Light distribution on the retina of a wide-angle theoretical eye

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In a theoretical eye with spherical and aspheric surfaces, the retinal illumination is calculated if a Ganzfeld luminance field is used. The resulting retinal light distribution is nearly homogeneous over the whole retina. The homogeneity is not much influenced by the size of the pupil or the shape of the optical surfaces. The corresponding retinal area and the luminous flux entering the eye are calculated as functions of the size of the visual field. The values of the length of the light path through the crystalline lens and of the angle of incidence on the retina are described as functions of the angle in the visual field.

INTRODUCTION

Ganzfeld illumination is used in electroretinography, dark-adaptation tests, and perimetry. Usually an internally illuminated globe with a homogeneous luminance is used,[1–3] but light-diffusing contact lenses are also suitable.[4,5] The following questions arise:

(1) What retinal illumination results from Ganzfeld illumination as a function of the angle of incident light relative to the optic axis?
(2) How does the light distribution on the retina depend on the size of the pupil?
(3) How strong is the influence of modifications in the shapes of the eye surfaces on the light distribution?

Although these questions look classical, it has not been possible to find all the answers in the literature. Recently, Bedell and Katz[6] deduced from the experimental results on pupillary area[7] and the calculated values of retinal area per solid angle of visual field[8] that the retinal illuminance of a target should remain essentially constant up to about 80°. Gouras[1] mentions that the pupillary aperture places restrictions on the homogeneity of the retinal illumination. Hoffman *et al.*[9] conclude that the pupil acts as a field stop that limits the visual angle for large fields, such as a Ganzfeld source. Gross[10] and Young[11] measured the luminance of the image of an object from the back of the excised rat eye as a function of eccentricity. None of these authors refers to calculations of the retinal light distribution. In this study a theoretical model is used to calculate retinal light distribution and the influence of some model parameters on the relative retinal illumination.

METHODS

Calculations are done on a schematic eye with optical surfaces that are surfaces of rotation of a spherical or aspheric conic section (Table 1 and Fig. 1). The dimensions of the schematic eye are based on the theoretical eye of Le Grand.[12]

A ray-tracing program in BASIC on a Commodore 3032 microcomputer was used to calculate the light rays in the meridional plane of the eye. The steps of the calculation are as follows:

(1) The apparent pupil diameter PD(φ) in the meridional plane is calculated as seen from oblique directions. The apparent pupil is the image of the physical pupil focused by the cornea in air. The physical pupil is assumed to be coincident with the anterior lens surface. The thickness of the iris edge is assumed to be zero in this study. A real thickness of the iris edge causes vignetting at all peripheral angles of incidence of 10° and greater but has an appreciable effect on the apparent pupil area only at angles of view greater than 70°.[7] The apparent pupil, as seen along the optic axis, has a circular shape. This shape changes if the eye is viewed obliquely. The apparent shape, seen from an angle φ with the optic axis, is defined as the shape of the cross section of the light pencil of parallel rays incident upon the eye at an angle φ with the optic axis and filling up the whole pupil after refraction by the cornea. This is the pupil as seen from the outside of the eye. An apparent pupil diameter observed along the optic axis corresponds to a smaller physical pupil diameter at the anterior surface of the lens. For apparent pupil diameters of 2 and 8 mm on the optic axis, these physical pupil diameters are calculated and are used to calculate the apparent pupil diameters for oblique directions. It is supposed that the apparent pupil diameter perpendicular to the meridional plane is independent of the angle φ. The apparent pupil area PA(φ) is calculated with the area equation of an ellipse:

$$\mathrm{PA}(\varphi) = \pi \times \mathrm{PD}(\varphi) \times \mathrm{PD}(0)/4.$$

The principal ray of the pencil of parallel rays with an angle φ with the optic axis passes through the center of the entrance pupil related to that pencil of rays (Ref. 13, p. 186). As a consequence of aberrations, the principal ray does not necessarily pass through the center of the physical pupil and can be different for large and small pupils.

(2) The retinal-area-per-steradian visual field is calculated as a function of φ. For that purpose the position and the length of the retinal arc that correspond to an increment of ±0.25° of the angle of the principal ray in the meridional plane are calculated as a function of φ. A ring-shaped retinal

Table 1. Schematic Eye[a]

	Refractive Index	Apical Radius (mm)	Conical Value p	Distance Corneal Vertex (mm)
Cornea	1.3771	7.8 (anterior) 6.5 (posterior)	1 or 0.75	0 0.55
Aqueous	1.3374	—	—	—
Lens	1.42	10.2 (anterior) −6 (posterior)	1 or −2.06 1 or 0	3.6 7.6
Vitreous	1.336	—	—	—
Retina	–	−10.8 or −14.1	1 or 1.346	24.2

[a] The theoretical eye of Le Grand has spherical surfaces ($p = 1$). In this study surfaces with an aspheric conical cross section are also used ($p \neq 1$). Calculations are done on the basic eye model with only spherical surfaces (model A) and on the basic eye model with only one of the following modifications: (model B) aspheric anterior and posterior corneal surfaces with an elliptical curvature, (model C) aspheric retinal surface with an elliptical curvature, (model D) aspheric anterior lens surface with a hyperbolic curvature, or (model E) aspheric posterior lens surface with a parabolic curvature.

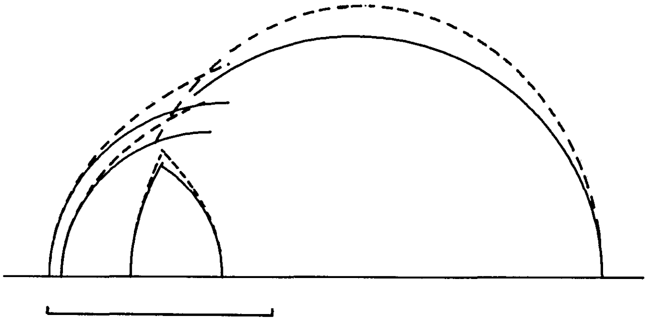

Fig. 1. Schematic eye. Solid lines are spherical surfaces (theoretical eye of Le Grand). Dashed lines are aspheric surfaces as used in this study.

area dRA(φ) around the optic axis is determined by this arc (Fig. 2). On this area fall the light rays that emerge from a ring-shaped solid angle dSA(φ) around the optic axis with a width of ±0.25° at a peripheral angle φ. The quotient RS(φ) = dRA(φ)/dSA(φ) is the retinal area per solid angle at eccentricity φ. For small and large pupils the principal rays are not identical to each other. Therefore dRA(φ) and RS(φ) are calculated for two pupil sizes.

(3) The retinal illumination RI(φ) with Ganzfeld luminance A in the object space is calculated as function of φ:

$$\mathrm{RI}(\varphi) = \frac{\mathrm{PA}(\varphi)}{\mathrm{RS}(\varphi)} \times A,$$

in which RI(φ) is the retinal illumination in lumens per square meter (lm m^2), PA(φ) is the apparent pupil area in square millimeters (mm^2), RS(φ) is the retinal area per solid angle in square millimeters per steradian (mm^2 sr^{-1}), and A is the Ganzfeld luminance in lumens per square meter × steradian (lm m^2 sr^{-1}).

Light losses in the eye media (from scattering, absorption, reflection, etc.) are not taken into account. Parallel incident light rays are assumed to focus perfectly at the retina at the point of incidence of the principal ray. This last assumption will overestimate inhomogeneities in the light distribution over the retina. Light spreading by spherical and comatic aberration will result in smaller differences in retinal illumination at various angles of incidence.

(4) The luminous flux L(φ) entering the eye is calculated for a luminance field with a limited size. The light source is assumed to be a homogenous field at infinity centered on the optic axis, subtending the solid angle SA(φ) bounded by the principal rays having an angle of incidence φ with the optic axis. The apparent pupil area PA(φ) (mm^2) multiplied by the solid angle dSA(φ) (sr^2) is the luminous flux dL(φ) (lm) that enters the eye in the ring-shaped solid angle dSA(φ) if the light source has a luminance of 1 lm/m^{-2} sr^{-1}. The luminous flux L(φ) is calculated with a discrete Simpson integration routine on dL(φ).

(5) The retinal area RA(φ) corresponding to the solid angle SA(φ) is calculated with the discrete Simpson integration routine on dRA(φ).

(6) The angle of incidence PI(φ) of the principal ray on the retina is obtained by these calculations. Because it may be relevant to discussions of photoreceptor orientation it is plotted in Fig. 3.[14,15]

(7) The path length PL(φ) of the principal ray through the crystalline lens can be used to estimate the light absorption. Light absorption depends on the age of the subject[16] and on the wavelength of the light.[17] Neither variable is considered in this paper. Therefore the absorption is not calculated, and only the path length in the lens is plotted (Fig. 4).

The calculations in points (1)–(3), (6), and (7) are done at angles of 0° to 100° with steps of 10°. In points (4) and (5), the Simpson integration routine is used on values calculated in points (1)–(3). Hence the values of the luminous flux L(φ) and the retinal area RA(φ) are obtained at 20°-interval steps. The curves in Figs. 3–9 are drawn by eye through the calculated values.

All calculations are done on a schematic eye with spherical surfaces only[12] and on schematic eyes with surfaces with

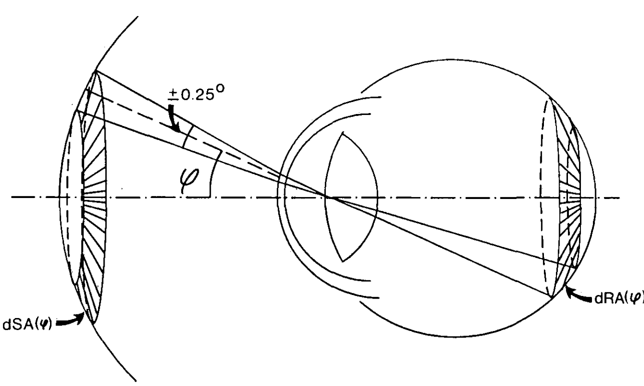

Fig. 2. Retinal area dRA(φ) corresponding to an annular solid angle of visual field dSA(φ) calculated for various values of φ and with an annular width of 0.5°.

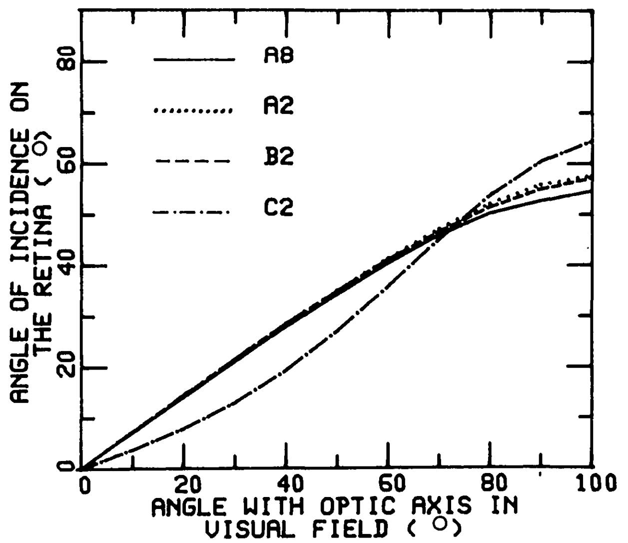

Fig. 3. Angle of incidence on the retina of the principal ray of a light pencil of parallel rays as a function of angle of incidence in the visual field. A, B, and C refer to the eye models A, B, and C in the caption of Table 1. The numbers 2 and 8 indicate that the eye model had a pupil diameter of 2 or 8 mm.

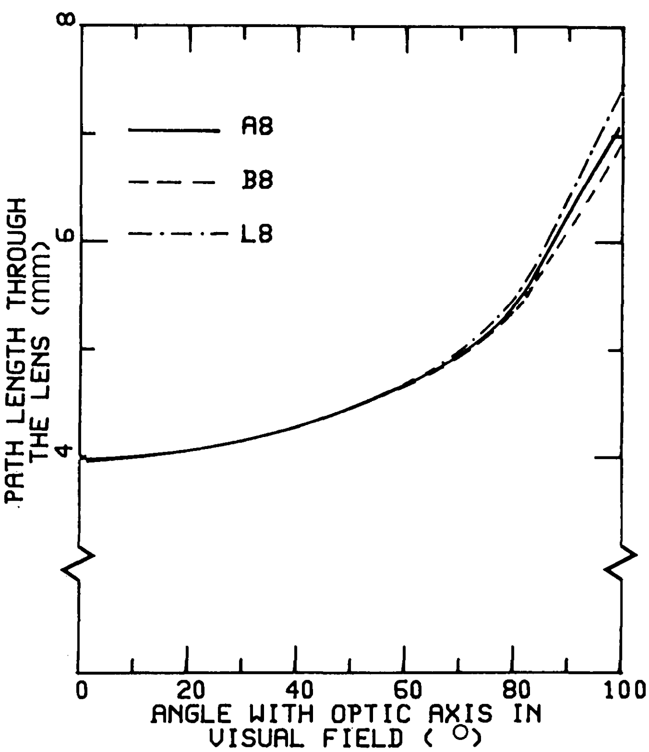

Fig. 4. Path length of the principal light ray through the lens. A and B refer to the eye models A and B in the caption of Table 1. L is calculated in an eye with aspheric anterior and posterior lens surfaces. It is the combination of models D and E in the caption of Table 1.

spherical and aspheric conic-section profiles (Table 1). The cross sections of these surfaces can be described by the conic equation (Ref. 18, p. 392)

$$\left(x - \frac{r}{p}\right)^2 + \frac{y^2}{p} = \frac{r^2}{p},$$

$$\therefore px^2 - 2xr + y^2 = 0,$$

with the x axis along the optic axis and the apex in the origin. The shape of the cross section depends on r and p. The apical radius of curvature r determines the paraxial optical power and is not influenced by the value of the conic constant p. The value of p is less than zero for a hyperbola, equal to zero for a parabola, between zero and one for an ellipse with the longer axis along the x axis, one for a circle, and larger than one for an ellipse with the shorter axis along the x axis. The relation between p and the elliptic eccentricity e (Refs. 8, 19, 20) is given by $p = 1 - e^2$. A typographical error in formula (4) of Ref. 19 confused Drasdo and Fowler[8] at this point but was corrected by Drasdo and Peaston.[20]

The expression $|(b/a) - 1|^{21,}$ with a and b the semiaxis in the x and the y direction, respectively, is a little bit confusing

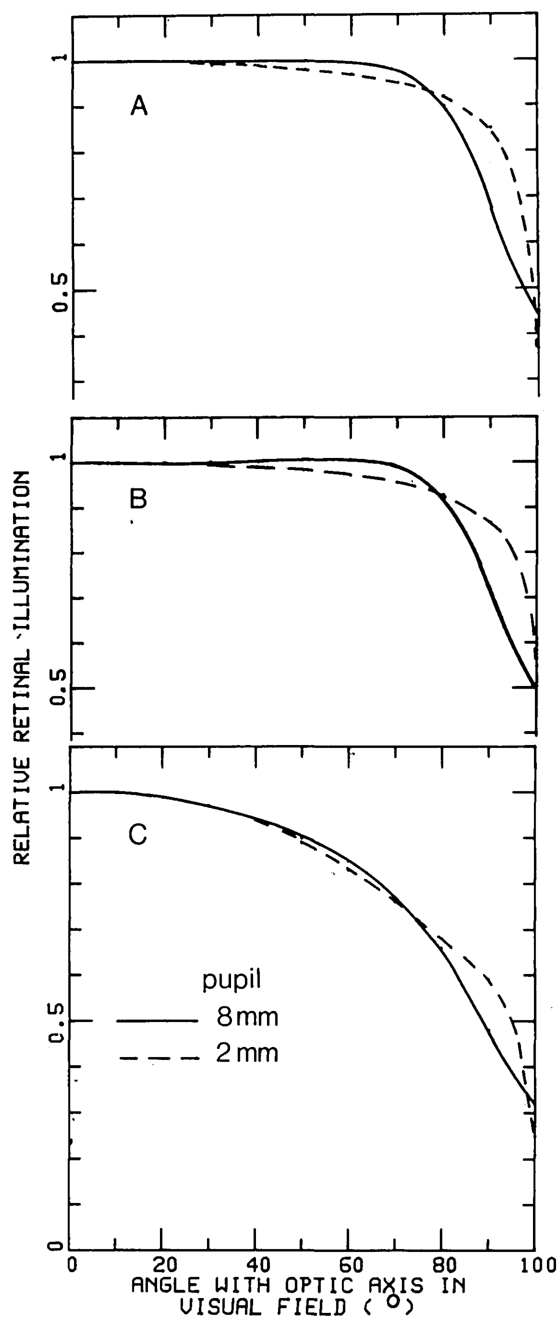

Fig. 5. Retinal illumination. A, B, and C are calculated in eye models A, B, and C as defined in the caption of Table 1.

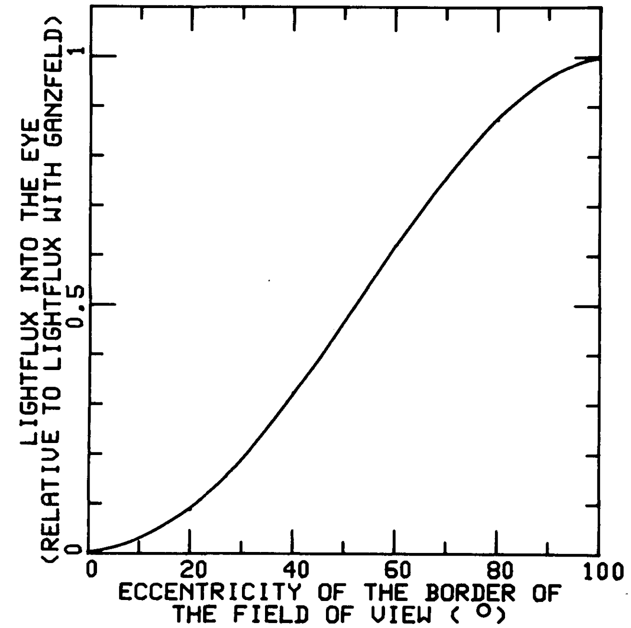

Fig. 6. Relative light flux into the eye with a homogeneous luminance field of limited size. All eye models (irrespective of whether the pupil diameter is 2 or 8 mm) in this study show the same relation between visual field and light flux. With a Ganzfeld with a luminance of 1 cd/m^2 the absolute light flux is 212 μlm with an 8-mm pupil and 13 μlm with a 2-mm pupil.

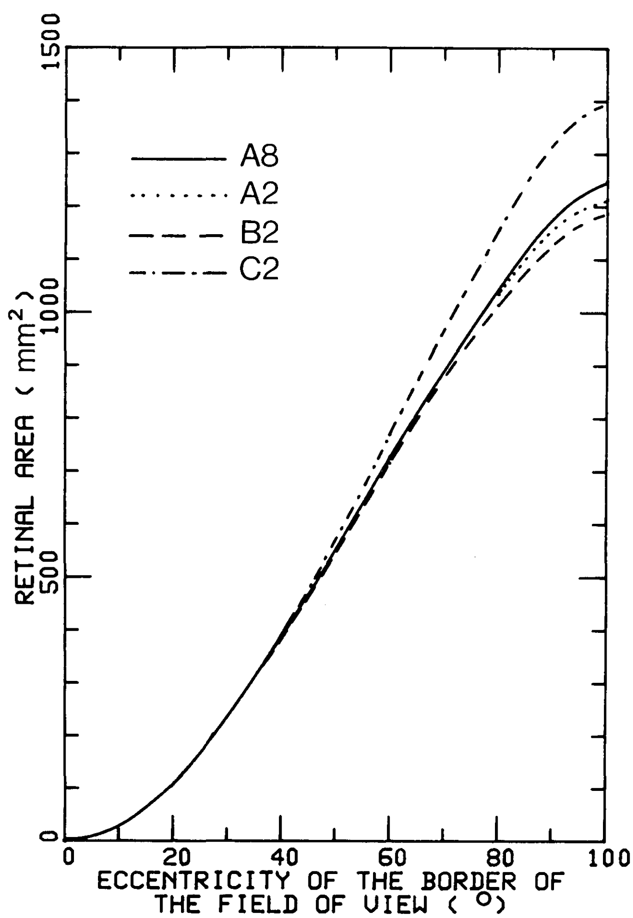

Fig. 7. Retinal area corresponding with angular field of view. A, B, and C refer to the eye models A, B, and C in the caption of Table 1. The numbers 2 and 8 indicate that the eye model had a pupil diameter of 2 or 8 mm.

because it holds only for the conic functions with real values of $b \leq a$ (half of the family of elliptical curves) and imaginary values of b (hyperbolic curves). The relation between $|(b/a) - 1|$ and the conic constant p is

$$\left.\begin{aligned} &\left|\frac{b}{a} - 1\right| \leqslant 1 \\ &p = \left(\frac{b}{a}\right)^2 = \left(1 - \left|\frac{b}{a} - 1\right|\right)^2 \end{aligned}\right\} \quad \text{if } a \text{ and } b \text{ are real and } b \leqslant a,$$

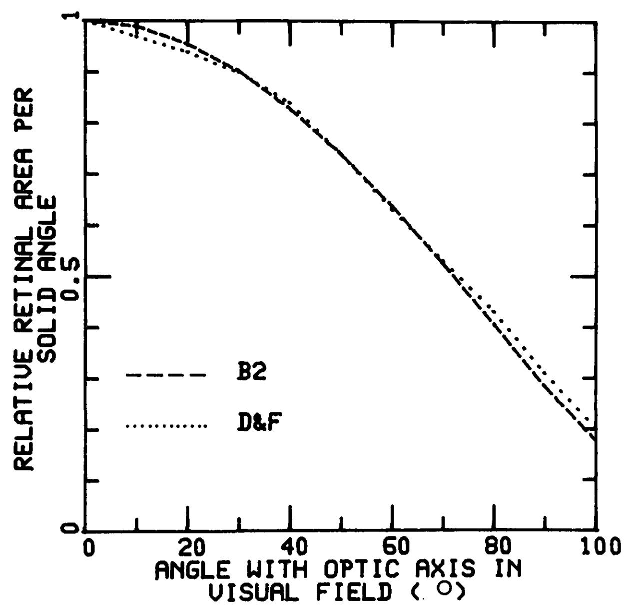

Fig. 8. Retinal area per solid degree in the visual field. B2: Calculated relation in the eye model with aspheric (conic constant $p = 0.75$) anterior and posterior corneal surfaces and spherical lens and retinal surfaces. Pupil diameter, 2 mm. D&F: Relative values calculated by Drasdo and Fowler[8] in an eye model with a single aspheric (conic constant $p = 0.5$) corneal surface and spherical lens and retinal surfaces.

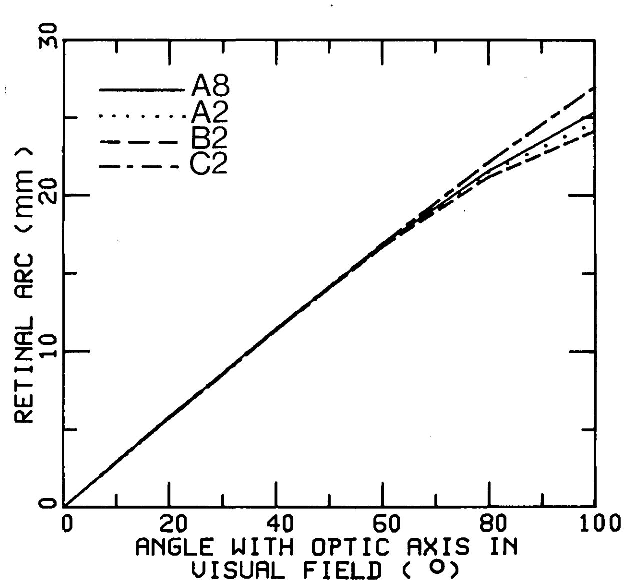

Fig. 9. Length of retinal arc between the optic axis and the point of incidence of the principal ray with eccentricity φ. A, B, and C refer to the eye models A, B, and C in the caption of Table 1. The numbers 2 and 8 indicate that the eye model had a pupil diameter of 2 or 8 mm.

$$\left.\begin{aligned} &\left|\frac{b}{a} - 1\right| > 1 \\ &p = \left(\frac{b}{a}\right)^2 = 1 - \left|\frac{b}{a} - 1\right|^2 \end{aligned}\right\} \quad \text{if } b \text{ is imaginary and } a \text{ is real.}$$

The value of p for the aspherical surfaces was taken from papers on measurements of the curvature of the cornea,[19] the lens,[21] and the retinal surface.[22]

An ellipsoid anterior and posterior corneal surface ($p = 0.75$), a hyperboloid anterior lens surface ($p = -2.06$), and a paraboloid posterior lens surface ($p = 0$) were used in the calculations. The apical radii of curvature were the same as in the calculations with spherical surfaces. Only for the retinal surface was the apical radius of curvature changed with a change in the value of p. In the front-to-back cross section of the eye the retinal surface is considered to be either a part of a circle with a radius of 10.8 mm or a part of an ellipse with $p = 1.346$ and an apical radius of curvature of 14.1 mm. The value of p is taken from the data of Krause.[22] The axial length values (19.72 and 22.88 mm) of Krause are too short to cross the posterior corneal surface near the position of the corneoscleral junction. Therefore slightly larger values are used in these calculations. The lengths of the short and long axes of the retinal surface are taken as 20.96 and 24.30 mm and are, respectively, along and perpendicular to the optic axis.

RESULTS

The results of the calculations on apparent pupil size and retinal area per solid angle are tabulated (Table 2). The apparent pupil area and the retinal area per solid angle are used to calculate retinal illumination (Table 2 and Fig. 5). The apparent pupil area is also used to calculate the luminous flux into the eye in a solid angle centered on the optic axis with various values of stimulus field size (Fig. 6). The retinal area $dRA(\varphi)$ is used to calculate the retinal area $RA(\varphi)$ corresponding to a solid angle around the optic axis and bounded by the principal rays with an angle φ with the optic axis (Fig. 7). The calculations with aspheric anterior or posterior lens surfaces yielded data on apparent pupil diameter, retinal area per solid angle, and retinal illumination, which were nearly identical to the data obtained with the spherical eye model (all differences <2%).

In the calculations no limitations were set on the diameter of the cornea. It appeared from the calculation that the extreme peripheral light rays ($\varphi = 100°$) hit the corneal surface at 6.91 mm (spherical cornea) and 7.32 mm (ellipsoid cornea) perpendicularly from the optic axis.

DISCUSSION

Spherical Eye Model

Calculations are done on an eye model with an iris without thickness. The apparent pupil areas calculated with 2- and 8-mm pupils for oblique directions are in good agreement with the experimentally measured values of Spring and Stiles.[23] In a model with an iris without thickness, vignetting occurs at angles of incidence of 90° and 100°. The incident light rays cannot reach the edge of the pupil at the other side of the optical axis. At that side the entrance pupil is limited by the light rays that are just refracted by the convex crystalline lens surface.

In a model with an iris with some thickness, vignetting occurs at all angles of incidence of 10° and above by the posterior and anterior edge of the iris. Jay[7] measured a smaller pupil area at angles of over 70° than did Spring and Stiles,[23] and he assumed that it was an effect of the thickness of the iris. It has a limited influence on the calculations on the retinal light distribution because the retinal illumination is nearly homogeneous up to 70°, and only at higher angles of incidence will the calculated retinal illumination be changed.

Drasdo and Fowler[8] calculated the retinal area per solid degree. The absolute values in the present study differ from the values of Drasdo and Fowler because of a difference in the axial lengths of the theoretical eye models. The relative

Table 2. Apparent Pupil Diameter, Retinal Area per Solid Angle, and Retinal Illumination Resulting from a Ganzfeld Light Source with a Luminance of 1 cd/m²

ANGLE WITH OPTIC AXIS		(A) SPHERIC EYE MODEL						(B) ASPHERIC CORNEA			(C) ASPHERIC RETINA		
		APPARENT PUPIL DIAMETER (mm)	RETINAL AREA (mm²/degree²)	RET. ILLUM. (lm/m²)	APPARENT PUPIL DIAMETER (mm)	RETINAL AREA (mm²/degree²)	RET. ILLUM. (lm/m²)	APPARENT PUPIL DIAMETER (mm)	RETINAL AREA (mm²/degree²)	RET. ILLUM. (lm/m²)	APPARENT PUPIL DIAMETER (mm)	RETINAL AREA (mm²/degree²)	RET. ILLUM. (lm/m²)
0	ABSOL.	2	.085	.011	8	.085	.18	2	.085	.011	2	.085	.011
0	RELATIVE VALUES	1	1	1	1	1	1	1	1	1		1	1
10		.99	.99	1	.99	.99	1	.99	.99	1		.99	1
20		.96	.96	1	.96	.96	1	.95	.96	1		.97	.99
30		.90	.91	.99	.91	.91	1	.90	.90	.99		.93	.97
40		.83	.84	.99	.84	.84	1	.82	.83	.99		.89	.94
50		.74	.76	.98	.75	.76	1	.73	.74	.98		.83	.89
60		.64	.66	.97	.66	.66	1	.62	.64	.97		.76	.83
70		.52	.55	.95	.54	.55	.98	.50	.53	.96		.68	.76
80		.39	.43	.92	.40	.44	.90	.38	.41	.93		.57	.68
90		.24	.29	.85	.23	.34	.68	.25	.28	.87		.41	.59
100		.07	.19	.37	.11	.25	.45	.12	.18	.55		.28	.25

retinal area per solid degree as a function of peripheral angle is in accord with the results of Drasdo and Fowler[8] (Fig. 8), who used an ellipsoid corneal surface with a conic constant $p = 0.5$. The use of $p = 0.5$ or $p = 0.75$ in the present study does change the curve less than the line width in the figure.

The apparent pupil area determines the light flux entering the eye. The distribution of this flux over the retina depends on the retinal area per solid degree. The relative values (Table 2) show that both change by nearly the same factor as a function of angle of incidence. The relative retinal illumination is the ratio between apparent pupil area and the retinal area per solid degree. The result is that the illumination is surprisingly flat over the retinal surface (Fig. 5A).

The relative light distribution over the retina is barely influenced by the size of the pupil. [One has to realize that a distinct angle of incidence does not necessarily correspond with the same retinal locus if the size of the pupil or the shape of any surface is changed (Fig. 9)].

Aspheric Eye Model

The schematic eye with spherical surfaces is a reasonable representation of a real emmetropic eye for the on-axis optical properties. The off-axis dimensions of real eyes can deviate considerably from the spherical shape. The cross sections of the cornea and the retina can be better described by ellipses.[19,22] The anterior and the posterior human lens surfaces have cross sections that can be described by a hyperbola and a parabola.[21] The sensitivity of the retinal light distribution to modifications in the model of the eye is calculated by using these surfaces in the model. To separate the influences of the surface modifications the calculations are done on the spherical eye model with only one or two surfaces at a time changed into an aspheric shape.

A realistic value of asphericity of the cornea results in a computed light distribution over the retina that is nearly as homogeneous as that calculated for the spherical eye model (Fig. 5B). A change of the anterior or the posterior lens surface into an aspherical shape has only a very small influence on the retinal light distribution (<1.5%). The modification of the anterior lens surface changes the position of the real pupil because it is assumed to be coincident with the anterior lens surface. This results in a small change of the apparent pupil area (<1%). The retinal area per solid angle is influenced (<2%) because the principal rays are a little displaced and cross the anterior lens surface at another point. Modification of the posterior lens surface leaves the apparent pupil size undisturbed and has only a slight influence on the retinal area per solid angle (<1%).

Modification of the shape of the retinal surface has the largest effect on the light distribution on the retina (Fig. 5C). Because at peripheral angles of incidence a larger retinal area is illuminated per solid angle of visual field in comparison with the area illuminated on a spherical retina, the resulting illumination decreases much more with eccentricity than in the models with a spherical retina.

The above theoretical results do not agree with experimental studies on retinal illumination with peripheral angle[10,11] and retinal light distribution with small and large pupil diameters.[9] Both Gross[10] and Young[11] measured the luminance of the retinal image of a small light source as a function of peripheral angle. But the quality of the imaging system deteriorates with peripheral angle, and therefore the

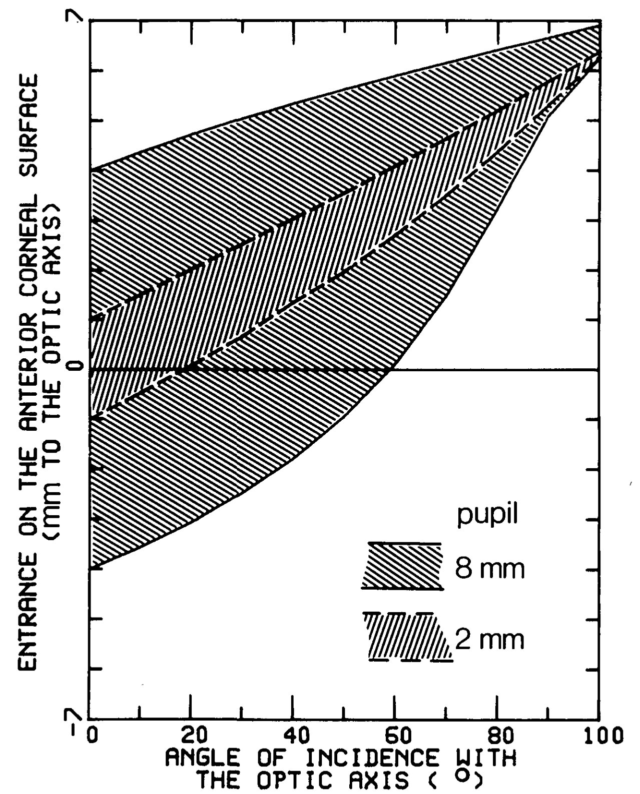

Fig. 10. Corneal arc in the meridional plane, covered by the light rays that pass through the pupil, depends on the angle of incidence and on the size of the pupil. These data are collected with the spherical eye model.

luminance of a small image decreases as the light is spread over a larger area.[24] I have calculated the retinal illumination with Ganzfeld illumination whereby image quality influences retinal illumination less than in the case in which the image of a small light source is being used. If the image of a small light source deteriorates, then the retinal illumination decreases as the light spreads. A Ganzfeld light source can be represented by many small light sources side by side, and the decrease of the illumination in each image will probably be compensated for in the most part by light spread from surrounding images. Bedell and Katz[6] recently combined the experimental results of Jay[7] on pupillary area with the calculations of Drasdo and Fowler[8] on retinal area per solid angle of visual field to calculate the retinal illumination. The conclusion of Bedell and Katz that retinal illumination is nearly constant up to about 80° in the peripheral field is identical to the calculated results in this study (Fig. 5). Hoffman *et al.*[9] measured electroretinogram (ERG) responses on Ganzfeld stimulation with large and small pupil diameters and concluded from their results that the pupil acts not only as an aperture but also as a field stop that influences the distribution of light on the retina. However, they used an ERG contact lens with a clear corneal lens (diameter 9 mm) and a black scleral rim. This configuration particularly influences the peripheral retinal illumination because light rays cannot enter the eye through the peripheral parts of the corneal surface (Fig. 10). The influence of this corneal diaphragm on the light input from peripheral angles is different for small and large pupils. This can be the cause of differences in the ERG responses obtained with small and large pupils.

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