Accommodation-dependent model of the human eye with aspherics

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We consider a schematic human eye with four centered aspheric surfaces. We show that by introducing recent experimental average measurements of cornea and lens into the Gullstrand–Le Grand model, the average spherical aberration of the actual eye is predicted without any shape fitting. The chromatic dispersions are adjusted to fit the experimentally observed chromatic aberration of the eye. The polychromatic point-spread function and modulation transfer function are calculated for several pupil diameters and show good agreement with previous experimental results. Finally, from this schematic eye an accommodation-dependent model is proposed that reproduces the increment of refractive power of the eye during accommodation. The variation of asphericity with accommodation is also introduced in the model and the resulting optical performance studied.

INTRODUCTION

Many experiments in physiological optics as well as in other fields, such as optical design, digital image processing, and robotics, must take into account the behavior and contribution of the optical system of the eye as a part of the whole system. In some cases a generalized behavior characterized by the optical transfer function (OTF) is sufficient, but a more detailed model of the optical system is often needed. Since the beginning of this century, many models have been proposed, progressively developed, and improved. Among them we shall point out those of Helmholtz[1] and Gullstrand.[2] The latter, after being revised by Le Grand,[3] has been widely used for first-order calculations, even though this model does not agree with the measured values of eye aberrations. To obtain better agreement, aspheric surfaces and a graded-index lens have been incorporated in theoretical eye models. In this way Lotmar[4] introduced asphericities for the cornea and the back surface of the lens in the Gullstrand–Le Grand model. He found a spherical aberration of the same order as that of the experimental findings, but the shell structure (or graded index) of the lens was required for off-axis aberrations to be predicted. El Hage and Berny[5] computed the aspheric shape of a two-surface lens that fitted their experimental measurements of the spherical aberration of the eye. Nakao *et al.*[6] showed that the spherical aberration of the lens can be predicted by the actual shell structure and proposed a theoretical eye model in accordance with it. Other eye models[7,8] also take into account the shell structure of the lens. However, because of the increasing complexity of these models (a large number of parameters are involved), attempts to model the optics of the eye by using a schematic two-surface lens are still of great interest. In this way Kooijman[9] has recently used a schematic model, similar to the one proposed in the present paper, to compute the retinal illumination corresponding to a Ganzfeld illuminance field.

It is not expected that a schematic model could predict the aberrations corresponding to the actual eye if the shell structure of the lens is not considered. It is generally agreed that such a model cannot predict off-axis aberrations accurately, but whether it can predict spherical abberation is not so clear. The research of Lotmar[4] and El Hage and Berny[5] indicates that, instead of the shell structure (or graded index) of the lens, an effective refractive index could be used to predict axial-spherical aberration. The resulting model, although not anatomical, is strongly simplified with respect to the shell of graded-index models, and the ray tracing becomes easier.

In the present paper a simple schematic eye, to be used in on-axis calculations, is proposed and tested. (See Fig. 1.) The model considers an optical system formed by four centered quadric refracting surfaces with rotational symmetry; each surface is defined by two parameters, the radius and the asphericity. The variations of the refractive indices with wavelength have been computed to fit the chromatic aberration of the eye. The optical performance of the resulting model has been tested by computing spherical aberration and the polychromatic modulation transfer function (MTF) and the point-spread function (PSF), and it shows good agreement with experimental findings. This fact indicates that the on-axis optical performance of the eye can be modeled without considering the shell structure of the lens.

Attempts to model the accommodation dependence of the eye have also been made by a small number of authors. Gullstrand[2] and Le Grand[3] give different lens parameters for the unaccommodated and for the fully accommodated theoretical eye but not for the more interesting intermediate states. A paraxial model that varies with accommodation was proposed by Blaker.[10] He considers the continuous variation of the shape and graded-index structure of the lens with accommodation. In the present paper an accommodation-dependent model based on our unaccommodated schematic eye is also proposed. Finally, a variation of the lens asphericity is also introduced in the model and tested.

SCHEMATIC EYE

The optimized version of the schematic-eye model has been obtained by introducing some recent average experimental data about the cornea and the lens into the Gullstrand–Le

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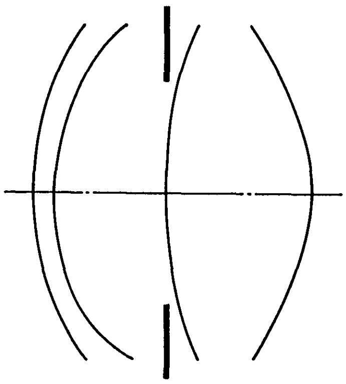

Fig. 1. Schematic eye.

Grand theoretical eye and by computing refractive indices to adjust the chromatic aberration of the eye. Here an aspheric surface is substituted for the spherical cornea of the Gullstrand–Le Grand eye. The radius and the asphericity correspond to an average human cornea obtained from measurements made *in vivo* by Kiely *et al.*[11] Aspherics are also used for the crystalline-lens surface. The asphericity parameters have been taken from the average data obtained *in vitro* by Howcroft and Parker.[12] Here, the Gullstrand–Le Grand radii have been conserved for two main reasons. First, excised lenses show smaller radii than those measured *in vivo*, perhaps because they are in an intermediate accommodation state. Second, the Gullstrand–Le Grand model does predict refractive power of the lens, and hence there is no *a priori* reason to change its radii of curvature. The effective refractive index of the lens has been also kept because it implies a strong simplification of the complex internal structure of the lens, and, as we will see, it allows the on-axis optical performance of the eye to be represented within a good approximation.

The schematic-eye parameters are shown in Table 1. Those of Gullstrand–Le Grand[3] and Kooijman[9] are also represented for comparison purposes. Note that our refractive index of the cornea is different. It has been taken from the original Gullstrand eye[2] to compensate partially for the greater refracting power introduced by a smaller radius (our corneal radius is also close to the original from Gullstrand). Compensation is not total, and the resulting refractive power is slightly greater than that of the Gullstrand–Le Grand eye. The image plane (retina) is then shifted 0.2 mm backward with respect to the Gullstrand–Le Grand eye, and thus the vitreous thickness is assumed to be 16.4 mm. The Kooijman model coincides with that of Gullstrand–Le Grand except for the asphericities. Our anterior lens surface asphericity differs slightly from that of Kooijman, although both of them have been taken from the same source.[12] Each quadric surface of the model is represented by the formula

$$x^2 + y^2 + (1 + Q)z^2 - 2Rz = 0, \qquad (1)$$

where R is the radius of curvature and Q the asphericity parameter; x, y, and z are spatial coordinates, the z axis being the optical axis.

The refractive indices of the ocular media for the different wavelengths have been determined to fit the experimental chromatic aberration[13,14] taking as the departure point the constringences of Polack,[15] cited by Le Grand.[16] Here there are two problems that make comparison with experiments difficult. First, there is a lack of experimental data about dispersions (or constringences) on human eyes,[17] and second, the dispersion of the effective refractive index assumed for the lens cannot be compared with the dispersions of the actual graded-index lens.[18] On the other hand, the Herzberger formula,[19] as a better description of optical media, has been applied here instead of the Cornu formula used by Le Grand to compute the refractive indices.[16] The Herzberger formula is expressed as follows:

$$n(\lambda) = a_1(\lambda)n^{**} + a_2(\lambda)n_F + a_3(\lambda)n_c + a_4(\lambda)n^{*}, \qquad (2)$$

where $n^{**} = n(0.365\ \mu m)$, $n_F = n(0.4861\ \mu m)$, $n_c = (0.6563\ \mu m)$, $n^* = n(1.014\ \mu m)$, and

$$a_1(\lambda) = 0.66147196 - 0.040352796\lambda^2 - \frac{0.2804679}{\lambda^2 - \lambda_0^2} + \frac{0.03385979}{(\lambda^2 - \lambda_0^2)^2},$$

$$a_2(\lambda) = -4.20146383 + 2.73508956\lambda^2 + \frac{1.50543784}{\lambda^2 - \lambda_0^2} - \frac{0.11593235}{(\lambda^2 - \lambda_0^2)^2},$$

$$a_3(\lambda) = 6.29834237 - 4.69409935\lambda^2 - \frac{1.5750865}{\lambda^2 - \lambda_0^2} + \frac{0.10293038}{(\lambda^2 - \lambda_0^2)^2},$$

$$a_4(\lambda) = 1.75835059 + 2.36253794\lambda^2 + \frac{0.35011657}{\lambda^2 - \lambda_0^2} - \frac{0.02085782}{(\lambda^2 - \lambda_0^2)^2},$$

where $\lambda_0^2 = 0.028\ \mu m^2$.

Table 1. Schematic Eye Parameters Compared with Those of Gullstrand–Le Grand and Kooijman

Parameters	Schematic Eye	Gullstrand–Le Grand	Kooijman[a]
Radius of curvature (mm)			
Anterior surface of cornea	7.72	7.8	7.8
Posterior surface of cornea	6.5	6.5	6.5
Anterior surface lens	10.2	10.2	10.2
Posterior surface lens	−6	−6	−6
Asphericity			
Cornea anterior	−0.26		−0.25(+1)
Cornea posterior	0		−0.25(+1)
Lens anterior	−3.1316		−3.06(+1)
Lens posterior	−1		−1(+1)
Thickness (mm)			
Cornea	0.55	0.55	0.55
Aqueous	3.05	3.05	3.05
Lens	4	4	4
Vitreous	16.4	16.6	16.6
Refractive index			
Cornea	1.367	1.3771	1.3771
Aqueous	1.3374	1.3374	1.3374
Lens	1.42	1.42	1.42
Vitreous	1.336	1.336	1.336
Resulting refractive power (diopters)	60.4	59.94	59.94

[a] Kooijman[10] gives conical parameters p instead of asphericity Q; $p = 1 + Q$.

Table 2. Refractive Indices of the Schematic Eye

Ocular Medium	n^{**} (λ = 0.365 μm)	n_F (λ = 0.4861 μm)	n_c (λ = 0.6563 μm)	n^* (λ = 1.014 μm)
Cornea	1.3975	1.3807	1.37405	1.3668
Aqueous	1.3593	1.3422	1.3354	1.3278
Lens	1.4492	1.42625	1.4175	1.4097
Vitreous	1.3565	1.3407	1.3341	1.3273

Table 3. Constringences of the Schematic Eye Compared with Data of Polack and Sivak and Mandelman

Ocular Medium	Theoretical Eye	Polack[a]	Sivak and Mandelman[b]	
Cornea	56.5	56	54 (cow)	61 (cat)
Aqueous	49.61	53.3	56 (water)	61 (cat)
Lens	48	52	29 (periphery)	35 (core)
Vitreous	50.9	53.3	56 (water)	60 (cat)

[a] Ref. 15.
[b] Ref. 17.

Table 2 shows the refractive indices n^{**}, n_F, n_c, and n^* of the ocular media of the schematic eye that we finally obtained, and Table 3 shows the constringences, defined as $(n_D - 1)/(n_F - n_C)$ compared with those of Polack[15] and those of Sivak and Mandelman[17] (from cat, cow, water, and the human lens). These indices agree only in their order of magnitude. As mentioned above, our refractive indices and constringences have been computed to fit chromatic aberration, and thus we have not tried to make them anatomical.

SPHERICAL AND CHROMATIC ABERRATIONS

In order to test the model, the on-axis aberrations of the schematic eye have been computed by ray tracing. The resulting longitudinal-spherical aberration (LSA) of the schematic eye as well as the contributions of the cornea and the lens are shown in Fig. 2. The parabolic adjustment made by Van Meeteren[20] from experimental data of several authors[13,21–23] and the spherical aberration of the Gullstrand–Le Grand eye are also included in the figure for comparison purposes. A good agreement with the Van Meeteren adjustment is obtained, whereas the Gullstrand–Le Grand eye predicts larger aberration. With respect to the contribution of the cornea and the lens to the spherical aberration, our results are in agreement with those of El Hage and Berny[5]; the lens has a spherical aberration of opposite sign from and smaller than the cornea, playing a compensatory role. There is agreement in average aberration values, but disagreement can be found in individuals' eyes because monochromatic aberrations show wide variations among subjects.[13,21–24] Chromatic aberration, the most important in foveal vision, has more similar values for different eyes than monochromatic aberration. This is probably because the refractive indices and chromatic dispersions, which depend on the composition of the media, are practically the same for all subjects (only Millodot and Newton[25] reported a possible change of refractive indices with age), whereas the shapes of the refracting surfaces have bigger variations. As has been pointed out above, the refractive indices of our schematic eye have been computed to fit the experimentally measured chromatic aberration, using the Herzberger formula. The chromatic aberration was computed by paraxial-ray tracing, and the refractive indices were progressively changed until the experimental aberration was obtained. Refractive indices are shown in Table 2, and the good agreement of chromatic aberration with the experimental findings of Ivanoff[13] and Wald and Griffin[14] can be seen in Fig. 3.

On the other hand, night myopia also appears in this eye model. With an 8-mm pupil diameter, the best image was found in a plane shifted 0.77 D from the paraxial-focus plane. This value, added to 0.6–0.8 D, which corresponds to the state of accommodation in darkness,[26,27] gives the mean value estimated for night myopia (about 1.5 D).

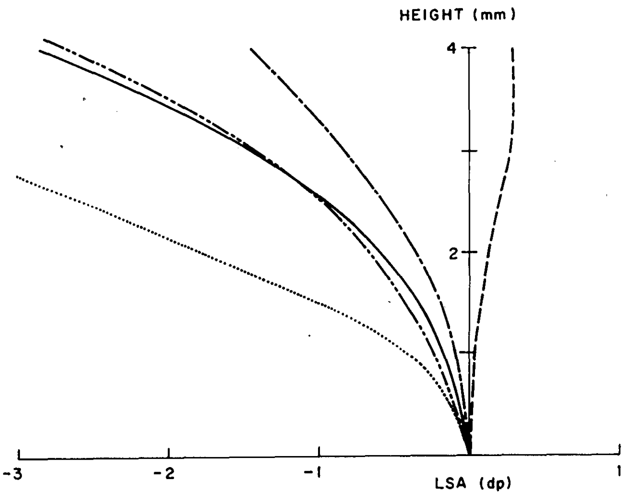

Fig. 2. LSA of the cornea (– — — —), of the lens (— —), and of the whole theoretical eye (—). The results corresponding to the Gullstrand–Le Grand eye (. . .) and to Van Meeteren's parabolic adjustment (– – – – –) are also represented. (dp, diopters.)

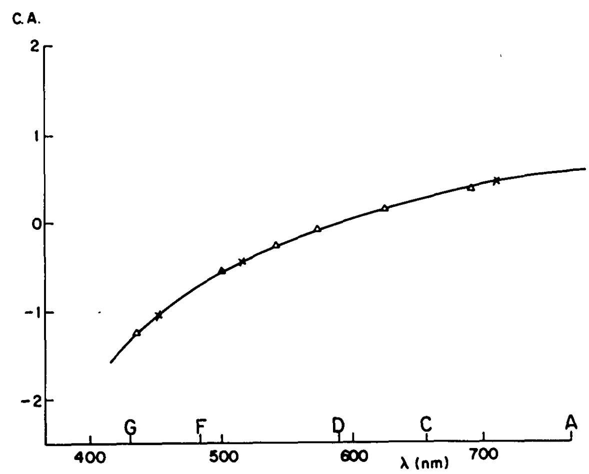

Fig. 3. Longitudinal chromatic aberration (C.A., in diopters) of the theoretical eye (—). Experimental results of Wald and Griffin (Δ) and of Ivanoff (X) are also represented.

DIFFRACTION PATTERNS AND THE MODULATION TRANSFER FUNCTION

The polychromatic PSF and the MTF corresponding to the schematic eye have been computed in the plane of maximum illuminance for several pupil diameters. The computing method is described in what follows:

The pupil function for each wavelength is obtained by polynomial fitting of the LSA, the wave aberration being obtained by integration. The chromatic aberration (treated as a defocusing) and the Stiles–Crawford appodizing effect are also introduced, resulting in the pupil function

$$P(r) = \exp(-0.05R_p{}^2r^2 \ln 10)[\exp ik(\delta_0 W_{20}r^2 + W_{40}r^4 + W_{60}r^6 + W_{80}r^8 + \ldots)], \qquad (3)$$

where $0 \leq r \leq 1$ is the normalized pupilar radial coordinate; R_p is the pupil radius (in millimeters); $\delta_0 W_{20}$ is a defocusing that accounts for the chromatic aberration and eventually for an additional shift of focus; and W_{40}, W_{60}, $W_{80}, \ldots$ are the resulting coefficients of the spherical-wave aberration. The first factor $\exp(-0.05R_p{}^2r^2 \ln 10)$ is the effective transmittance produced by the Stiles–Crawford effect.[20]

The monochromatic PSF and MTF are computed by Fourier transform and autocorrelation of the pupil function, respectively. Then the polychromatic functions are computed by integration of the monochromatic ones along the visible spectrum sampled in 40 intervals. Integration is made for the D_{65} spectral distribution and for the spectral-sensitivity functions of the eye to obtain the tristimulus values.[28] The maximum-illuminance plane is found by progressive shifts of focus.

The considerable influence of the Stiles–Crawford apodization is shown in Fig. 4. The figure represents the monochromatic MTF for a 4-mm pupil diameter with and without considering the Stiles–Crawford effect. The resulting distribution of illuminance and chromaticity along a radius of the polychromatic PSF and the polychromatic MTF are represented in Fig. 5 for several pupil diameters.

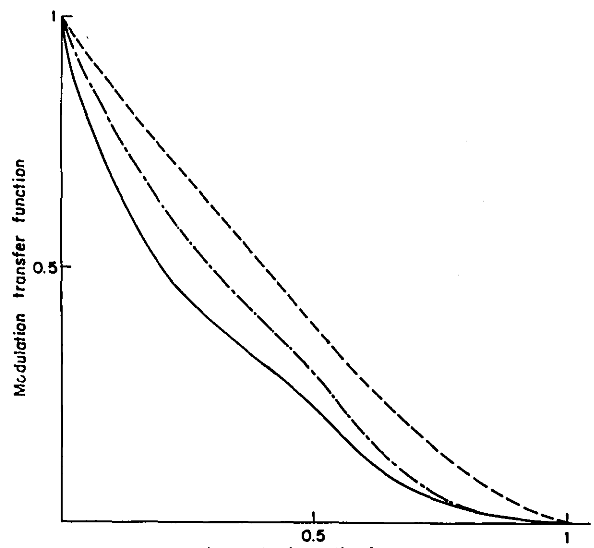

Fig. 4. Influence of Stiles–Crawford effect on the monochromatic MTF of a 4-mm pupil-diameter schematic eye: aberration-free system (– –), eye model without apodization (—) and with apodization (– – – –).

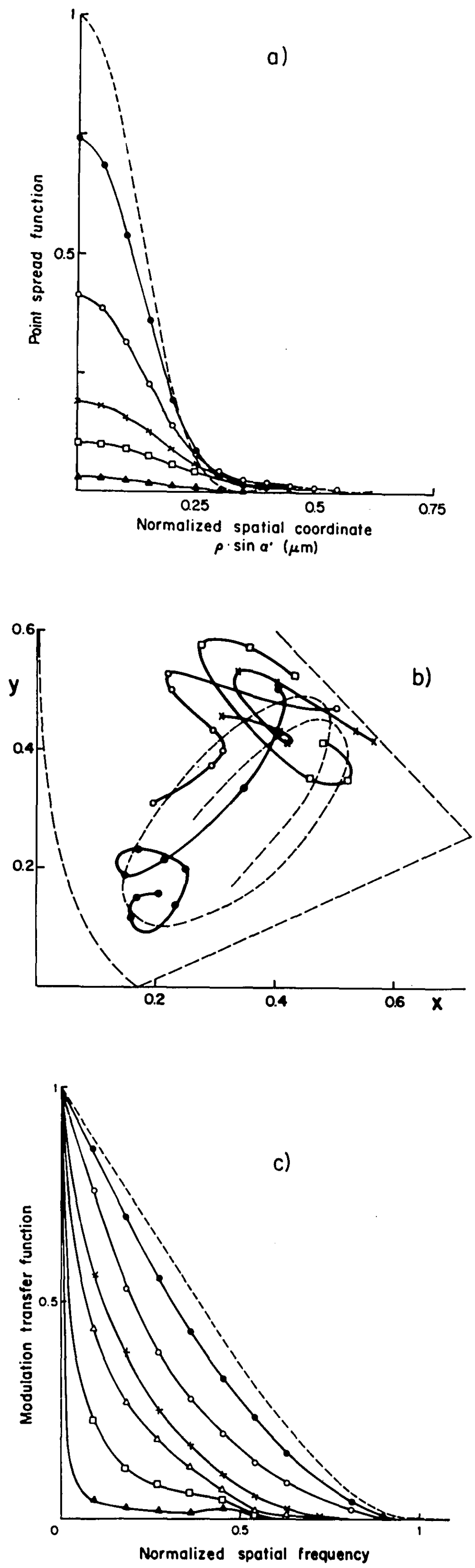

Fig. 5. Distribution of a) illuminance and b) chromaticity coordinates (x, y) of the polychromatic PSF and c) the polychromatic MTF for the schematic eye with the following pupil diameters: 2 mm (●), 3 mm (○), 4 mm (×), 5 mm (△), 6 mm (□), and 8 mm (▲). $\nu = 1$ corresponds to the cutoff frequency at $\lambda = 500$ nm, ρ is the radial coordinate on the image plane, and α' is the semiaperture angle. Dashed curves correspond to the aberration-free system.

A comparison with experimental measurements is made in Figs. 6 and 7. Figure 6 shows the theoretical MTF for a 4-mm pupil diameter compared with the experimental findings of Arnulf,[29] who measured the MTF directly, and those of Campbell and Gubisch,[30] who computed it by Fourier transform from their experimental line-spread function. The theoretical MTF lies between the two experimental curves. The Strehl ratio, defined as the ratio of the maximum illuminance in the PSF formed by a real system to the maximum illuminance in the PSF formed by an ideal optical system working at the same aperture and with the same spectral composition of light, is represented versus pupil diameter in Fig. 7. The figure shows the Strehl ratio for the schematic eye (theoretical) and for the experimental data of Gubisch.[31]

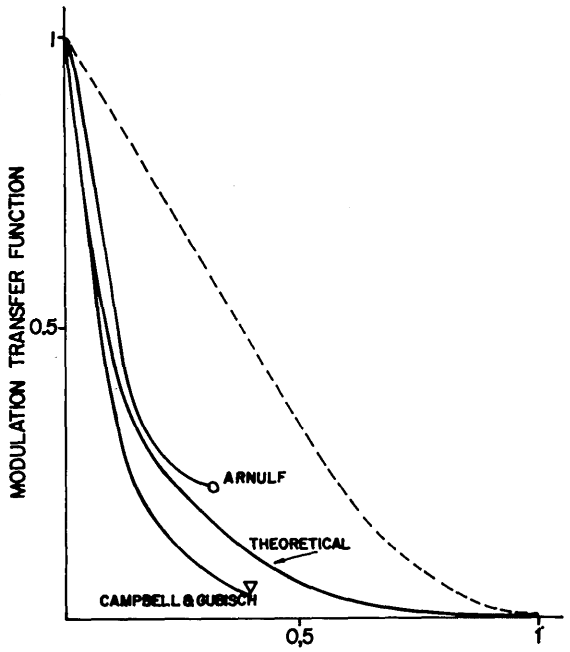

Fig. 6. Polychromatic MTF of the schematic eye (theoretical) computed for 4-mm pupil diameter, compared with experimental curves from Arnulf (○)[29] and Campbell and Gubisch (▽).[30] (The last corresponds to 3.8-mm pupil diameter.)

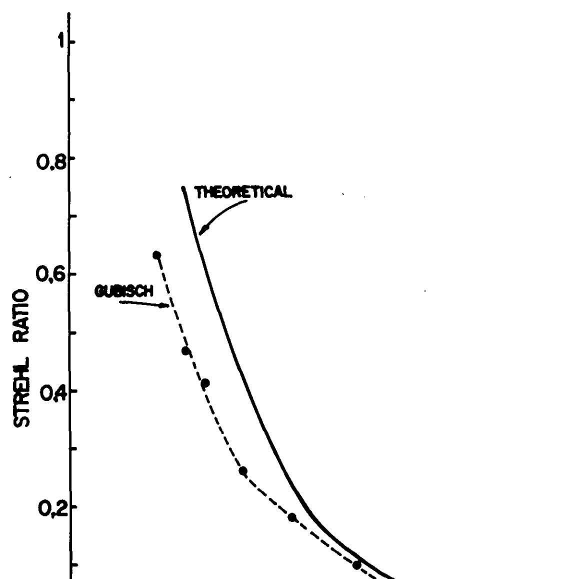

Fig. 7. Strehl ratio versus pupil diameter for the schematic eye (theoretical) compared with experimental findings from Gubisch (●).[31] They differ by an almost constant factor of the order of 0.7.

The theoretical Strehl ratio is higher than the experimental one, and both of them are related by an almost constant factor (about 0.7). This discrepancy can be explained in the following way: The experiment was made by a double pass through the optical system of the eye[30,31] that includes a diffuse reflection on the retina, and thus the actual retinal image is expected to have a little better quality. Moreover, irregular aberrations[32,33] have not been considered in the model. According to Van Meeteren,[20] their influence on the image is not greater than a defocusing of 0.15 D. We should remember that the PSF and the MTF have been obtained in the plane of maximum illuminance (maximum Strehl ratio), and the inclusion of a small defocusing of about 0.15 D in the computations would be enough to produce a better fitting.

ACCOMMODATION-DEPENDENT MODEL

As we have shown above, a model with four aspheric surfaces can predict the on-axis performance of the average eye. Thus, if a paraxial accommodation-dependent model with four surfaces is available, the shapes (asphericities) of both anterior and posterior lens surfaces could be adjusted to fit spherical aberration for each accommodation state in the same way as was done by El Hage and Berny[5] to fit the aberration of the unaccommodated lens. In what follows, a paraxial model that varies continuously with accommodation is proposed first. Then, instead of computing asphericities for each accommodation state, a functional variation of the asphericities with accommodation is introduced in the model and tested.

It is well known that the change in the anterior radius of the lens is the major factor in the increment of refracting power of the lens with accommodation, the posterior radius and the thickness having a secondary role. This continuous change of the anterior radius can be represented by a mathematical function expressed by Blaker,[10] who used a linear function. Other authors have used a variation of the radius inversely proportional to the accommodation in theoretical computations.[34] However, experimental measurements[13,26,35,36] show wide variation of the form of the dependence. We looked for a simple mathematical function that would connect both unaccommodated and fully accommodated anterior radii of the Gullstrand–Le Grand schematic eye and would have a similar shape to the Ivanoff experimental average curve.[13] The function finally adopted was logarithmic. Comparison of Ivanoff's mean curve and the adopted function is made in Fig. 8, showing the similarity of the shapes. The same function has also been used for the variation of the posterior radius and the thickness of the lens by fitting averaged values from Brown.[36] The aqueous thickness is decreased by half of the increase of the lens thickness. The resulting functions are given in Table 4.

These changes during accommodation do not predict the total increment in refractive power of the lens, which is given by the change of the internal graded-index structure of the

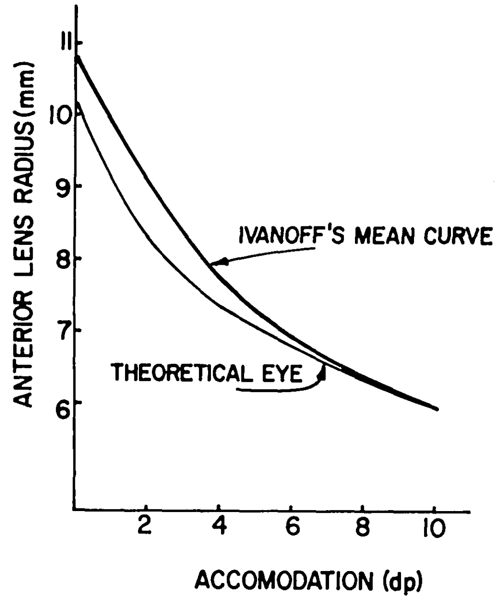

Fig. 8. Variation of the anterior lens radius with accommodation. The figure shows the experimental mean curve from Ivanoff[13] and that adopted for the schematic theoretical eye. They have similar shapes, but the theoretical eye curve differs somewhat from that of Ivanoff because our schematic eye starts from the Gullstrand–Le Grand eye, whose anterior unaccommodated radius differs from that of Ivanoff. (dp, diopters.)

Table 4. Accommodation Dependence of the Lens Parameters on Accommodation A (in diopters)

Lens Parameter	Accommodation Dependence
Anterior lens radius	$R_3(A) = 10.2 - 1.75 \cdot \ln(A + 1)$
Posterior lens radius	$R_4(A) = -6 + 0.2294 \cdot \ln(A + 1)$
Aqueous thickness	$D_2(A) = 3.05 - 0.05 \cdot \ln(A + 1)$
Lens thickness	$D_3(A) = 4 + 0.1 \cdot \ln(A + 1)$
Lens refractive index	$n_3(A) = 1.42 + 9 \times 10^{-5} \cdot (10 \cdot A + A^2)$

lens with accommodation. However, an effective refractive index instead of the graded-index structure has been used in the model. In order to account for the additional necessary increment in refractive power, the intracapsular mechanism of accommodation (postulated by Gullstrand[2]), by which the effective refractive index of the lens also changes with accommodation, has been introduced in the model. This mechanism is fictitious and must be considered not as an actual fact but as a mathematical artifice that permits a strong simplification of the modeling. As a first approximation, a parabolic adjustment has been made to fit the refractive powers for 5 and 10 D of accommodation. The resulting expression is also shown in Table 4. Figure 9 shows the resulting refractive power of the eye, computed by paraxial-ray tracing, versus accommodation. Starting from this paraxial model and from the asphericities of the unaccommodated schematic eye, the case in which the asphericities also have a logarithmic dependence on accommodation, with an amount of change of the same order of magnitude than experimental findings,[36] has been considered. The adopted functions are in Table 5. With these functions the spherical aberration has been computed for several accommodations, and the results are represented in Fig. 10. Agreement with published experimental results has been found in an individual eye[37] (0 D and 3.4 D of accommodation).

The effective refractive indices have been computed in order to keep the chromatic aberration for each accommodation state (in longitudinal units) equal to that of the unaccommodated eye. This implies that the chromatic aberration, expressed in diopters, increases slightly with accommodation,

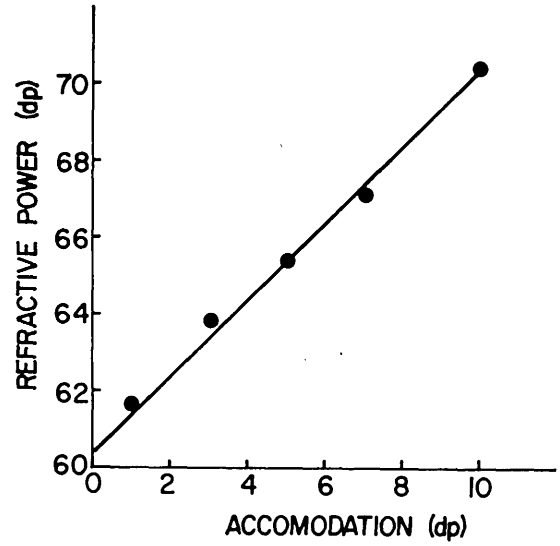

Fig. 9. Refractive power of the schematic eye versus accommodation (●). The straight line corresponds to the ideal fitting. (dp, diopters.)

Table 5. Test Parameters Used in Optical Performance Computations

Asphericities				
Anterior lens asphericity	$Q_3(A) = -3.1316 - 0.34 \cdot \ln(A + 1)$			
Posterior lens asphericity	$Q_4(A) = -1 - 0.125 \cdot \ln(A + 1)$			
Refractive Indices of the Lens				
Accommodation (diopters)	n^{**}	n_F	n_c	n^*
3	1.4511	1.4298	1.421	1.4134
5	1.4533	1.43313	1.42423	1.4162

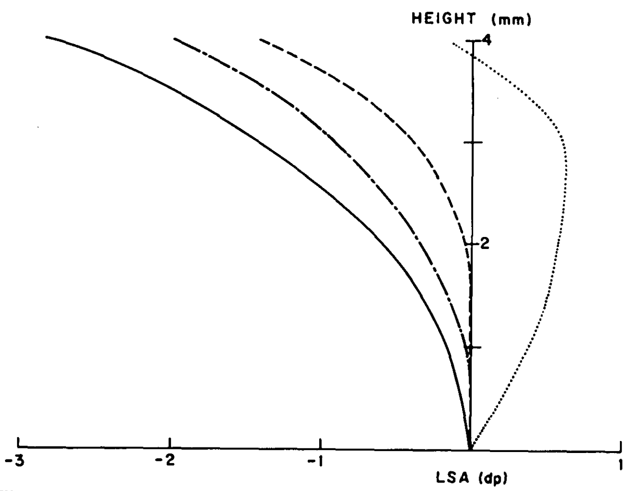

Fig. 10. LSA obtained for several accommodation states: unaccommodated (—), 3 D (- — — —), 5 D (- -), and 10 D (. . .). (dp, diopters.)

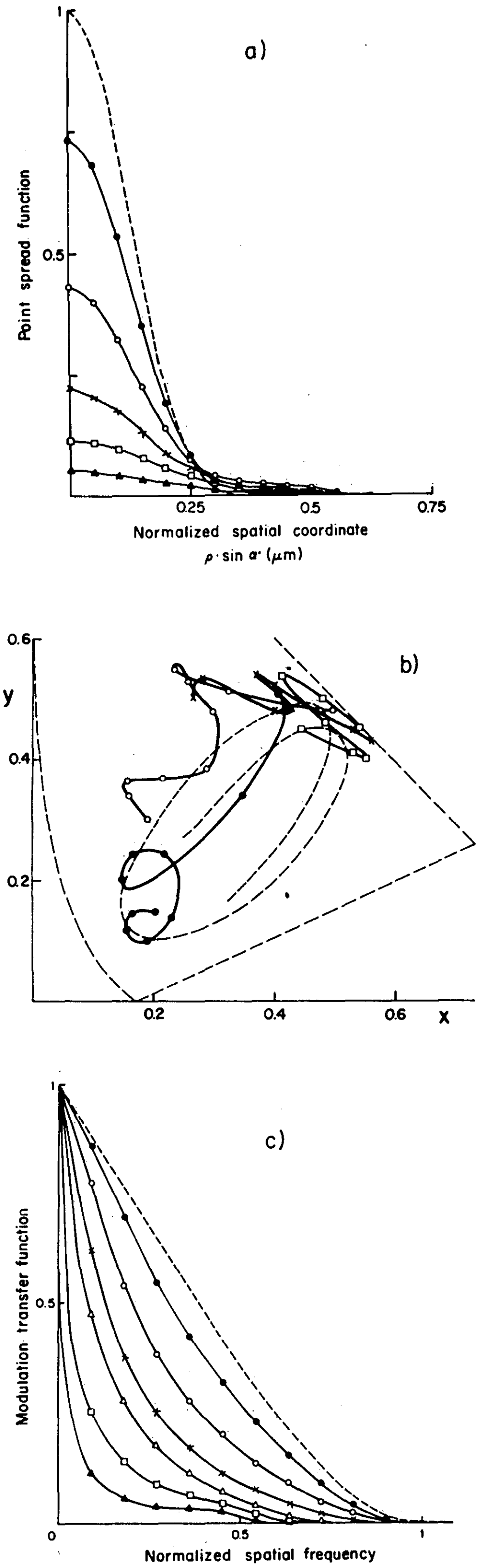

Fig. 11. Distribution of a) illuminance and b) chromaticity coordinates (x, y) of the polychromatic PSF and the c) polychromatic MTF obtained for the 3 D accommodated eye with the following pupil diameters: 2 mm (●), 3 mm (O), 4 mm (X), 5 mm (Δ), 6 mm (□), and 8 mm (▲). $\nu = 1$ corresponds to the cutoff frequency at $\lambda = 500$ nm, ρ is the radial coordinate on the image plane, and α' is the semiaperture angle. Dashed curves correspond to the aberration-free system.

in accordance (in a first approximation) with the effect observed experimentally.[38] The resulting refractive indices for 3 and 5 D are shown in Table 5. Finally, the polychromatic PSF and MTF have also been computed for several pupil diameters and accommodations, and the results for the 3-D accommodated eye are shown in Fig. 11 (the results for 4 and 5 D are very similar). This simple model, although it does not accurately predict average spherical aberration, does predict the small improvement in optical performance for intermediate accommodation states (near vision) that is usually observed.

DISCUSSION AND CONCLUSIONS

Unaccommodated Schematic Eye

A schematic eye with four aspheric surfaces is proposed in this paper. It is shown that, by introducing recent average experimental measurements about the cornea and the lens into the Gullstrand–Le Grand eye, the experimentally observed average spherical aberration is predicted. The main difference between our model and the previous work of El Hage and Berny[5] is that they computed the shape of the lens surfaces to fit the experimental spherical aberration, whereas the asphericities introduced in our schematic eye have been taken from anatomical measurements.[12] The resulting model does predict average spherical aberration without any shape fitting. This fact seems to be in disagreement with previous findings[6] that show that the graded-index structure of the lens, not considered in our schematic eye, has an important role on the state of spherical correction of the lens. In this way a schematic eye that does not include the graded-index lens is not expected to reproduce the spherical aberration of the actual eye. However, Lotmar[4] pointed out that not only the graded-index structure of the lens but the asphericities (shapes) have an important role in the spherical aberration of the eye. In fact, he found order-of-magnitude agreement in spherical aberration just by introducing asphericities in the cornea and in the back surface of the lens. He also pointed out that, on the contrary, off-axis aberrations cannot be predicted without considering the graded-index lens. Therefore it appears that asphericities play the major role in spherical aberration, whereas off-axis behavior is managed by the graded-index structure of the lens. On the other hand, it must be taken into account that the asphericities of the lens surfaces used in our model were measured from excised lenses,[12] the corresponding radii of curvature being smaller than those measured *in vivo*. Perhaps both radii and asphericities should correspond to an intermediate state of accommodation, and consequently the actual asphericities of the unaccommodated eye could differ somewhat from those used in the model. Such a difference (expected to be small) could explain the apparent contradiction of predicted spherical aberration with anatomically determined asphericities and without considering the graded-index lens.

The schematic eye proposed in this paper is similar to the one described by Kooijman in Ref. 9, which was published when our work was almost finished, and therefore a similar monochromatic optical performance must also be expected. He used his model as a wide-angle eye to compute retinal illumination produced by uniform field but not to reproduce the optical performance of the eye. Nevertheless he found better results with his aspheric model than with the Gullstrand–Le Grand model.

It must be pointed out that the refractive indices proposed (Table 2) differ from those of Le Grand[16] and are not anatomical. They have been computed to fit the chromatic aberration of the whole eye, and so the model could fail to predict chromatic aberration of the different components of the eye. The contribution of each ocular medium to the chromatic aberration has been calculated by Millodot and Newton[25] using Le Grand's model.

The model predicts the on-axis optical performance of an average eye. Disagreements could be found if it is compared with individual eyes because of the wide variation in the spherical aberration from different eyes. Some multilayered or graded-index models can fit individual spherical aberration,[7] but they need a large number of parameters (a greater number if the polychromatic performance also has to be modeled). In spite of their complexity they do not predict irregular aberrations or intraocular scattering. Therefore simplified eye models are still useful, mainly in those cases in which on-axis performance is required. Irregular aberrations are caused by the lack of rotational symmetry, decentering of the surfaces, etc., and they have an influence on the image no greater than a defocusing of 0.15 D, according to Van Meeteren.[20] However, more recent work on monochromatic aberrations of the eye[33] attributes a more important role to the irregular aberrations, even more important than the spherical aberration. This seems reasonable for individual subjects, because the eye is not a rotationally symmetric system and, on the other hand, the fovea is placed off the optical axis of the eye. However, when an average eye is modeled, the average aberration is then composed mainly of a rotationally symmetric aberration (spherical) and a residual aberration (irregular). Also, the Stiles–Crawford effect and the chromatic aberration soften the relative influence of the monochromatic irregular aberrations.

Accommodation-Dependent Model

Based on the unaccommodated eye model, a schematic eye that varies continuously with accommodation is also proposed. A logarithmic variation for the radius and the thickness of the lens has been adopted instead of the linear variation proposed by Blaker.[10] Both kinds of variation are approaches to the actual changes during accommodation. Linear variation has been found in a few individual cases, while logarithmic variation appears to be a better approach to average variation. The intracapsular mechanism of accommodation has been introduced in the model to account for the increment of refractive power produced by the change in the graded-index structure of the lens during accommodation. The results obtained in the accommodation-dependent case show that it is a better approach than previous paraxial models. In fact, although spherical aberration of the accommodated eye is not accurately predicted, the resulting polychromatic optical performance seems reasonable. Further improvements on the accommodation-dependent model can be made by computing more adequate asphericities and constringences, although more experimental data are needed to permit effective modeling.

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