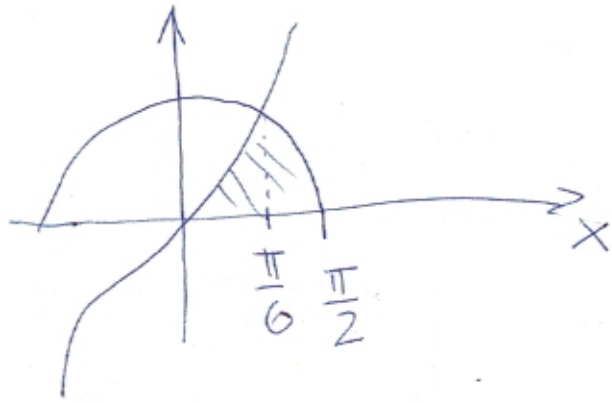
1) OBSAH:

$y = \operatorname{tg} x$

$y = \frac{2}{3}\cos x$

$y = 0$



PRŮSEČÍK

$\operatorname{tg} x = \frac{\sin x}{\cos x} = \frac{2}{3}\cos x$

$\sin x = \frac{2}{3}\cos^2 x = \frac{2}{3}(1 - \sin^2 x)$

$2\sin^2 x + 3\sin x - 2 = 0 \qquad \sin x = t$

$2t^2 + 3t - 2 = 0$

$t_1 = -2$ ×

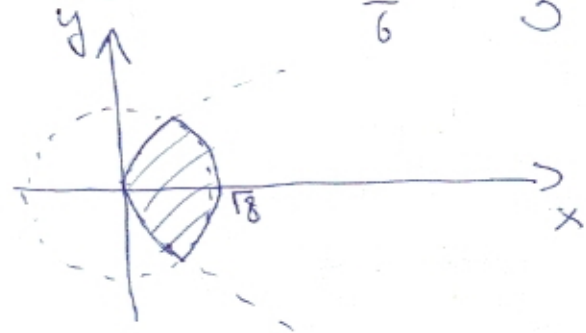
$t_2 = \frac{1}{2} \Rightarrow \sin x = \frac{1}{2} \quad x = \frac{\pi}{6}$

$S_1 = \int_0^{\frac{\pi}{6}} \operatorname{tg} x \, dx = \int_0^{\frac{\pi}{6}} \frac{\sin x}{\cos x} = \left[-\ln|\cos x|\right]_0^{\frac{\pi}{6}} = 0{,}14$

$S_2 = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{2}{3}\cos x \, dx = \frac{2}{3}\left[\sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \frac{1}{3}$

2) $x^2 + y^2 = 8$

$y^2 = 2x$



PRŮSEČÍK:

$x^2 + y^2 - 8 = y^2 - 2x$

$x^2 + 2x - 8 = 0$

$x_1 = +2$ ✓

$x_2 = -4$ ×

$S = 2(S_1 + S_2)$

$S_1 = \int_0^2 \sqrt{2x}\, dx = \left[\sqrt{2} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2 = \frac{8}{3}$

$S_2 = \int_2^{\sqrt{8}} \sqrt{8 - x^2}\, dx = \left| \begin{array}{l} x = \sqrt{8}\sin t \\ dx = \sqrt{8}\cos t\, dt \\ x^2 = 8\sin^2 t\, dt \\ x = \sqrt{8} \ldots t = \pi/2 \\ x = 2 \ldots t = \frac{\pi}{4} \end{array} \right| = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 8\sqrt{1 - \sin^2 t} \cdot \cos t \, dt$

$$L_2 = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 8\cos^2 t\,dt = 8\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1+\cos 2t}{2}\,dt = 8\cdot\left[\frac{1}{2}t + \frac{1}{4}\sin 2t\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \left[4t + 2\sin 2t\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = 2\pi - \pi - 2 = \pi - 2$$

$$S = 2\left(\frac{8}{3} + \pi - 2\right) = \frac{4}{3} + 2\pi$$

3) OBJEM

$y = x^2$

$y = x$

$$V = \pi\int_0^1 (x^2 - x^4)\,dx = \pi\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1 = \frac{2\pi}{15}$$