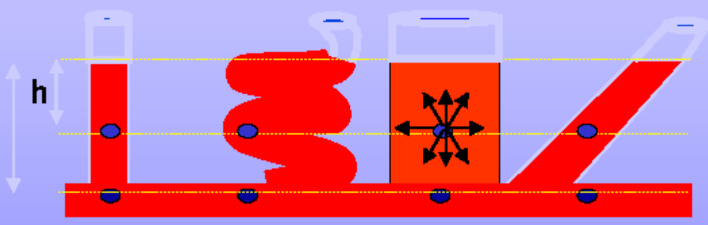
Rheology of blood circulation

1. Basic physical laws of liquids

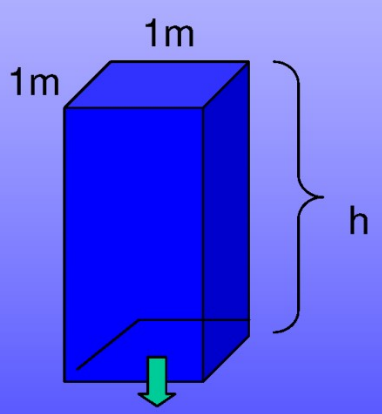
Law of Pascal

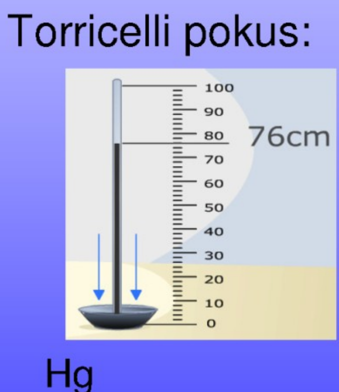
Liquid column causes a pressure (hydrostatic pressure) that is directly proportional to the height of the liquid column (h), density of the liquid (and gravitational acceleration (g).

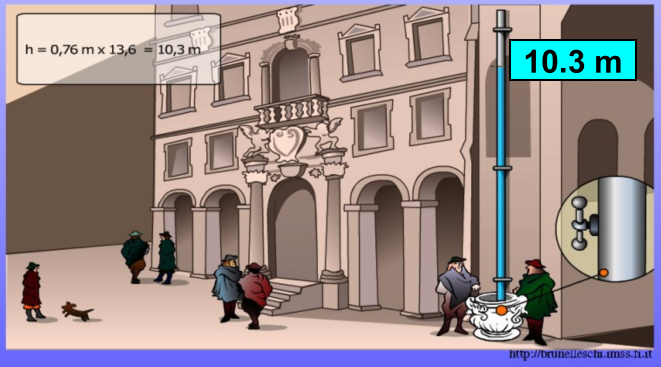


$p = h.\rho.g$

h =height
ρ =density
g = gravitational acceleration







H_2O

Pa

mm Hg

mm H_2O

133,322 Pa = 1 mm Hg

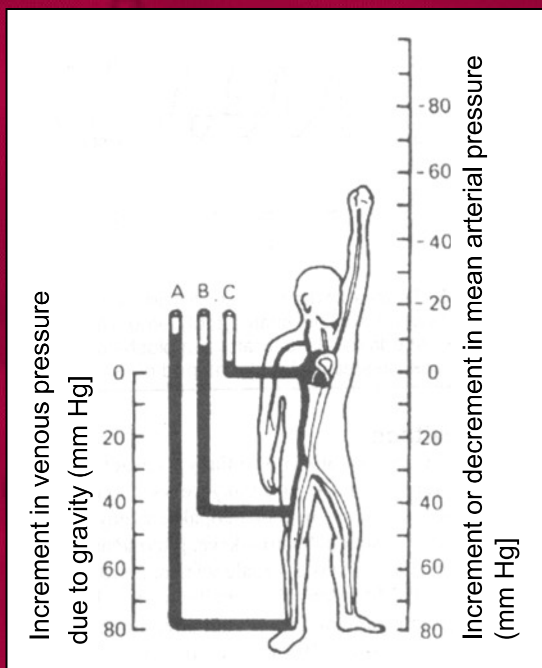
760 mmHg= 1 atm = 10.3 m H_2O

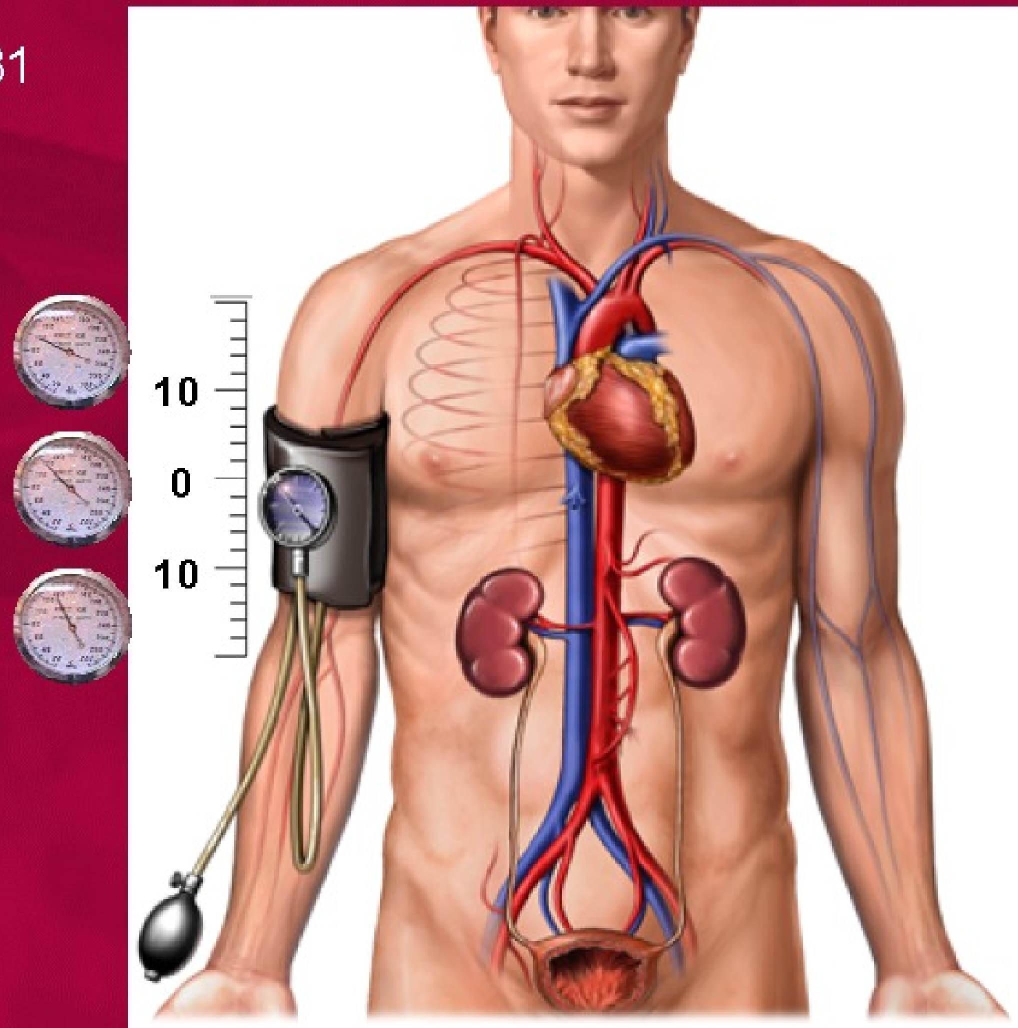
Effect of gravity on arterial and venous pressure

Per each 10 cm

$$\Delta p = \Delta h . \rho_{krve} . g = 0,1 \, . \, 1\,065 \, . \, 9,81$$

$$= 1\,045 Pa = 7.8 \text{ mm Hg}$$





Law of Laplace

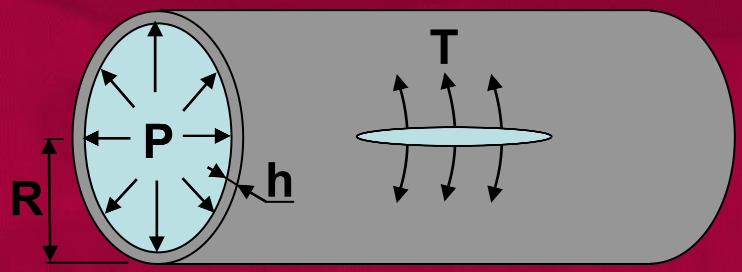
Relation between distending pressure (P [N/m²]) and tension in the wall of hollow object (T [N/m]) :

$$T = \frac{P}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

R_1 and R_2 are the biggest and the smallest radii of curvature

For vessel:

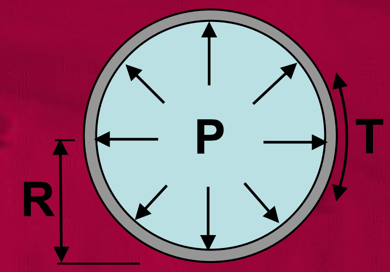
$R_2 =$ $T = P \cdot R$



Considering thickness of vessel wall (h [m]): $T = P \cdot R/h$ [N/m²]

For sphere:

$R_1 = R_2$ $T = P \cdot R/2$



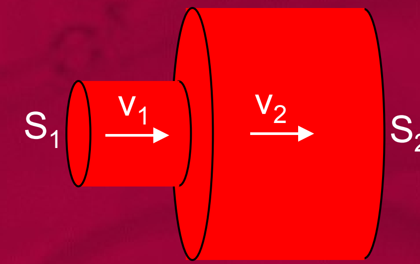
Characteristics of vessels

vessel	P [kPa]	radius	tension (N/m)	wall thickness	tension (N/m^2)
	P	R	P•R	h	P•R/h
aorta	13,3	13 mm nebo méně	170	2 mm	85000
arteries	12	5 mm	60	1 mm	60000
arterioles	8	150–62 μm	1,2–0,5	20 μm	40000
capillaries	4	4 μm	$1{,}6.10^{-2}$	1 μm	16000
venules	2,6	10 μm	$2{,}6.10^{-2}$	2 μm	13000
veins	2	200 μm a více	0,4	0,5 mm	800
vena cava	1,33	16 mm	21	1,5 mm	14000

Continuity equation

The volume of fluid flowing through a tube (vessel) per unit of time (Q [l/s]) is constant.

$$Q = S_1 . v_1 = S_2 \cdot v_2 = \text{constant}$$



v – velocity S – area

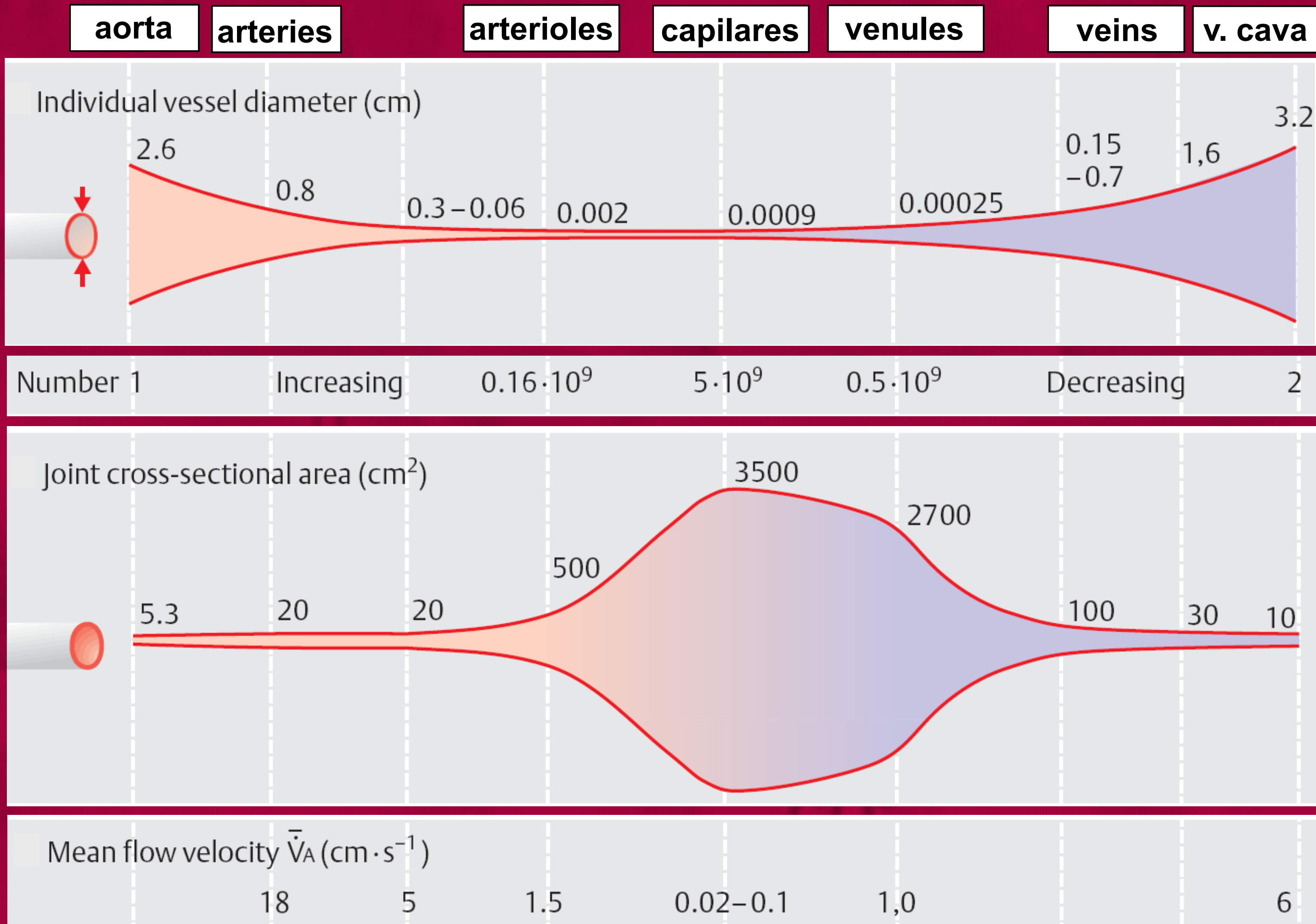
Average blood velocity in vessels

$$v = \frac{Q}{S}$$

$Q_{rest} \approx 5.6$ l/min

vessel	diameter	number	total area	velocity
aorta	2.6 cm	1	5.3 cm²	~ 18 cm/s
arterioles	20-50			~ 1.5 cm/s
capilaries	4-9 μm	5×10^9	2000 cm²	~ 0.04 cm/s
venules				~ 1 cm/s
vena cava		2		~ 7 cm/s

Relation between total cross-sectional area of vessels and mean blood flow velocity



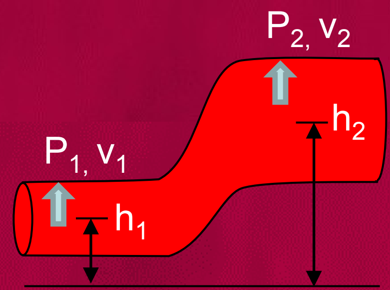
Bernoulli’s principle

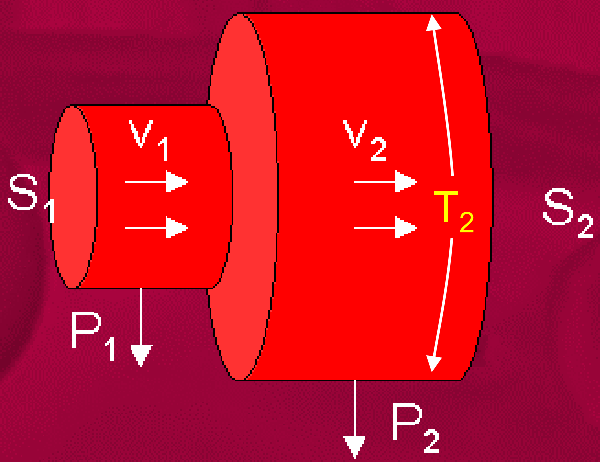
Law of energy conservation for fluid :

$$\frac{1}{2}\rho v^2 + h.\rho.g + P = \textbf{costant}$$

dynamic pressure

static pressure





$T_2 = P_2 \cdot R_2$

Implication for aortic aneurysm

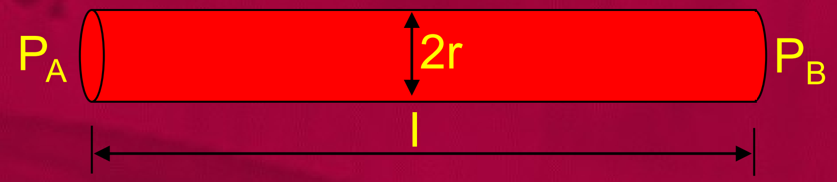
$S_1v_1 = S_2v_2$ a je-li $S_1<S_2$, musí platit: $v_1>v_2$

$$\frac{1}{2}\rho v_1^2 + \cancel{h.\rho.g} + P_1 = \frac{1}{2}\rho v_2^2 + \cancel{h.\rho.g} + P_2$$

$$\frac{1}{2}\rho v_1^2 + P_1 = \frac{1}{2}\rho v_2^2 + P_2$$

For $v_2 < v_1 \Rightarrow P_2 > P_1$

Poiseuille – Hagen equation



$$Q = \frac{\pi \cdot \Delta P \cdot r^4}{8 \cdot l \cdot \eta}$$

The flow of liquid in the cylindrical tube (Q) is directly proportional to the pressure difference between two ends of the tube ($\Delta P = P_A - P_B$), to the fourth power of the tube radius (r) and inversely proportional to tube length (*l*) and to the viscosity of liquid (η)

Limitation:

- For stationary flow in Newtonian fluids where viscosity is constant and independent on flow velocity.

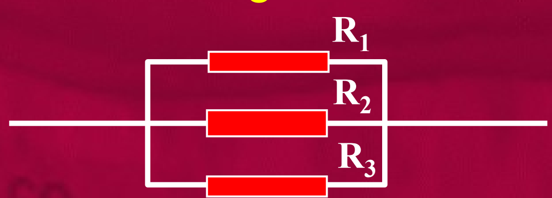
$$Q = \frac{\pi \cdot \Delta P \cdot r^4}{8 \cdot l \cdot \eta}$$

$$Q = \frac{\Delta P}{R_v}$$

Vascular resistance (R_v): a consequence of the friction between fluid and vessel wall.

$$R_v = \frac{\Delta P}{Q} = \frac{8 \cdot l \cdot \eta}{\pi \cdot r^4}$$

Parallel arrangement of vessels

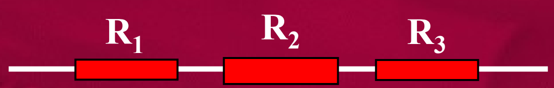


$$\frac{1}{R_c} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$$

pro $R_1 = R_2 = R_3 = R_n$

$R_c = R/n$

Series arrangement of vessels

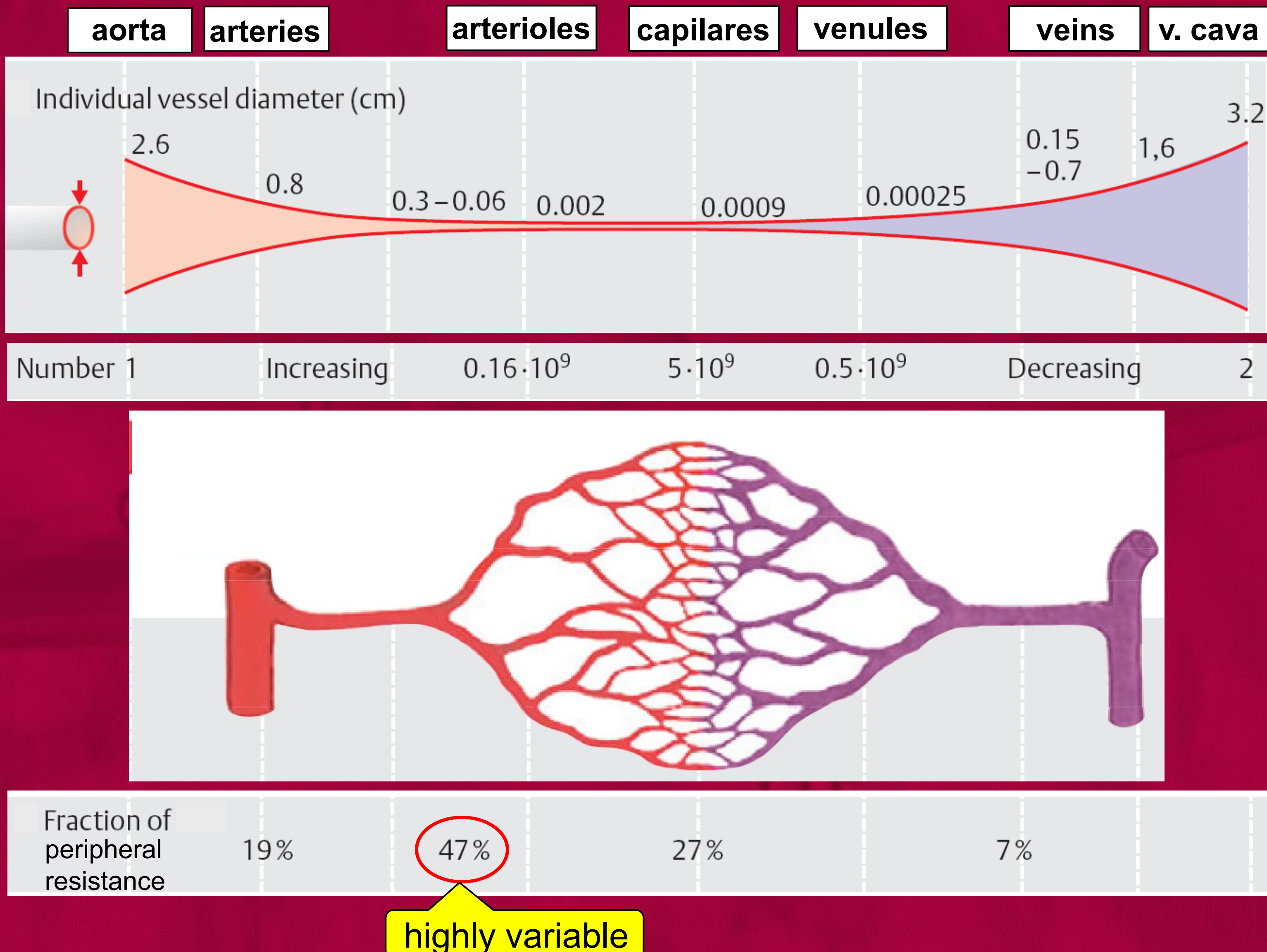


$$R_c = R_1 + R_2 + \ldots$$

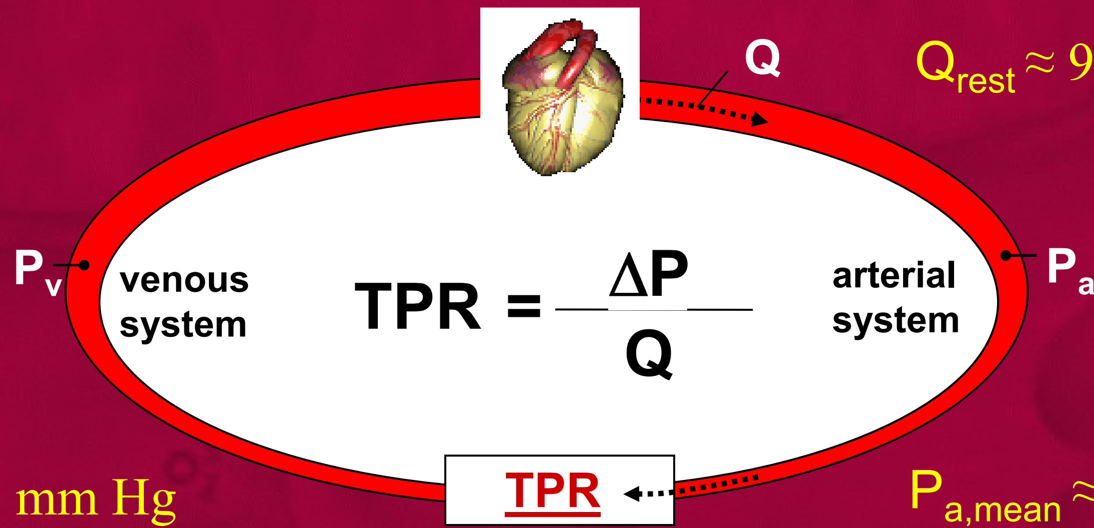
pro $R_1 = R_2 = R_3 = R_n$

$R_c = R \cdot n$

Relation between vessel radius and peripheral resistance



Total peripheral resistance (TPR) of vascular system



$Q_{rest} \approx 90$ ml/s

$P_{v,mean} \approx 3$ mm Hg

$P_{a,mean} \approx 93$ mm Hg

$$TPR = \frac{\Delta P}{Q} = \frac{P_a - P_v}{Q} \approx \frac{P_a}{Q} = \frac{93}{90} \approx 1 \frac{\text{mmHg s}}{\text{ml}}$$

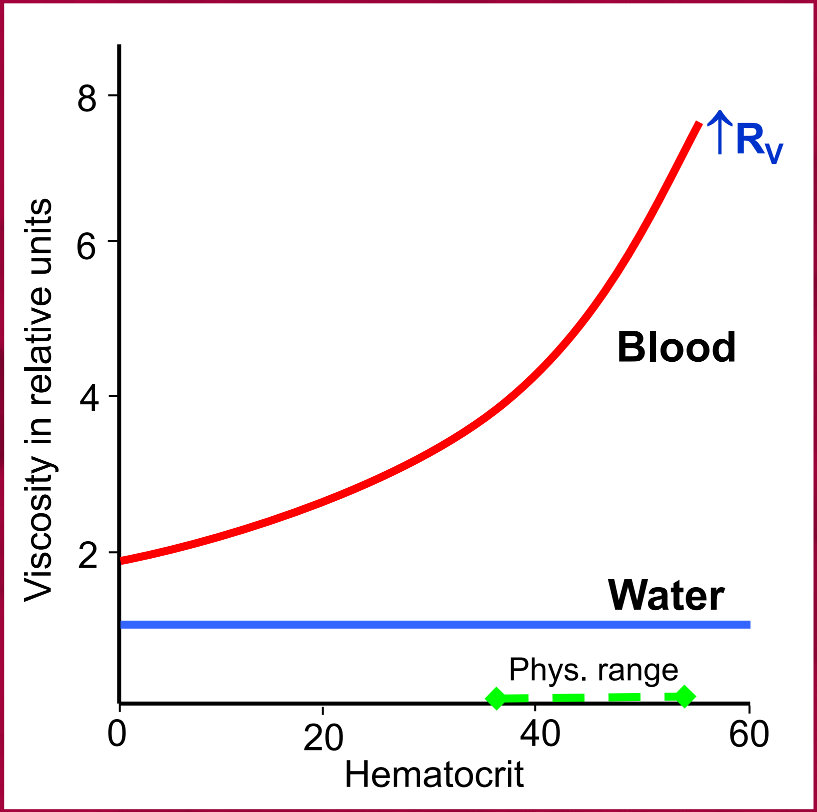
For constant Q: PR P_a ypertension,…cardiac disease.

2. Rheological features of blood and vessels

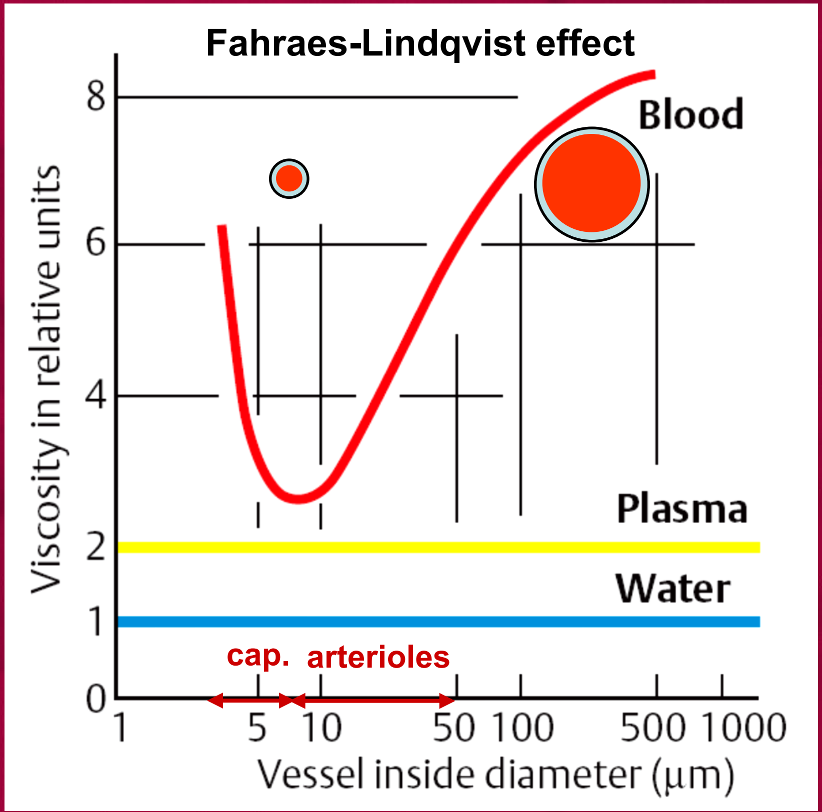
Blood viscosity

$$R_v = 8 \cdot l \cdot \eta / (\pi \cdot r^4)$$

Effect of hematocrit



Effect of diameter in small vessels

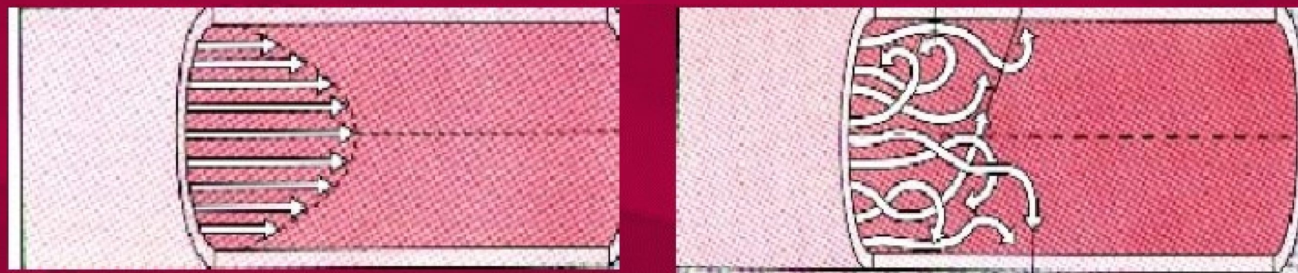


Other factors causing increase of viscosity:

- decrease of blood flow velocity
- elevation of plasma proteins

Laminar and turbulent flow

Velocity profile in laminar and turbulent flow



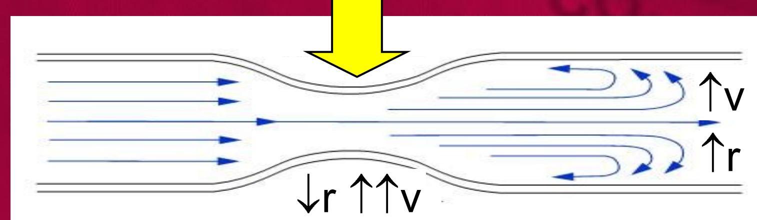
The character of the flow is determined by Reynolds number

$$R_e = \frac{v \cdot \rho \cdot r}{\eta}$$

laminar flow
Re<2000

turbulent flow
Re>3000

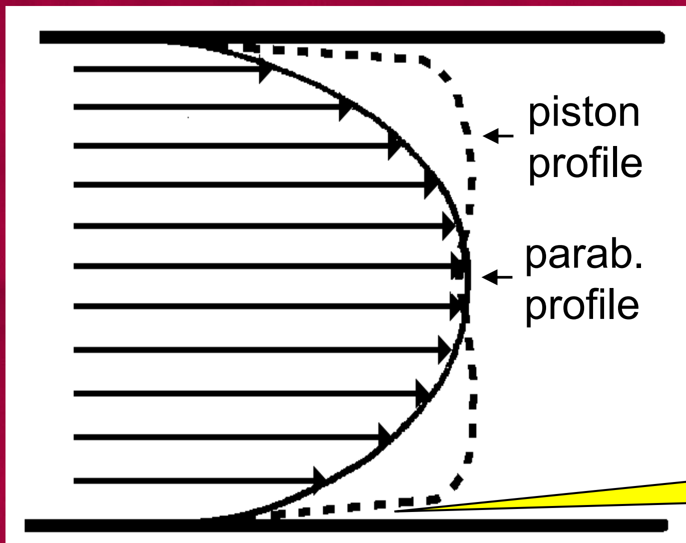
Sudden change of vessel diameter

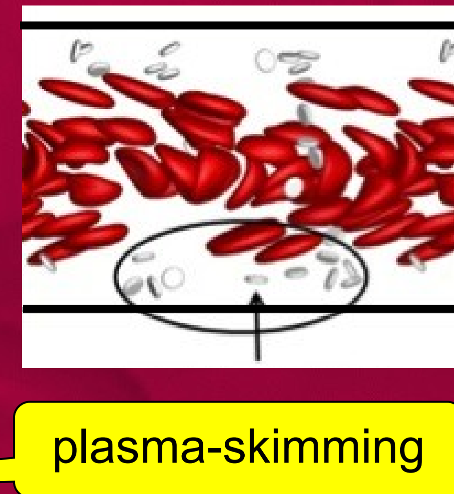


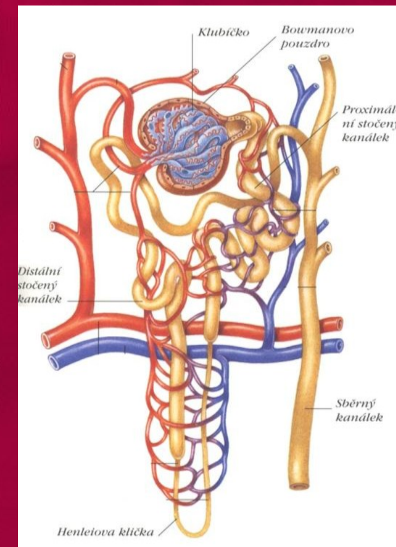
$\uparrow R_e \Rightarrow$ `R_V

Pathological states causing turbulent flow: aneurisma, stenosis, arteriosclerosis, decreased blood viscosity, .

Velocity profile of the blood flow in vessels

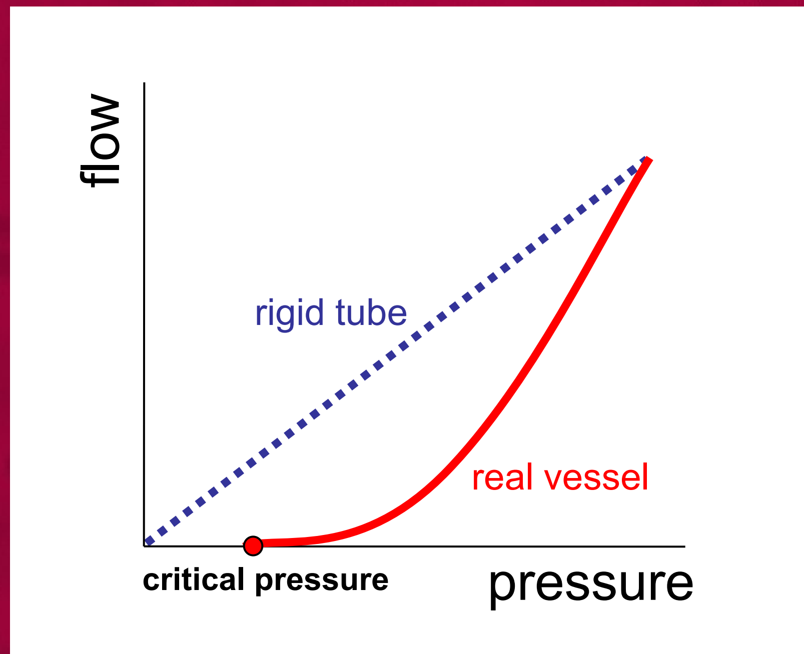


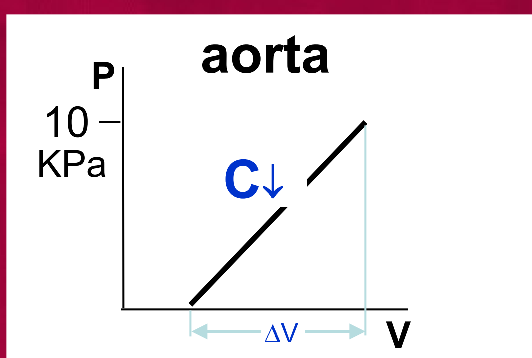




- In small arteries (at r < 100 μm), the central movement of erytrocytes causes a piston-like profile of the blood flow.
- In bigger arteries (at r > 500 μm), the laminar flow prevails and the velocity profile of the blood flow has a parabolic shape.
- In big arteries (especially in aorta), a higher cardiac output causes a turbulent flow (R_e > 3000) and the parabolic profile of the blood flow changes to the piston one.

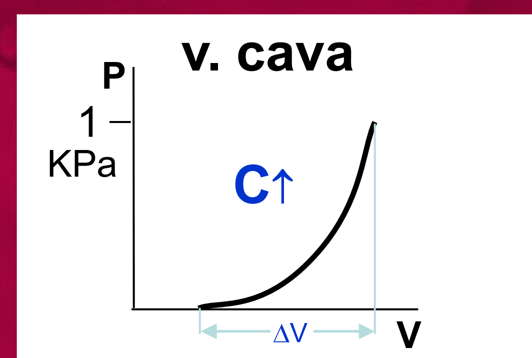
Elasticity of vessels



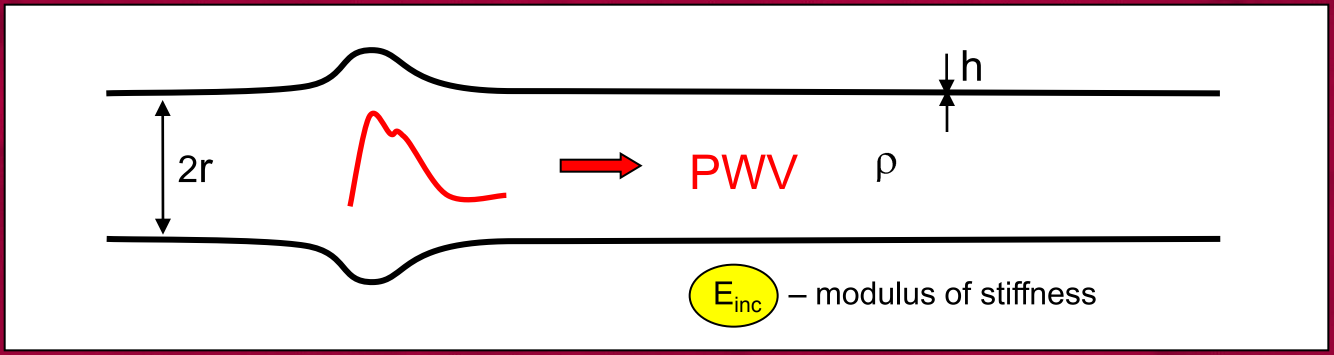


compliance

$$C = \frac{\Delta V}{\Delta P}$$



Pulse wave velocity (PWV)

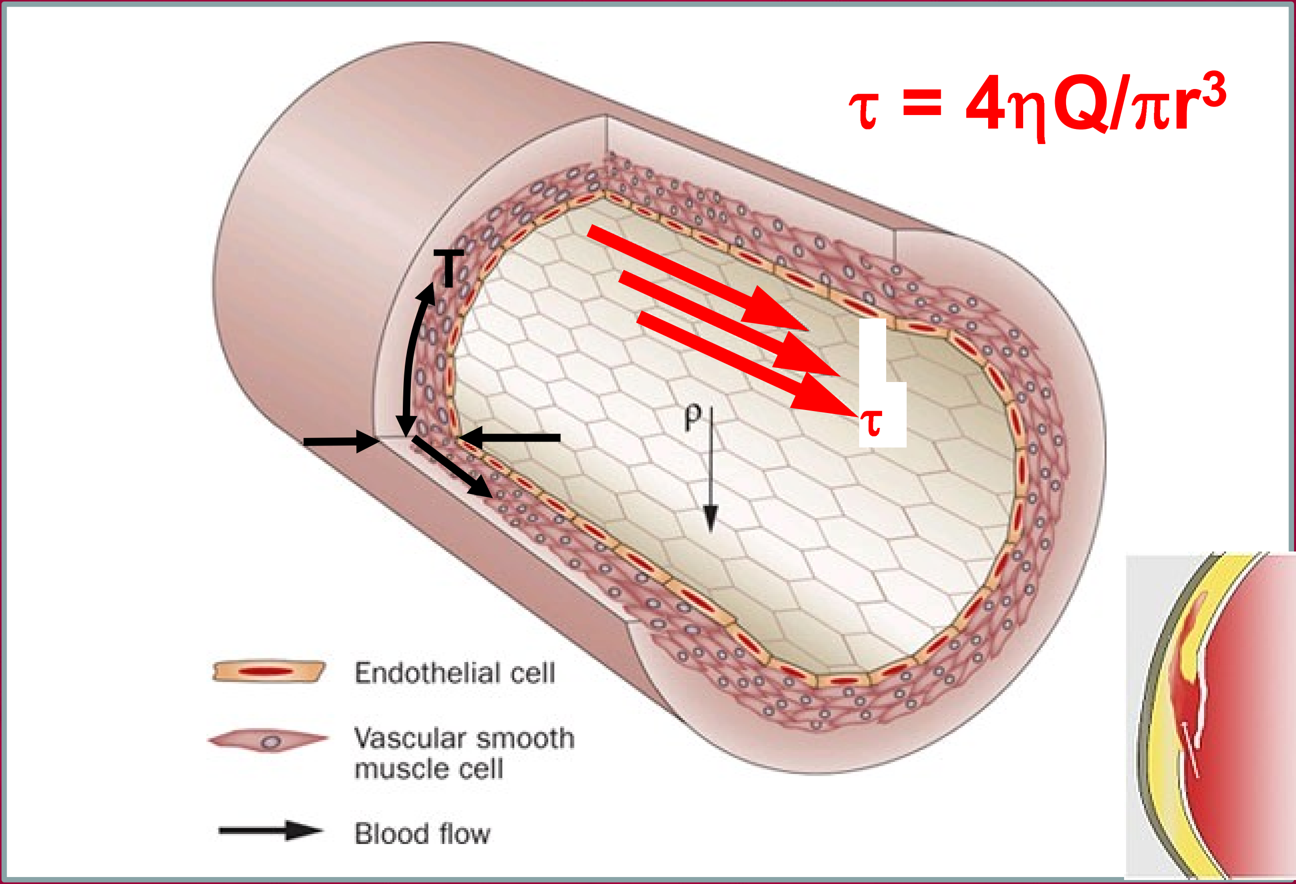


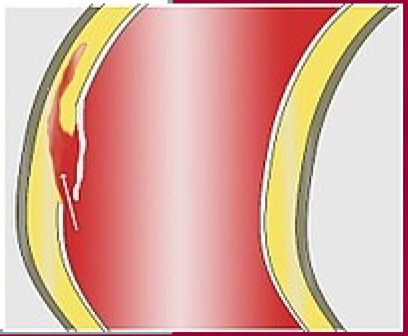
Moens-Korteweg (1878)

$$PWV = \sqrt{\frac{E_{inc} \cdot h}{2 \cdot r \cdot \rho}}$$

In aorta PWV = 4 - 6 m/s

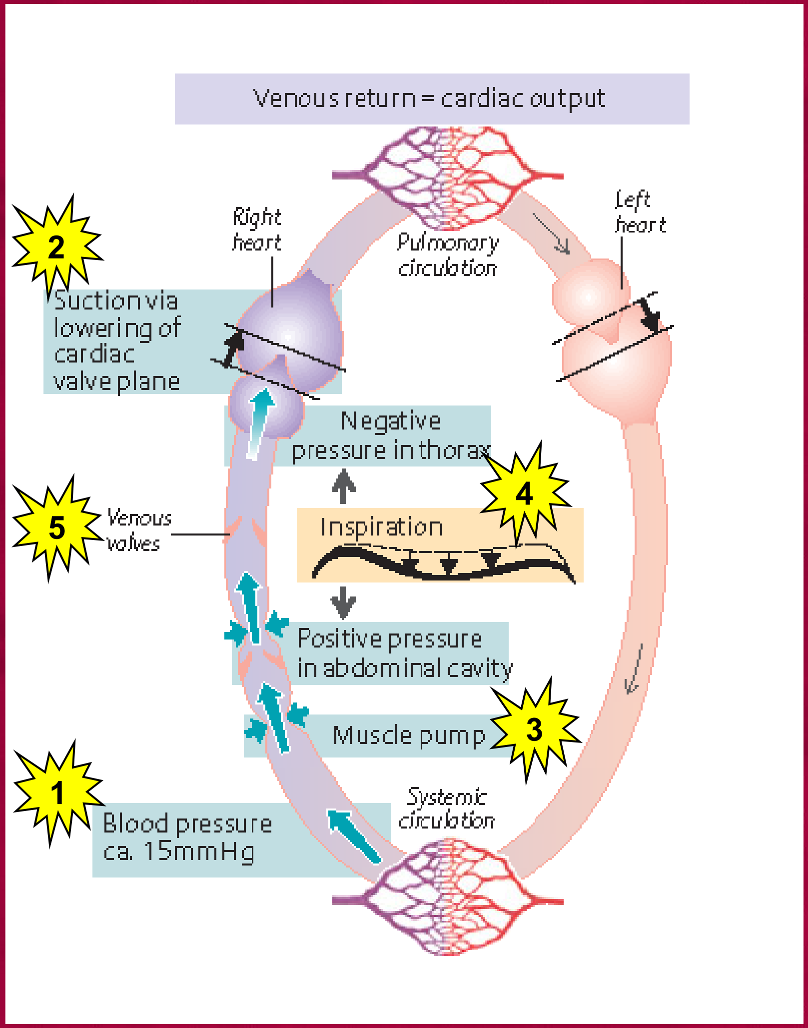
Share stress in vessel wall





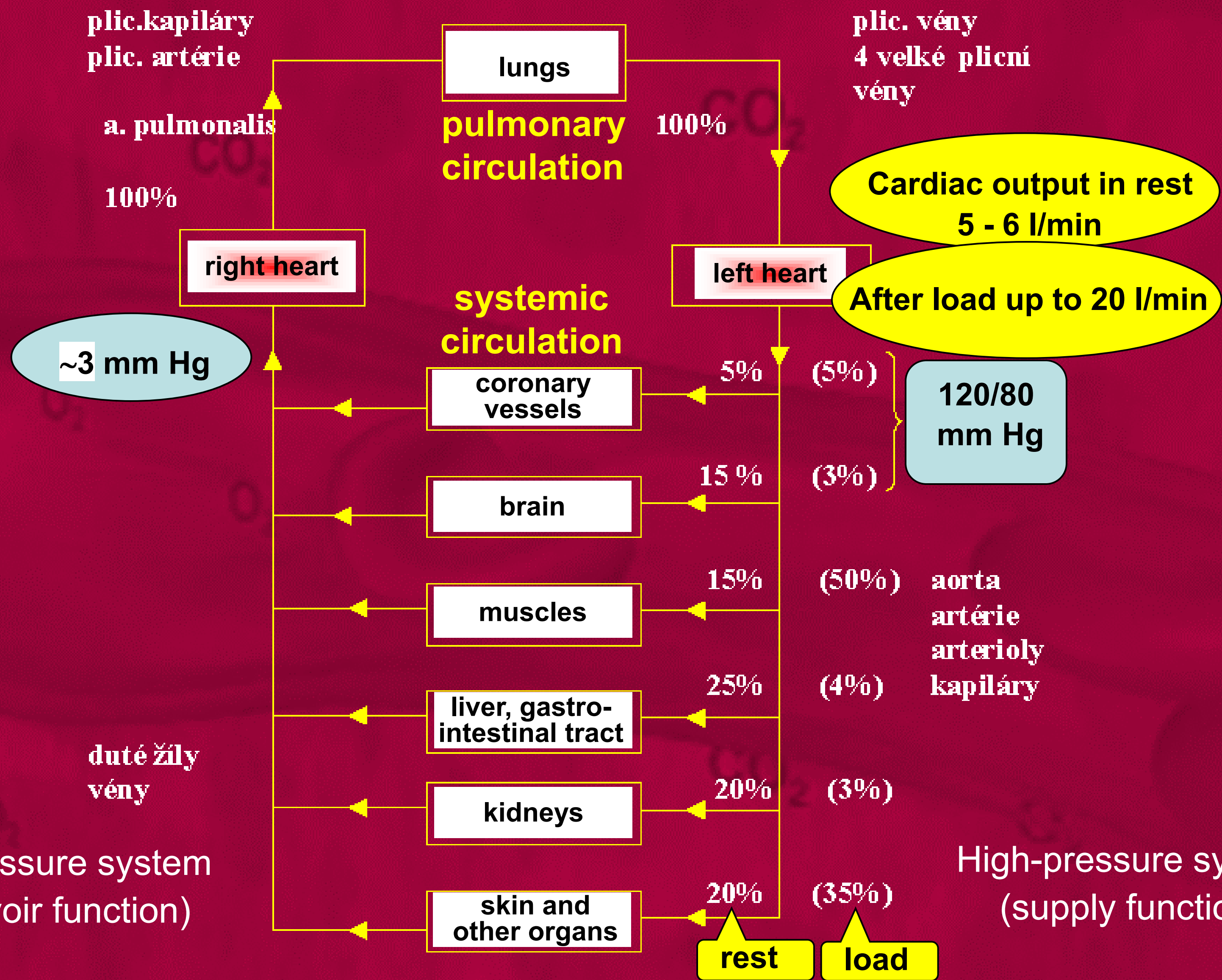
- Share stress in vessel wall may lead to a tear in endothelial layer and to arterial dissection.

Mechanisms of venous return



3. Blood circulation and pressure

Blood circulation

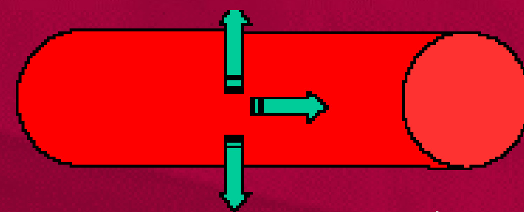


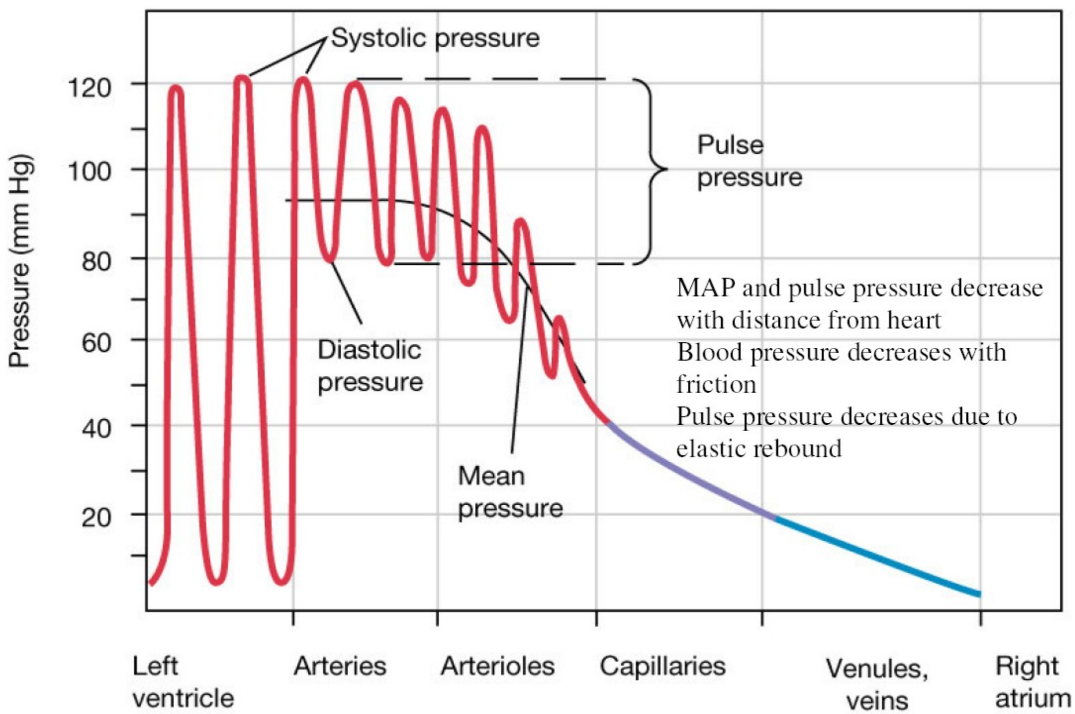
Low-pressure system
(reservoir function)

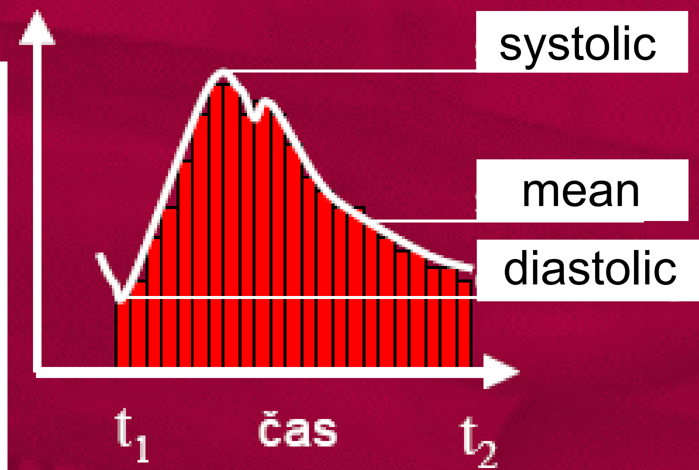
High-pressure system
(supply function)

Blood pressure

Blood pressure (BP) is the pressure exerted by circulating blood upon the walls of blood vessels.





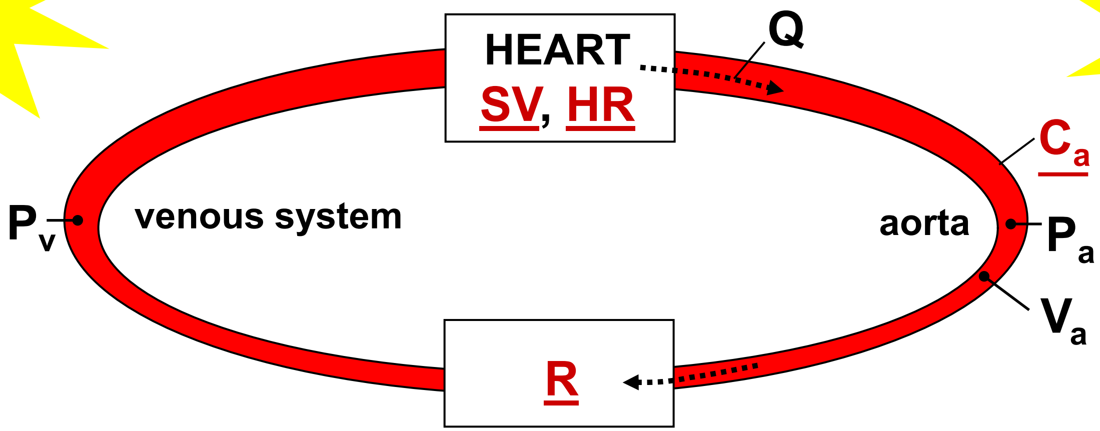


$$P_{mean} = \int_{t_1}^{t_2} \frac{P\,dt}{t_2 - t_1}$$

$$P_{mean} \cong P_d + \frac{1}{3}\left(P_s - P_d\right)$$

Dependence of blood pressure on cardiac output and vascular parameters

$$Q = \frac{\Delta P}{R}$$



$$C = \frac{\Delta V}{\Delta P}$$

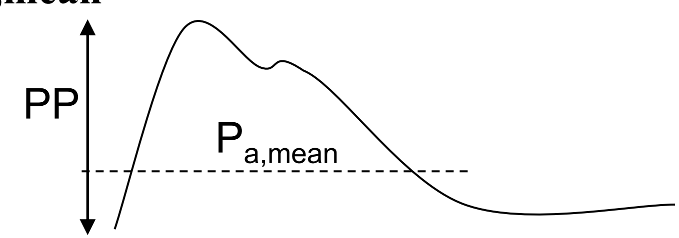
$$P_{a,mean} - P_{v,mean} = Q \cdot R$$

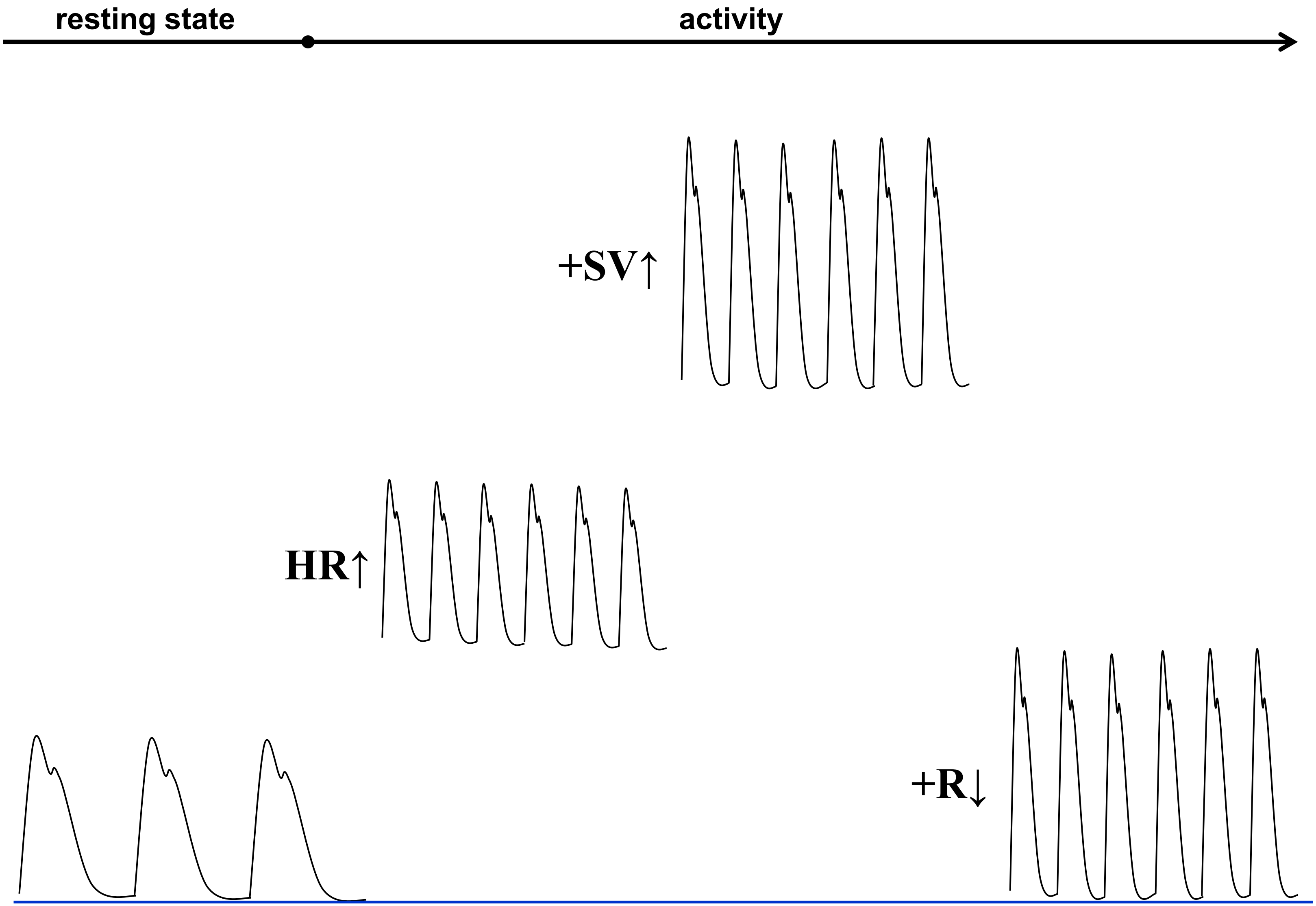
$$\Delta V \cong SV$$

$$P_{a,mean} = SV \cdot HR \cdot R + P_{v,mean}$$

$$PP \cong \frac{SV}{C}$$

$$P_{a,mean} \cong SV \cdot HR \cdot R$$





$$P_{a,mean} \cong SV \cdot HR \cdot R$$

$$PP \cong \frac{SV}{C}$$

Model of blood pressure changes in aorta