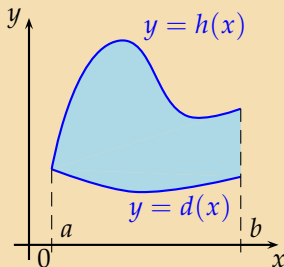


Obsah rovinného útvaru mezi dvěma křivkami

Robert Mařík a Lenka Příbylová

31. července 2006

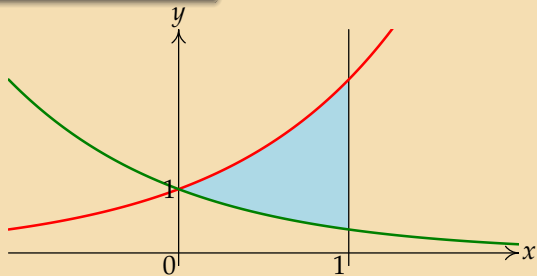
Obsah rovinné plochy omezené spojitými funkcemi $y = d(x)$ a $y = h(x)$, které na intervalu $\langle a, b \rangle$ splňují $d(x) \leq h(x)$, a přímkami $x = a$ a $x = b$:



$$S = \int_a^b h(x) - d(x) \, dx$$

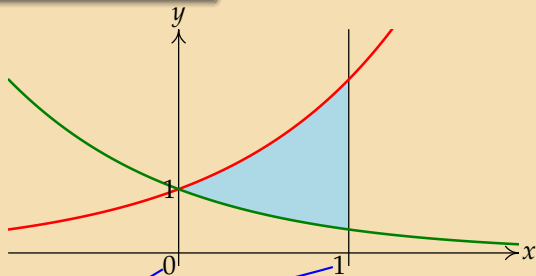
Určete obsah množiny mezi křivkami $y = e^x$ a $y = e^{-x}$ pro $x \in [0, 1]$.

$$y = e^x, y = e^{-x}, x \in [0, 1], S = ?.$$



Zakreslíme křivky.

$$y = e^x, y = e^{-x}, x \in [0, 1], S = ?.$$

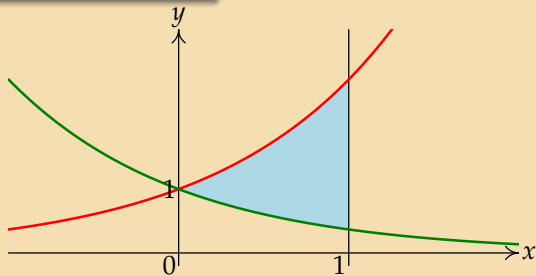


$$S = \int_0^1 e^x - e^{-x} dx$$

Vyjádříme obsah plochy jako určitý integrál.

$$h(x) = e^x, d(x) = e^{-x}$$

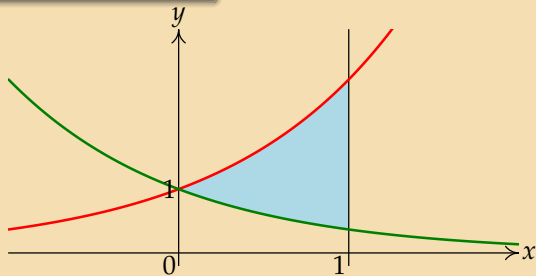
$$y = e^x, y = e^{-x}, x \in [0, 1], S = ?.$$



$$S = \int_0^1 e^x - e^{-x} dx = [e^x + e^{-x}]_0^1$$

Vypočteme neurčitý integrál.

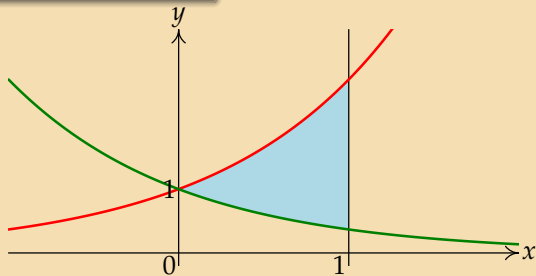
$$y = e^x, y = e^{-x}, x \in [0, 1], S = ?.$$



$$S = \int_0^1 e^x - e^{-x} dx = [e^x + e^{-x}]_0^1 = e^1 + e^{-1} - [e^0 + e^0]$$

Vypočítáme určitý integrál pomocí Newtonovy–Leignizovy formule. Dosadíme tedy meze.

$$y = e^x, y = e^{-x}, x \in [0, 1], S = ?.$$



$$S = \int_0^1 e^x - e^{-x} dx = [e^x + e^{-x}]_0^1 = e^1 + e^{-1} - [e^0 + e^0] = e + \frac{1}{e} - 2$$

Dopočítáme.

Určete obsah množiny mezi křivkami $y = 1 - (x - 1)^2$ a $x + y = 0$.

Určete obsah množiny mezi křivkami $y = 1 - (x - 1)^2$ a $x + y = 0$.

$$1 - (x - 1)^2 = -x$$

- První z křivek je parabola, druhá z křivek je přímka $y = -x$.
- Křivky se protínají v bodě, jehož x -ová splňuje rovnici

$$1 - (x - 1)^2 = -x$$

Určete obsah množiny mezi křivkami $y = 1 - (x - 1)^2$ a $x + y = 0$.

$$1 - (x - 1)^2 = -x$$

$$1 - (x^2 - 2x + 1) = -x$$

$$1 - x^2 + 2x - 1 = -x$$

$$3x - x^2 = 0$$

$$(3 - x)x = 0$$

Průsečíky křivek jsou body $[0, 0]$ a $[3, -3]$.

Určete obsah množiny mezi křivkami $y = 1 - (x - 1)^2$ a $x + y = 0$.

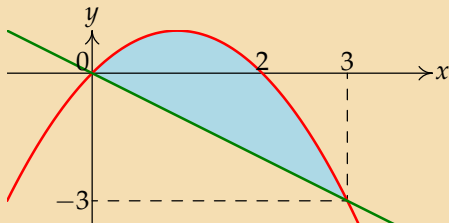
$$1 - (x - 1)^2 = -x$$

$$1 - (x^2 - 2x + 1) = -x$$

$$1 - x^2 + 2x - 1 = -x$$

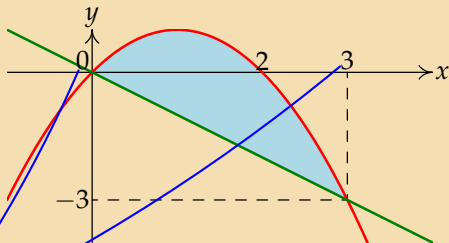
$$3x - x^2 = 0$$

$$(3 - x)x = 0$$



$$y = 1 - (x - 1)^2 = 1 - (x^2 - 2x + 1) = 2x - x^2 = x(2 - x)$$

Určete obsah množiny mezi křivkami $y = 1 - (x - 1)^2$ a $x + y = 0$.

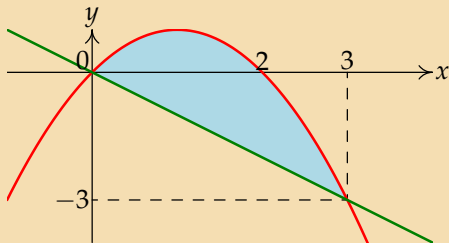


$$S = \int_0^3 1 - (x - 1)^2 - (-x) dx$$

$$h(x) = 1 - (x - 1)^2$$

$$d(x) = -x, \text{ protože } x + y = 0 \iff y = -x$$

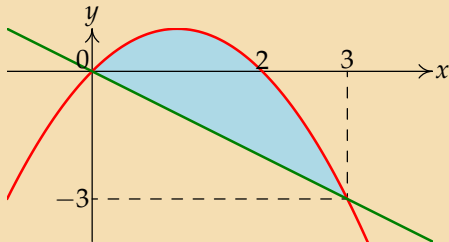
Určete obsah množiny mezi křivkami $y = 1 - (x - 1)^2$ a $x + y = 0$.



$$S = \int_0^3 1 - (x - 1)^2 - (-x) dx = \int_0^3 1 - (x^2 - 2x + 1) + x dx$$

Umocníme.

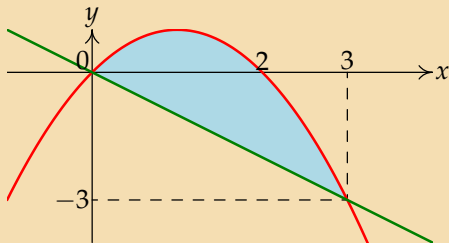
Určete obsah množiny mezi křivkami $y = 1 - (x - 1)^2$ a $x + y = 0$.



$$\begin{aligned} S &= \int_0^3 1 - (x - 1)^2 - (-x) \, dx = \int_0^3 1 - (x^2 - 2x + 1) + x \, dx \\ &= \int_0^3 -x^2 + 3x \, dx \end{aligned}$$

Upravíme integrand.

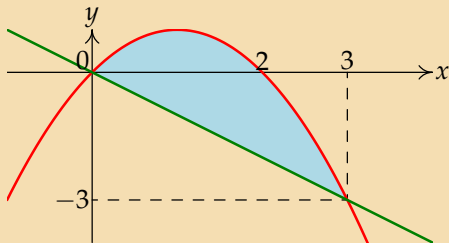
Určete obsah množiny mezi křivkami $y = 1 - (x - 1)^2$ a $x + y = 0$.



$$\begin{aligned} S &= \int_0^3 1 - (x - 1)^2 - (-x) \, dx = \int_0^3 1 - (x^2 - 2x + 1) + x \, dx \\ &= \int_0^3 -x^2 + 3x \, dx = \left[-\frac{x^3}{3} + 3\frac{x^2}{2} \right]_0^3 \end{aligned}$$

$$\int_a^b f(x) \, dx = [F(x)]_a^b = F(b) - F(a)$$

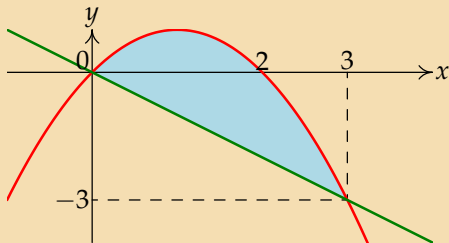
Určete obsah množiny mezi křivkami $y = 1 - (x - 1)^2$ a $x + y = 0$.



$$\begin{aligned} S &= \int_0^3 1 - (x - 1)^2 - (-x) dx = \int_0^3 1 - (x^2 - 2x + 1) + x dx \\ &= \int_0^3 -x^2 + 3x dx = \left[-\frac{x^3}{3} + 3\frac{x^2}{2} \right]_0^3 = \left[-\frac{3^3}{3} + 3\frac{3^2}{2} \right] - \left[-\frac{0^3}{3} + 3\frac{0^2}{2} \right] \end{aligned}$$

$$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$$

Určete obsah množiny mezi křivkami $y = 1 - (x - 1)^2$ a $x + y = 0$.



$$\begin{aligned} S &= \int_0^3 1 - (x - 1)^2 - (-x) dx = \int_0^3 1 - (x^2 - 2x + 1) + x dx \\ &= \int_0^3 -x^2 + 3x dx = \left[-\frac{x^3}{3} + 3\frac{x^2}{2} \right]_0^3 = \left[-\frac{3^3}{3} + 3\frac{3^2}{2} \right] - \left[-\frac{0^3}{3} + 3\frac{0^2}{2} \right] \\ &= -9 + \frac{27}{2} = \frac{9}{2} \end{aligned}$$

Dopočítáme obsah množiny.

KONEC