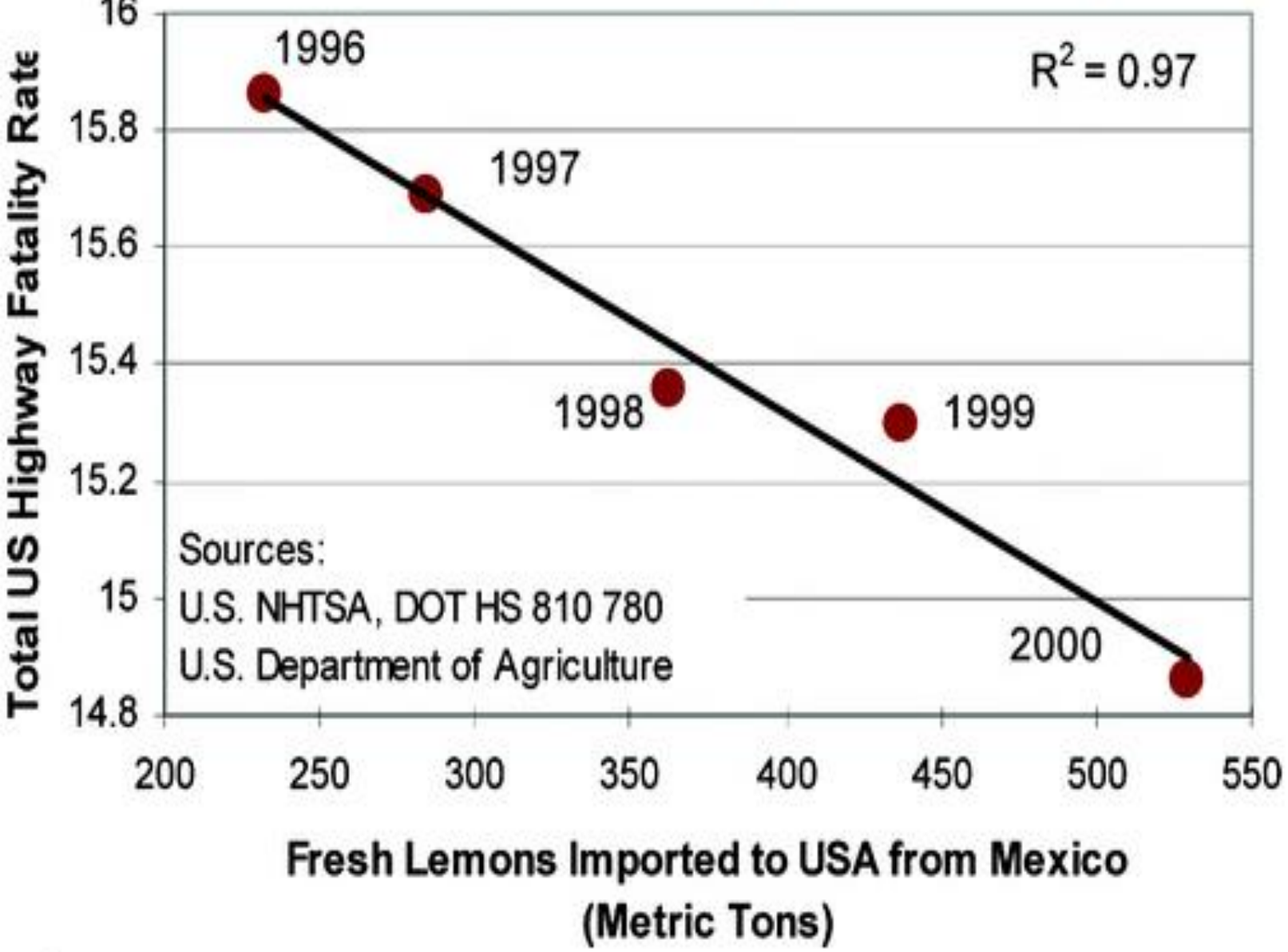
Correlation: 0.666004

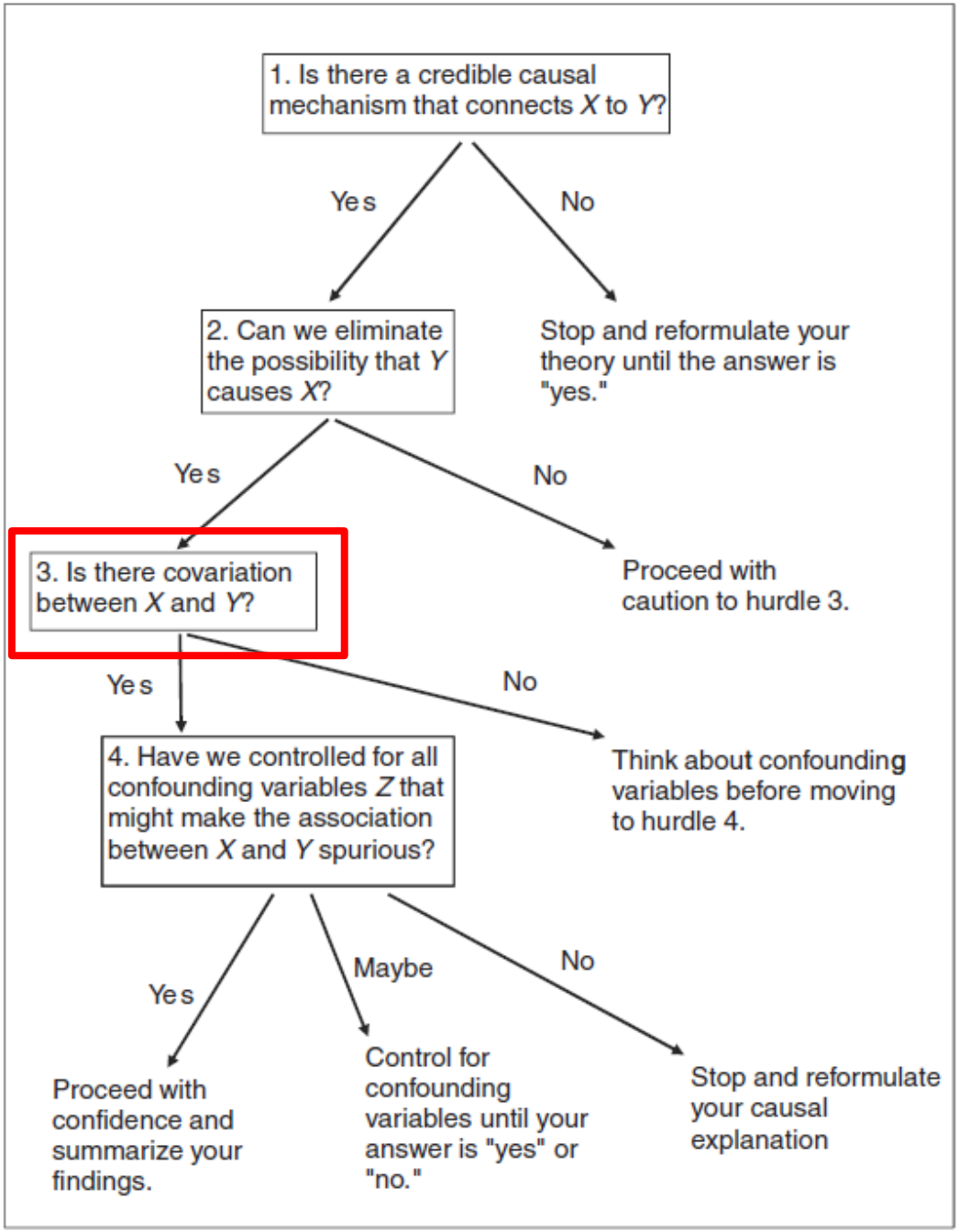
4. Eating organic food causes autism.



Sources: Organic Trade Association, 2011 Organic Industry Survey; U.S. Department of Education, Office of Special Education Programs, Data Analysis System (DANS), OMB# 1820-0043: "Children with Disabilities Receiving Special Education Under Part B of the Individuals with Disabilities Education Act

7. Mexican lemon imports prevent highway deaths.





Kellstedt & Whitten 2013: 58

Test hypotézy: Pearsonovo r

- H0: Mezi proměnnými X a Y *neexistuje* vztah lineární závislosti (korelace *r*)
- HA: Mezi proměnnými X a Y *existuje* vztah lineární závislosti (korelace *r*)

- H0: $r = 0$; $r >= 0$; $r <= 0$
- HA: $r \neq 0$ (oboustranná hypotéza)
- HA: $r > 0$ (jednostranná hypotéza, pozitivní korelace)
- HA: $r < 0$ (jednostranná hypotéza, negativní korelace)

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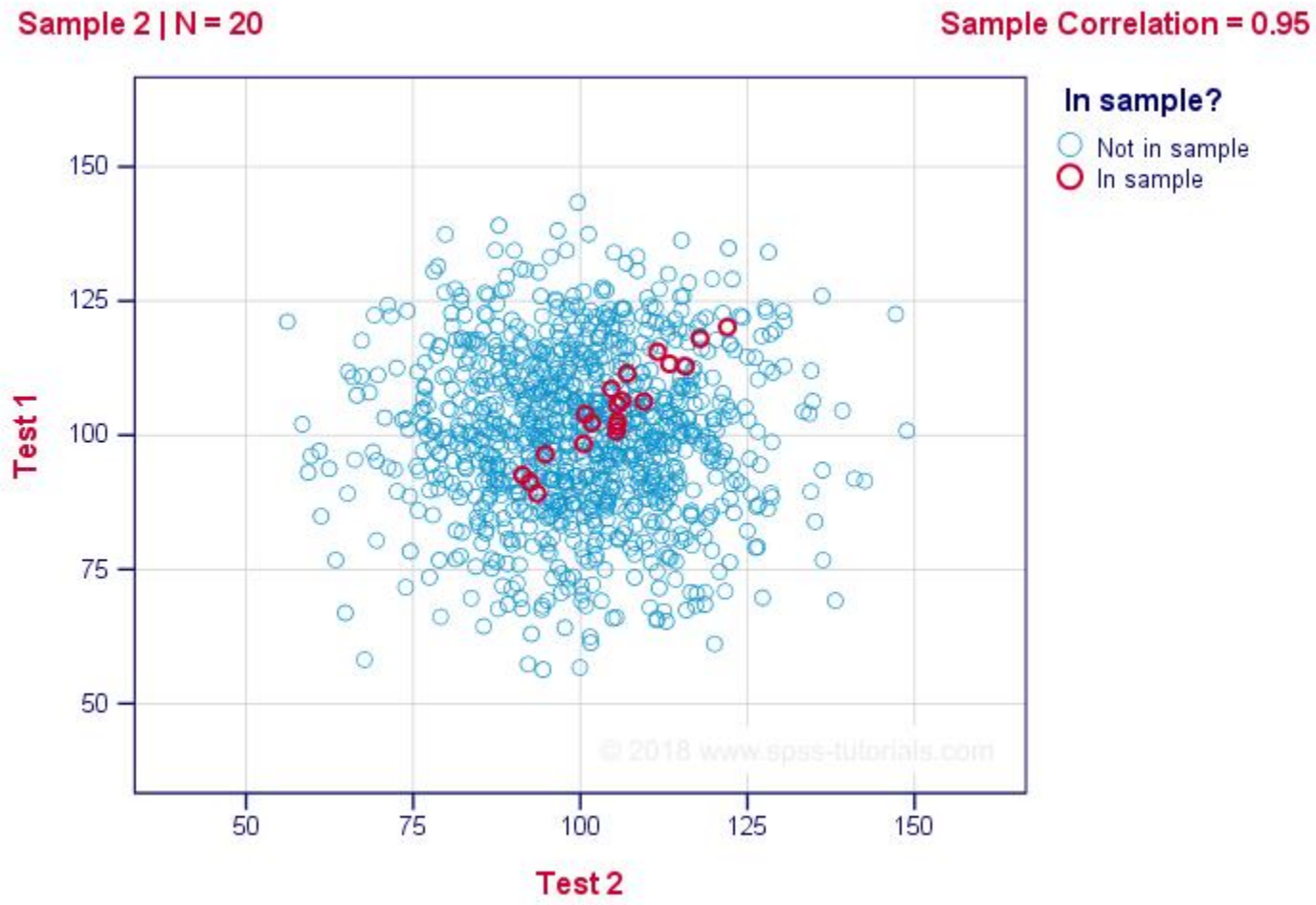
H0 vždy zahrnuje nulu; v literatuře se pro jednostranné testy objevují dvě varianty zápisu: se znaménkem nerovnosti, nebo bez něj

Test hypotézy: Pearsonovo r

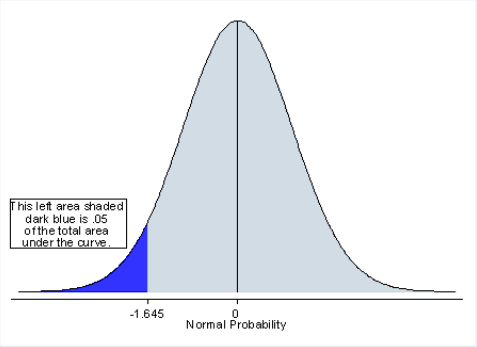


SPSS tutorial 2018

Test hypotézy: Pearsonovo r

- **Teoretický rámec:** volební chování je do určité míry určeno sociokulturními štěpnými liniemi (Norris & Inglehart 2019; Lipset & Rokkan 1967)

- **H0:** Mezi volebním výsledkem Obamy v roce 2012 (X) a podílem protestantů (Y) *není* korelace; $r(x, y) >= 0$
- **HA:** Mezi volebním výsledkem Obamy v roce 2012 (X) a podílem protestantů (Y) *je negativní* korelace; $r(x, y) < 0$



Test hypotézy: Pearsonovo r

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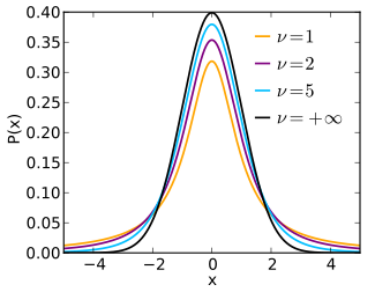
- **HA:** Mezi volebním výsledkem Obamy v roce 2012 (X) a podílem protestantů (Y) *je negativní* korelace; $r(x, y) < 0$

- **Data:** 50 pozorování (státy USA)

- Stanovení **stupňů volnosti** pro *r*: **n - 2**, tj. 50 - 2 = **48**

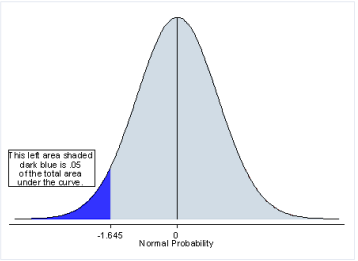
- Stanovení **hladiny testu** = 0.05 (5 %) a kritické hodnoty ***t*** pro jednostrannou hypotézu (0.05, 48) = -**1.677**



Appendix: Critical Values Tables

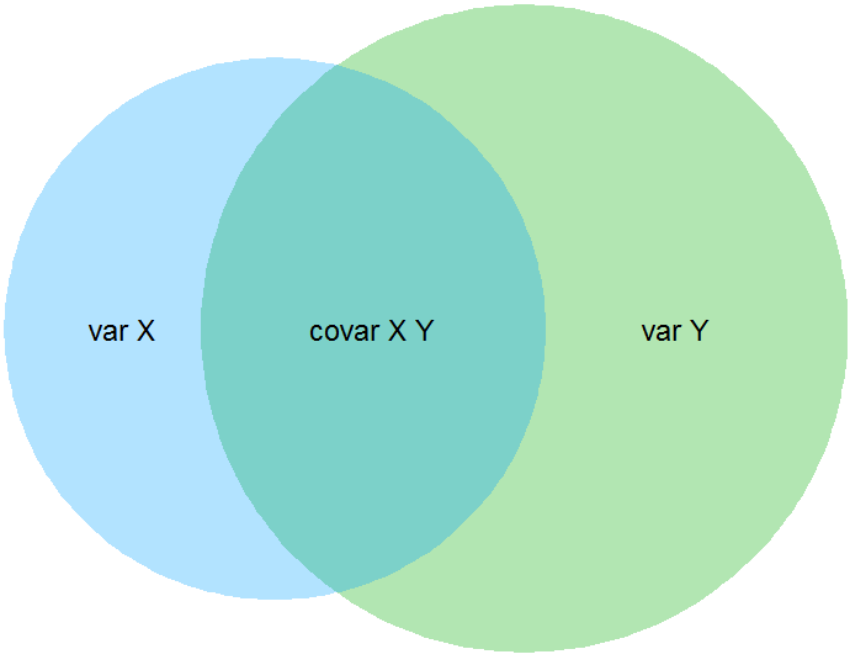
Degrees of Freedom (*df*)	80%	90%	95%	98%	99%
41	1.303	1.683	2.020	2.421	2.701
42	1.302	1.682	2.018	2.418	2.698
43	1.302	1.681	2.017	2.416	2.695
44	1.301	1.680	2.015	2.414	2.692
45	1.301	1.679	2.014	2.412	2.690
46	1.300	1.679	2.013	2.410	2.687
47	1.300	1.678	2.012	2.408	2.685
48	1.299	1.677	2.011	2.407	2.682
49	1.299	1.677	2.010	2.405	2.680
50	1.299	1.676	2.009	2.403	2.678
51	1.298	1.675	2.008	2.402	2.676
52	1.298	1.675	2.007	2.400	2.674
53	1.298	1.674	2.006	2.399	2.672
54	1.297	1.674	2.005	2.397	2.670
55	1.297	1.673	2.004	2.396	2.668
56	1.297	1.673	2.003	2.395	2.667
57	1.297	1.672	2.002	2.394	2.665
58	1.296	1.672	2.002	2.392	2.663
59	1.296	1.671	2.001	2.391	2.662
60	1.296	1.671	2.000	2.390	2.660
61	1.296	1.670	2.000	2.389	2.659
62	1.295	1.670	1.999	2.388	2.657
63	1.295	1.669	1.998	2.387	2.656
64	1.295	1.669	1.998	2.386	2.655
65	1.295	1.669	1.997	2.385	2.654
66	1.295	1.668	1.997	2.384	2.652
67	1.294	1.668	1.996	2.383	2.651
68	1.294	1.668	1.995	2.382	2.650
74	1.293	1.666	1.993	2.378	2.644
75	1.293	1.665	1.992	2.377	2.643
76	1.293	1.665	1.992	2.376	2.642
77	1.293	1.665	1.991	2.376	2.641
78	1.292	1.665	1.991	2.375	2.640
79	1.292	1.664	1.990	2.374	2.640
80	1.292	1.664	1.990	2.374	2.639
81	1.292	1.664	1.990	2.373	2.638
82	1.292	1.664	1.989	2.373	2.637
83	1.292	1.663	1.989	2.372	2.636

90% hodnota pro oboustranný test, tj. 95% pro jednostranný



V tabulce jsou pouze pozitivní hodnoty ***t***. Protože *t* rozdělení je symetrické, je nadbytečné uvádět tutéž hodnotu pro negativní a pozitivní ***t***. Pro HA: r < 0 tedy ***t*** předřadíme znaménko minus.

- r = kovariance X, Y / kombinovaný rozptyl X, Y



$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \times \sum (y_i - \bar{y})^2}}$$

- Korelace mezi Obama2012 a relig_prot: r = **-0.413**

Testovací statistika *t*

- Abychom zjistili, zda se *r* významně liší od nuly, užijeme **t-test** pro korelační koeficient *r***.**
- *Je hodnota r významně odlišná od předpokládaného populačního průměru 0* (H0: r >= 0)*?*

- $t = \frac{signal}{noise}$; $t = \frac{r * \sqrt{n-2}}{\sqrt{1-r^2}}$; $\mathrm{n} = \text{velikost vzorku}$

- $t = \frac{-0.413 * \sqrt{50-2}}{\sqrt{1-(-0.413)^2}} = \mathbf{-3.14}$

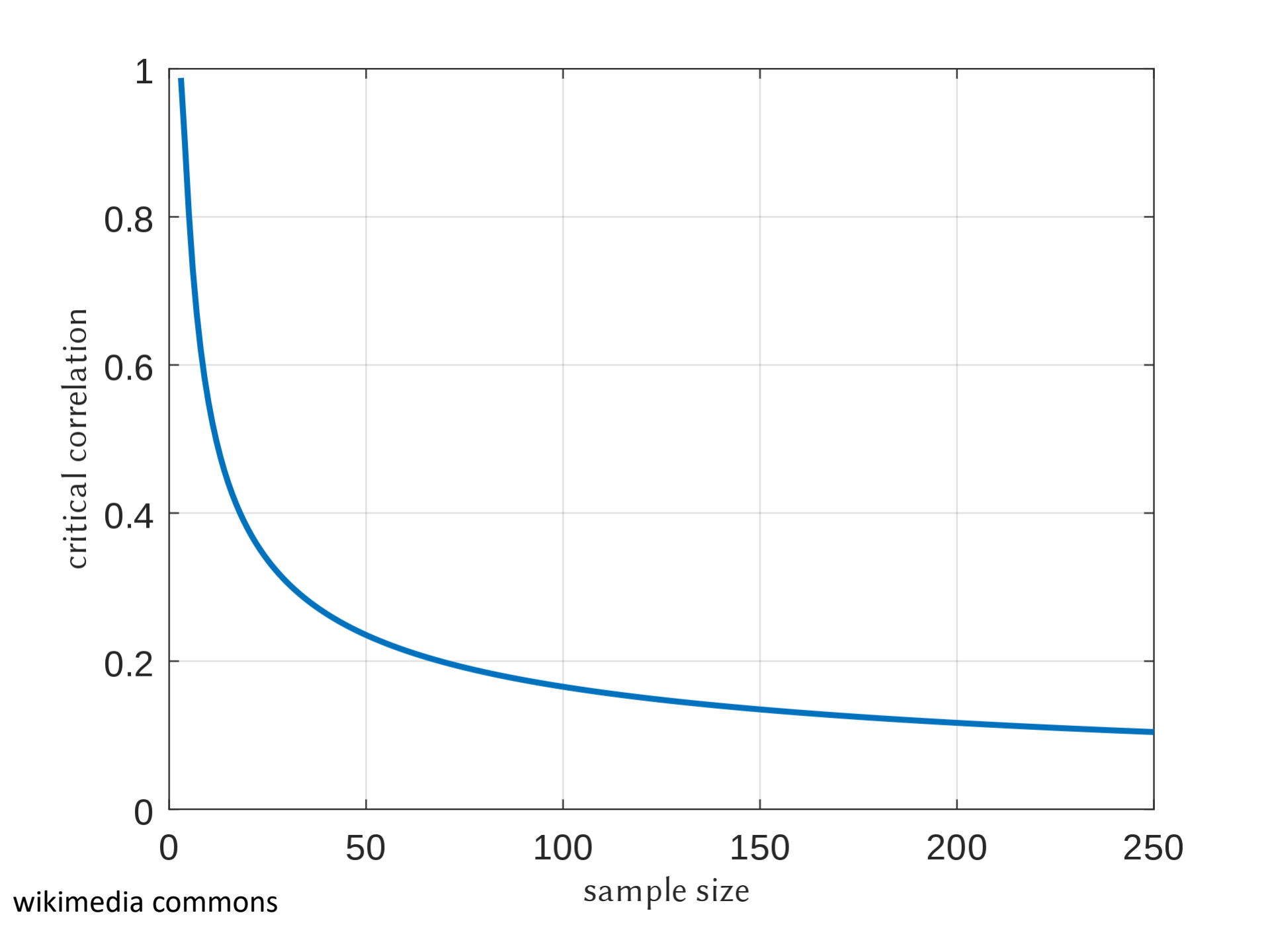
- **t-hodnota** Pearsonova *r* je **testovací statistikou**

Rozhodnutí o H0

- **H0:** Mezi volebním výsledkem Obamy v roce 2012 (X) a podílem protestantů (Y) *není* korelace; $r(x, y) >= 0$
- **HA:** Mezi volebním výsledkem Obamy v roce 2012 (X) a podílem protestantů (Y) *je negativní* korelace; $r(x, y) < 0$

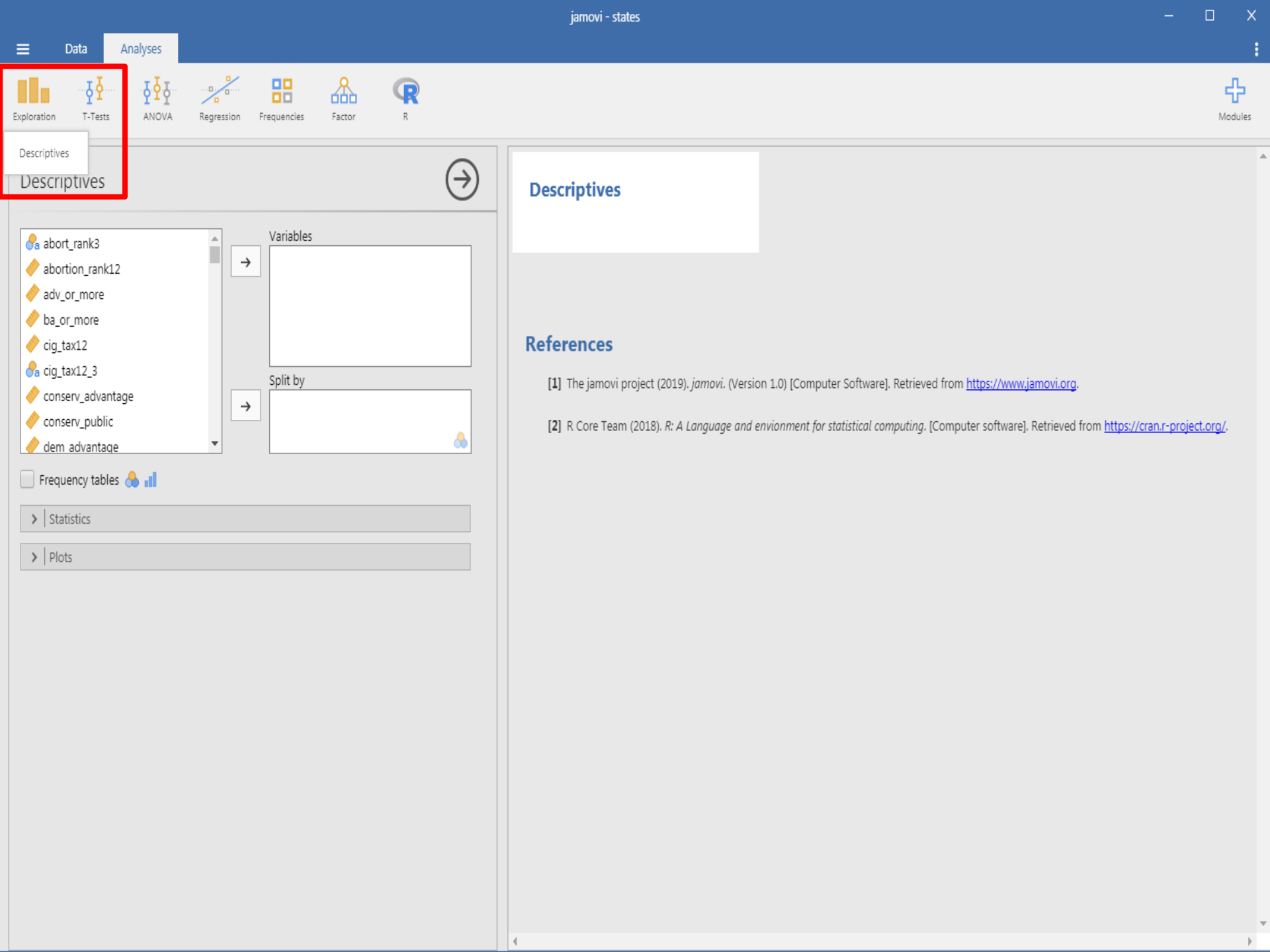
- Pearsonovo ***r*** = **-0.413**
- Testovací statistika ***t*** **=** **-3.14;** kritická hodnota ***t*** (α **=** 0.05) = **-1.677**
- Protože ***t*** = **-3.14** **< (*α* = 0.05, 48)** = **-1.677**, **odmítáme H0:** r >= 0 a přijímáme **HA:** $r < 0$.
- **p-hodnota** = 0.003 (tj. 0.3 %) značí pravděpodobnost pozorování dané, či ještě extrémnější, hodnoty **testovací statistiky** (***t*** = **-3.14**) při **platnosti H0**.

- Tj. mezi volebním výsledkem Obamy (X) a podílem protestantů (Y) je, na 5% hladině st. významnosti, lineární závislost (korelace *r*).



wikimedia commons

Pearsonovo r v Jamovi


jamovi - states
Data
Analyses
Exploration
T-Tests
ANOVA
Regression
Frequencies
Factor
R
Modules
Descriptives
Descriptives
abort_rank3
abortion_rank12
adv_or_more
ba_or_more
cig_tax12
cig_tax12_3
conserv_advantage
conserv_public
dem_advantage
Variables
Split by
Frequency tables
Statistics
Plots
Descriptives
References
[1] The jamovi project (2019). jamovi. (Version 1.0) [Computer Software]. Retrieved from https://www.jamovi.org.
[2] R Core Team (2018). R: A Language and envionment for statistical computing. [Computer software]. Retrieved from https://cran.r-project.org/.

Descriptives

popchng0010
popchngpct
pot_policy
prochoice
prolife
relig_cath
relig_high
relig_low
religiosity3

Variables
obama2012
relig_prot

Split by

Frequency tables

Statistics

Sample Size
N Missing

Percentile Values
Quartiles
Cut points for 4 equal groups

Central Tendency
Mean
Median
Mode
Sum

Dispersion
Std. deviation Minimum
Variance Maximum
Range S. E. Mean

Distribution
Skewness
Kurtosis

Normality
Shapiro-Wilk

Plots

Descriptives

Descriptives

	obama2012	relig_prot
N	50	50
Missing	0	0
Mean	48.2	52.4
Median	50.2	51.6
Standard deviation	10.3	14.8
Minimum	24.7	12.3
Maximum	70.5	80.3
25th percentile	40.0	42.6
50th percentile	50.2	51.6
75th percentile	55.4	61.7

References

[1] The jamovi project (2019). *jamovi*. (Version 1.0) [Computer Software]. Retrieved from https://www.jamovi.org.

[2] R Core Team (2018). *R: A Language and envionment for statistical computing*. [Computer software]. Retrieved from https://cran.r-project.org/.

jamovi - states

Data | Analyses

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Mean, Median, Mode, Sum

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Plots

Histograms
Histogram, Density

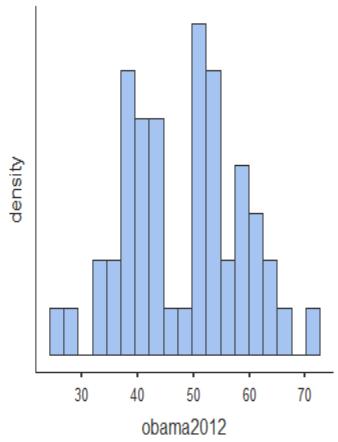
Box Plots
Box plot, Violin, Data
Jittered

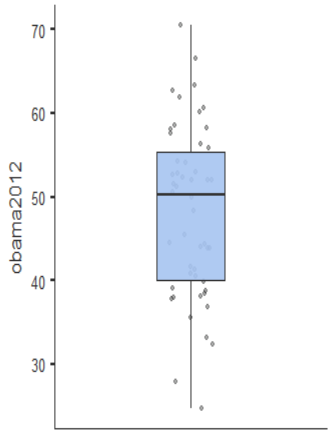
Bar Plots
Bar plot

Q-Q Plots
Q-Q

Plots

obama2012





relig_prot

jamovi - states

Data | Analyses

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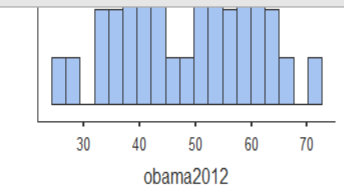
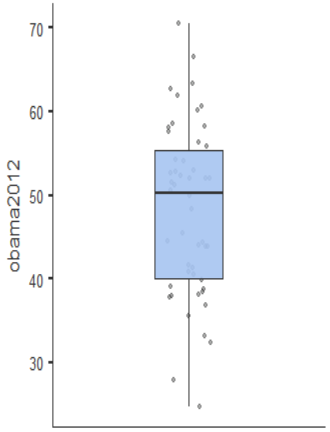
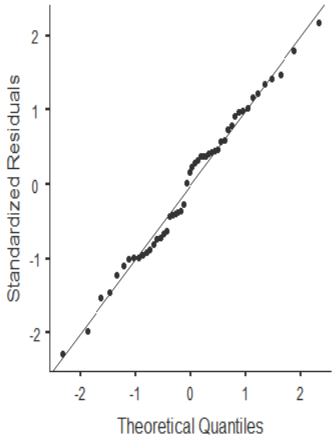
Bar Plots
Bar plot

Q-Q Plots
Q-Q







jamovi - states

Data | Analyses

Exploration | T-Tests | ANOVA | Regression | Frequencies | Factor | R | Modules

Correlation Matrix
Linear Regression
Logistic Regression
2 Outcomes — Binomial
N Outcomes — Multinomial
Ordinal Outcomes

Descriptives

popchng0010
popchngpct
pot_policy
prochoice
prolife
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relig_low
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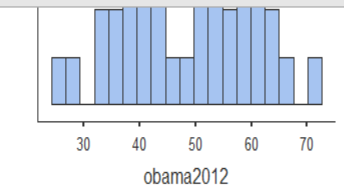
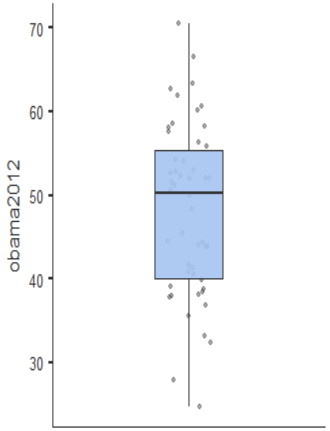
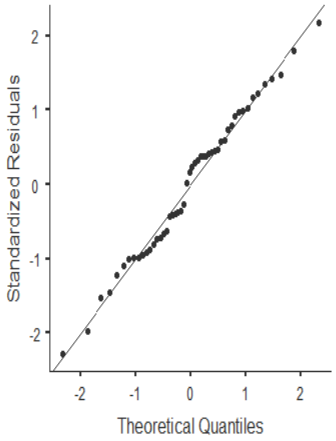
Bar plot

Q-Q Plots
Q-Q







Correlation Matrix

pop2000
pop2010
pop2010_hun_thou
popchng0010
popchngpct
relig_cath
relig_high
relig_low

obama2012
relig_prot

Correlation Coefficients

- [x] Pearson
- [ ] Spearman
- [ ] Kendall's tau-b

Additional Options

- [x] Report significance
- [x] Flag significant correlations
- [x] Confidence intervals

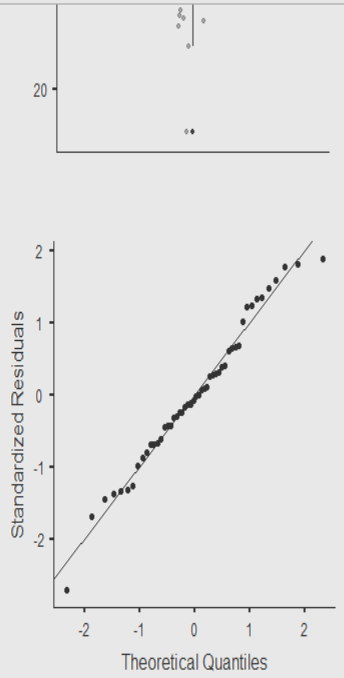
Interval 95 %

Hypothesis

- ( ) Correlated
- ( ) Correlated positively
- (•) Correlated negatively

Plot

- [ ] Correlation matrix
- [ ] Densities for variables
- [ ] Statistics



Correlation Matrix

Correlation Matrix

		obama2012	relig_prot
obama2012	Pearson's r	—	
	p-value	—	
	95% CI Upper	—	
	95% CI Lower	—	
relig_prot	Pearson's r	−0.413 **	—
	p-value	0.001	—
	95% CI Upper	−0.197	—
	95% CI Lower	−1.000	—

Note. H_a is negative correlation

Note. * p < .05, ** p < .01, *** p < .001, one-tailed

Correlation Matrix

modpct_m
obama08
over64
pop_18_24
pop_18_24_10
reppct_m
to_0004
to_0408
trnout00

obama2012
relig_prot
urban
prcapinc

Correlation Coefficients

- [x] Pearson
- [ ] Spearman
- [ ] Kendall's tau-b

Additional Options

- [x] Report significance
- [x] Flag significant correlations
- [x] Confidence intervals

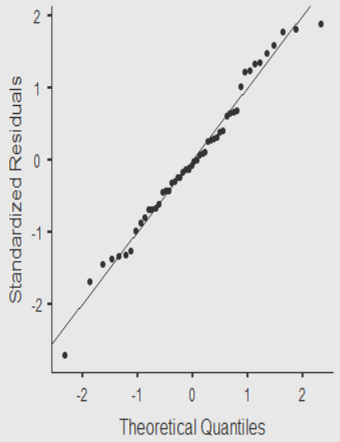
Interval 95 %

Hypothesis

- (•) Correlated
- ( ) Correlated positively
- ( ) Correlated negatively

Plot

- [ ] Correlation matrix
- [ ] Densities for variables
- [ ] Statistics



Correlation Matrix

Correlation Matrix

		obama2012	relig_prot	urban	prcapinc
obama2012	Pearson's r	—			
	p-value	—			
	95% CI Upper	—			
	95% CI Lower	—			
relig_prot	Pearson's r	−0.413**	—		
	p-value	0.003	—		
	95% CI Upper	−0.152	—		
	95% CI Lower	−0.620	—		
urban	Pearson's r	0.406**	−0.577***	—	
	p-value	0.003	< .001	—	
	95% CI Upper	0.615	−0.356	—	
	95% CI Lower	0.144	−0.737	—	
prcapinc	Pearson's r	0.610***	−0.546***	0.526***	—
	p-value	< .001	< .001	< .001	—
	95% CI Upper	0.759	−0.315	0.701	—
	95% CI Lower	0.400	−0.715	0.290	—

Note. * p < .05, ** p < .01, *** p < .001

Correlation Matrix

modpct_m
obama08
over64
pop_18_24
pop_18_24_10
reppct_m
to_0004
to_0408
trnout00

→

obama2012
relig_prot
urban
prcapinc

Correlation Coefficients

- [x] Pearson
- [ ] Spearman
- [ ] Kendall's tau-b

Additional Options

- [x] Report significance
- [x] Flag significant correlations
- [x] Confidence intervals

Interval 95 %

Hypothesis

- (•) Correlated
- ( ) Correlated positively
- ( ) Correlated negatively

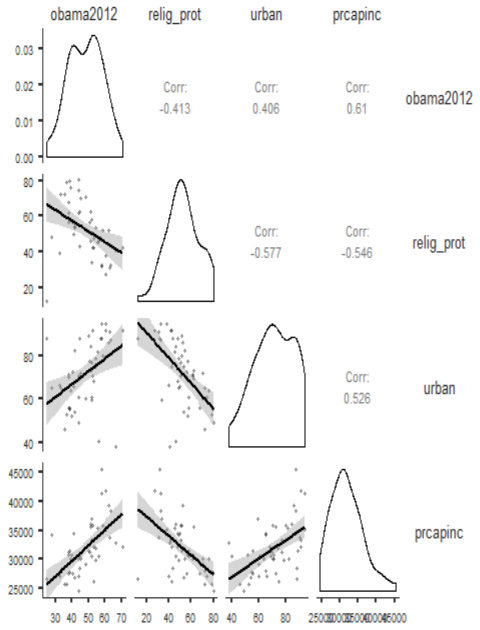
Plot

- [x] Correlation matrix
  - [x] Densities for variables
  - [x] Statistics

	95% CI Lower	—			
relig_prot	Pearson's r	−0.413 **	—		
	p-value	0.003	—		
	95% CI Upper	−0.152	—		
	95% CI Lower	−0.620	—		
urban	Pearson's r	0.406 **	−0.577 ***	—	
	p-value	0.003	< .001	—	
	95% CI Upper	0.615	−0.356	—	
	95% CI Lower	0.144	−0.737	—	
prcapinc	Pearson's r	0.610 ***	−0.546 ***	0.526 ***	—
	p-value	< .001	< .001	< .001	—
	95% CI Upper	0.759	−0.315	0.701	—
	95% CI Lower	0.400	−0.715	0.290	—

Note. * p < .05, ** p < .01, *** p < .001

Plot



Seminář

Pearsonovo r: výpočet

- Dvě proměnné: X a Y

X	Y
1	0
2	1
1	4
6	8
7	4

- (1) výpočet **rozptylu proměnných**
- *mean(x)* = 3.4; *mean(y)* = 3.4

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$

X	**(x – m)**	**dev.**	**dev.^2**
1	(1 – 3.4)	-2.4	5.76
2	(2 – 3.4)	-1.4	1.96
1	(1 – 3.4)	-2.4	5.76
6	(6 – 3.4)	2.6	6.76
7	(7 – 3.4)	3.6	12.96
sum	0	0	33.2

Y	**(y – m)**	**dev.**	**dev.^2**
0	(0 – 3.4)	-3.4	11.56
1	(1 – 3.4)	-2.4	5.76
4	(4 – 3.4)	0.6	0.36
8	(8 – 3.4)	4.6	21.16
4	(4 – 3.4)	0.6	0.36
sum	0	0	39.2

- **s^2(X)** = 33.2 / 4 = **8.3**; **s^2(Y)** = 39.2 / 4 = **9.8**

- (2) výpočet **kovariance proměnných**
- **Kovariance** je součet produktů odchylek proměnných vydělený n–1

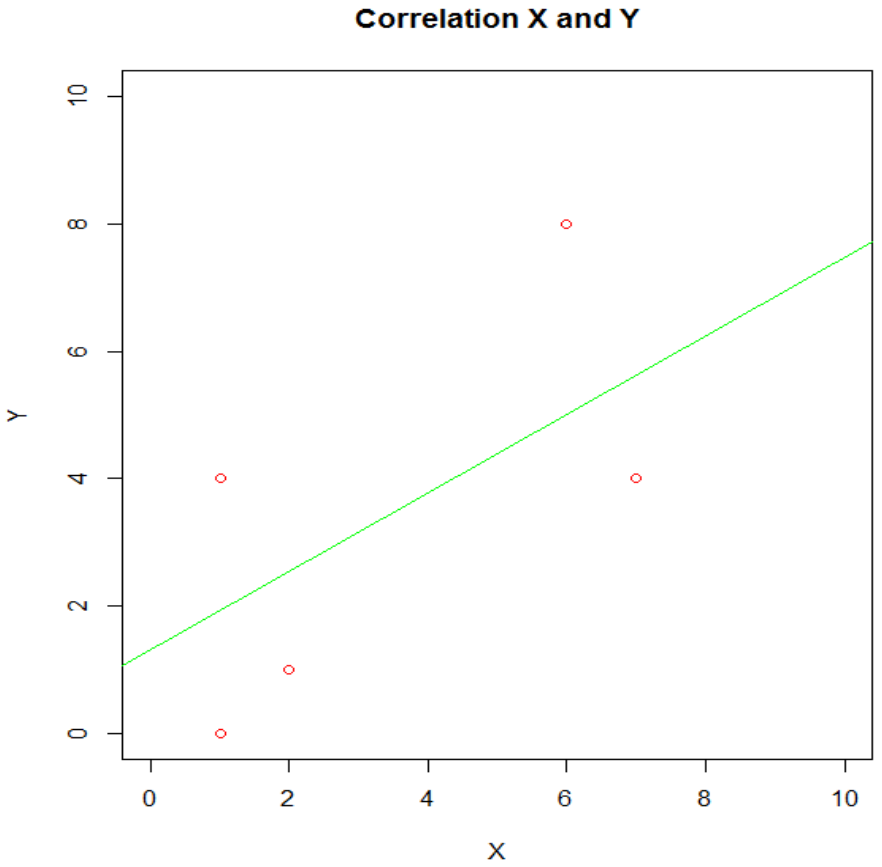
(x – m)	**(y – m)**	**cross-prod.**
(1 – 3.4)	(0 – 3.4)	8.16
(2 – 3.4)	(1 – 3.4)	3.36
(1 – 3.4)	(4 – 3.4)	-1.44
(6 – 3.4)	(8 – 3.4)	11.96
(7 – 3.4)	(4 – 3.4)	2.16
0	0	**24.2**

cov(X, Y) = 24.2 / 4 = **6.05**

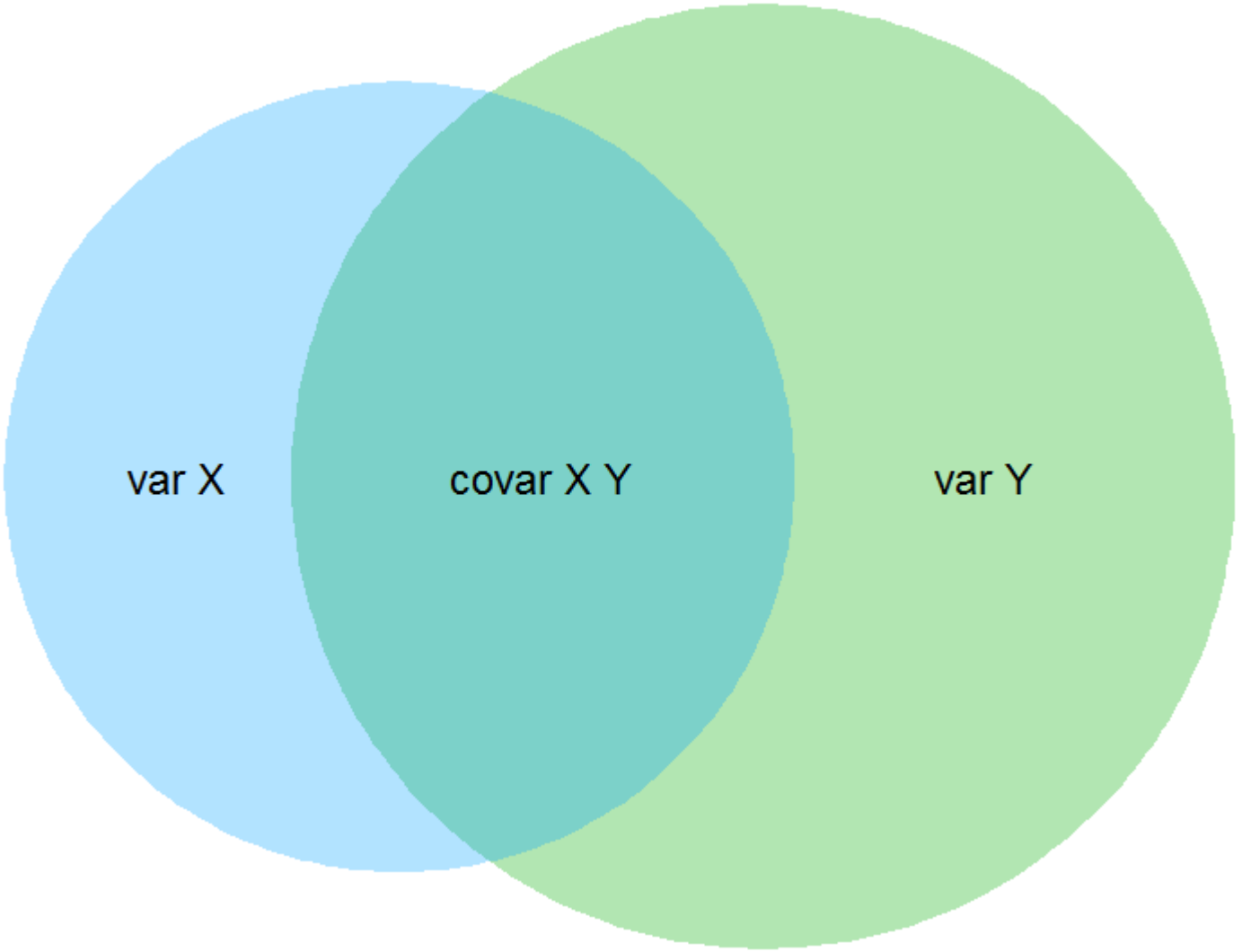
$$COV(x,y) = \frac{\sum_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})}{n-1}$$

- (3) Kovarianci vydělíme odmocninou produktu rozptylů X a Y
  - **r = cov(X, Y) / sqrt(var(X) * var(Y))**
  - **r** = 6.05 / sqrt(8.3 * 9.8) = **0.67**

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \times \sum (y_i - \bar{y})^2}}$$



- r = kovariance X, Y / kombinovaný rozptyl X, Y



Critical Values for Pearson's Correlation Coefficient

	Proportion in ONE Tail					
	.25	.10	.05	.025	.01	.005
	Proportion in TWO Tails					
DF	.50	.20	.10	.05	.02	.01
1	.7071	.9511	.9877	.9969	.9995	.9999
2	.5000	.8000	.9000	.9500	.9800	.9900
3	.4040	.6870	.8054	.8783	.9343	.9587
4	.3473	.6084	.7293	.8114	.8822	.9172
5	.3091	.5509	.6694	.7545	.8329	.8745
6	.2811	.5067	.6215	.7067	.7887	.8343
7	.2596	.4716	.5822	.6664	.7498	.7977
8	.2423	.4428	.5494	.6319	.7155	.7646
9	.2281	.4187	.5214	.6021	.6851	.7348
10	.2161	.3981	.4973	.5760	.6581	.7079
11	.2058	.3802	.4762	.5529	.6339	.6835
12	.1968	.3646	.4575	.5324	.6120	.6614
13	.1890	.3507	.4409	.5140	.5923	.6411
14	.1820	.3383	.4259	.4973	.5742	.6226
15	.1757	.3271	.4124	.4821	.5577	.6055
16	.1700	.3170	.4000	.4683	.5425	.5897
17	.1649	.3077	.3887	.4555	.5285	.5751
18	.1602	.2992	.3783	.4438	.5155	.5614
19	.1558	.2914	.3687	.4329	.5034	.5487
20	.1518	.2841	.3598	.4227	.4921	.5368
21	.1481	.2774	.3515	.4132	.4815	.5256
22	.1447	.2711	.3438	.4044	.4716	.5151
23	.1415	.2653	.3365	.3961	.4622	.5052
24	.1384	.2598	.3297	.3882	.4534	.4958
25	.1356	.2546	.3233	.3809	.4451	.4869
26	.1330	.2497	.3172	.3739	.4372	.4785
27	.1305	.2451	.3115	.3673	.4297	.4705
28	.1281	.2407	.3061	.3610	.4226	.4629
29	.1258	.2366	.3009	.3550	.4158	.4556
30	.1237	.2327	.2960	.3494	.4093	.4487
31	.1217	.2289	.2913	.3440	.4032	.4421
32	.1197	.2254	.2869	.3388	.3972	.4357
33	.1179	.2220	.2826	.3338	.3916	.4296
34	.1161	.2187	.2785	.3291	.3862	.4238
35	.1144	.2156	.2746	.3246	.3810	.4182
36	.1128	.2126	.2709	.3202	.3760	.4128
37	.1113	.2097	.2673	.3160	.3712	.4076
38	.1098	.2070	.2638	.3120	.3665	.4026
39	.1084	.2043	.2605	.3081	.3621	.3978
40	.1070	.2018	.2573	.3044	.3578	.3932
41	.1057	.1993	.2542	.3008	.3536	.3887
42	.1044	.1970	.2512	.2973	.3496	.3843
43	.1032	.1947	.2483	.2940	.3457	.3801
44	.1020	.1925	.2455	.2907	.3420	.3761
45	.1008	.1903	.2429	.2876	.3384	.3721
46	.0997	.1883	.2403	.2845	.3348	.3683
47	.0987	.1863	.2377	.2816	.3314	.3646
48	.0976	.1843	.2353	.2787	.3281	.3610
49	.0966	.1825	.2329	.2759	.3249	.3575
50	.0956	.1806	.2306	.2732	.3218	.3542