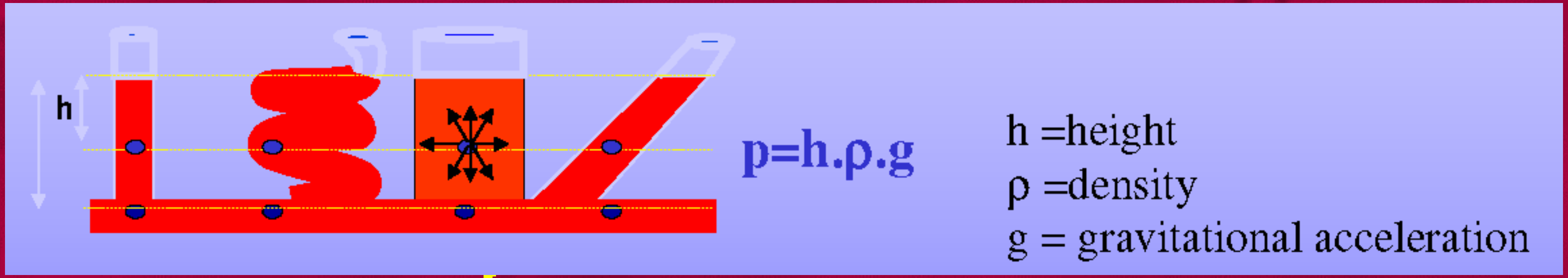
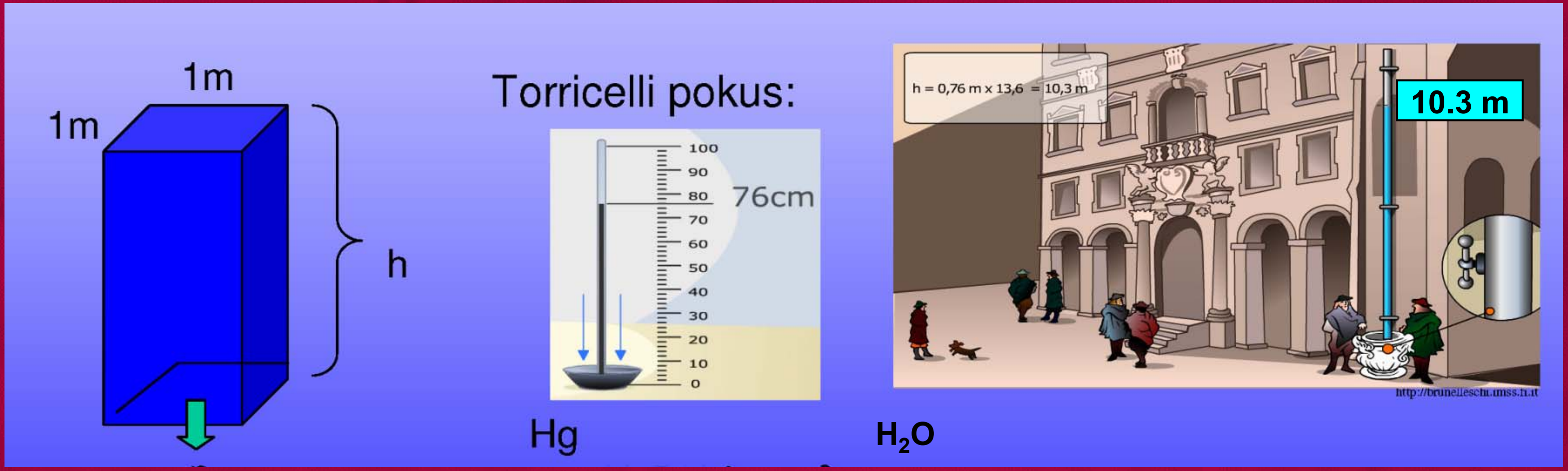
Rheology of blood circulation

1. Basic physical laws of liquids

Law of Pascal

Liquid column causes a pressure (hydrostatic pressure) that is directly proportional to the height of the liquid column (h), density of the liquid (ρ) and gravitational acceleration (g).





Pa

mm Hg

mm H_2O

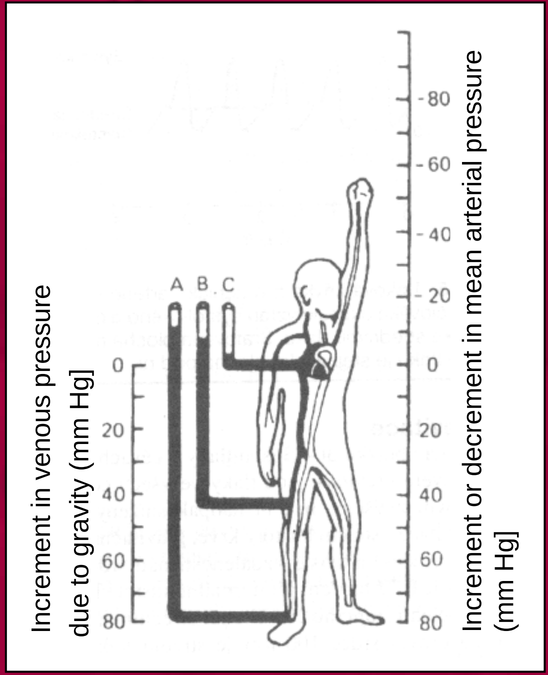
133,322 Pa = 1 mm Hg

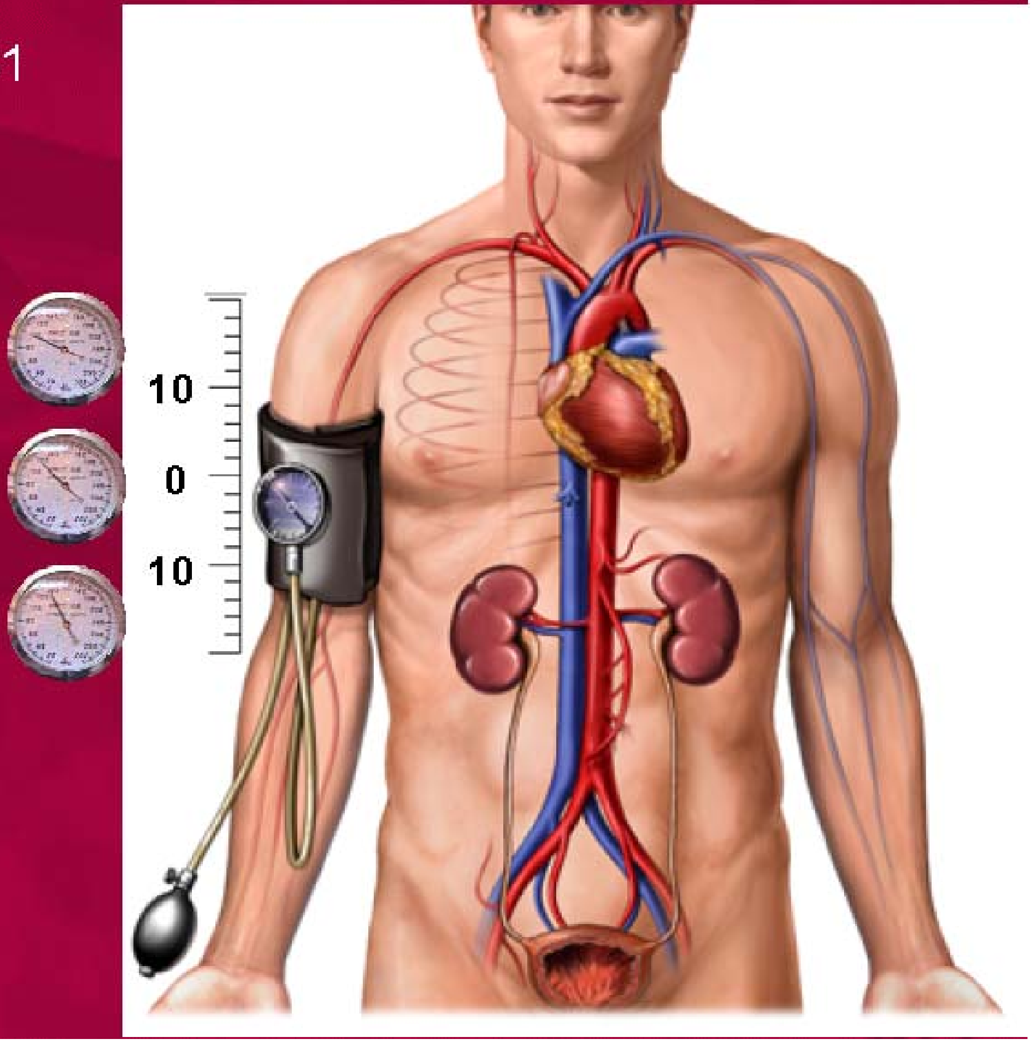
760 mmHg= 1 atm = 10.3 m H_2O

Effect of gravity on arterial and venous pressure

Per each 10 cm

$$\Delta p = \Delta h . \rho_{krve} . g = 0{,}1 \ . \ 1\,065 \ . \ 9{,}81$$
$$= 1\,045 Pa = 7.8 \text{ mm Hg}$$





Law of Laplace

Relation between distending pressure (P [N/m²]) and tension in the wall of hollow object (T [N/m]) :

$$T = \frac{P}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

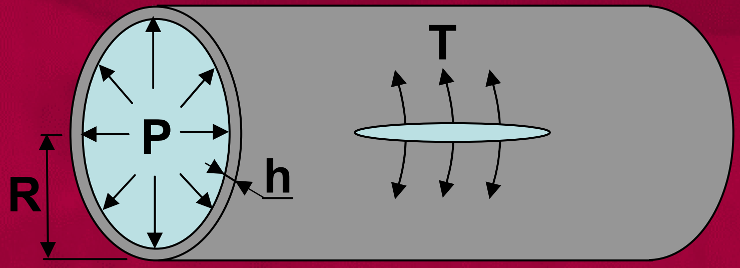
R_1 and **R_2** are the biggest and the smallest radii of curvature

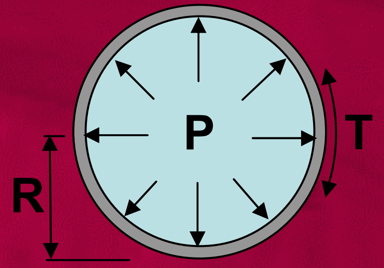
For vessel:

$R_2 = \infty \Rightarrow$ **$T = P \cdot R$**

For sphere:

$R_1 = R_2 \Rightarrow$ **$T = P \cdot R/2$**





Considering thickness of vessel wall (h [m]): **$T = P \cdot R/h$ [N/m²]**

Characteristics of vessels

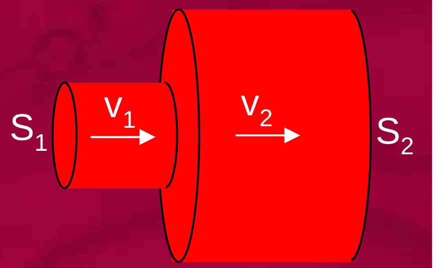
vessel	P [kPa]	radius	tension (N/m)	wall thickness	tension (N/m²)
	P	R	P.R	h	P.R/h
aorta	13,3	13 mm nebo méně	170	2 mm	85000
arteries	12	5 mm	60	1 mm	60000
arterioles	8	150–62 μm	1,2–0,5	20 μm	40000
capillaries	4	4 μm	$1{,}6.10^{-2}$	1 μm	16000
venules	2,6	10 μm	$2{,}6.10^{-2}$	2 μm	13000
veins	2	200 μm a více	0,4	0,5 mm	800
vena cava	1,33	16 mm	21	1,5 mm	14000

Continuity equation

The volume of fluid flowing through a tube (vessel) per unit of time (Q [l/s]) is constant.

$$Q = S_1 . v_1 = S_2 . v_2 = \text{constant}$$

v – velocity S – area



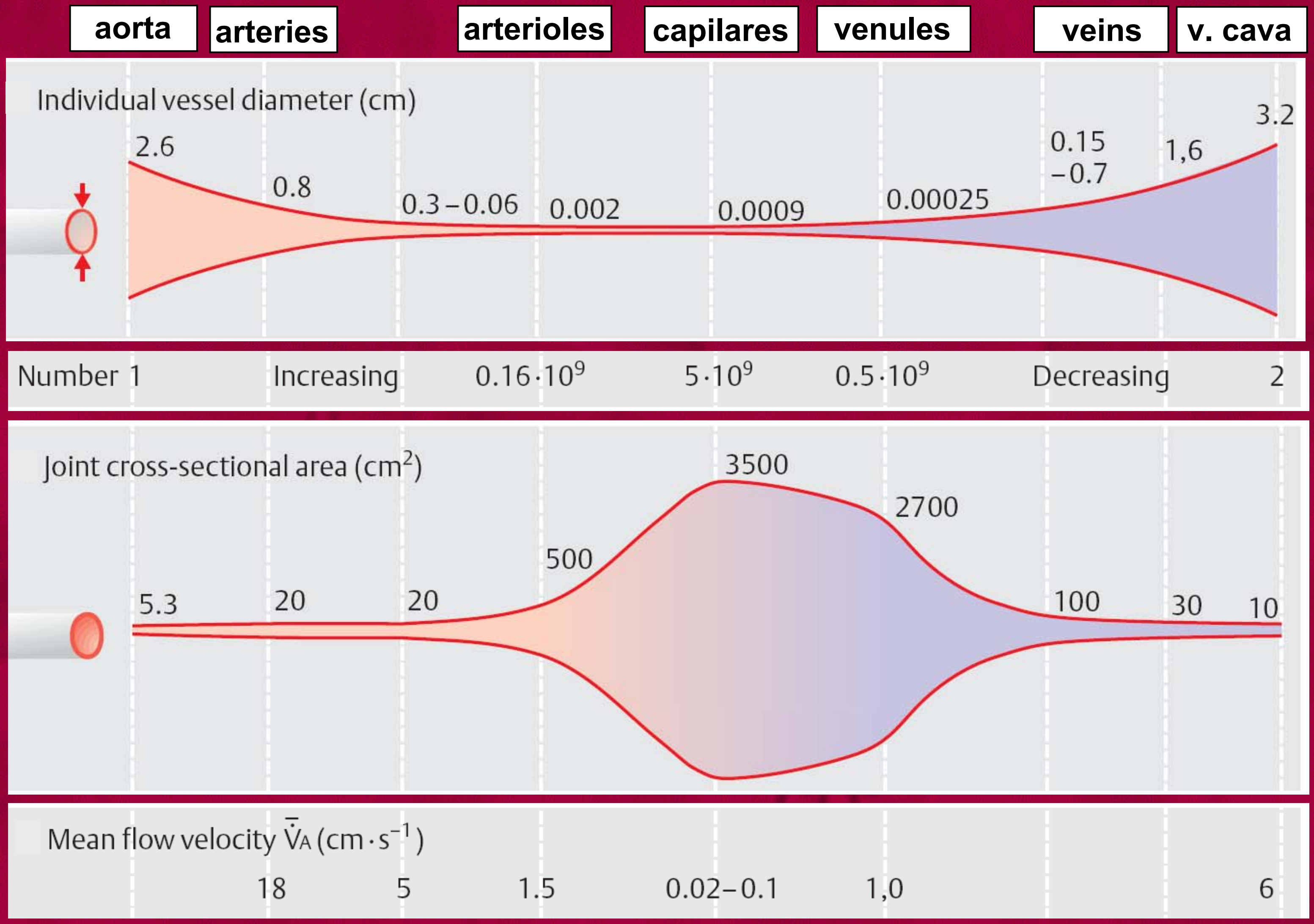
Average blood velocity in vessels

$$v = \frac{Q}{S}$$

$Q_{rest} \approx 5.6$ l/min

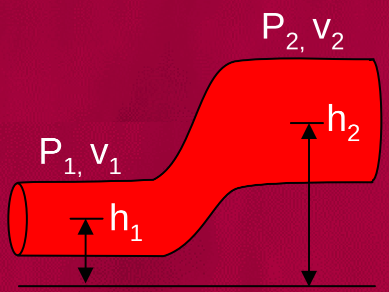
vessel	diameter	number	total area	velocity
aorta	~ 2.6 cm	1	~ 5.3 cm^2	~ 18 cm/s
arterioles	20-50 μm	~ 5×10^6	~ 60 cm^2	~ 1.5 cm/s
capilaries	4-9 μm	~ 5×10^9	~ 2000 cm^2	~ 0.04 cm/s
venules	~ 20 μm	~ 32×10^6	~100 cm^2	~ 1 cm/s
vena cava	~ 3 cm	2	~ 14 cm^2	~ 7 cm/s

Relation between total cross-sectional area of vessels and mean flow velocity

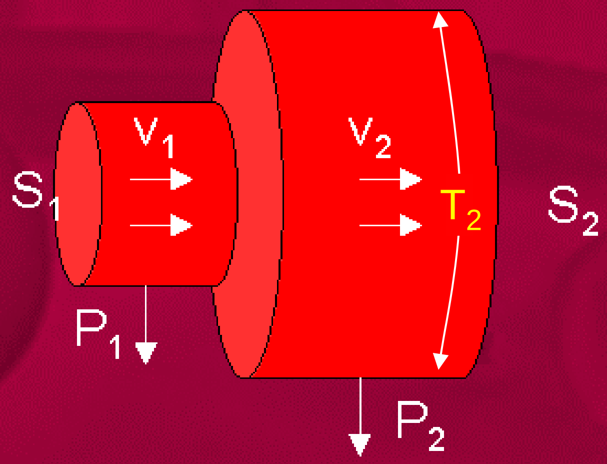


Bernoulli's principle

Law of energy conversation for fluid :



$$\frac{1}{2}\rho v^2 + h.\rho.g + P = \textbf{costant}$$



$T_2 = P_2 \cdot R_2$

Implication at aortic aneurysm

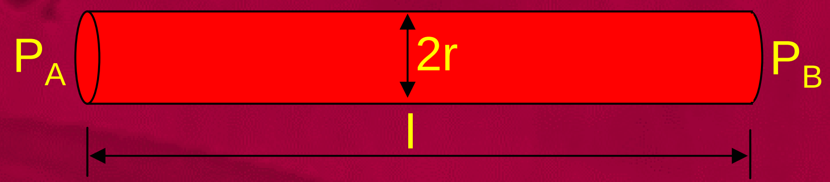
$S_1v_1 = S_2v_2$ a je-li $S_1 < S_2$, musí platit: $v_1 > v_2$

$$\frac{1}{2}\rho v_1^2 + \cancel{h.\rho.g} + P_1 = \frac{1}{2}\rho v_2^2 + \cancel{h.\rho.g} + P_2$$

$$\frac{1}{2}\rho v_1^2 + P_1 = \frac{1}{2}\rho v_2^2 + P_2$$

For $v_2 < v_1 \Rightarrow P_2 > P_1$

Poiseuille – Hagen equation



$$Q = \frac{\pi \cdot \Delta P \cdot r^4}{8 \cdot l \cdot \eta}$$

The flow of liquid in the cylindrical tube (Q) is directly proportional to the pressure difference between two ends of the tube ($\Delta P = P_A - P_B$), to the fourth power of the tube radius (r) and inversely proportional to tube length (*l*) and to the viscosity of liquid (η).

Limitation:

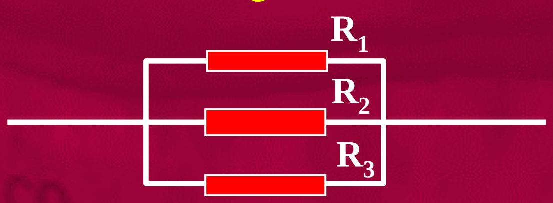
- For stationary flow in Newtonian fluids where viscosity is constant and independent on flow velocity.

$$Q = \frac{\pi \cdot \Delta P \cdot r^4}{8 \cdot l \cdot \eta} \Leftrightarrow Q = \frac{\Delta P}{R_v}$$

Vascular resistance (R_v): a consequence of the friction between fluid and vessel wall.

$$R_v = \frac{\Delta P}{Q} = \frac{8 \cdot l \cdot \eta}{\pi \cdot r^4}$$

Parallel arrangement of vessels

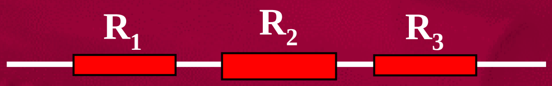


$$R_c = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

pro $R_1=R_2=R_3=R_n$

$R_c=R/n$

Series arrangement of vessels

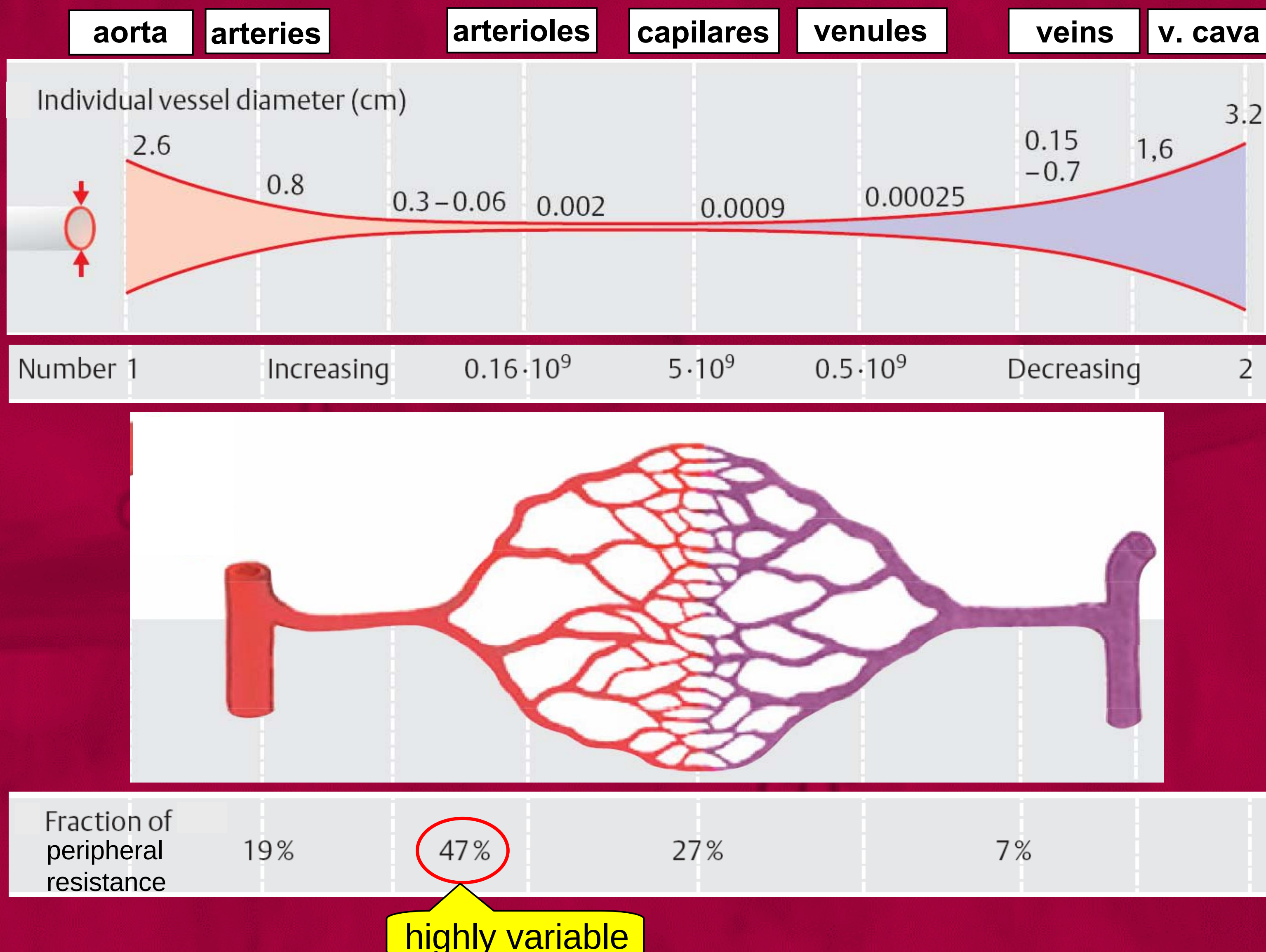


$$R_c = R_1 + R_2 + \dots$$

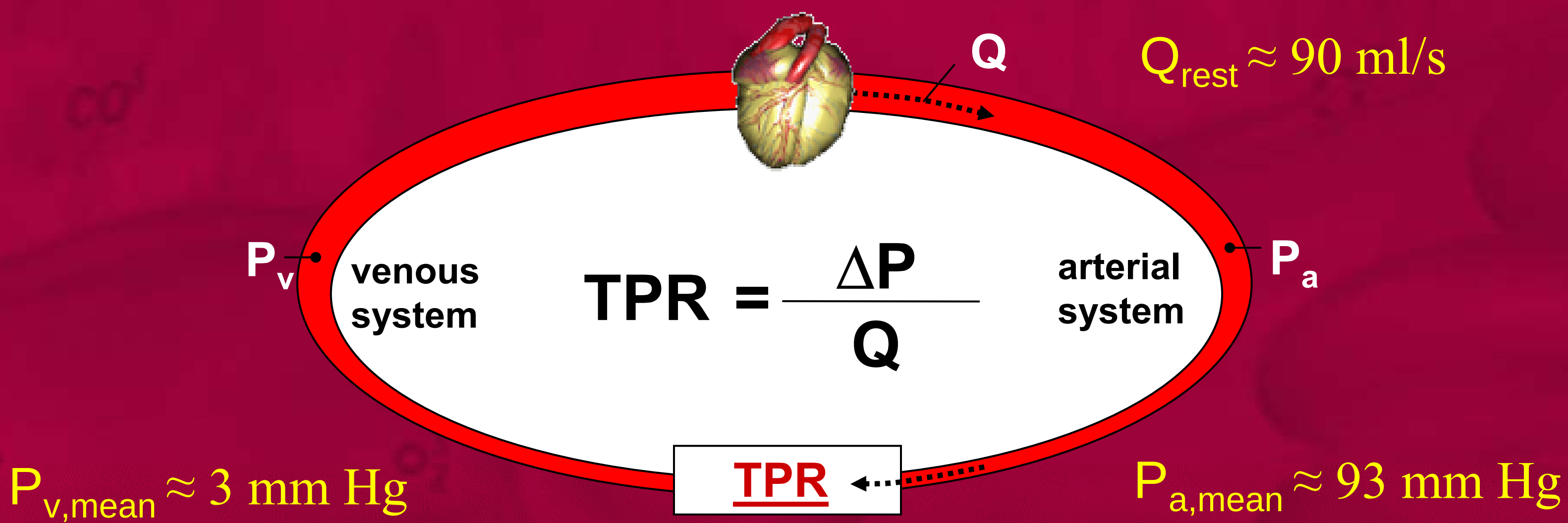
pro $R_1=R_2=R_3=R_n$

$R_c=R \cdot n$

Relation between vessel radius and peripheral resistance



Total peripheral resistance (TPR) of vascular system



$$TPR = \frac{\Delta P}{Q} = \frac{P_a - P_v}{Q} \approx \frac{P_a}{Q} = \frac{93}{90} \approx 1 \frac{\text{mmHg s}}{\text{ml}}$$

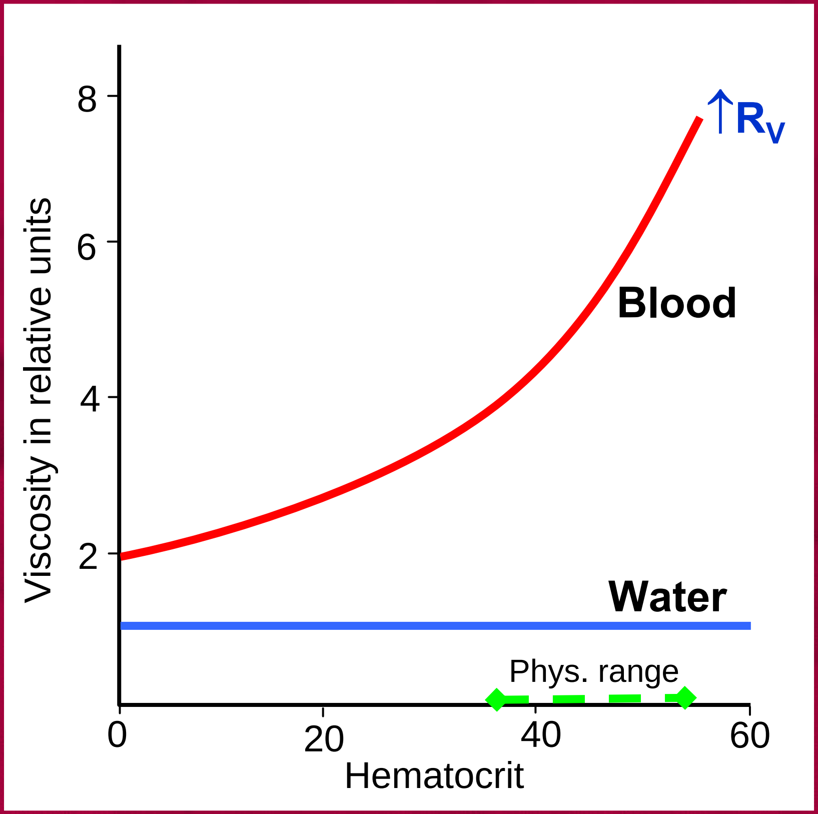
For constant Q: $\uparrow$TPR $\Rightarrow \uparrow P_a \Rightarrow$ hypertension,….

2. Rheological features of blood and vessels

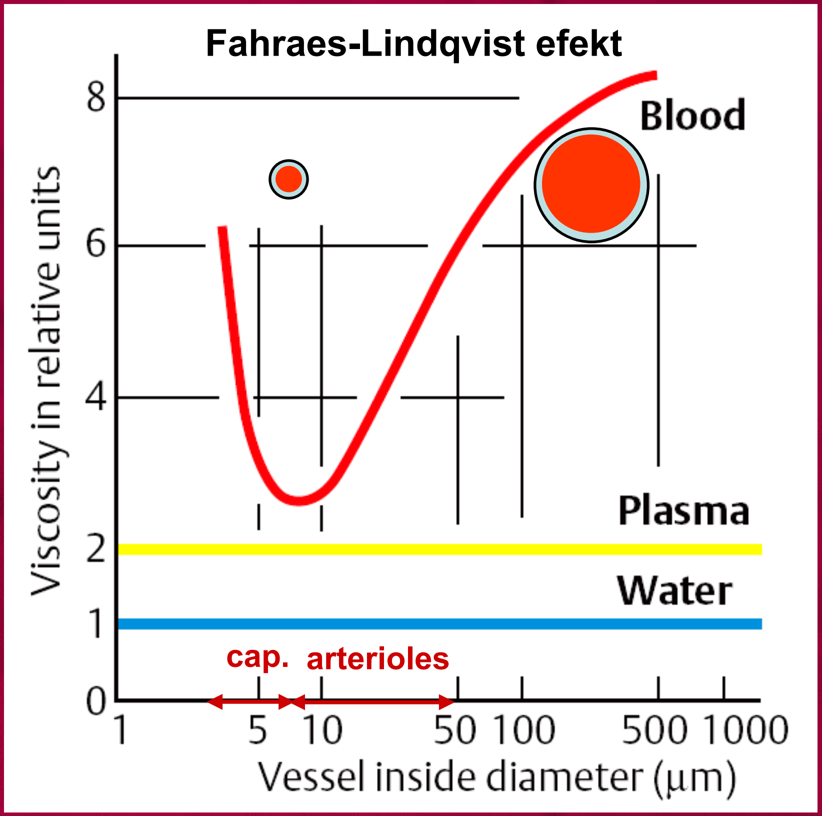
Blood viscosity

$$R_v = 8 \cdot l \cdot \eta / (\pi \cdot r^4)$$

Effect of hematocrit



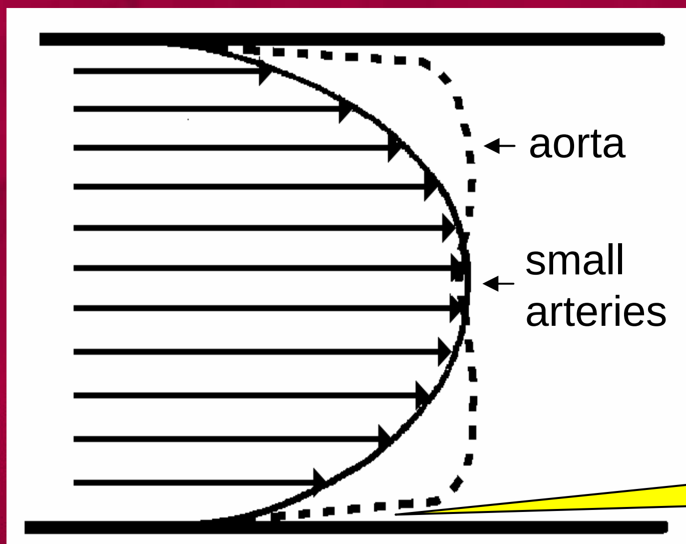
Effect of diameter in small vessels

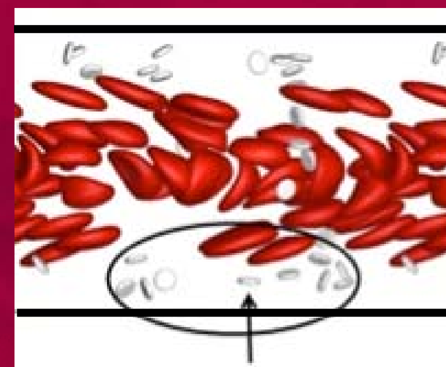


Other factors causing increase of viscosity:

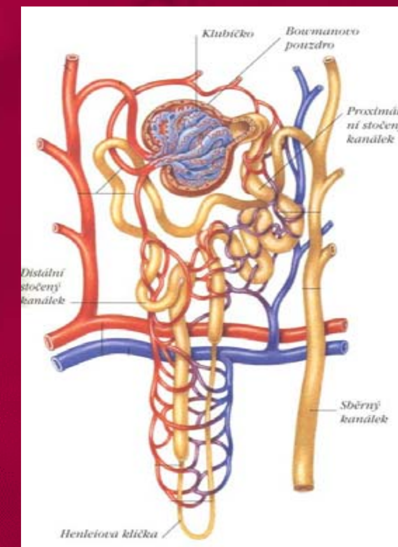
- decrease of blood flow velocity
- elevation of plasma proteins

Velocity profile of the blood flow in vessels





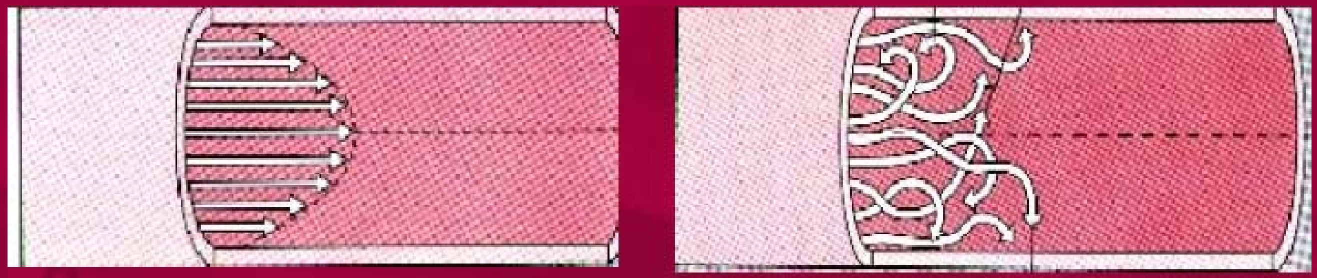
plasma-skimming



- In small arteries the velocity profile of the flowing blood has a parabolic shape. In the bigger arteries it has a piston shape.
- The layer close to vessel wall is poor of erythrocytes.

Laminar and turbulent flow

Velocity profile in laminar and turbulent flow



The character of the flow is determined by Reynolds number

$$R_e = \frac{v \cdot \rho \cdot r}{\eta}$$

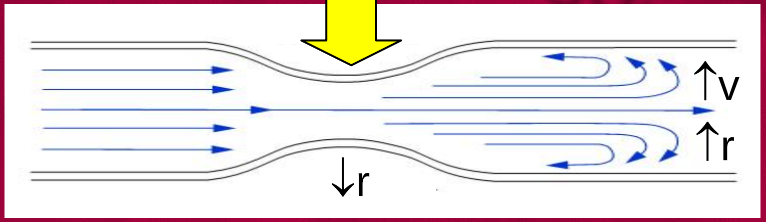
laminar flow

Re<1000

turbulent flow

Re>1000

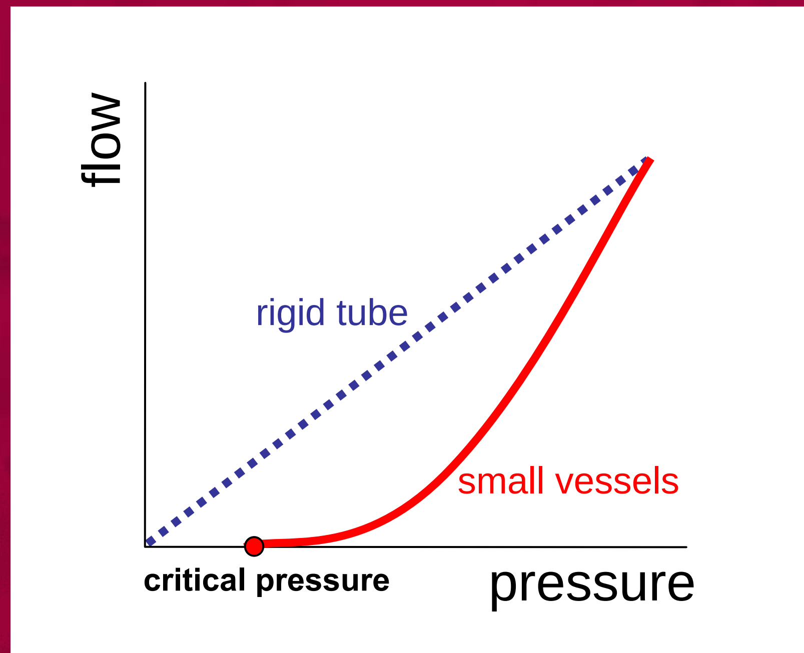
Sudden change of vessel diameter

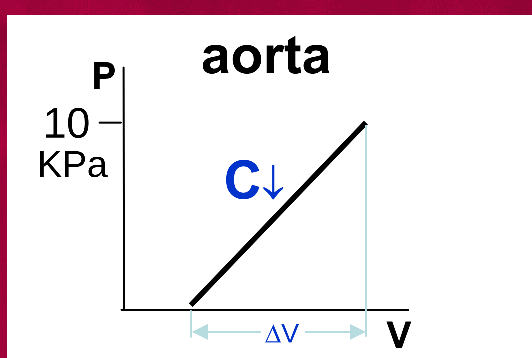


$\uparrow R_e \Rightarrow \uparrow R_V$

Pathological states causing turbulent flow: decreased blood viscosity, aneurisma, stenosis, arteriosclerosis.

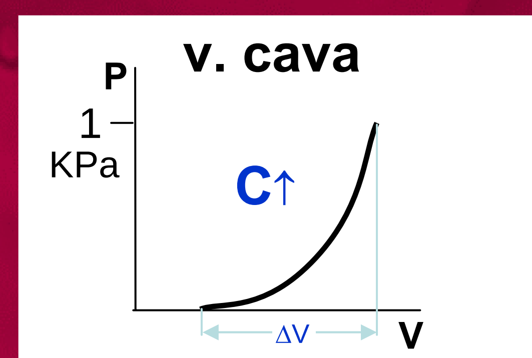
Elasticity of vessels



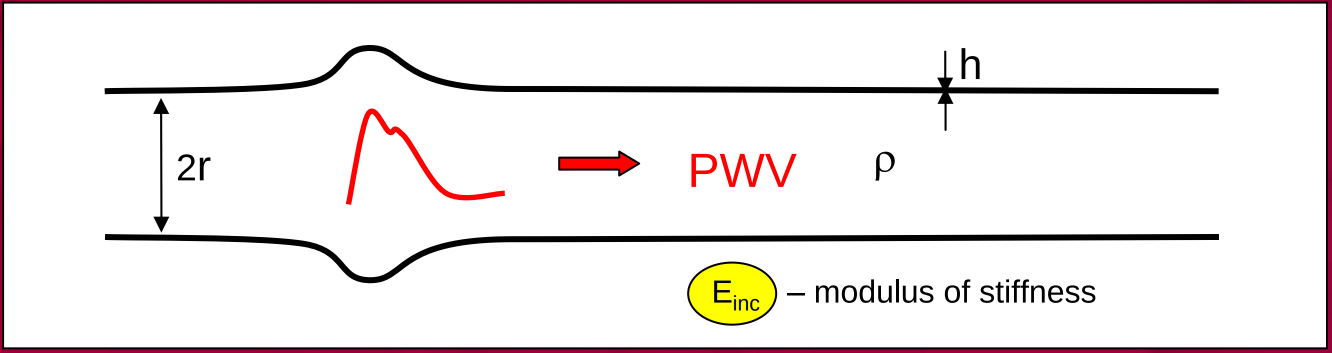


compliance

$$C = \frac{\Delta V}{\Delta P}$$



Pulse wave velocity (PWV)

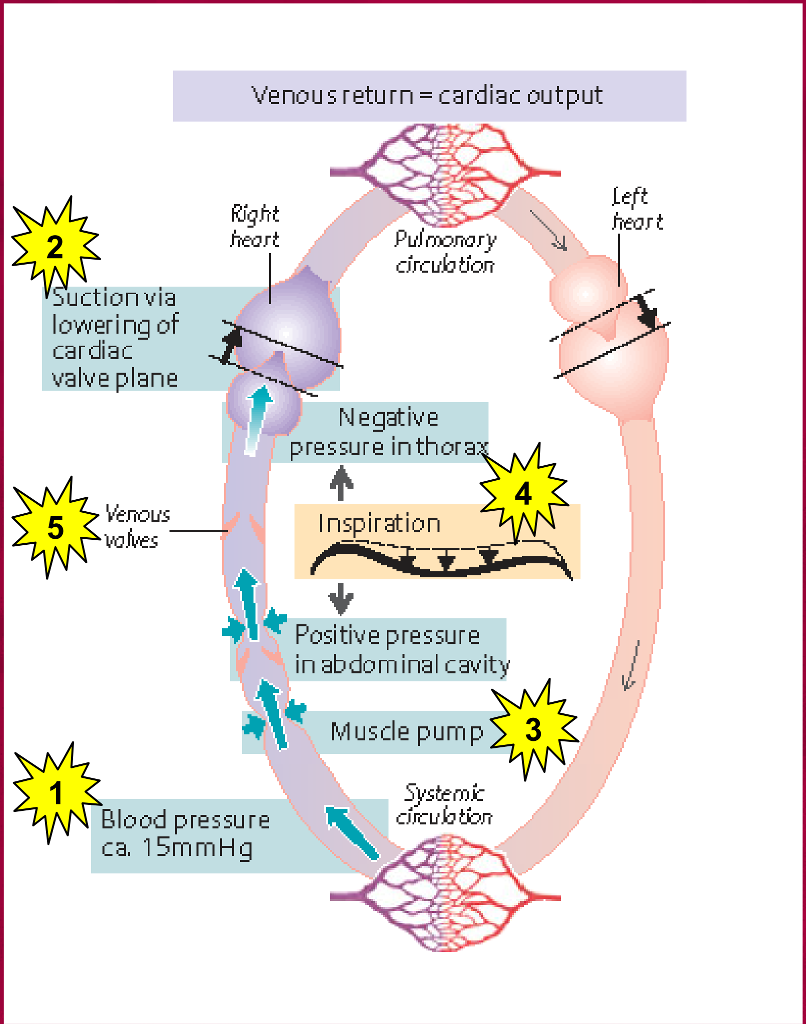


Moens-Korteweg (1878)

$$PWV = \sqrt{\frac{E_{inc} \cdot h}{2 \cdot r \cdot \rho}}$$

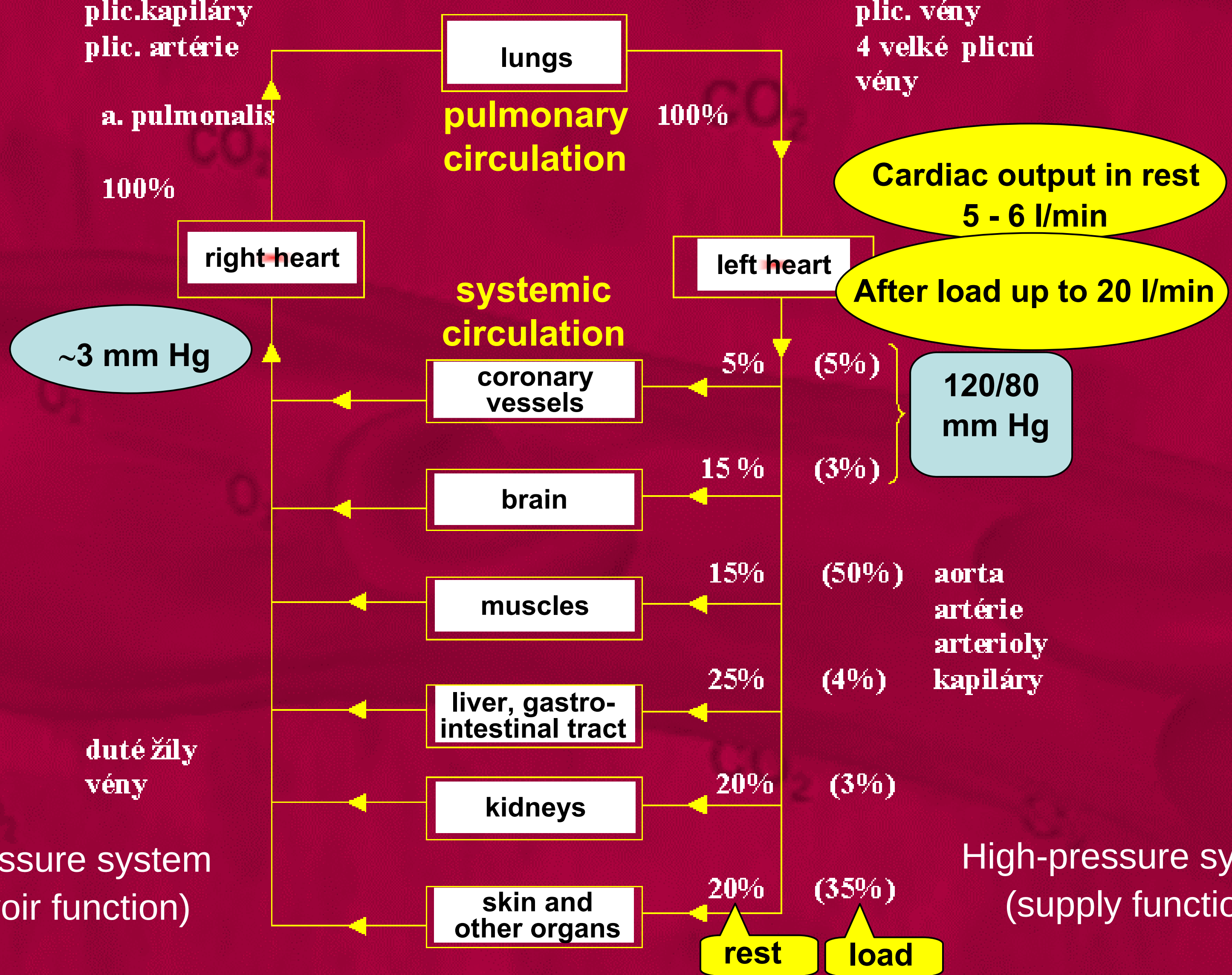
In aorta PWV = 4 - 6 m/s

Mechanisms of venous return



3. Blood circulation and pressure

Blood circulation

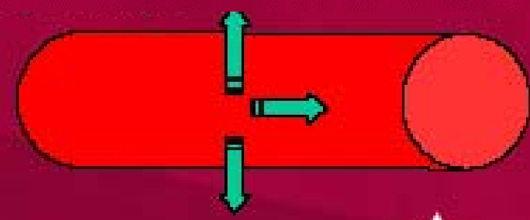


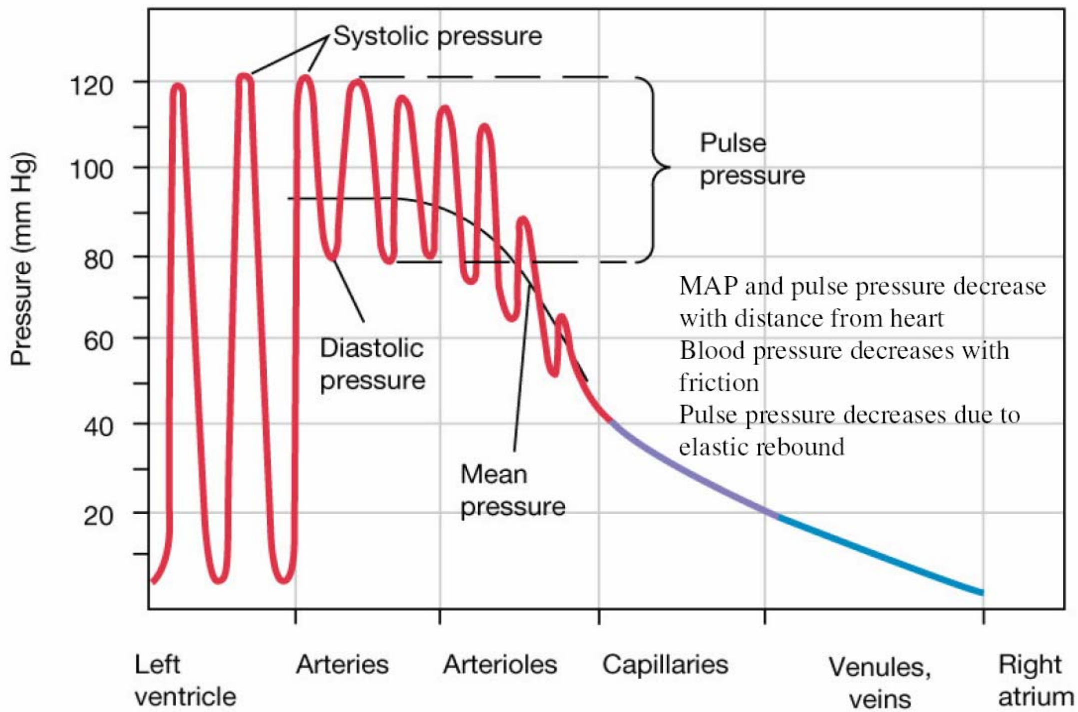
Low-pressure system
(reservoir function)

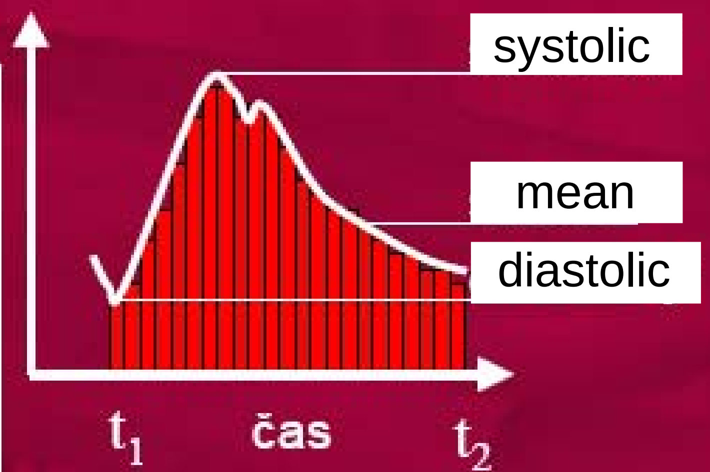
High-pressure system
(supply function)

Blood pressure

Blood pressure (BP) is the pressure exerted by circulating blood upon the walls of blood vessels.



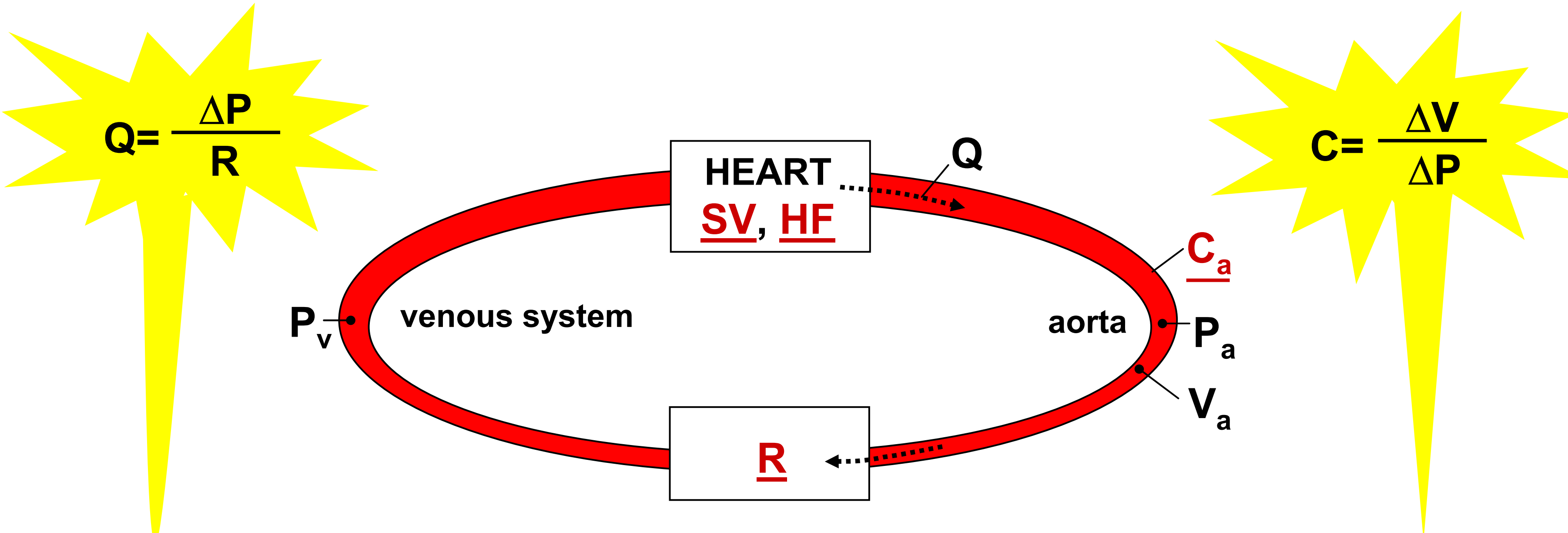




$$P_{mean} = \int_{t_1}^{t_2} \frac{P dt}{t_2 - t_1}$$

$$P_{mean} \cong Pd + \frac{1}{3}(Ps - Pd)$$

Dependence of blood pressure on cardiac output and vascular parameters

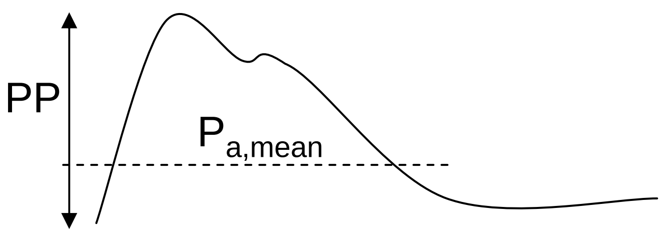


$$P_{a,mean} - P_{v,mean} = Q \cdot R$$

$$\Delta V \cong SV$$

$$P_{a,mean} = SV \cdot HF \cdot R + P_{v,mean}$$

$$P_{a,mean} \cong SV \cdot HF \cdot R$$



$$PP \cong \frac{SV}{C}$$

Model of blood pressure changes in aorta