

Integral calculus

Integrální počet

Robert Mařík
Mendel University Brno

16. května 2005

[Test1](#)

[Test2](#)

[Test3](#)

[Test4](#)

[Home Page](#)

[Print](#)

[Title Page](#)



Page 1 of 50

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)



- Write appropriate functions or numbers into blank fields and press **Enter**.
- Use functions and mathematical notation as explained in the file [instrukce.pdf](#).
- The green boundary indicates correct answer, the red boundary indicates wrong answer.
- If you cannot solve the problem, click **Ans** to see the correct answer. If there are more fields to be filled, click repeatedly.



- Vepište do políček co tam patří a stiskněte **Enter**.
- Zápis funkcí provádějte tak, jak je vysvětleno v nápovědě v souboru [instrukce.pdf](#).
- Zelený okraj obélníku znamená správnou odpověď, červený špatnou.
- Kliknutím na **Ans** se zobrazí správný výsledek – s případě že problém nejste schopni vyřešit. Je-li v otázce více políček, klikněte na **Ans** opakovaně.



Test1

Test2

Test3

Test4

Home Page

Print

Title Page



Page 2 of 50

Go Back

Full Screen

Close

Quit

1. Test1

🇬🇧 Indefinite integrals by formulas

🇨🇪 Užití vzorců

Quiz

1. $\int e^x dx =$ $+ C$
2. $\int e^{2x} dx =$ $+ C$
3. $\int (1 + 3e^{-x}) dx =$ $+ C$
4. $\int (e^x + 1)^2 dx =$
5. $\int \frac{1}{2}(e^x + e^{-x}) dx =$ $+ C$
6. $\int \left(\frac{1 + 2e^x}{e^x} \right) dx =$ $+ C$
7. $\int \frac{e^x}{1 + e^x} dx =$ $+ C$
8. $\int \frac{e^{-2x}}{1 + e^{-2x}} dx =$ $+ C$

Test1

Test2

Test3

Test4

Home Page

Print

Title Page



Page 3 of 50

Go Back

Full Screen

Close

Quit



Test1

Test2

Test3

Test4

Home Page

Print

Title Page



Page 4 of 50

Go Back

Full Screen

Close

Quit

$$9. \int 3 \cdot 2^x dx = \quad + C$$

$$10. \int \frac{x^2 + x + 4}{x} dx = \quad + C$$

$$11. \int \frac{\sqrt{x} + 1}{x} dx = \quad + C$$

$$12. \int (2x^2 - x + 4) dx = \quad + C$$

$$13. \int \sqrt{x}(1 - \sqrt{x}) dx = \quad + C$$

$$14. \int \frac{(x+1)(x-1)}{x^2} dx = \quad + C$$

$$15. \int \frac{x}{x^2 + 6} dx =$$

$$16. \int \frac{1}{x^2 + 6} dx = \quad + C$$

$$17. \int \frac{x^2 + 2}{x^2 + 1} dx = \quad + C$$

$$18. \int \frac{x + 5}{x^2 + 4} dx = \quad + C$$

$$19. \int \frac{1 - \cos^2 x}{\cos^2 x} dx = \quad + C$$



Test1

Test2

Test3

Test4

Home Page

Print

Title Page

◀◀ ▶▶

◀ ▶

Page 5 of 50

Go Back

Full Screen

Close

Quit

$$20. \int \frac{\sin x}{\cos x} dx = \quad + C$$

$$21. \int 2 \sin x \cos x dx = \quad + C$$

$$22. \int \sin\left(x - \frac{\pi}{2}\right) dx = \quad + C$$

$$23. \int \sin(\pi - x) dx = \quad + C$$

$$24. \int e^{-x} dx = \quad + C$$

$$25. \int e^{3x+1} dx = \quad + C$$

$$26. \int 2e^{x-2} dx = \quad + C$$

$$27. \int e^{5-3x} dx = \quad + C$$

$$28. \int \frac{1}{3+x^2} dx = \quad + C$$

$$29. \int \frac{1}{\sqrt{3+x^2}} dx = \quad + C$$

$$30. \int \frac{-4}{\cos^2(2x)} dx = \quad + C$$



Test1

Test2

Test3

Test4

Home Page

Print

Title Page



Page 6 of 50

Go Back

Full Screen

Close

Quit

$$31. \int \left(\frac{6}{x^3} + x \right) dx = \quad + C$$

$$32. \int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) dx = \quad + C$$

$$33. \int (x + 1)^2 dx = \quad + C$$

$$34. \int \frac{1}{3x + 5} dx =$$

35. Write correct numbers inside the small colored rectangles and then write the primitive function (white field).

Vepište správná čísla do malých podbarvených políček a potom nalezněte primitivní funkci (bílé políčko).

$$\text{(a)} \int \frac{x^2}{x^3+1} dx = \int \frac{(x^3+1)'}{x^3+1} dx$$

$$= \quad \quad \quad + C$$

$$\text{(b)} \int \frac{3x}{x^2+4} dx = \int \frac{(x^2+4)'}{x^2+4} dx$$

$$= \quad \quad \quad + C$$

$$\text{(c)} \int \frac{x^2-1}{x^2+1} dx = \int \quad + \frac{\quad}{x^2+1}$$

$$= \quad \quad \quad dx + C$$

$$\text{(d)} \int \frac{x^2-2x+1}{x^2+2x+1} dx = \int \quad + \frac{x+}{x^2+2x+1} dx$$

$$= \int \quad + \frac{2x+2}{x^2+2x+1} + \frac{\quad}{x^2+2x+1}$$

$$= \quad \quad \quad + C$$

Test1

Test2

Test3

Test4

Home Page

Print

Title Page

◀◀

▶▶

◀

▶

Page 7 of 50

Go Back

Full Screen

Close

Quit

Test1

Test2

Test3

Test4

Home Page

Print

Title Page

◀◀ ▶▶

◀ ▶

Page 8 of 50

Go Back

Full Screen

Close

Quit

$$(e) \int \frac{x+5}{x^2+4} dx = \int \left(\frac{2x}{x^2+4} + \frac{1}{x^2+4} \right) dx$$

$$= \frac{1}{2} \ln|x^2+4| + \frac{1}{2} \arctan\left(\frac{x}{2}\right) + C$$

$$(f) \int \frac{1}{x^2+2x+5} dx = \int \frac{1}{(x+1)^2+4} dx$$

$$= \frac{1}{2} \arctan\left(\frac{x+1}{2}\right) + C$$

$$(g) \int \frac{1}{x^2-3x+4} dx = \int \frac{1}{(x-\frac{3}{2})^2+\frac{7}{4}} dx$$

$$= \frac{2}{\sqrt{7}} \arctan\left(\frac{2x-3}{\sqrt{7}}\right) + C$$

$$(h) \int \frac{1}{\sqrt{x^2+x+1}} dx = \int \frac{1}{\sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}} dx$$

$$= \ln\left|x+\frac{1}{2}+\sqrt{(x+\frac{1}{2})^2+\frac{3}{4}}\right| + C$$

$$(i) \int \frac{x+1}{x^2+4x+6} dx = \int \frac{2x+4}{x^2+4x+6} dx + \int \left(\frac{1}{(x+2)^2+2} \right) dx$$

$$= \ln|x^2+4x+6| - \frac{1}{\sqrt{2}} \arctan\left(\frac{x+2}{\sqrt{2}}\right) + C$$

$$(j) \int \sin x \cos x dx = \int \sin \left(\frac{x}{2} \right) dx + C$$



Test1

Test2

Test3

Test4

Home Page

Print

Title Page



Page 9 of 50


Go Back


Full Screen

Close

Quit

2. Test2

 Integration by parts

 When integrating by parts we use the formula

 Integrate per-partés

 Pro integraci per-partés používáme následující vzorec

$$\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx. \quad (\text{Eq:1})$$

Test1

Test2

Test3

Test4

Home Page

Print

Title Page



Page 10 of 50

Go Back

Full Screen

Close

Quit

Quiz

We use the integration by parts especially for integrals of the type

$$\int p(x)f(ax+b)dx, \quad (\text{Eq:2})$$

where $p(x)$ is a polynomial and

Typicky používáme integraci per-partés pro integrály typu

kde $p(x)$ je polynom a

$$f(x) \in \{e^x, \sin x, \cos x, \text{atan } x, \ln^m x\}$$

Here $\text{atan}(x)$ is the usual arctangent functions.

Zde $\text{atan}(x)$ je obvyklá funkce arkustangens.

Question: Are the following integrals like (Eq:2)? Are the integral convenient for integration by parts?

Otázka: Jsou následující integrály typu (Eq:2)? Je vhodné je integrovat metodou per-partés?

1. $\int e^{-x^2} dx$

Yes No

2. $\int xe^{x^2} dx$

Yes No

3. $\int x^2 e^x dx$

Yes No

4. $\int (3x+1)e^{-x+1} dx$

Yes No



Test1

Test2

Test3

Test4

Home Page

Print

Title Page



Page 12 of 50

Go Back

Full Screen

Close

Quit

5. $\int (x + 4) \operatorname{atan} \frac{x}{2} dx$

Yes No

6. $\int x \sin x^2 dx$

Yes No

7. $\int x^2 \ln x dx$

Yes No

8. $\int \operatorname{atan} x dx$

Yes No

9. $\int x \ln x \cos x dx$

Yes No

10. $\int x \cos^3 x dx$

Yes No

11. $\int (2 + x) \cos(2x) dx$

Yes No

12. $\int (x^3 - 1) \sin \left(\frac{\pi}{2} - x \right) dx$

Yes No

Quiz Integrate

Integrujte

$$I = \int (x^2 + x - 2) \sin x dx.$$

1. $u =$ $u' =$

$v' =$ $v =$

2. $I =$ $- \int$ dx

Prev. Page

Next Page

Home Page

Print

Title Page



Page 13 of 50

Go Back

Full Screen

Close

Quit

3.

$$I = -(x^2 + 2x + 1) \cos x + 2 \int (x + 1) \cos x dx.$$

Now we have an expression which can be written as above (check it yourself). We integrate by parts in $\int (x + 1) \cos x dx$.

Nyní máme něco, co se dá přepsat do výše uvedeného tvaru (zkontrolujte si) do tvaru. Integrujeme výraz $\int (x + 1) \cos x dx$. Použijeme opět metodu per-partés.

$$u = \quad \quad \quad u' =$$

$$v' = \quad \quad \quad v =$$

4.

$$I = -(x^2 + 2x + 1) \cos x + 2 \left(\quad \quad \quad - \int \quad \quad \quad dx \right)$$

Prev. Page

Next Page

Test1

Test2

Test3

Test4

Home Page

Print

Title Page

◀◀ ▶▶

◀ ▶

Page 14 of 50

Go Back

Full Screen

Close

Quit

5.

$$\begin{aligned}
 I &= -(x^2 + 2x + 1) \cos x + 2 \left((x + 1) \sin x - \int \sin x dx \right) \\
 &= -(x^2 + 2x + 1) \cos x + 2 \left((x + 1) \sin x - \right. \\
 &= \left(\right) \sin x + \left(\right) \cos x + C
 \end{aligned}$$

Prev. Page

Next Page



Test1

Test2

Test3

Test4

Home Page

Print

Title Page

◀◀ ▶▶

◀ ▶

Page 15 of 50

Go Back

Full Screen

Close

Quit

Quiz Integrate

Integrujte

$$I = \int a \tan x dx.$$

1. $u =$ $u' =$

$v' =$ $v =$

2. $I =$ $- \int$ dx

Prev. Page

Next Page

Test1

Test2

Test3

Test4

Home Page

Print

Title Page




Page 16 of 50


Go Back

Full Screen

Close

Quit

3.  Now we have an expression which can be written in the form (check it yourself). Find out the number which has to be in the first colored field. When you find out this number, the integration is easy.

 Nyní máme něco, co se dá přepsat (zkontrolujte si) do tvaru. Zjistíte-li, jaké číslo je potřeba zapsat do prvního podbarveného obdélníčku, je integrace snadná.

$$\begin{aligned}
 I &= x \operatorname{atan} x - \int \frac{x}{x^2 + 1} dx \\
 &= x \operatorname{atan} x - \left(\quad \right) \int \frac{2x}{x^2 + 1} dx \\
 &= x \operatorname{atan} x -
 \end{aligned}$$

[Prev. Page](#)

[Next Page](#)

[Test1](#)

[Test2](#)

[Test3](#)

[Test4](#)

[Home Page](#)

[Print](#)

[Title Page](#)



Page 17 of 50


[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)



4.  The result is

$$\int \operatorname{atan} x dx = x \operatorname{atan} x - \frac{1}{2} \ln(1 + x^2) + C$$

 Výsledek je

Test1

Test2

Test3

Test4

Home Page

Print

Title Page



Page 18 of 50

Go Back

Full Screen


Close

Quit

Quiz Integrate


Integrujte

$$I = \int (x^2 - 1)e^x dx$$


1.  We integrate by parts with $u(x) = (x^2 - 1)$. With this notation we have (use zero constant of integration in responses)


$$u = x^2 - 1 \quad u' =$$

$$v' = \quad v =$$

2.  Integration by parts gives ...

$$I = \underbrace{\hspace{10em}} - \int \underbrace{\hspace{10em}} dx$$

 Integrujeme per-partés při volbě $u(x) = x^2 - 1$

 Po použití vzorce pro integraci per-partés máme ...

Test1

Test2

Test3

Test4

Home Page

Print

Title Page

◀◀ ▶▶

◀ ▶

Page 19 of 50

Go Back

Full Screen

Close

Quit

3. We integrate once more by parts

Budeme integrovat ještě jednou per-partés

$$u = 2x$$

$$u' =$$

$$v' =$$

$$v =$$

4. The second integration by parts gives ...

Opětovné použití vzorce per-partés dává ...

$$I = \quad - \left[\quad - \int \quad dx \right]$$

5. The result after the last integration and simplifications is ...

Po poslední integraci a po snadné úpravě obdržíme ...

$$I = \quad + C$$

Test1

Test2

Test3

Test4

Home Page

Print

Title Page

◀◀ ▶▶

◀ ▶

Page 20 of 50

Go Back

Full Screen


Close


Quit

Quiz Integrate

Integrujte


$$I = \int x \ln(x+1) dx$$

1.  We integrate by parts with $u(x) = \ln(x+1)$.

 Budeme integrovat per-partés při volbě $u(x) = \ln(x+1)$.

$$u = \ln(x+1) \quad u' =$$

$$v' = \quad v =$$

2.  Integration by parts gives ...

 Aplice vzorce per-partés dává ...

$$I = \underbrace{\quad} - \int \underbrace{\quad}_A dx$$

Prev. Page

Next Page

Home Page

Print

Title Page



Page 21 of 50

Go Back

Full Screen

Close

Quit

3. The expression denoted by A is a rational function which is not proper. Divide the numerator by the denominator and write this function as a sum of polynomial and proper function. Write the polynomial into the first field and the proper function into the second one.

$$A = \underbrace{\hspace{10em}}_{\text{polynomial}} + \underbrace{\hspace{10em}}_{\text{remainder}}$$

4. The integration and simplification give ...

$$I =$$

Výraz označený jako A je racionální funkce a je nutno ji integrovat tak, že nejprve vydělíme čítelel jmenovatelem. Napište do prvního políčka podíl a do druhého zbytek po dělení.

Finální integraci a úpravou získáme ...

$$+ C$$

[Test1](#)[Test2](#)[Test3](#)[Test4](#)[Home Page](#)[Print](#)[Title Page](#)[◀◀](#)[▶▶](#)[◀](#)[▶](#)[Page 23 of 50](#)[Go Back](#)[Full Screen](#)[Close](#)[Quit](#)

Quiz Find the following integral: $I = \int (x+1)e^{-x} dx$

1. We integrate by parts with $u(x) = (x+1)$. With this notation we have (use zero constant of integration in responses):

$$u'(x) =$$

$$v'(x) =$$

$$v(x) =$$

2. Integration by parts gives

$$I = \quad - \int \quad dx$$

3. Integration gives the indefinite integral

$$I = \quad + C$$



Quiz Find the following integral: $I = \int (x^2 - 1) \sin x dx$

1. We integrate by parts with $u(x) = (x^2 - 1)$. With this notation we have (use zero constant of integration in responses):

$$u'(x) =$$

$$v'(x) =$$

$$v(x) =$$

2. Integration by parts gives

$$I = \quad - \int \quad dx$$

[Go to the next page.](#)

Test1

Test2

Test3

Test4

Home Page

Print

Title Page



Page 24 of 50

Go Back

Full Screen

Close

Quit

[Test1](#)[Test2](#)[Test3](#)[Test4](#)[Home Page](#)[Print](#)[Title Page](#)

Page 25 of 50

[Go Back](#)[Full Screen](#)[Close](#)[Quit](#)

3. Now you have $I = -(x^2 - 1) \cos(x) + 2 \int x \cos(x) dx$. We integrate by parts with $u(x) = x$. With this notation we have (use zero constant of integration in responses):

$$u'(x) =$$

$$v'(x) =$$

$$v(x) =$$

4. Integration by parts gives

$$I = -(x^2 - 1) \cos x + 2 \left[\quad - \int \quad dx \right]$$

5. Integration gives the indefinite integral

$$I = \quad + C$$

[Test1](#)[Test2](#)[Test3](#)[Test4](#)[Home Page](#)[Print](#)[Title Page](#)[◀◀](#)[▶▶](#)[◀](#)[▶](#)[Page 26 of 50](#)[Go Back](#)[Full Screen](#)[Close](#)[Quit](#)

Quiz Find the following integral: $I = \int \ln x dx$

1. We integrate by parts with $u(x) = \ln x$. With this notation we have (use zero constant of integration in responses):

$$u'(x) =$$

$$v'(x) =$$

$$v(x) =$$

2. Integration by parts gives

$$I = \quad - \int \quad dx$$

3. Integration gives the indefinite integral

$$I = \quad + C$$

[Test1](#)[Test2](#)[Test3](#)[Test4](#)[Home Page](#)[Print](#)[Title Page](#)[Page 27 of 50](#)[Go Back](#)[Full Screen](#)[Close](#)[Quit](#)

Quiz Find the following integral: $I = \int x^2 \operatorname{atan} x dx$
(we use “ $\operatorname{atan}(x)$ ” for the usual arctangent function).

1. We integrate by parts with $u(x) = \operatorname{atan} x$. With this notation we have (use zero constant of integration in responses):

$$u'(x) =$$

$$v'(x) =$$

$$v(x) =$$

2. Integration by parts gives

$$I = \quad - \int \quad dx$$

3. Integration gives the indefinite integral

$$I = \quad + C$$

[Test1](#)[Test2](#)[Test3](#)[Test4](#)[Home Page](#)[Print](#)[Title Page](#)[◀◀](#)[▶▶](#)[◀](#)[▶](#)[Page 28 of 50](#)[Go Back](#)[Full Screen](#)[Close](#)[Quit](#)

Quiz Find the following integral: $I = \int (x + 3)e^{2x} dx$

1. We integrate by parts with $u(x) = (x + 3)$. With this notation we have (use zero constant of integration in responses):

$$u'(x) =$$

$$v'(x) =$$

$$v(x) =$$

2. Integration by parts gives

$$I = \quad - \int \quad dx$$

3. Integration gives the indefinite integral

$$I = \quad + C$$

3. Test3

Integration by substitution

Integrate substitucí

When integrating by substitution we use the formula

$$\int f(\phi(x))\phi'(x)dx = \int f(t)dt \quad (\text{Eq:3})$$

(i.e. we substitute $\phi(x) = t$ and $\phi'(x)dx = dt$) or

$$\int f(x)dx = \int f(\phi(t))\phi'(t)dt \quad (\text{Eq:4})$$

(i.e. we substitute $x = \phi(t)$ and $dx = \phi'(t)dt$).

Pro integraci pomocí substituce používáme výše uvedené vzorce.

Quiz Find the following integral: $I = \int \frac{x + \sqrt{x-4}}{(x+1)\sqrt{x-4}} dx$

1. We use the substitution $x - 4 =$

2. With this substitution we have

$$dx = \quad dt \quad x =$$

$$t =$$

3. Substitution gives

$$I = \int \quad dt$$

4. We have to divide the numerator by the denominator. This gives a sum of polynomial and proper rational fraction (which is also a partial fraction). Write this polynomial into the first and the partial fraction into the second field.

$$I = \int \quad + \quad dt$$

5. Integration in t gives

$$I =$$

6. The back substitution gives the result in the variable x

$$I = \quad + C$$

Quiz Find the following integral: $I = \int \frac{\sin(x) \cos(x)}{\sin(x) + 1} dx$

1. We use the substitution $t =$

2. With this substitution we have

$$dt = \quad dx$$

3. Substitution gives

$$I = \int \quad dt$$

4. We have to divide the numerator by the denominator. This gives a sum of polynomial and proper rational fraction (which is also a partial fraction in our particular example). Write this polynomial into the first and the partial fraction into the second field.

$$I = \int \quad + \quad dt$$

5. Integration in t gives

$$I =$$

6. The back substitution gives the result in the variable x

$$I = \quad + C$$

Quiz Convert the following integral by substitution into an integral of rational

function: $I = \int x \sqrt{\frac{x+1}{x-1}} dx$

1. We use the substitution $t^2 = \frac{x+1}{x-1}$ With this substitution we have

$$x =$$

$$dx = \quad \quad \quad dx$$

2. Substitution and simplification give

$$I = \int \quad \quad \quad dt$$

[Test1](#)[Test2](#)[Test3](#)[Test4](#)[Home Page](#)[Print](#)[Title Page](#)[Page 33 of 50](#)[Go Back](#)[Full Screen](#)[Close](#)[Quit](#)

Quiz Find the following integral: $I = \int x^2 e^{-x^3} dx$

1. With substitution $-x^3 = t$ we have
 $\cdot dx = dt$

2. Substitution gives

$$I = \int \quad dt$$

3. Integration in t gives the indefinite integral

$$I = \quad + C$$

4. In the original variable x we have

$$I = \quad + C$$



Test1

Test2

Test3

Test4

Home Page

Print

Title Page

◀◀ ▶▶

◀ ▶

Page 34 of 50

Go Back

Full Screen

Close

Quit

Quiz Find the following integral: $I = \int \sin^5 x dx$

1. With substitution $\cos x = t$ we have
 $\cdot dx = dt$

2. Substitution gives

$$I = \int \quad dt$$

3. Integration in t gives the indefinite integral

$$I = \quad + C$$

4. In the original variable x we have

$$I = \quad + C$$

Quiz Substitute $\tan x = t$ in the integral $I = \int \frac{\sin x - \cos x}{\sin^3 x + \cos^3 x} dx$

1. With substitution $\tan x = t$ we have (write expression in t)

$x =$

2. Differentiating we get

$dx =$ $\cdot dt$

3. From the right triangle with angle x , opposite side t , adjacent side 1 and hypotenuse $\sqrt{1+t^2}$ (draw such an triangle) we have the following relations between $\sin(x)$, $\cos(x)$ and new variable t :

$\sin(x) =$ (write expression in t)

$\cos(x) =$ (write expression in t)

4. Substitution gives

$I = \int$ dt

5. Now we stop. However, you can evaluate this integral using partial fractions.

[Test1](#)[Test2](#)[Test3](#)[Test4](#)[Home Page](#)[Print](#)[Title Page](#)[Page 36 of 50](#)[Go Back](#)[Full Screen](#)[Close](#)[Quit](#)

Quiz Evaluate integral $I = \int \frac{1}{1+e^x} dx$ by substitution.

1. Differentiating $e^x = t$ we get
 $\cdot dx = dt$

2. From $e^x = t$ we have (write x as a function of t)
 $x =$

Differentiating this relation we have
 $dx = \cdot dt$

3. After substitution we have

$$I = \int \cdot dt$$


4. Decomposition into partial fraction and integration give the integral in the variable t :


$$I = \cdot + C$$

5. We return to the original variable x . We have

$$I = \cdot + C$$

4. Test4

 Definite integral in geometry

 Aplikace v geometrii

Test1

Test2

Test3

Test4

Home Page

Print

Title Page



Page 37 of 50

Go Back

Full Screen

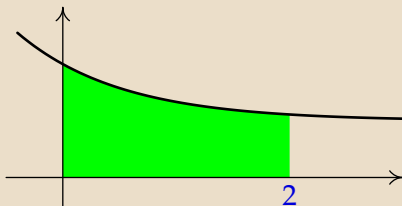
Close

Quit

Quiz

The function on the picture is the function $y = e^x$ reflected about the y -axis and moved by one above. (In notation of this document the function e^x can be written as $\exp(x)$, or $e^{\wedge}(x)$.) The green region corresponds to the interval $x \in [0, 2]$.

Na obrázku je funkce $y = e^x$ převrácená okolo osy y a posunutá o jedničku nahoru. (V notaci tohoto dokumentu je možno funkci e^x zapsat jako $\exp(x)$, nebo $e^{\wedge}(x)$.) Označený region odpovídá intervalu $x \in [0, 2]$.



- Write an analytical formula for the function.

$y =$

Napište analytický tvar funkce.

2. Express the area of the green region as the definite integral.

$$S = \int \quad \quad \quad dx$$

- Vyjádřete obsah vybarveného regionu jako určitý integrál.

3. Complete the following formula. This formula may be used later for integration.

$$\int e^{-x} dx = \quad \quad \quad + C.$$

- Doplňte vzorec, který potom použijte pro integraci.

4. Integrate and use the Newton-Leibniz formula.

$$S = \left[\quad \quad \quad \right]$$

- Integrujte a použijte Newtonovu-Leibnizovu formuli.

5. Substitute the limits and evaluate the integral.

$$S = \quad \quad \quad .$$

- Dosad'te meze a dopočíteje integrál.

Prev. Page

Next Page

Test1

Test2

Test3

Test4

Home Page

Print

Title Page




Page 39 of 50

Go Back


Full Screen

Close


Quit

6.  Write the volume of the of the solid of revolution formed by revolving the green region about the x -axis as a definite integral.


$$V = \pi \int$$

7.  Simplifying and integrating we get (use zero constant of integration) ...


$$V = \pi \left[\right] .$$


8.  The volume is ...

$$V = \pi .$$

 Vyjádřete jako určitý integrál objem tělesa, které vznikne rotací tohoto obrazce okolo osy x .

dx .

 Po umocnění integrandu a po integraci (volte nulovou integrační konstantu) máme pro objem vztah ...

 Výsledný objem je ...

Test1

Test2

Test3

Test4

Home Page

Print

Title Page

◀◀ ▶▶

◀ ▶

Page 40 of 50

Go Back

Full Screen

Close

Quit

Prev. Page

Next Page

Test1

Test2

Test3

Test4

Home Page

Print

Title Page

◀ ▶

◀ ▶

Page 41 of 50

Go Back

Full Screen

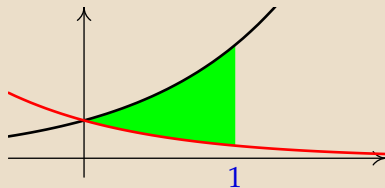
Close

Quit

Quiz

The functions on the picture are $y = e^x$ and $y = e^{-x}$ (In notation of this document we can write the function e^x as $\exp(x)$ or $e^{\wedge}(x)$ and the function e^{-x} as $\exp(-x)$ or $e^{\wedge}(-x)$.) The green region corresponds to $x \in [0, 1]$.

Na obrázku jsou funkce $y = e^x$ a $y = e^{-x}$ (V notaci tohoto dokumentu je možno funkci e^x zapsat jako $\exp(x)$, nebo $e^{\wedge}(x)$ a funkci e^{-x} jako $\exp(-x)$, nebo $e^{\wedge}(-x)$.) Označený region odpovídá intervalu $x \in [0, 1]$.



1. The black curve is

$y =$

Černá funkce je

2. The red curve is

$y =$

Červená funkce je

Prev. Page

Next Page

3. Area of the green region can be evaluated as a definite integral ...

$$S = \int \quad \quad \quad dx$$

Obsah vybarveného regionu je možno vyjádřit jako určitý integrál ...

4. Integration gives

$$S = \left[\quad \quad \quad \right]$$

Po integraci dostaneme

5. Substituting limits and simplifying we obtain

$$S = \quad \quad \quad .$$

Po dosazení mezí a výpočtu dostáváme

[Prev. Page](#)

[Next Page](#)

[Home Page](#)

[Print](#)

[Title Page](#)



Page 42 of 50

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)

Test1

Test2

Test3

Test4

Home Page

Print

Title Page

◀ ▶

◀ ▶

Page 43 of 50

Go Back

Full Screen

Close

Quit

6. The volume of the solid of revolution which can be obtained by revolving the green region about the x -axis can be evaluated as the definite integral ...

$$V = \pi \int$$

7. Algebraic simplifications and integration give (use a zero constant of integration) ...

$$V = \pi \left[\right] .$$

8. The volume is ...

$$V = \pi.$$

Objem tělesa, které vznikne rotací tohoto obrazce okolo osy x je možno vyjádřit jako určitý integrál ...

dx .

Po umocnění integrandu a po integraci (volte nulovou integrační konstantu) máme pro objem vztah ...

Výsledný objem je ...

Prev. Page

Next Page

Test1

Test2

Test3

Test4

Home Page

Print

Title Page

◀ ▶

◀ ▶

Page 44 of 50

Go Back

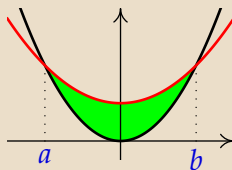
Full Screen

Close

Quit

Quiz The functions on the picture are $y = x^2$ and $y = \frac{x^2}{2} + 2$ (In the notation of this document you can write something like $y=x^2$ and $y=x^2/2+2$).

Na obrázku jsou funkce $y = x^2$ a $y = \frac{x^2}{2} + 2$ (v notaci tohoto dokumentu lze tyto funkce zapsat např. jako $y=x^2$ a $y=x^2/2+2$).



1. The black curve is:

$y =$.

Černá křivka je grafem funkce:

2. The red curve is:

$y =$.

Červená křivka je grafem funkce:

3. Find the intercepts of both curves.

$a =$, $b =$.

Najděte průsečíky křivek.

Prev. Page

Next Page

4. Express the area of the shaded region as an definite integral.

$$S = \int \quad \quad \quad dx.$$

5. The function inside integral is a polynomial. Find the coefficients of this polynomial.

$$S = \int \left(\quad x^2 + \quad \right) dx.$$

6. Integrate and use the Newton-Leibniz formula.

$$S = \left[\quad \right] = \quad .$$

- Vyjádřete obsah vyšrafované plochy pomocí určitého integrálu.

- Integrand lze zapsat jako polynom. Doplňte koeficienty tohoto polynomu.

- Integrujte a použijte Newtonovu-Leibnizovu formuli

Prev. Page

Next Page

Home Page

Print

Title Page



Page 45 of 50

Go Back

Full Screen

Close

Quit

7. Write the integral which express the volume of the solid obtained by a revolution of the shaded region about the x -axis.

$$V = \pi \int$$

8. The function in the integral can be expressed as a polynomial. Complete the coefficients of the polynomial.

Rotuje-li vyšrafovaná plocha okolo osy x , získáme rotační těleso, jehož objem je možno zapsat ve tvaru určitého integrálu. Napište tento integrál.

dx .

Integrand lze vyjádřit jako polynom (doplňte čísla)

[Prev. Page](#)

[Next Page](#)

[Home Page](#)

[Print](#)

[Title Page](#)



Page 46 of 50

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)

$$V = \pi \int (x^4 + x^2 +) dx.$$

9. Integrate and use the Newton-Leibniz formula.

Integrujte a použijte Newtonovu-Leibnizovu formuli.

$$V = \pi \left[\right] .$$

10. Substitute the limits and evaluate the integral.

Dosad'te horn'ı a doln'ı mez a vy-po'c'te'te integr'al.

$$V = \pi.$$

[Prev. Page](#)

[Next Page](#)

[Home Page](#)

[Print](#)

[Title Page](#)



Page 47 of 50

[Go Back](#)

[Full Screen](#)

[Close](#)

[Quit](#)

Test1

Test2

Test3

Test4

Home Page

Print

Title Page

◀ ▶

◀ ▶

Page 48 of 50

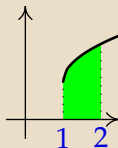
Go Back

Full Screen

Close

Quit

Quiz Na obrázku je funkce $y = \sqrt{x}$ posunutá o jedničku nahoru a o jedničku doprava. (V notaci tohoto dokumentu je možno funkci \sqrt{x} zapsat jako `sqrt(x)`, nebo $x^{1/2}$.)



1. Analytický tvar funkce je $y =$.
2. Obsah vybarveného regionu je možno vyjádřit jako určitý integrál

$$S = \int \quad dx$$

3. Pro integraci lze použít vzorec

$$\int \sqrt{x} dx = \int x^{\frac{1}{2}} dx = \quad + C.$$

4. Po aplikaci tohoto vzorečku dostáváme

$$S = \left[\quad \right]$$

Prev. Page

Next Page

5. Po dosazení mezí a výpočtu dostáváme $S =$.

6. Objem tělesa, které vznikne rotací tohoto obrazce je možno vyjádřit jako určitý integrál

$$V = \pi \int \quad dx.$$

7. Po umocnění integrandu a po integraci (volte nulovou integrační konstantu) máme pro objem vztah

$$V = \pi \left[\quad \right] .$$

8. Výsledný objem je $V =$ π .

Test1

Test2

Test3

Test4

Home Page

Print

Title Page

◀◀ ▶▶

◀ ▶

Page 49 of 50

Go Back

Full Screen

Close

Quit



Test1

Test2

Test3

Test4

Home Page

Print

Title Page



Page 50 of 50

Go Back

Full Screen

Close

Quit

That's all. The user is kindly asked to send his comments to these quizzes to my E-mail address.

Tot' vše. Prosím uživatele, aby své případné komentáře a náměty zasílali na moji E-mailovou adresu.