

# Radiologická fyzika

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podzim 2013

# Fyzikální základy radiologie

## Jak to dělají jinde?

Terminologie. Záření: částice a vlny.

Hmota se skládá z atomů Atomy. Zmínka o kvantové teorii.

Zákony zachování. Soustava SI. Jednotky v radiologii.

Radioaktivita.

Rentgenové záření a  $\gamma$  záření.

Jaderná magnetická rezonance.

Vytváření obrazu při MRI a CT.

Ultrazvuková diagnostika.

Ultrazvuk – vlnové vlastnosti.

Repetitorium.

# Chapter 1: Basic Radiation Physics

Slide set of 195 slides based on the chapter authored by  
E.B. Podgorsak  
of the IAEA publication (ISBN 92-0-107304-6):

*Review of Radiation Oncology Physics:  
A Handbook for Teachers and Students*

Obdélníkový výstřižek

## Objective:

To familiarize the student with basic principles of radiation physics and modern physics used in radiotherapy.



**IAEA**

International Atomic Energy Agency

Slide set prepared in 2006  
by E.B. Podgorsak (Montreal, McGill University)  
Comments to S. Vatnitsky:  
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**Version 2012**

- 1.1. Introduction
- 1.2. Atomic and nuclear structure
- 1.3. Electron interactions
- 1.4. Photon interactions

# 1.1 INTRODUCTION

## 1.1.1 Fundamental physical constants

- ❑ Avogadro's number:  $N_A = 6.022 \times 10^{23}$  atom/mol
- ❑ Speed of light in vacuum:  $c = 3 \times 10^8$  m/s
- ❑ Electron charge:  $e = 1.6 \times 10^{19}$  As
- ❑ Electron rest mass:  $m_e = 0.511$  MeV/c<sup>2</sup>
- ❑ Proton rest mass:  $m_p = 938.2$  MeV/c<sup>2</sup>
- ❑ Neutron rest mass:  $m_n = 939.3$  MeV/c<sup>2</sup>
- ❑ Atomic mass unit:  $u = 931.5$  MeV/c<sup>2</sup>

# 1.1 INTRODUCTION

## 1.1.2 Derived physical constants

- ☐ Reduced Planck's constant  $\times$  speed of light in vacuum

$$\hbar c = 197 \text{ MeV} \cdot \text{fm} \approx 200 \text{ MeV} \cdot \text{fm}$$

- ☐ Fine structure constant

$$\alpha = \frac{e^2}{4\pi\epsilon_0} \frac{1}{\hbar c} = \frac{1}{137}$$

- ☐ Classical electron radius

$$r_e = \frac{e^2}{4\pi\epsilon_0} \frac{1}{m_e c^2} = 2.818 \text{ MeV}$$

# 1.1 INTRODUCTION

## 1.1.2 Derived physical constants

□ Bohr radius:

$$a_0 = \frac{\hbar c}{\alpha m_e c^2} = \frac{4\pi\epsilon_0 (\hbar c)^2}{e^2 m_e c^2} = 0.529 \text{ \AA}$$

□ Rydberg energy:

$$E_R = \frac{1}{2} m_e c^2 \alpha^2 = \frac{1}{2} \left[ \frac{e^2}{4\pi\epsilon_0} \right]^2 \frac{m_e c^2}{(\hbar c)^2} = 13.61 \text{ eV}$$

□ Rydberg constant:

$$R_\infty = \frac{E_R}{2\pi\hbar c} = \frac{m_e c^2 \alpha^2}{4\pi\hbar c} = 109\,737 \text{ cm}^{-1}$$

# 1.1 INTRODUCTION

## 1.1.3 Physical quantities and units

- **Physical quantities** are characterized by their numerical value (magnitude) and associated unit.
- **Symbols** for physical quantities are set in *italic type*, while symbols for units are set in roman type.

*For example:*  $m = 21$  kg;  $E = 15$  MeV



# 1.1 INTRODUCTION

## 1.1.3 Physical quantities and units

- Numerical value and the unit of a physical quantity must be separated by space.

*For example:*

21 kg and **NOT 21kg**; 15 MeV and **NOT 15MeV**

- The currently used metric system of units is known as the **Système International d'Unités** (International system of units) or the **SI system**.

# 1.1 INTRODUCTION

## 1.1.3 Physical quantities and units

The SI system of units is founded on base units for seven physical quantities:

Quantity	SI unit
length $l$	meter (m)
mass $m$	kilogram (kg)
time $t$	second (s)
electric current ( $I$ )	ampère (A)
temperature ( $T$ )	kelvin (K)
amount of substance	mole (mol)
luminous intensity	candela (cd)

# 1.1 INTRODUCTION

## 1.1.4 Classification of forces in nature

There are **four distinct forces** observed in interaction between various types of particles

Force	Source	Transmitted particle	Relative strength
Strong	Strong charge	Gluon	1
EM	Electric charge	Photon	1/137
Weak	Weak charge	$W^+$ , $W^-$ , and $Z^0$	$10^{-6}$
Gravitational	Energy	Graviton	$10^{-39}$

# 1.1 INTRODUCTION

## 1.1.5 Classification of fundamental particles

Two classes of fundamental particles are known:

- **Quarks** are particles that exhibit strong interactions

Quarks are constituents of hadrons with a fractional electric charge ( $2/3$  or  $-1/3$ ) and are characterized by one of three types of strong charge called color (**red**, **blue**, **green**).

- **Leptons** are particles that do not interact strongly.

Electron, muon, tau, and their corresponding neutrinos.

# 1.1 INTRODUCTION

## 1.1.6 Classification of radiation

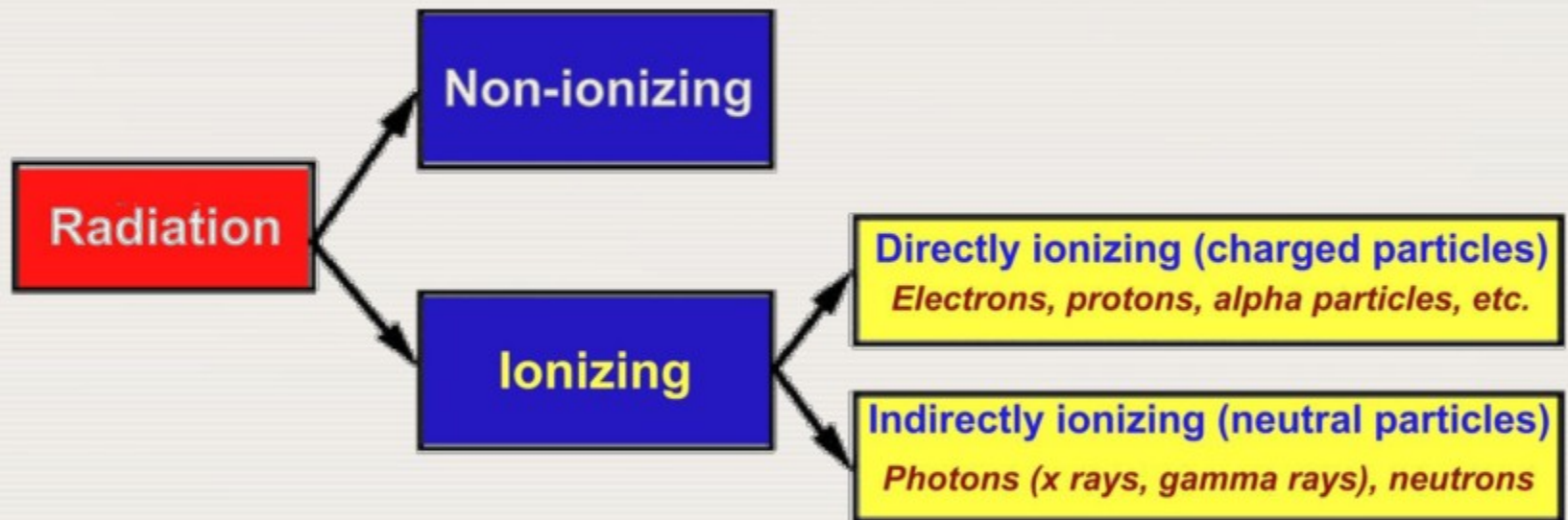
**Radiation** is classified into two main categories:

- ❑ Non-ionizing radiation (cannot ionise matter).
- ❑ **Ionizing radiation** (can ionize matter).
  - **Directly ionizing radiation** (charged particles)  
electron, proton, alpha particle, heavy ion
  - **Indirectly ionizing radiation** (neutral particles)  
photon (x ray, gamma ray), neutron

# 1.1 INTRODUCTION

## 1.1.6 Classification of radiation

Radiation is classified into two main categories:



# 1.1 INTRODUCTION

## 1.1.7 Classification of ionizing photon radiation

**Ionizing photon radiation** is classified into four categories:

- ☐ **Characteristic x ray**

Results from electronic transitions between atomic shells.

- ☐ **Bremsstrahlung**

Results mainly from electron-nucleus Coulomb interactions.

- ☐ **Gamma ray**

Results from nuclear transitions.

- ☐ **Annihilation quantum (annihilation radiation)**

Results from positron-electron annihilation.

# 1.1 INTRODUCTION

## 1.1.8 Einstein's relativistic mass, energy, and momentum

□ Mass:

$$m(v) = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{m_0}{\sqrt{1 - \beta^2}} = \gamma m_0$$

□ Normalized mass:

$$\frac{m(v)}{m_0} = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1 - \beta^2}} = \gamma$$

where

$$\beta = \frac{v}{c} \quad \text{and} \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

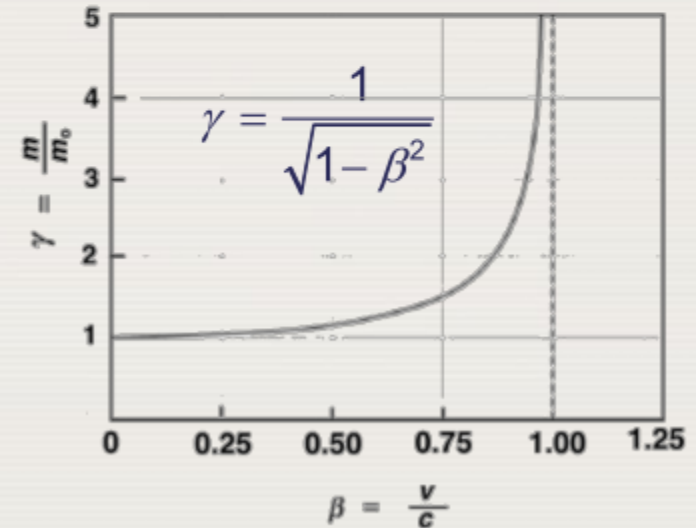
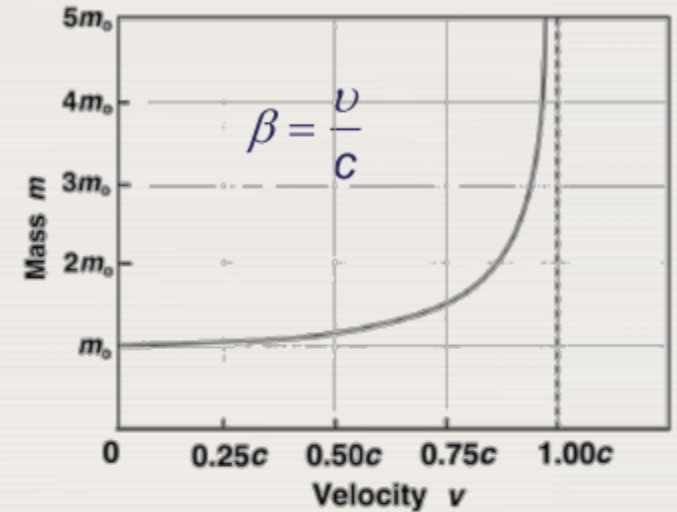


# 1.1 INTRODUCTION

## 1.1.8 Einstein's relativistic mass, energy, and momentum

$$m(v) = \frac{m_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{m_0}{\sqrt{1 - \beta^2}} = \gamma m_0$$

$$\frac{m(v)}{m_0} = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1 - \beta^2}} = \gamma$$



# 1.1 INTRODUCTION

## 1.1.8 Einstein's relativistic mass, energy, and momentum

□ Total energy:  $E = m(v)c^2$

□ Rest energy:  $E_0 = m_0c^2$

□ Kinetic energy:  $E_K = E - E_0 = (\gamma - 1)E_0$

□ Momentum:  $p = \frac{1}{c}\sqrt{E^2 - E_0^2}$

with

$$\beta = \frac{v}{c} \quad \text{and} \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

# 1.1 INTRODUCTION

## 1.1.9 Radiation quantities and units

Quantity	Definition	SI unit	Old unit	Conversion
Exposure $X$	$X = \frac{\Delta Q}{\Delta m_{\text{air}}}$	$\frac{2.58 \times 10^{-4} \text{ C}}{\text{kg air}}$	$1 \text{ R} = \frac{1 \text{ esu}}{\text{cm}^3 \text{ air}_{\text{STP}}}$	$1 \text{ R} = \frac{2.58 \times 10^{-4} \text{ C}}{\text{kg air}}$
Dose $D$	$D = \frac{\Delta E_{\text{ab}}}{\Delta m}$	$1 \text{ Gy} = 1 \frac{\text{J}}{\text{kg}}$	$1 \text{ rad} = 100 \frac{\text{erg}}{\text{g}}$	$1 \text{ Gy} = 100 \text{ rad}$
Equivalent dose $H$	$H = Dw_{\text{R}}$	$1 \text{ Sv}$	$1 \text{ rem}$	$1 \text{ Sv} = 100 \text{ rem}$
Activity $\mathcal{A}$	$\mathcal{A} = \lambda N$	$1 \text{ Bq} = 1 \text{ s}^{-1}$	$1 \text{ Ci} = 3.7 \times 10^{10} \text{ s}^{-1}$	$1 \text{ Bq} = \frac{1 \text{ Ci}}{3.7 \times 10^{10}}$

# 1.2 ATOMIC AND NUCLEAR STRUCTURE

## 1.2.1 Basic definitions for atomic structure

□ Constituent particles forming an atom are:

- Proton
- Neutron
- Electron

Protons and neutrons are known as **nucleons** and form the **nucleus**.

□ **Atomic number  $Z$**

Number of protons and number of electrons in an atom.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.1 Basic definitions for atomic structure

#### □ Atomic mass number $A$

Number of nucleons ( $Z + N$ ) in an atom,

where

- $Z$  is the number of protons (atomic number) in an atom.
- $N$  is the number of neutrons in an atom.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.1 Basic definitions for atomic structure

- There is no basic relation between the atomic mass number  $A$  and atomic number  $Z$  of a nucleus but the empirical relationship:

$$Z = \frac{A}{1.98 + 0.0155A^{2/3}}$$

furnishes a good approximation for stable nuclei.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.1 Basic definitions for atomic structure

- **Atomic mole** is defined as the **number of grams** of an atomic compound that contains exactly one Avogadro's number of atoms, i.e.,

$$N_A = 6.022 \times 10^{23} \text{ atom/mol}$$

- Atomic mass number  $A$  of all elements is defined such that  $A$  grams of every element contain exactly  $N_A$  atoms.
- *For example:*
  - 1 mole of cobalt-60 is 60 g of cobalt-60.
  - 1 mole of radium-226 is 226 g of radium-226.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.1 Basic definitions for atomic structure

- ❑ **Molecular mole** is defined as the number of grams of a molecular compound that contains exactly one Avogadro's number of molecules, i.e.,

$$N_A = 6.022 \times 10^{23} \text{ molecule/mol}$$

- ❑ Mass of a molecule is the sum of masses of all atoms that make up the molecule.
- ❑ *For example:*
  - 1 mole of water ( $\text{H}_2\text{O}$ ) is 18 g of water.
  - 1 mole of carbon dioxide ( $\text{CO}_2$ ) is 44 g of carbon dioxide.



## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.1 Basic definition for atomic structure

- Atomic mass  $\mathcal{M}$  is expressed in atomic mass units  $u$ :
  - 1  $u$  is equal to 1/12th of the mass of the carbon-12 atom or 931.5 MeV/ $c^2$ .
  - Atomic mass  $\mathcal{M}$  is smaller than the sum of the individual masses of constituent particles because of the intrinsic energy associated with binding the particles (nucleons) within the nucleus.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.1 Basic definition for atomic structure

- Nuclear mass  $M$  is defined as the atomic mass with the mass of atomic orbital electrons subtracted, i.e.,

$$M = \mathcal{M} - Zm_e$$

where  $\mathcal{M}$  is the atomic mass. Binding energy of orbital electrons to the nucleus is neglected.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.1 Basic definitions for atomic structure

In nuclear physics the convention is to designate a nucleus

X as  ${}^A_Z\text{X}$ ,

where

A is the atomic mass number.

Z is the atomic number.

*For example:*

- **Cobalt-60** nucleus with  $Z = 27$  protons and  $A = 60$  nucleons is identified as  ${}^{60}_{27}\text{Co}$ .
- **Radium-226** nucleus with 88 protons and 138 neutrons is identified as  ${}^{226}_{88}\text{Ra}$ .

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.1 Basic definitions for atomic structure

- Number of atoms  $N_a$  per mass  $m$  of an element:

$$\frac{N_a}{m} = \frac{N_A}{A}$$

- Number of electrons  $N_e$  per mass  $m$  of an element:

$$\frac{N_e}{m} = Z \frac{N_a}{m} = Z \frac{N_A}{A}$$

- Number of electrons  $N_e$  per volume  $V$  of an element:

$$\frac{N_e}{V} = \rho Z \frac{N_a}{m} = \rho Z \frac{N_A}{A}$$

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.1 Basic definitions for atomic structure

□ For all elements the ratio  $Z/A \approx 0.5$  with two notable exceptions:

- Hydrogen-1 for which  $Z/A = 1.0$ .
- Helium-3 for which  $Z/A = 0.67$ .

□ Actually, the ratio  $Z/A$  gradually decreases:

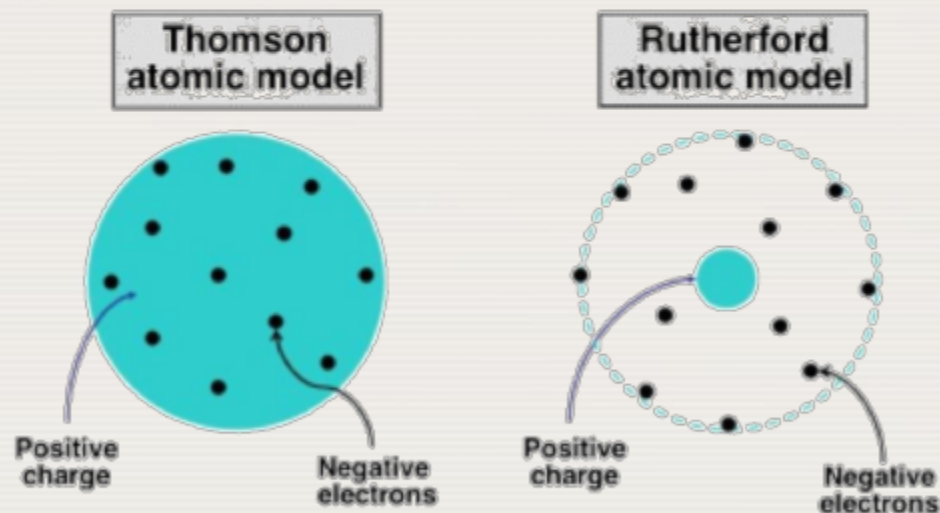
- From 0.5 for low atomic number  $Z$  elements.
- To  $\sim 0.4$  for high atomic number  $Z$  elements.

□ *For example:*  $Z/A = 0.50$  for  ${}^4_2\text{He}$   
 $Z/A = 0.45$  for  ${}^{60}_{27}\text{Co}$   
 $Z/A = 0.39$  for  ${}^{235}_{92}\text{U}$

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.2 Rutherford's model of the atom

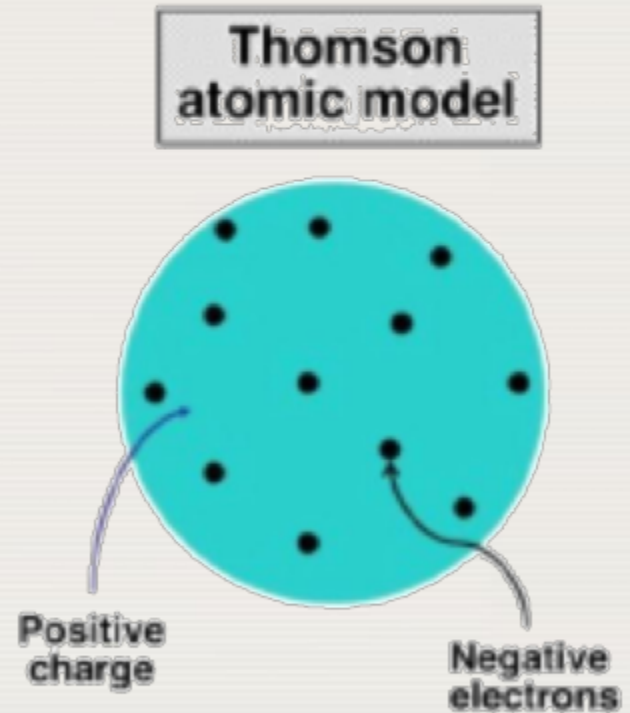
- Rutherford's atomic model is based on results of the Geiger-Marsden experiment of 1909 with 5.5 MeV alpha particles scattered on thin gold foils with a thickness of the order of  $10^{-6}$  m.



## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.2 Rutherford's model of the atom

- ❑ At the time of the Geiger-Marsden experiment in 1909 Thomson atomic model was the prevailing atomic model.
- ❑ Thomson model was based on an assumption that the positive and the negative (electron) charges of the atom were distributed uniformly over the atomic volume (“plum-pudding model of the atom”).



## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.2 Rutherford's model of the atom

- Geiger and Marsden found that **more than 99 %** of the alpha particles incident on the gold foil were scattered at scattering **angles less than  $3^\circ$**  and that the distribution of scattered alpha particles followed a Gaussian shape.
- Geiger and Marsden also found that roughly **one in  $10^4$**  alpha particles was scattered with a scattering **angle exceeding  $90^\circ$**  (probability  $10^{-4}$ )
- This finding was in drastic disagreement with the theoretical prediction of **one in  $10^{3500}$**  resulting from the Thomson's atomic model (probability  $10^{-3500}$ ).

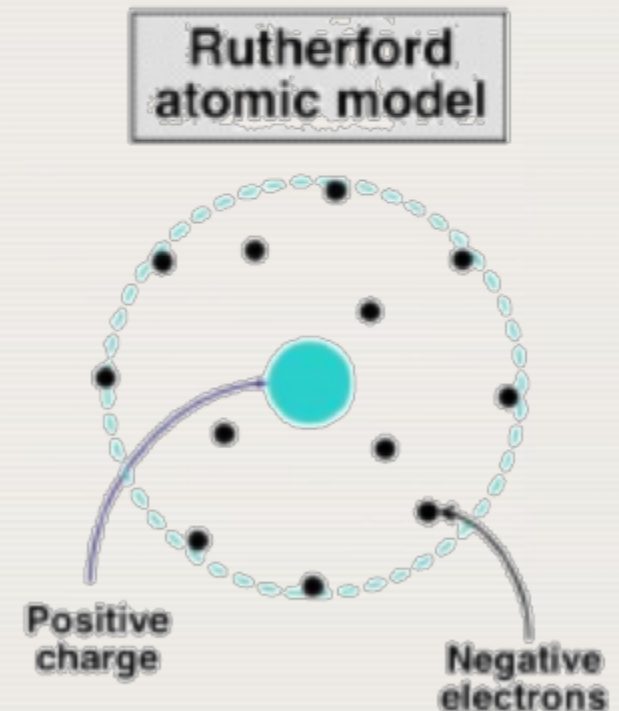


## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.2 Rutherford's model of the atom

□ Ernest Rutherford concluded that the peculiar results of the Geiger-Marsden experiment did not support the Thomson's atomic model and proposed **the currently accepted atomic model** in which:

- Mass and positive charge of the atom are concentrated in the **nucleus** the size of which is **of the order of  $10^{-15}$  m.**
- Negatively charged electrons revolve about the nucleus in a spherical cloud on the periphery of the **Rutherford atom with a radius of the order of  $10^{-10}$  m.**



## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.2 Rutherford's model of the atom

- ❑ Based on his model and **four additional assumptions**, Rutherford derived the kinematics for the scattering of alpha particles on gold nuclei using basic principles of classical mechanics.
  
- ❑ The **four assumptions** are related to:
  - Mass of the gold nucleus.
  - Scattering of alpha particles.
  - Penetration of the nucleus.
  - Kinetic energy of the alpha particles.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.2 Rutherford's model of the atom

□ The four assumptions are:

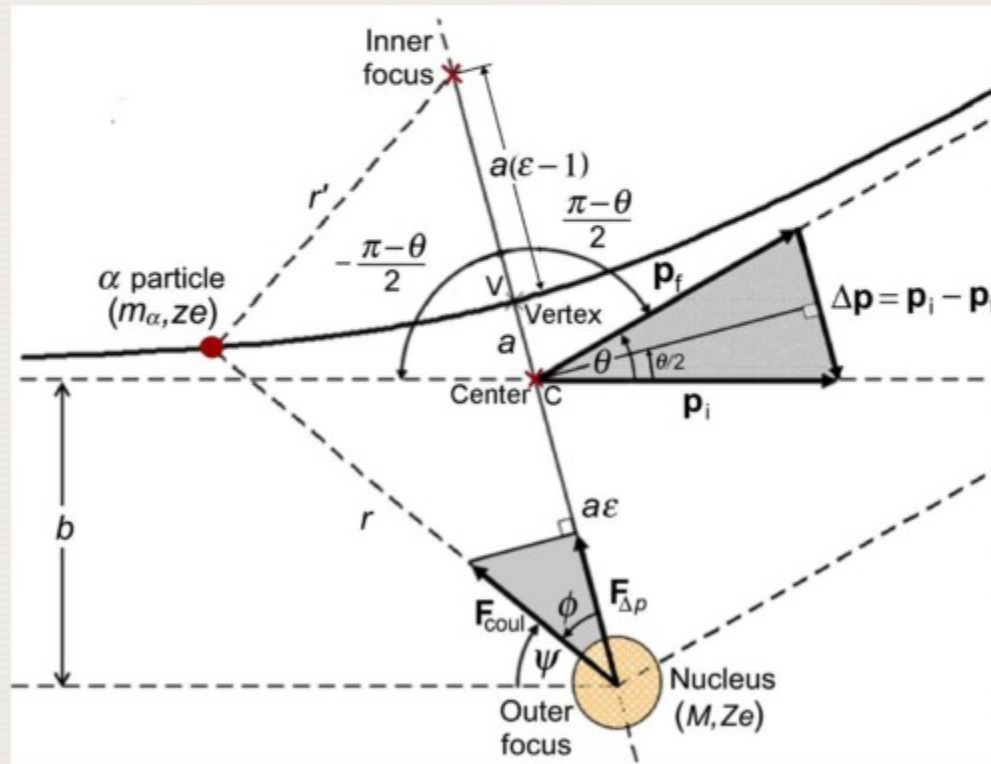
- Mass of the gold nucleus  $M \gg$  mass of the alpha particle  $m_\alpha$ .
- Scattering of alpha particles on atomic electrons is negligible.
- Alpha particle does not penetrate the nucleus, i.e., there are no nuclear reactions occurring.
- Alpha particles with kinetic energies of the order of a few MeV are **non-relativistic** and the simple classical relationship for the kinetic energy  $E_K$  of the alpha particle is valid:

$$E_K = \frac{m_\alpha v^2}{2}$$

# 1.2 ATOMIC AND NUCLEAR STRUCTURE

## 1.2.2 Rutherford's model of the atom

As a result of the **repulsive Coulomb interaction** between the alpha particle (charge  $+2e$ ) and the nucleus (charge  $+Ze$ ) the alpha particle follows a hyperbolic trajectory.



## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.2 Rutherford's model of the atom

- Shape of the **hyperbolic trajectory** and the scattering angle  $\theta$  depend on the impact parameter  $b$ .

The limiting case is a direct hit with  $b = 0$  and  $\theta = \pi$  (backscattering) that, assuming conservation of energy, determines the **distance of closest approach**  $D_{\alpha-N}$  in a direct hit (backscattering) interaction.

$$E_K = \frac{2Z_N e^2}{4\pi\epsilon_0 D_{\alpha-N}} \Rightarrow D_{\alpha-N} = \frac{2Z_N e^2}{4\pi\epsilon_0 E_K}$$

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.2 Rutherford's model of the atom

- Shape of the **hyperbolic trajectory** and the scattering angle  $\theta$  are a function of the impact parameter  $b$ .
- **Repulsive Coulomb force** between the alpha particle (charge  $ze$ ,  $z = 2$ ) and the nucleus (charge  $Ze$ ) is governed by  $1/r^2$  dependence:

$$F_{\text{coul}} = \frac{2Ze^2}{4\pi\epsilon_0 r^2}$$

where  $r$  is the separation between the two charged particles.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.2 Rutherford's model of the atom

- Relationship between the impact parameter  $b$  and the scattering angle  $\theta$  follows from the conservation of energy and momentum considerations:

$$b = \frac{1}{2} D_{\alpha-N} \cot \frac{\theta}{2}$$

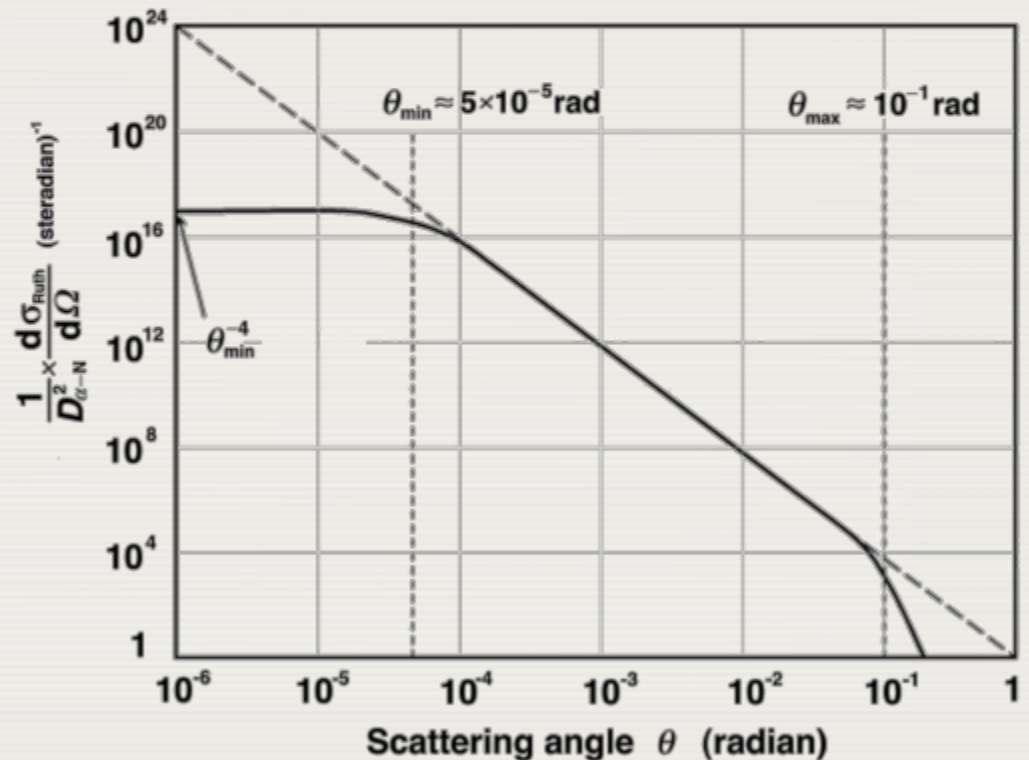
- This expression is derived using:
  - Classical relationship for the kinetic energy of the  $\alpha$  particle:
$$E_K = m_\alpha v^2 / 2$$
  - Definition of  $D_{\alpha-N}$  in a direct hit head-on collision for which the impact parameter  $b = 0$  and the scattering angle  $\theta = \pi$ .

# 1.2 ATOMIC AND NUCLEAR STRUCTURE

## 1.2.2 Rutherford's model of the atom

Differential Rutherford scattering cross section is:

$$\frac{d\sigma}{d\Omega} = \left[ \frac{D_{\alpha-N}}{4} \right]^2 \frac{1}{\sin^4(\theta/2)}$$



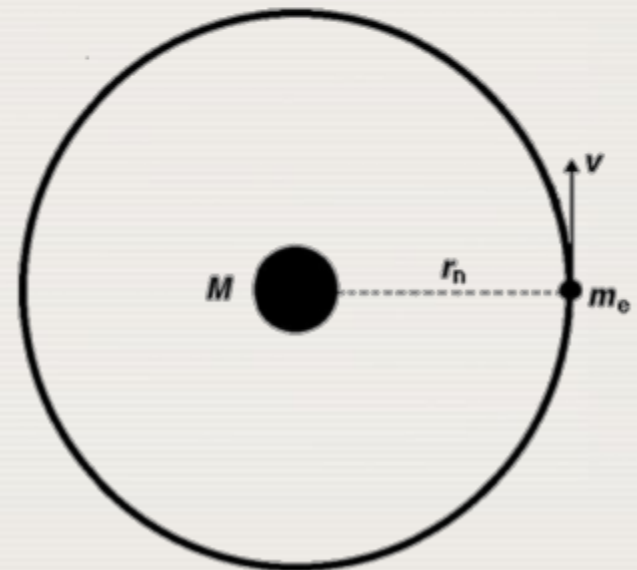


## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.3 Bohr's model of the hydrogen atom

- Niels Bohr in 1913 combined Rutherford's concept of nuclear atom with Planck's idea of quantized nature of the radiation process and developed an atomic model that successfully deals with one-electron structures, such as hydrogen atom, singly ionized helium, etc.

- $M$  nucleus with mass  $M$
- $m_e$  electron with mass  $m_e$
- $r_n$  radius of electron orbit



## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.3 Bohr's model of the hydrogen atom

□ Bohr's atomic model is based on four postulates:

- *Postulate 1:* Electrons revolve about the Rutherford nucleus in well-defined, allowed orbits (**planetary-like motion**).
- *Postulate 2:* While in orbit, the electron does not lose any energy despite being constantly accelerated (**no energy loss while electron is in allowed orbit**).
- *Postulate 3:* The angular momentum of the electron in an allowed orbit is quantized (**quantization of angular momentum**).
- *Postulate 4:* An atom emits radiation only when an electron makes a transition from one orbit to another (**energy emission during orbital transitions**).

# 1.2 ATOMIC AND NUCLEAR STRUCTURE

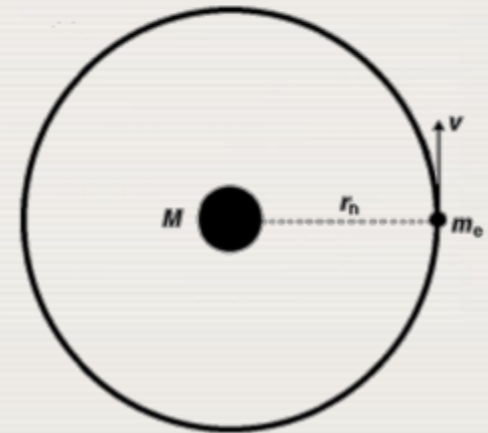
## 1.2.3 Bohr's model of the hydrogen atom

Bohr's atomic model is based on four postulates:

### Postulate 1: Planetary motion of electrons

- Electrons revolve about the Rutherford nucleus in well-defined, allowed orbits.
- Coulomb force of attraction between the electron and the positively charged nucleus is balanced by the centrifugal force

$$F_{\text{coul}} = \frac{1}{4\pi\epsilon_0} \frac{Ze^2}{r_e^2} \equiv F_{\text{cent}} = \frac{m_e v_e^2}{r_e}$$



## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.3 Bohr's model of the hydrogen atom

Bohr's atomic model is based on four postulates:

**Postulate 2: No energy loss while electron is in orbit.**

- While in orbit, the electron does not lose any energy despite being constantly accelerated.
- This is a direct contravention of the basic law of nature (Larmor's law) which states that:  
“Any time a charged particle is accelerated or decelerated part of its energy is emitted in the form of photon (bremsstrahlung)”.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.3 Bohr's model of the hydrogen atom

Bohr's atomic model is based on four postulates:

#### **Postulate 3: Quantization of angular momentum**

- Angular momentum  $L = m_e v r$  of the electron in an allowed orbit is quantized and given as  $L = n\hbar$ , where  $n$  is an integer referred to as the **principal quantum number** and  $\hbar = h/2\pi$ .
- Lowest possible angular momentum of electron in an allowed orbit is  $L = \hbar$ .
- All atomic orbital electron angular momenta are integer multiples of  $\hbar$ .

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.3 Bohr's model of the hydrogen atom

Bohr's atomic model is based on four postulates:

**Postulate 4:** Emission of photon during atomic transition.

- Atom emits radiation only when an electron makes a transition from an initial allowed orbit with quantum number  $n_i$  to a final orbit with quantum number  $n_f$ .
- Energy of the emitted photon equals the difference in energy between the two atomic orbits.

$$h\nu = E_i - E_f$$

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.3 Bohr's model of the hydrogen atom

- Radius  $r_n$  of a one-electron Bohr atom is:

$$r_n = a_0 \left[ \frac{n^2}{Z} \right] = (0.53 \text{ \AA}) \times \left[ \frac{n^2}{Z} \right]$$

- Velocity  $v_n$  of the electron in a one-electron Bohr atom is:

$$v_n = \alpha c \left[ \frac{Z}{n} \right] = \frac{c}{137} \left[ \frac{Z}{n} \right] \approx 7 \times 10^{-3} c \left[ \frac{Z}{n} \right]$$

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.3 Bohr's model of the hydrogen atom

- Energy levels  $E_n$  of orbital electron shells in a one-electron Bohr atom are:

$$E_n = -E_R \left[ \frac{Z}{n} \right]^2 = (-13.6 \text{ eV}) \times \left[ \frac{Z}{n} \right]^2$$

- Wave number  $k$  for transition from shell  $n_i$  to shell  $n_f$ :

$$k = R_\infty Z^2 \left\{ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right\} = 109\,737 \text{ cm}^{-1}$$



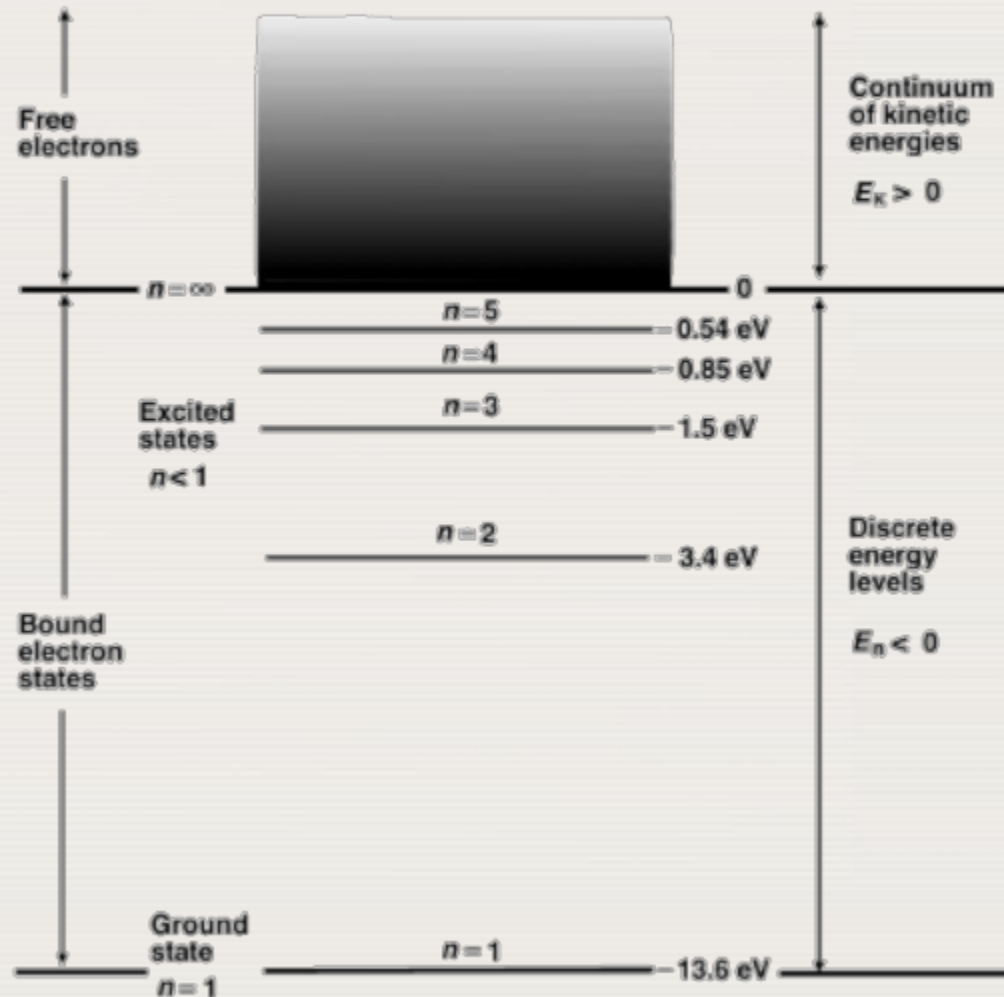
# 1.2 ATOMIC AND NUCLEAR STRUCTURE

## 1.2.3 Bohr's model of the hydrogen atom

- Energy levels  $E_n$  of orbital electron shells in a one-electron Bohr atom are:

$$E_n = -E_R \left[ \frac{Z}{n} \right]^2$$
$$= (-13.6 \text{ eV}) \times \left[ \frac{Z}{n} \right]^2$$

- $E_R$  = Rydberg energy



## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.3 Bohr's model of the hydrogen atom

Velocity of the orbital electron in the ground state  $n = 1$  is less than 1 % of the speed of light for the hydrogen atom with  $Z = 1$ .

$$\frac{v_n}{c} = \alpha \left[ \frac{Z}{n} \right] = \frac{1}{137} \left[ \frac{Z}{n} \right] \approx (7 \times 10^{-3}) \times \left[ \frac{Z}{n} \right]$$

Therefore, the use of classical mechanics in the derivation of the kinematics of the Bohr atom is justified.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.3 Bohr's model of the hydrogen atom

- ❑ Both Rutherford and Bohr **used classical mechanics** in their discoveries of the atomic structure and the kinematics of the electronic motion, respectively.
- ❑ Rutherford introduced the idea of atomic nucleus that contains most of the atomic mass and is 5 orders of magnitude smaller than the atom.
- ❑ Bohr introduced the idea of electronic angular momentum quantization.

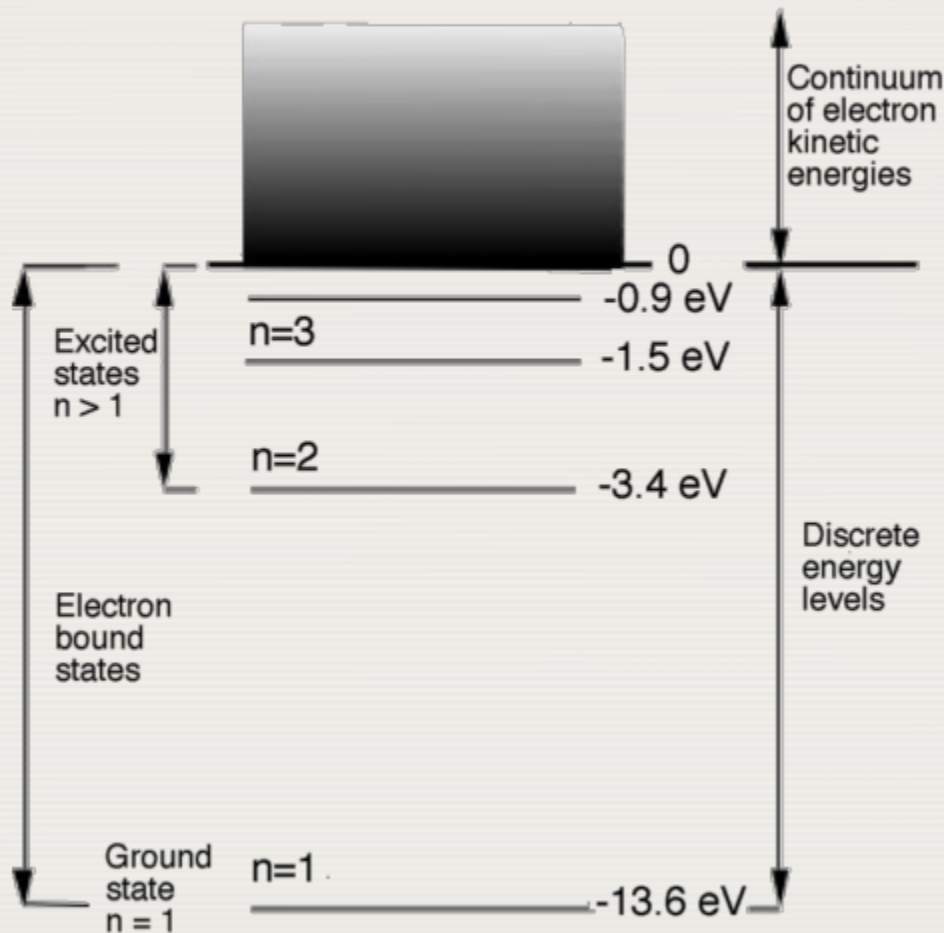
## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.3 Bohr's model of the hydrogen atom

- ❑ Nature provided Rutherford with an **atomic probe** (naturally occurring alpha particles) having just the appropriate energy (few MeV) to probe the atom without having to deal with relativistic effects and nuclear penetration.
- ❑ Nature provided Bohr with the **hydrogen one-electron atom** in which the electron can be treated with simple classical relationships.

# 1.2 ATOMIC AND NUCLEAR STRUCTURE

## 1.2.3 Bohr's model of the hydrogen atom



Energy level diagram for the hydrogen atom.

$n = 1$  ground state

$n > 1$  excited states

Wave number of emitted photon

$$k = \frac{1}{\lambda} = R_{\infty} Z^2 \left\{ \frac{1}{n_f^2} - \frac{1}{n_i^2} \right\}$$

$$R_{\infty} = 109\,737 \text{ cm}^{-1}$$

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.4 Multi-electron atom

- ❑ Bohr theory works well for one-electron structures but does not apply directly to multi-electron atoms because of the repulsive Coulomb interactions among the atomic electrons.
- ❑ Electrons occupy allowed shells; however, **the number of electrons per shell is limited to  $2n^2$ .**
- ❑ Energy level diagrams of multi-electron atoms resemble those of one-electron structures, except that **inner shell electrons are bound with much larger energies than  $E_R$ .**

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.4 Multi-electron atoms

- **Douglas Hartree** proposed an approximation that predicts the energy levels and radii of multi-electron atoms reasonably well despite its inherent simplicity.
- Hartree assumed that the potential seen by a given atomic electron is

$$V(r) = -\frac{Z_{\text{eff}} e^2}{4\pi\epsilon_0} \frac{1}{r}$$

where  $Z_{\text{eff}}$  is the effective atomic number that accounts for the potential screening effects of orbital electrons ( $Z_{\text{eff}} < Z$ ).

- $Z_{\text{eff}}$  for K-shell ( $n = 1$ ) electrons is  $Z - 2$ .
- $Z_{\text{eff}}$  for outer shell electrons is approximately equal to  $n$ .

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.4 Multi-electron atom

Hartree's expressions for atomic radii and energy level

#### □ Atomic radius

**In general**

$$r_n = a_0 \frac{n^2}{Z_{\text{eff}}}$$

**For the K shell**

$$r(\text{K shell}) = r_1 = a_0 \frac{n^2}{Z-2}$$

**For the outer shell**

$$r_{\text{outer shell}} \approx na_0$$

#### □ Binding energy

**In general**

$$E_n = -E_R \frac{Z_{\text{eff}}^2}{n^2}$$

**For the K shell**

$$E(\text{K shell}) = E_1 = -E_R (Z-2)^2$$

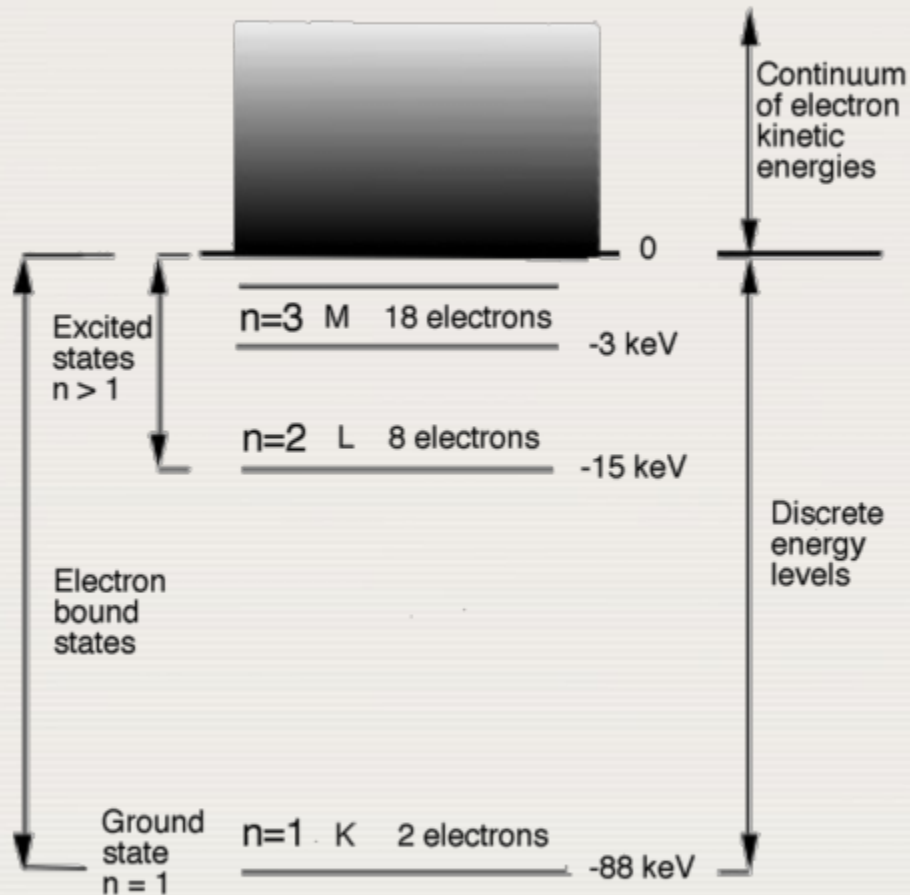
**For outer shell**

$$E_{\text{outer shell}} \approx -E_R$$



# 1.2 ATOMIC AND NUCLEAR STRUCTURE

## 1.2.4 Multi-electron atom



Energy level diagram for multi-electron atom (lead)

Shell (orbit) designations:

$n = 1$  K shell (2 electrons)

$n = 2$  L shell (8 electrons)

$n = 3$  M shell (18 electrons)

$n = 4$  N shell (32 electrons)

.....

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.5 Nuclear structure

- ❑ Most of the **atomic mass** is concentrated in the **atomic nucleus** consisting of  $Z$  protons and  $A - Z$  neutrons where  $Z$  is the atomic number and  $A$  the atomic mass number (Rutherford-Bohr atomic model).
- ❑ **Protons and neutrons** are commonly called **nucleons** and are bound to the nucleus with the strong force.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.5 Nuclear structure

- In contrast to the electrostatic and gravitational forces that are inversely proportional to the square of the distance between two particles, the **strong force between two particles is a very short range force**, active only at distances of the order of a few femtometers.
- **Radius  $r$  of the nucleus** is estimated from:  $r = r_0 \sqrt[3]{A}$ , where  $r_0$  is the nuclear radius constant (1.25 fm).

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.5 Nuclear structure

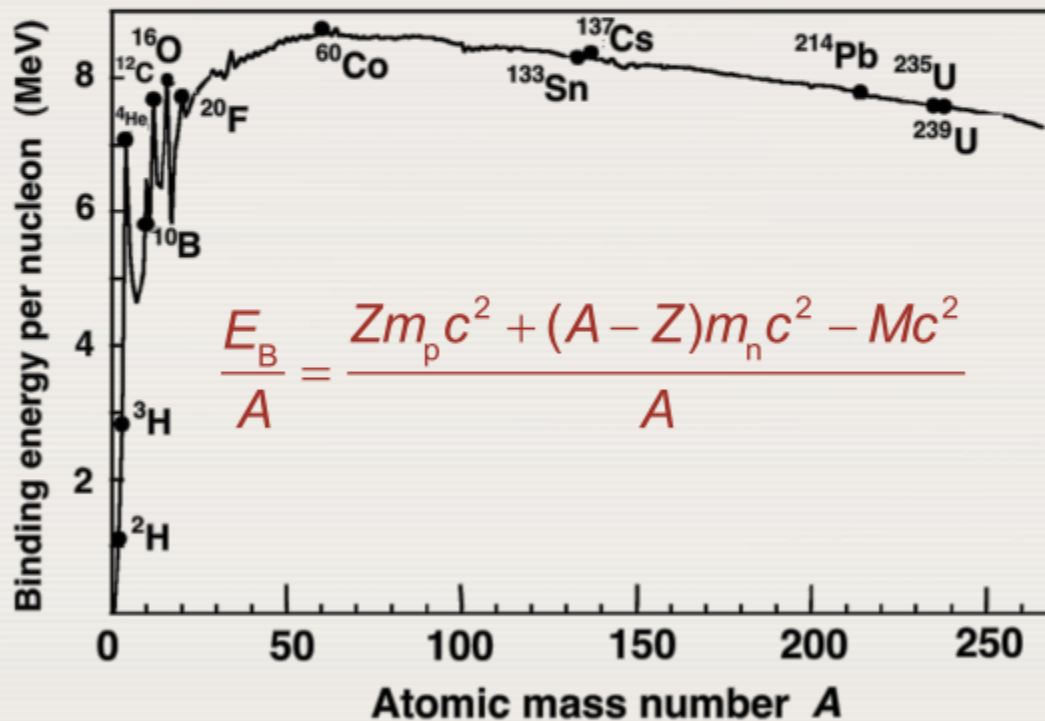
- Sum of masses of the individual components of a nucleus that contains  $Z$  protons and  $(A - Z)$  neutrons is larger than the mass of the nucleus  $M$ .
- This difference in masses is called the **mass defect** (deficit)  $\Delta m$  and its energy equivalent  $\Delta mc^2$  is called the **total binding energy**  $E_B$  of the nucleus:

$$E_B = Zm_p c^2 + (A - Z)m_n c^2 - Mc^2$$

# 1.2 ATOMIC AND NUCLEAR STRUCTURE

## 1.2.5 Nuclear structure

Binding energy per nucleon ( $E_B/A$ ) in a nucleus varies with the number of nucleons  $A$  and is of the order of 8 MeV per nucleon.



Nucleus	$E_B/A$ (MeV)
$^2_1\text{H}$	1.1
$^3_1\text{H}$	2.8
$^3_1\text{He}$	2.6
$^4_2\text{He}$	7.1
$^{60}_{27}\text{Co}$	8.8
$^{238}_{92}\text{U}$	7.3

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.6 Nuclear reactions

- ❑ Nuclear reaction:  $A + a = B + b$  or  $A(a,b)B$   
Projectile  $a$  bombards target  $A$   
which is transformed into reactants  $B$  and  $b$ .
- ❑ The most important physical quantities that are conserved in a nuclear reaction are:
  - Charge
  - Mass number
  - Linear momentum
  - Mass-energy

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.6 Nuclear reactions

- Threshold kinetic energy  $(E_K)_{\text{thr}}^a$  for a nuclear reaction is calculated from the **relativistic invariant** and is the smallest value of projectile's kinetic energy at which the reaction will take place:

$$(E_K)_{\text{thr}}^a = \frac{(m_B c^2 + m_b c^2)^2 - (m_A c^2 + m_a c^2)^2}{2m_A c^2}$$

- Threshold total energy  $E_{\text{thr}}^a$  for a nuclear reaction to occur is:

$$E_{\text{thr}}^a = \frac{(m_B c^2 + m_b c^2)^2 - (m_A^2 c^4 + m_a^2 c^4)}{2m_A c^2}$$

$m_A$ ,  $m_a$ ,  $m_B$ , and  $m_b$

are rest masses of A, a, B, and b, respectively.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.7 Radioactivity

- ❑ **Radioactivity** is a process by which an unstable nucleus (parent) decays into a new nuclear configuration (daughter) that may be stable or unstable.
- ❑ If the daughter is unstable, it will decay further through a chain of decays until a stable configuration is attained.



## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.7 Radioactivity

- ❑ Henri Becquerel discovered natural radioactivity in 1896.
- ❑ Other names used for radioactive decay are:
  - Nuclear decay.
  - Nuclear disintegration.
  - Nuclear transformation.
  - Nuclear transmutation.
  - Radioactive decay.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.7 Radioactivity

- ❑ **Radioactive decay** involves a transition from the quantum state of the parent  $P$  to a quantum state of the daughter  $D$ .
- ❑ Energy difference between the two quantum states is called the **decay energy  $Q$** .
- ❑ Decay energy  $Q$  is emitted:
  - In the form of **electromagnetic radiation** (gamma rays)  
or
  - In the form of **kinetic energy of the reaction products**.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.7 Radioactivity

□ All radioactive processes are governed by the same formalism based on:

- Characteristic parameter called the **decay constant**  $\lambda$
- **Activity**  $\mathcal{A}(t)$  defined as  $\lambda N(t)$  where  $N(t)$  is the number of radioactive nuclei at time  $t$

$$\mathcal{A}(t) = \lambda N(t)$$

□ **Specific activity**  $a$  is the parent's activity per unit mass:

$$a = \frac{\mathcal{A}(t)}{M} = \frac{\lambda N(t)}{M} = \frac{\lambda N_A}{A}$$

$N_A$  is Avogadro's number.

$A$  is atomic mass number.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.7 Radioactivity

- ❑ Activity represents the total number of disintegrations (decays) of parent nuclei per unit time.
- ❑ SI unit of activity is the becquerel ( $1 \text{ Bq} = 1 \text{ s}^{-1}$ ).

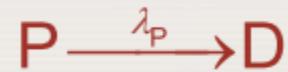
Both becquerel and hertz correspond to  $\text{s}^{-1}$  yet hertz expresses frequency of periodic motion, while **becquerel expresses activity**.
- ❑ The older unit of activity is the curie ( $1 \text{ Ci} = 3.7 \times 10^{10} \text{ s}^{-1}$ ), originally defined as the activity of 1 g of radium-226.

Currently, the **activity of 1 g of radium-226 is 0.988 Ci**.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.7 Radioactivity

- Decay of radioactive parent P into stable daughter D:



- Rate of depletion of the number of radioactive parent nuclei  $N_P(t)$  is equal to the activity  $\mathcal{A}_P(t)$  at time  $t$ :

$$\frac{dN_P(t)}{dt} = -\mathcal{A}_P(t) = -\lambda_P N_P(t) \qquad \int_{N_P(0)}^{N_P(t)} \frac{dN_P(t)}{N_P} = -\int_0^t \lambda_P dt$$

where  $N_P(0)$  is the initial number of parent nuclei at time  $t = 0$ .

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.7 Radioactivity

- Number of radioactive parent nuclei  $N_p(t)$  as a function of time  $t$  is:

$$N_p(t) = N_p(0)e^{-\lambda_p t}$$

- Activity of the radioactive parent  $\mathcal{A}_p(t)$  as a function of time  $t$  is:

$$\mathcal{A}_p(t) = \lambda_p N_p(t) = \lambda_p N_p(0)e^{-\lambda_p t} = \mathcal{A}_p(0)e^{-\lambda_p t}$$

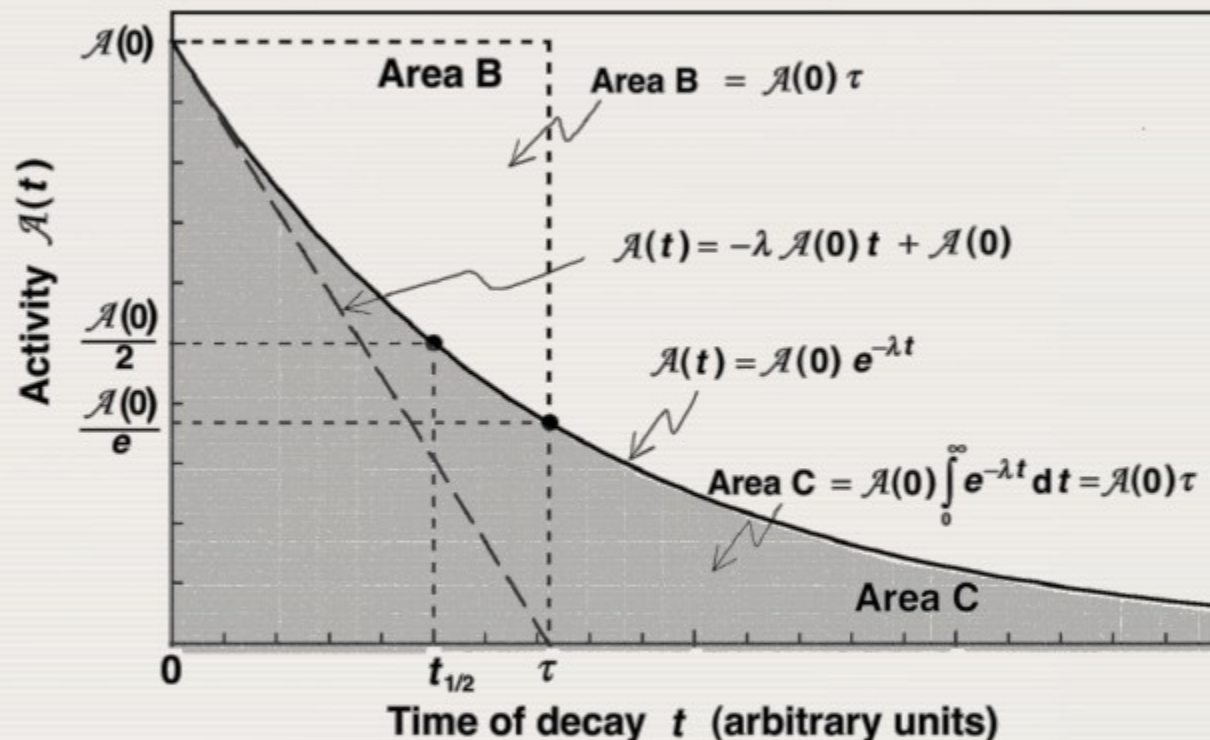
where  $\mathcal{A}_p(0)$  is the initial activity at time  $t = 0$ .

# 1.2 ATOMIC AND NUCLEAR STRUCTURE

## 1.2.7 Radioactivity

Parent activity  $\mathcal{A}_p(t)$  plotted against time  $t$  illustrating:

- Exponential decay of the activity.
- Concept of half life.
- Concept of mean life.



## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.7 Radioactivity

- **Half life**  $(t_{1/2})_P$  of radioactive parent P is the time during which the number of radioactive parent nuclei decays from the initial value  $N_P(0)$  at time  $t = 0$  to half the initial value

$$N_P(t = t_{1/2}) = (1/2)N_P(0) = N_P(0)e^{-\lambda_P(t_{1/2})_P}$$

- Decay constant  $\lambda_P$  and the half life  $(t_{1/2})_P$  are related as follows

$$\lambda_P = \frac{\ln 2}{(t_{1/2})_P}$$



## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.7 Radioactivity

- Decay of radioactive parent P into unstable daughter D which in turn decays into granddaughter G:



- Rate of change  $dN_D/dt$  in the number of daughter nuclei D equals to supply of new daughter nuclei through decay of P given as  $\lambda_P N_P(t)$  and the loss of daughter nuclei D from the decay of D to G given as  $-\lambda_D N_D(t)$

$$\frac{dN_D}{dt} = \lambda_P N_P(t) - \lambda_D N_D(t) = \lambda_P N_P(0) e^{-\lambda_P t} - \lambda_D N_D(t)$$

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.7 Radioactivity

- Number of daughter nuclei is:

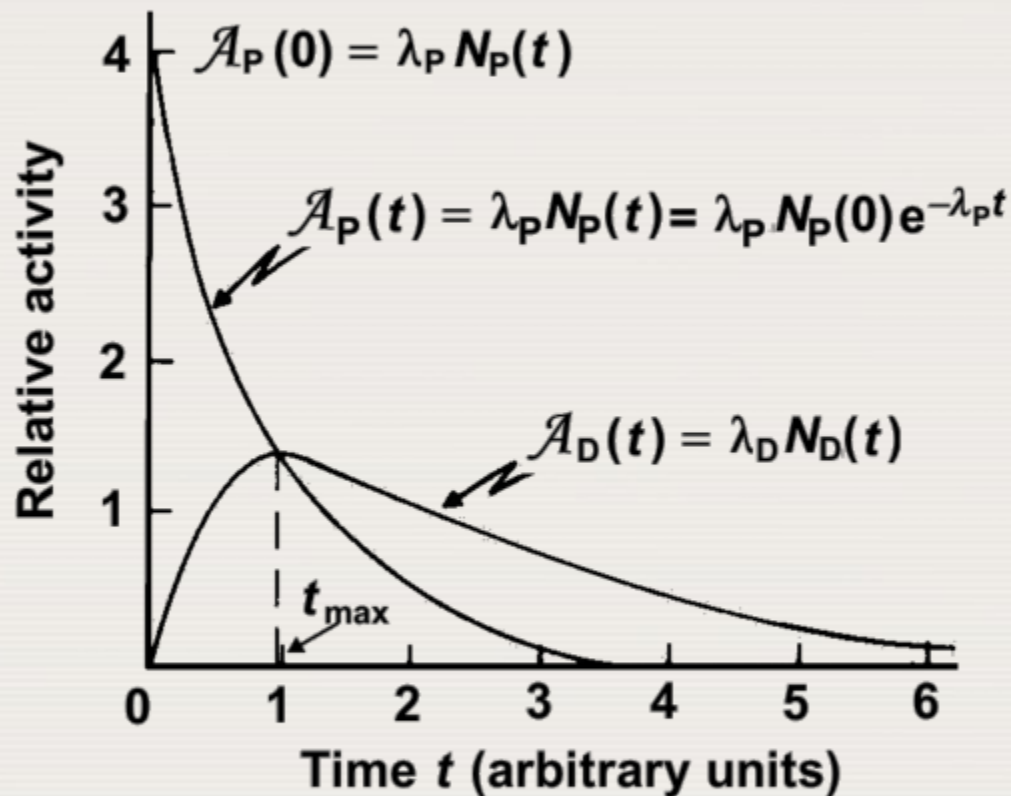
$$N_D(t) = N_P(0) \frac{\lambda_P}{\lambda_D - \lambda_P} \left\{ e^{-\lambda_P t} - e^{-\lambda_D t} \right\}$$

- Activity of the daughter nuclei is:

$$\begin{aligned} \mathcal{A}_D(t) &= \frac{N_P(0) \lambda_P \lambda_D}{\lambda_D - \lambda_P} \left\{ e^{-\lambda_P t} - e^{-\lambda_D t} \right\} = \mathcal{A}_P(0) \frac{\lambda_D}{\lambda_D - \lambda_P} \left\{ e^{-\lambda_P t} - e^{-\lambda_D t} \right\} = \\ &= \mathcal{A}_P(0) \frac{1}{1 - \frac{\lambda_P}{\lambda_D}} \left\{ e^{-\lambda_P t} - e^{-\lambda_D t} \right\} = \mathcal{A}_P(t) \frac{\lambda_D}{\lambda_D - \lambda_P} \left\{ 1 - e^{-(\lambda_D - \lambda_P)t} \right\}, \end{aligned}$$

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.7 Radioactivity



At  $t = t_{\max}$   
parent and daughter  
activities are equal and  
the daughter activity reaches  
its maximum

$$\left. \frac{d\mathcal{A}_D}{dt} \right|_{t=t_{\max}} = 0$$

and

$$t_{\max} = \frac{\ln \frac{\lambda_D}{\lambda_P}}{\lambda_D - \lambda_P}$$

Parent and daughter activities against time for  $P \xrightarrow{\lambda_P} D \xrightarrow{\lambda_D} G$

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.7 Radioactivity

Special considerations for the  $P \xrightarrow{\lambda_P} D \xrightarrow{\lambda_D} G$  relationship:

□ For  $\lambda_D < \lambda_P$  or  $(t_{1/2})_D > (t_{1/2})_P$

General relationship (no equilibrium)

$$\frac{A_D}{A_P} = \frac{\lambda_D}{\lambda_D - \lambda_P} \left\{ 1 - e^{-(\lambda_D - \lambda_P)t} \right\}$$

□ For  $\lambda_D > \lambda_P$  or  $(t_{1/2})_D < (t_{1/2})_P$

Transient equilibrium for  $t \gg t_{\max}$

$$\frac{A_D}{A_P} = \frac{\lambda_D}{\lambda_D - \lambda_P}$$

□ For  $\lambda_D \gg \lambda_P$  or  $(t_{1/2})_D \ll (t_{1/2})_P$

Secular equilibrium

$$\frac{A_D}{A_P} \approx 1$$

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.8 Activation of nuclides

- **Radioactivation** of nuclides occurs when a parent nuclide P is bombarded with thermal neutrons in a nuclear reactor and transforms into a radioactive daughter nuclide D that decays into a granddaughter nuclide G.



- Probability for radioactivation to occur is governed by the **cross section**  $\sigma$  for the nuclear reaction and the **neutron fluence rate**  $\dot{\phi}$ .
  - Unit of  $\sigma$  is barn per atom where  $1 \text{ barn} = 1 \text{ b} = 10^{-24} \text{ cm}^2$ .
  - Unit of  $\dot{\phi}$  is  $\text{cm}^{-2} \cdot \text{s}^{-1}$ .

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.8 Activation of nuclides

- Daughter activity  $\mathcal{A}_D(t)$  in radioactivation is described by an expression similar to that given for the series decay except that  $\lambda_p$  is replaced by the product  $\sigma\dot{\phi}$

$$\mathcal{A}_D(t) = \frac{\sigma \dot{\phi} \lambda_D}{\lambda_D - \sigma \dot{\phi}} N_P(0) \left[ e^{-\sigma \dot{\phi} t} - e^{-\lambda_D t} \right]$$

- Time at which the daughter activity reaches its maximum value is given by

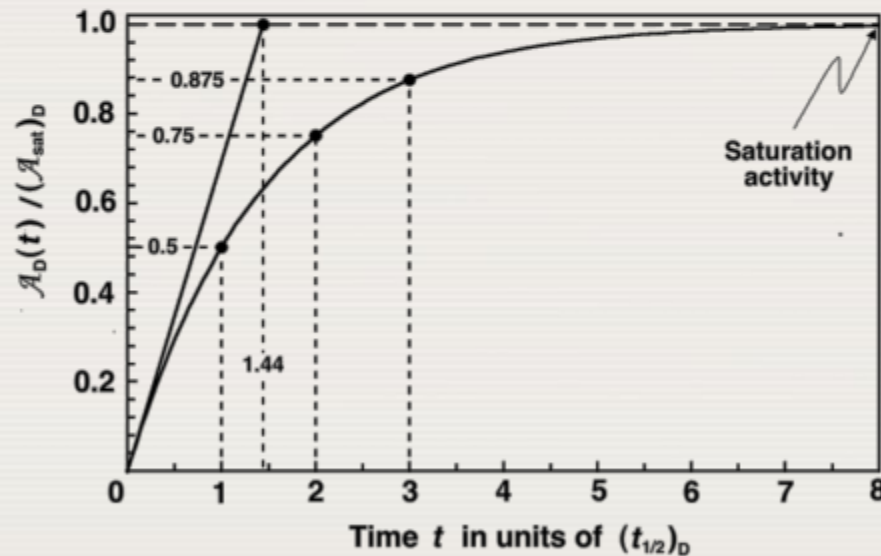
$$t_{\max} = \frac{\ln[\lambda_D / (\sigma \dot{\phi})]}{\lambda_D - \sigma \dot{\phi}}$$

# 1.2 ATOMIC AND NUCLEAR STRUCTURE

## 1.2.8 Activation of nuclides

- When  $\sigma\dot{\phi} \ll \lambda_D$ , the daughter activity expression transforms into a simple exponential growth expression

$$\mathcal{A}_D(t) = \sigma \dot{\phi} N_P(0) \left\{ 1 - e^{-\lambda_D t} \right\} = \mathcal{A}_{\text{sat}} \left\{ 1 - e^{-\lambda_D t} \right\}$$



## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.8 Activation of nuclides

- An important example of nuclear activation is the production of the **cobalt-60 radionuclide** through bombarding stable cobalt-59 with thermal neutrons



- For cobalt-59 the cross section  $\sigma$  is 37 b/atom.
- Typical reactor fluence rates  $\dot{\phi}$  are of the order of  $10^{14} \text{ cm}^{-2} \cdot \text{s}^{-1}$ .



## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.9 Modes of radioactive decay

- ❑ Radioactive decay is a process by which unstable nuclei reach a more stable configuration.
- ❑ There are four main modes of radioactive decay:
  - Alpha decay
  - Beta decay
    - Beta plus decay
    - Beta minus decay
    - Electron capture
  - Gamma decay
    - Pure gamma decay
    - Internal conversion
  - Spontaneous fission

# 1.2 ATOMIC AND NUCLEAR STRUCTURE

## 1.2.9 Modes of radioactive decay

☐ Nuclear transformations are usually accompanied by emission of energetic particles (charged particles, neutral particles, photons, neutrinos)

☐ **Radioactive decay**

- Alpha decay
- Beta plus decay
- Beta minus decay
- Electron capture
- Pure gamma decay
- Internal conversion
- Spontaneous fission

**Emitted particles**

$\alpha$  particle

$\beta^+$  particle (positron), neutrino

$\beta^-$  particle (electron), antineutrino

Neutrino

Photon

Orbital electron

Fission products

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.9 Modes of radioactive decay

- ❑ In each nuclear transformation a number of physical quantities must be conserved.
  
- ❑ The most important conserved physical quantities are:
  - Total energy
  - Momentum
  - Charge
  - Atomic number
  - Atomic mass number (number of nucleons)

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.9 Modes of radioactive decay

- Total energy of particles released by the transformation process is equal to the net decrease in the rest energy of the neutral atom, from parent P to daughter D.
- Decay energy (Q value) is given as:

$$Q = \{M(P) - [M(D) + m]\} c^2$$

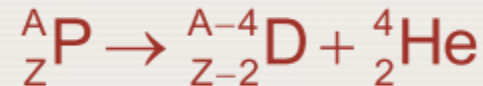
$M(P)$ ,  $M(D)$ , and  $m$  are the **nuclear** rest masses of the parent, daughter and emitted particles.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.9 Modes of radioactive decay

□ **Alpha decay** is a nuclear transformation in which:

- Energetic alpha particle (helium-4 ion) is emitted.
- Atomic number  $Z$  of the parent decreases by 2.
- Atomic mass number  $A$  of the parent decreases by 4.



## 1.2 ATOMIC AND NUCLEAR STRUCTURE

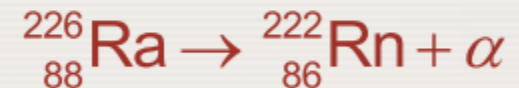
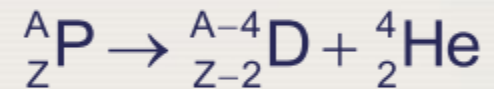
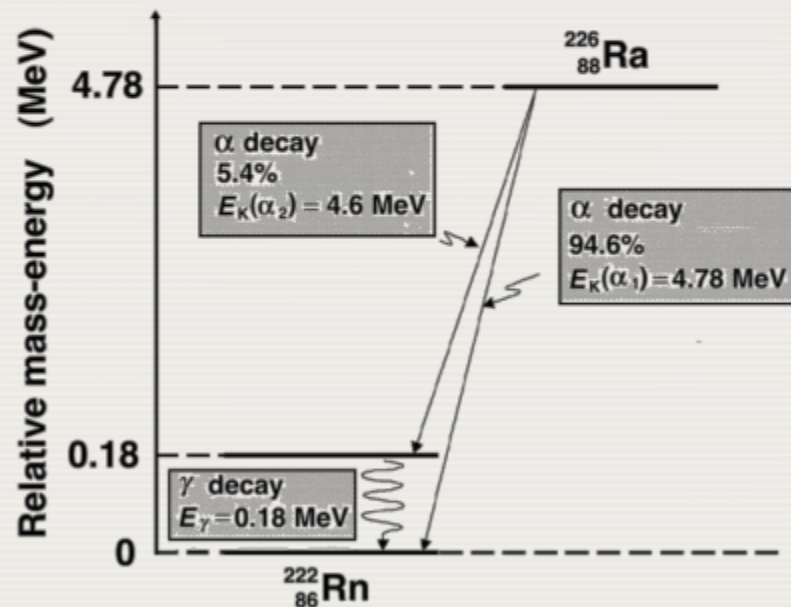
### 1.2.9 Modes of radioactive decay

- ❑ **Henri Becquerel** discovered alpha decay in **1896**; **George Gamow** explained its exact nature in 1928 using the quantum mechanical effect of tunneling.
- ❑ **Hans Geiger** and **Ernest Marsden** used 5.5 MeV alpha particles emitted by radon-222 in their experiment of alpha particle scattering on a gold foil.
- ❑ Kinetic energy of alpha particles released by naturally occurring radionuclides is **between 4 MeV and 9 MeV**.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.9 Modes of radioactive decay

- Best known example of **alpha decay** is the transformation of **radium-226** into **radon-222** with a half life of 1602 years.

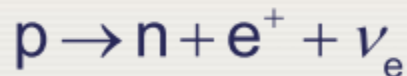


## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.9 Modes of radioactive decay

□ **Beta plus decay** is a nuclear transformation in which:

- Proton-rich radioactive parent nucleus transforms a proton into a neutron.
- Positron and neutrino, sharing the available energy, are ejected from the parent nucleus.
- Atomic number  $Z$  of the parent decreases by one; the atomic mass number  $A$  remains the same.
- Number of nucleons and total charge are conserved in the beta decay process and the daughter  $D$  can be referred to as an isobar of the parent  $P$ .

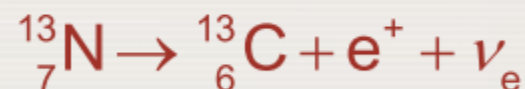
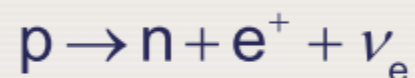
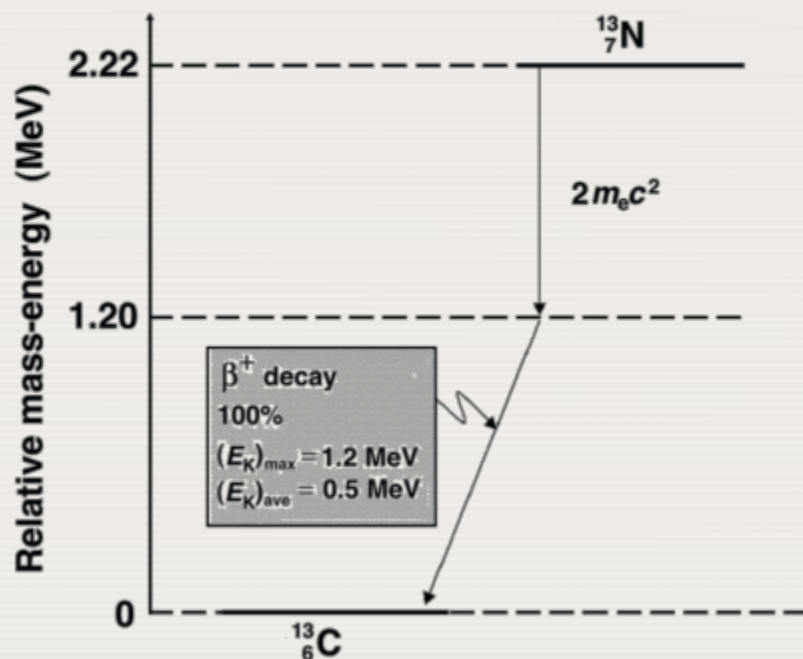




## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.9 Modes of radioactive decay

- Example of a beta plus decay is the transformation of nitrogen-13 into carbon-13 with a half life of 10 min.

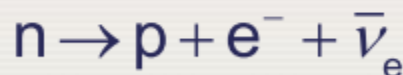


## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.9 Modes of radioactive decay

□ **Beta minus decay** is a nuclear transformation in which:

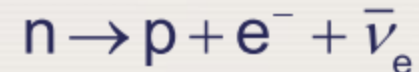
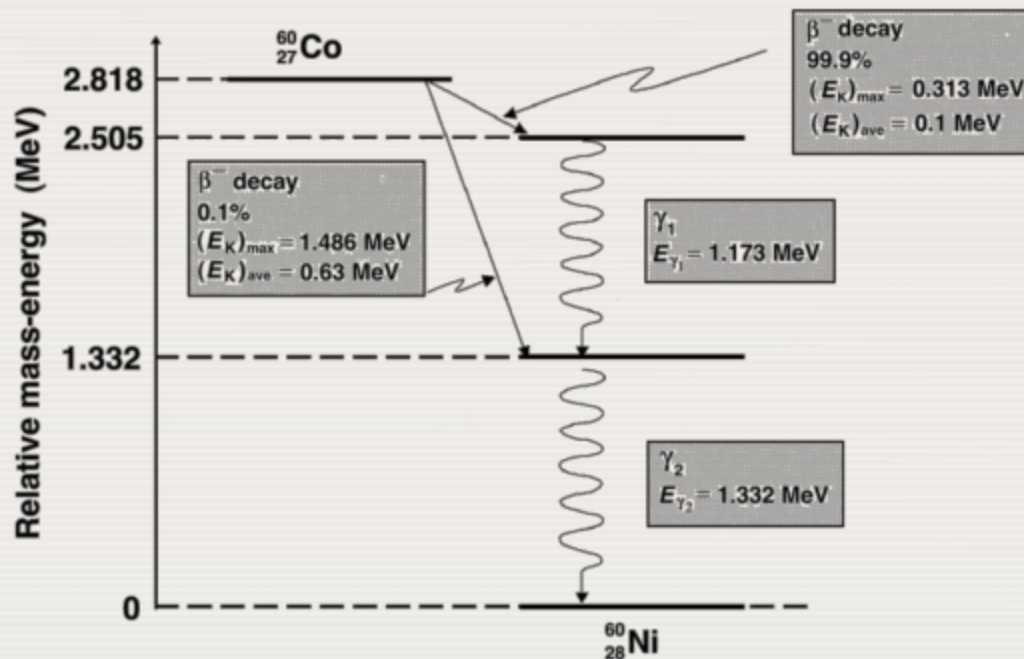
- Neutron-rich radioactive parent nucleus transforms a neutron into a proton.
- Electron and anti-neutrino, sharing the available energy, are ejected from the parent nucleus.
- Atomic number  $Z$  of the parent increases by one; the atomic mass number  $A$  remains the same.
- Number of nucleons and total charge are conserved in the beta decay process and the daughter  $D$  can be referred to as an isobar of the parent  $P$ .



# 1.2 ATOMIC AND NUCLEAR STRUCTURE

## 1.2.9 Modes of radioactive decay

- Example of beta minus decay is the transformation of cobalt-60 into nickel-60 with a half life of 5.26 y.

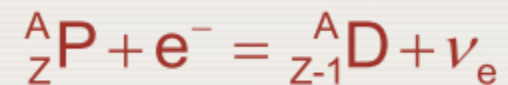
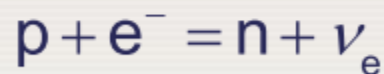


## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.9 Modes of radioactive decay

□ Electron capture decay is a nuclear transformation in which:

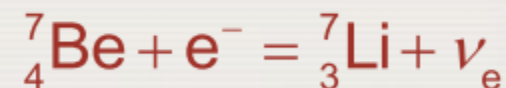
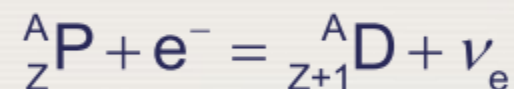
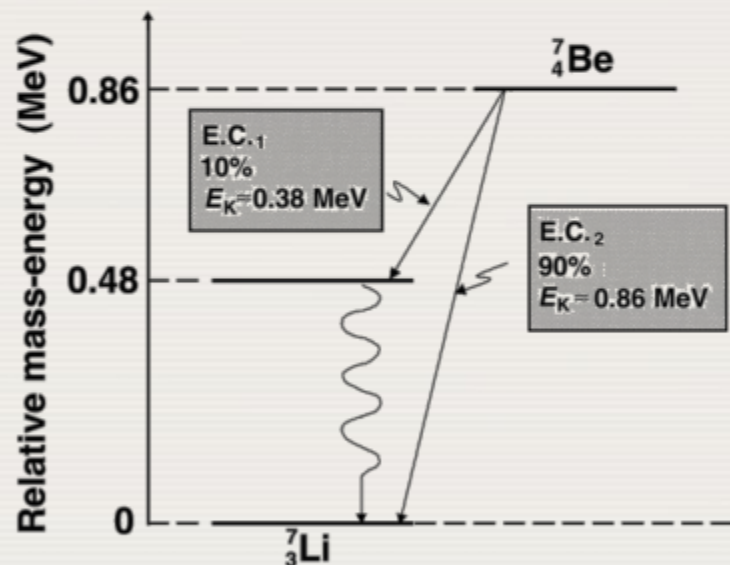
- Nucleus captures an atomic orbital electron (usually K shell).
- Proton transforms into a neutron.
- Neutrino is ejected.
- Atomic number  $Z$  of the parent decreases by one; the atomic mass number  $A$  remains the same.
- Number of nucleons and total charge are conserved in the beta decay process and the daughter  $D$  can be referred to as an isobar of the parent  $P$ .



## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.9 Modes of radioactive decay

- Example of nuclear decay by electron capture is the transformation of **berillium-7** into **lithium-7**



## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.9 Modes of radioactive decay

- **Gamma decay** is a nuclear transformation in which an excited parent nucleus  $P$ , generally produced through alpha decay, beta minus decay or beta plus decay, attains its ground state through **emission of one or several gamma photons**.
- **Atomic number  $Z$  and atomic mass number  $A$  do not change in gamma decay.**

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

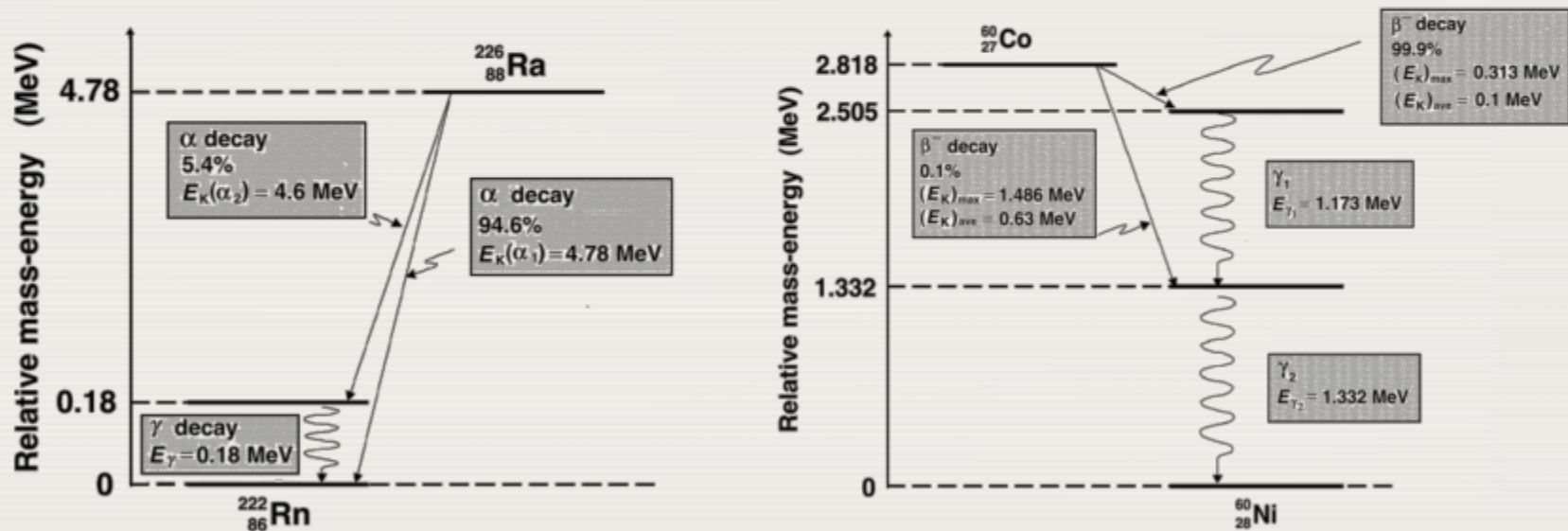
### 1.2.9 Modes of radioactive decay

- ❑ In most alpha and beta decays the daughter de-excitation occurs instantaneously, so that we refer to the emitted gamma rays as if they were produced by the parent nucleus.
- ❑ If the daughter nucleus de-excites with a time delay, the excited state of the daughter is referred to as a **metastable state** and process of de-excitation is called an **isomeric transition**.

# 1.2 ATOMIC AND NUCLEAR STRUCTURE

## 1.2.9 Modes of radioactive decay

- Examples of gamma decay are the transformation of cobalt-60 into nickel-60 by beta minus decay, and transformation of radium-226 into radon-222 by alpha decay.





## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.9 Modes of radioactive decay

□ **Internal conversion** is a nuclear transformation in which:

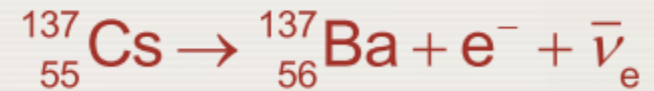
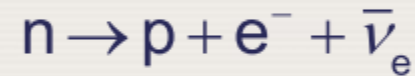
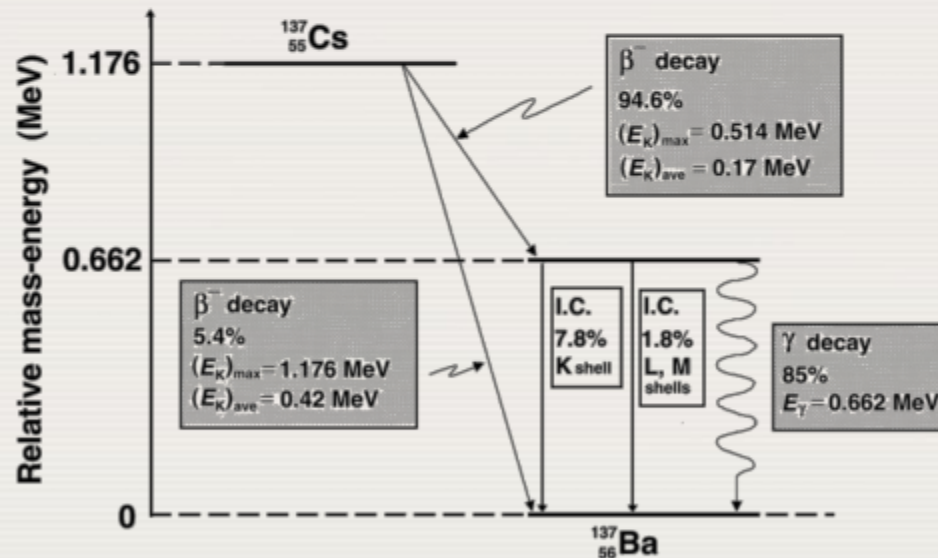
- Nuclear de-excitation energy is transferred to an orbital electron (usually K shell) .
- Electron is emitted from the atom with a kinetic energy equal to the de-excitation energy less the electron binding energy.
- Resulting shell vacancy is filled with a higher-level orbital electron and the transition energy is emitted in the form of characteristic photons or Auger electrons.



# 1.2 ATOMIC AND NUCLEAR STRUCTURE

## 1.2.9 Modes of radioactive decay

- Example for both the emission of gamma photons and emission of conversion electrons is the beta minus decay of cesium-137 into barium-137 with a half life of 30 years.



## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.9 Modes of radioactive decay

- ❑ **Spontaneous fission** is a nuclear transformation by which a high atomic mass **nucleus spontaneously splits into two nearly equal fission fragments**.
- ❑ Two to four neutrons are emitted during the spontaneous fission process.
- ❑ Spontaneous fission follows the same process as nuclear fission except that it is not self-sustaining, since it does not generate the neutron fluence rate required to sustain a “**chain reaction**”.

## 1.2 ATOMIC AND NUCLEAR STRUCTURE

### 1.2.9 Modes of radioactive decay

- In practice, spontaneous fission is only energetically feasible for nuclides with atomic masses above 230 u or with  $Z^2/A \geq 235$ .
- Spontaneous fission is a competing process to alpha decay; the higher is  $A$  above uranium-238, the more prominent is the spontaneous fission in comparison with the alpha decay and the shorter is the half-life for spontaneous fission.

## 1.3 ELECTRON INTERACTIONS

- As an energetic electron traverses matter, it undergoes **Coulomb interactions** with absorber atoms, i.e., with:
  - Atomic orbital electrons.
  - Atomic nuclei.
  
- Through these collisions the electrons may:
  - Lose their kinetic energy (**collision and radiation loss**).
  - Change direction of motion (**scattering**).

## 1.3 ELECTRON INTERACTIONS

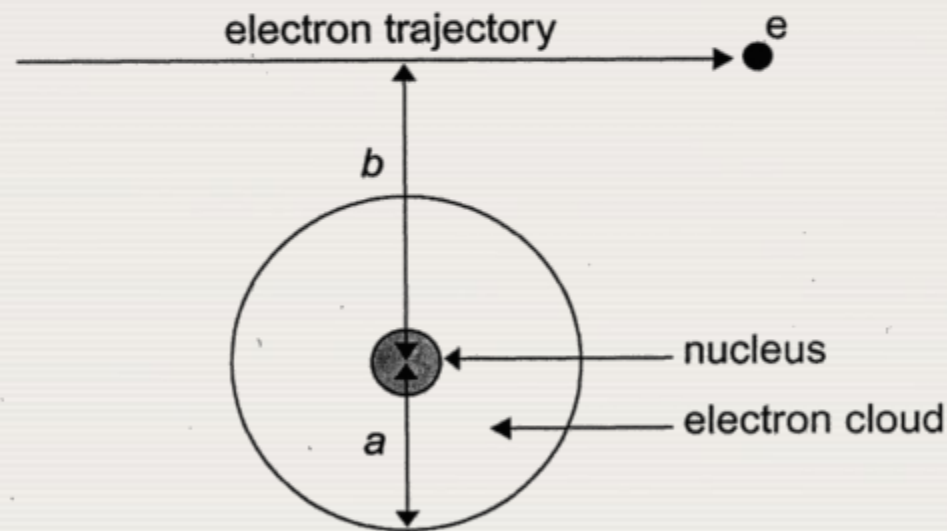
- ❑ Energy losses are described by **stopping power**.
- ❑ Scattering is described by **angular scattering power**.
- ❑ Collision between incident electron and absorber atom may be:
  - Elastic
  - Inelastic

## 1.3 ELECTRON INTERACTIONS

- ❑ In an **elastic collision** the incident electron is deflected from its original path but no energy loss occurs.
- ❑ In an **inelastic collision** with orbital electron the incident electron is deflected from its original path and loses part of its kinetic energy.
- ❑ In an **inelastic collision** with nucleus the incident electron is deflected from its original path and loses part of its kinetic energy in the form of **bremsstrahlung**.

## 1.3 ELECTRON INTERACTIONS

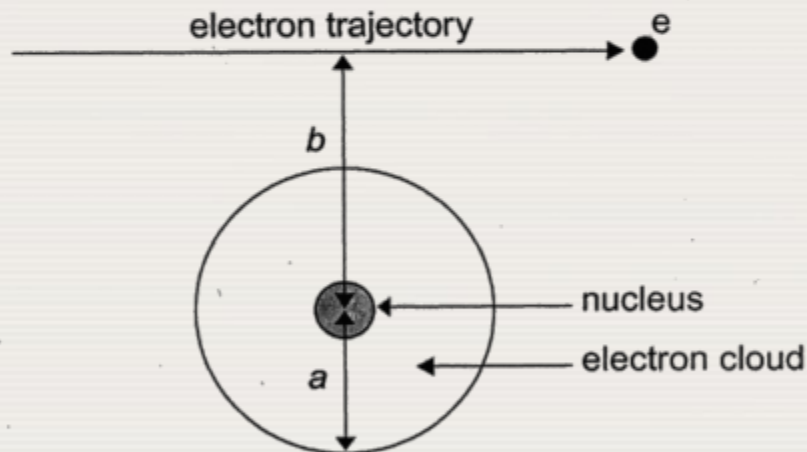
Type of inelastic interaction that an electron undergoes with a particular atom of radius  $a$  depends on the **impact parameter**  $b$  of the interaction.





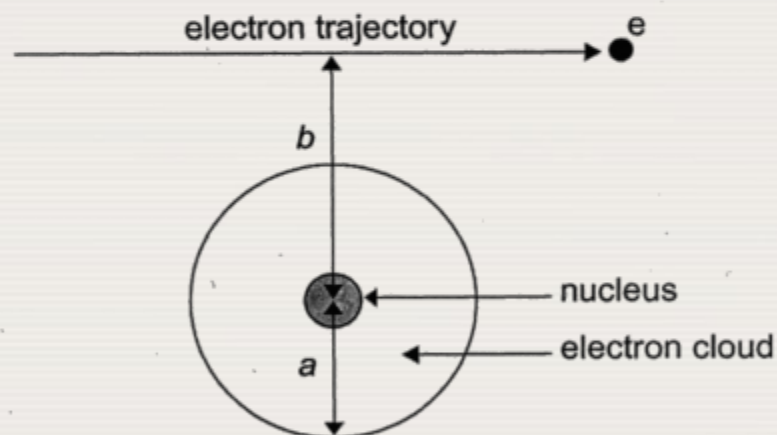
## 1.3 ELECTRON INTERACTIONS

- For  $b \gg a$ , incident electron will undergo a **soft collision** with the whole atom and only a small amount of its kinetic energy (few %) will be transferred from the incident electron to orbital electron.



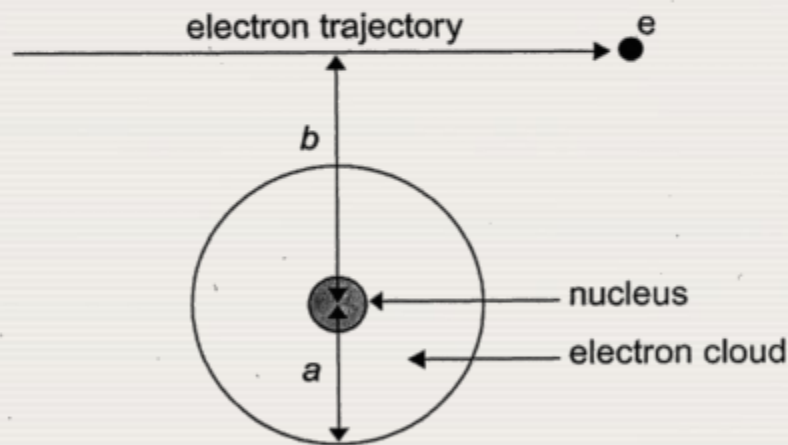
## 1.3 ELECTRON INTERACTIONS

- For  $b \approx a$ , incident electron will undergo a **hard collision** with an orbital electron and a significant fraction of its kinetic energy (up to 50 %) will be transferred to the orbital electron.



## 1.3 ELECTRON INTERACTIONS

- For  $b \ll a$ , incident electron will undergo a **radiation collision** with the atomic nucleus and emit a bremsstrahlung photon with energy between 0 and the incident electron kinetic energy.



## 1.3 ELECTRON INTERACTIONS

### 1.3.1 Electron-orbital electron interactions

- ❑ Inelastic collisions between the incident electron and an orbital electron are Coulomb interactions that result in:
  - **Atomic ionization:**  
Ejection of the orbital electron from the absorber atom.
  - **Atomic excitation:**  
Transfer of an atomic orbital electron from one allowed orbit (shell) to a higher level allowed orbit.
  
- ❑ Atomic ionizations and excitations result in collision energy losses experienced by the incident electron and are characterized by **collision (ionization) stopping power**.

## 1.3 ELECTRON INTERACTIONS

### 1.3.2 Electron-nucleus interaction

- Coulomb interaction between the incident electron and an absorber nucleus results in:
  - Electron scattering and no energy loss (elastic collision):  
characterized by **angular scattering power**.
  - Electron scattering and some loss of kinetic energy in the form of bremsstrahlung (radiation loss):  
characterized by **radiation stopping power**.

## 1.3 ELECTRON INTERACTIONS

### 1.3.2 Electron-nucleus interaction

- **Bremsstrahlung production** is governed by the Larmor relationship:

$$P = \frac{q^2 a^2}{6\pi\epsilon_0 c^3} .$$

**Power  $P$  emitted in the form of bremsstrahlung photons** from a charged particle with charge  $q$  accelerated with acceleration  $a$  is proportional to:

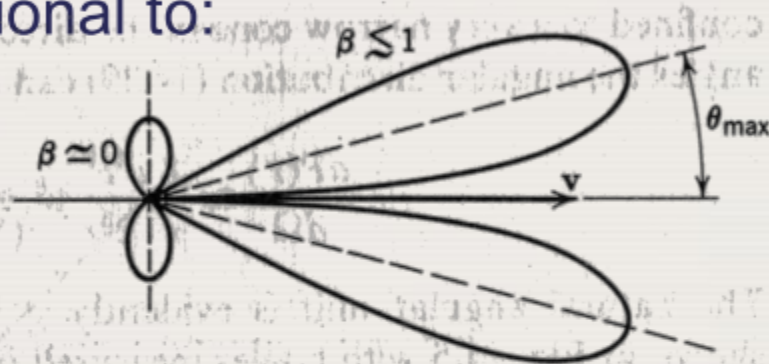
- Square of the particle acceleration  $a$ .
- Square of the particle charge  $q$ .

## 1.3 ELECTRON INTERACTIONS

### 1.3.2 Electron-nucleus interactions

- Angular distribution of the emitted bremsstrahlung photons is in general proportional to:

$$\frac{\sin^2 \theta}{(1 - \beta \cos \theta)^5}$$



- At small particle velocity ( $v \ll c$ , i.e.,  $\beta = (v/c) \rightarrow 0$ ) the angular distribution of emitted photons is proportional to  $\sin^2 \theta$ .
- Angle  $\theta_{\max}$  at which the photon intensity is maximum is:

$$\theta_{\max} = \arccos \left[ \frac{1}{3\beta} (\sqrt{1 + 15\beta} - 1) \right]$$

## 1.3 ELECTRON INTERACTIONS

### 1.3.3 Stopping power

- Energy loss by incident electron through inelastic collisions is described by **total linear stopping power**  $S_{\text{tot}}$  which represents kinetic energy  $E_K$  loss by the electron per unit path length  $x$ :

$$S_{\text{tot}} = \frac{dE_K}{dx} \quad \text{in MeV/cm}$$



## 1.3 ELECTRON INTERACTIONS

### 1.3.3 Stopping power

- Total mass stopping power  $(S/\rho)_{\text{tot}}$  is defined as the linear stopping power divided by the density of the absorbing medium.

$$\left(\frac{S}{\rho}\right)_{\text{tot}} = \frac{1}{\rho} \frac{dE_{\text{K}}}{dx} \quad \text{in} \quad \text{MeV} \cdot \text{cm}^2 / \text{g}$$

## 1.3 ELECTRON INTERACTIONS

### 1.3.3 Stopping power

- Total mass stopping power  $(S/\rho)_{\text{tot}}$  consists of two components:
  - Mass collision stopping power  $(S/\rho)_{\text{col}}$   
resulting from electron-orbital electron interactions  
(atomic ionizations and atomic excitations)
  - Mass radiation stopping power  $(S/\rho)_{\text{rad}}$   
resulting mainly from electron-nucleus interactions  
(bremsstrahlung production)
  - Total mass stopping power is the sum of the two components

$$\left(\frac{S}{\rho}\right)_{\text{tot}} = \left(\frac{S}{\rho}\right)_{\text{col}} + \left(\frac{S}{\rho}\right)_{\text{rad}}$$

## 1.3 ELECTRON INTERACTIONS

### 1.3.3 Stopping power

- For **heavy charged particles** the radiation stopping power  $(S/\rho)_{\text{rad}}$  is negligible thus  $(S/\rho)_{\text{tot}} \approx (S/\rho)_{\text{col}}$
- For **light charged particles** both components contribute to the total stopping power thus  $(S/\rho)_{\text{tot}} = (S/\rho)_{\text{col}} + (S/\rho)_{\text{rad}}$ 
  - Within a broad range of kinetic energies below 10 MeV collision (ionization) losses are dominant  $(S/\rho)_{\text{col}} > (S/\rho)_{\text{rad}}$ ; however, the situation is reversed at high kinetic energies.
  - Cross over between the two modes occurs at a critical kinetic energy  $(E_K)_{\text{crit}}$  where the two stopping powers are equal

$$(E_K)_{\text{crit}} \approx \frac{800 \text{ MeV}}{Z}$$

## 1.3 ELECTRON INTERACTIONS

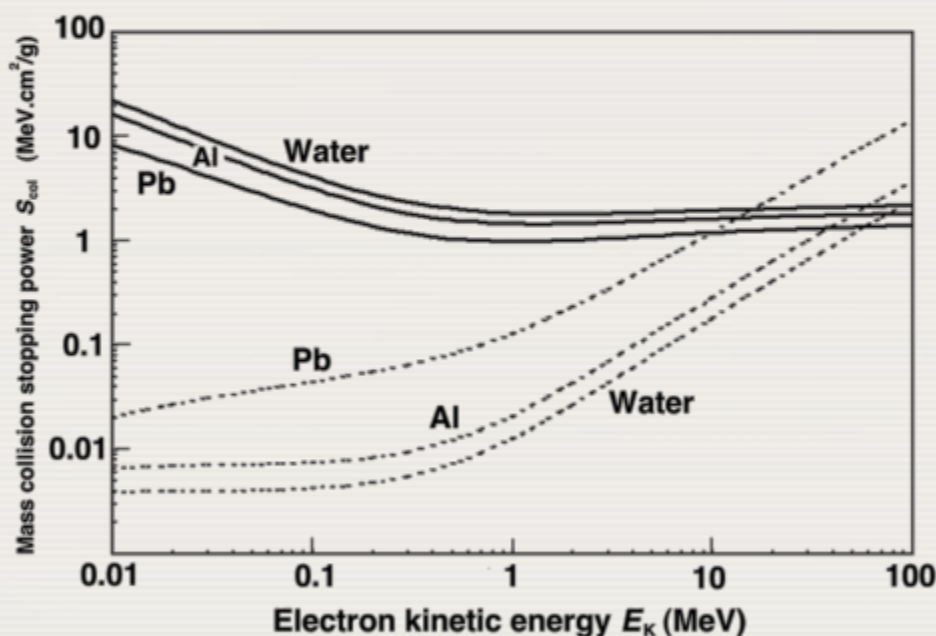
### 1.3.3 Stopping power

- ❑ Electrons traversing an absorber lose their kinetic energy through **ionization collisions** and **radiation collisions**.
- ❑ Rate of energy loss per gram and per  $\text{cm}^2$  is called the mass stopping power and it is a sum of two components:
  - Mass collision stopping power
  - Mass radiation stopping power
- ❑ **Rate of energy loss** for a therapy electron beam in water and water-like tissues, averaged over the electron's range, is about **2 MeV/cm**.

# 1.3 ELECTRON INTERACTIONS

## 1.3.3 Stopping power

- Rate of energy loss for **collision interactions** depends on:
  - Kinetic energy of the electron.
  - Electron density of the absorber.



Rate of **collision energy loss** is greater for low atomic number  $Z$  absorbers than for high  $Z$  absorbers, because high  $Z$  absorbers have lower electron density (fewer electrons per gram).

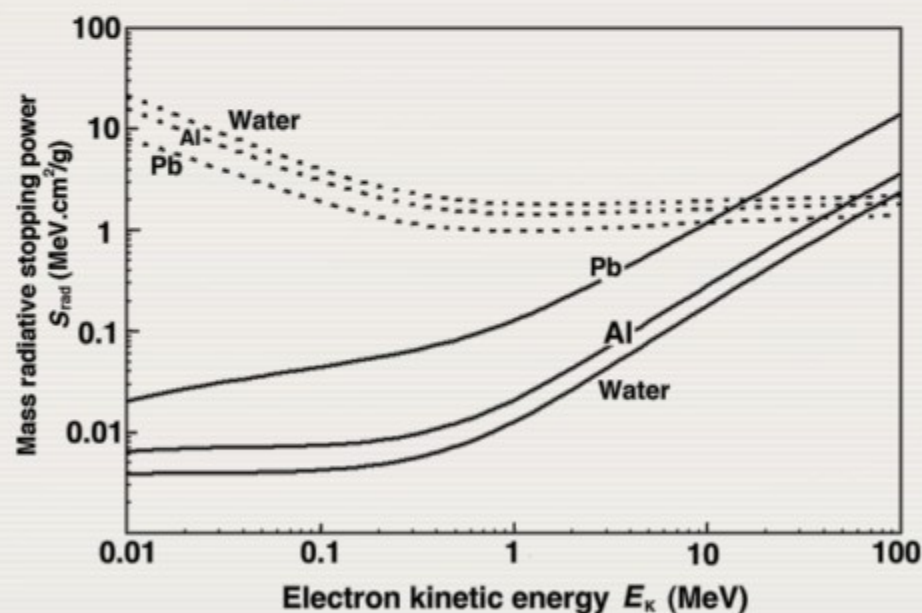
Solid lines: mass collision stopping power  
Dotted lines: mass radiation stopping power

# 1.3 ELECTRON INTERACTIONS

## 1.3.3 Stopping power

□ Rate of energy loss for **radiation interactions** (bremsstrahlung) is approximately proportional to:

- Kinetic energy of the electron.
- Square of the atomic number of the absorber.



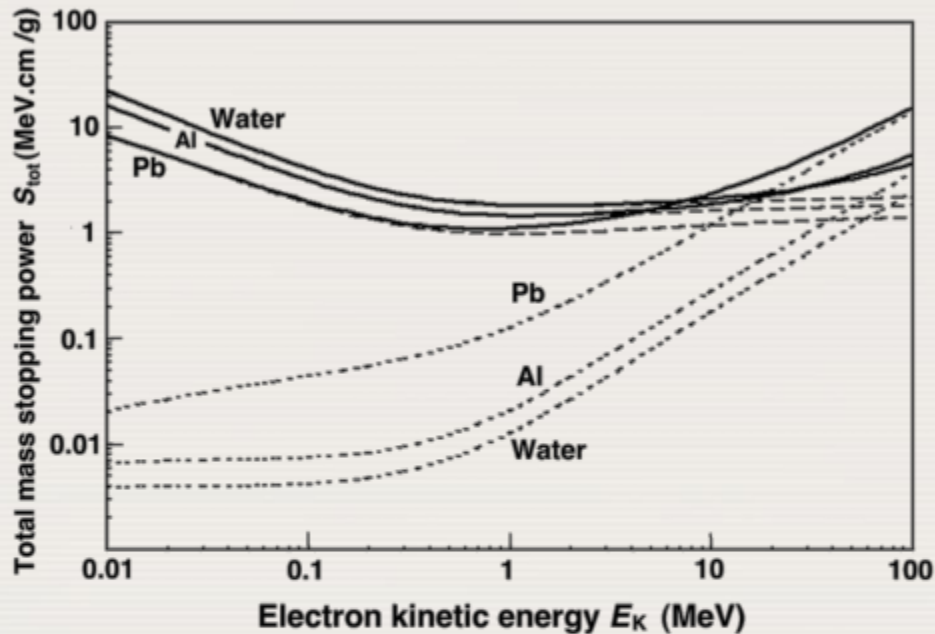
**Bremsstrahlung production** through radiation losses is more efficient for higher energy electrons and higher atomic number absorbers

**Solid lines:** mass radiation stopping power

**Dotted lines:** mass collision stopping power

# 1.3 ELECTRON INTERACTIONS

## 1.3.3 Stopping power



Total energy loss by electrons traversing an absorber depends upon:

- Kinetic energy of the electron
- Atomic number of the absorber
- Electron density of the absorber

Total mass stopping power is the sum of mass collision and mass radiation stopping powers

Solid lines: total mass stopping power

Dashed lines: mass collision stopping power

Dotted lines: mass radiation stopping power

$$\left(\frac{S}{\rho}\right)_{tot} = \left(\frac{S}{\rho}\right)_{col} + \left(\frac{S}{\rho}\right)_{rad}$$

# 1.3 ELECTRON INTERACTIONS

## 1.3.3 Stopping power

- Total mass stopping power  $(S/\rho)_{\text{tot}}$  for electrons in water, aluminum and lead against the electron kinetic energy (solid curves).

Solid lines:

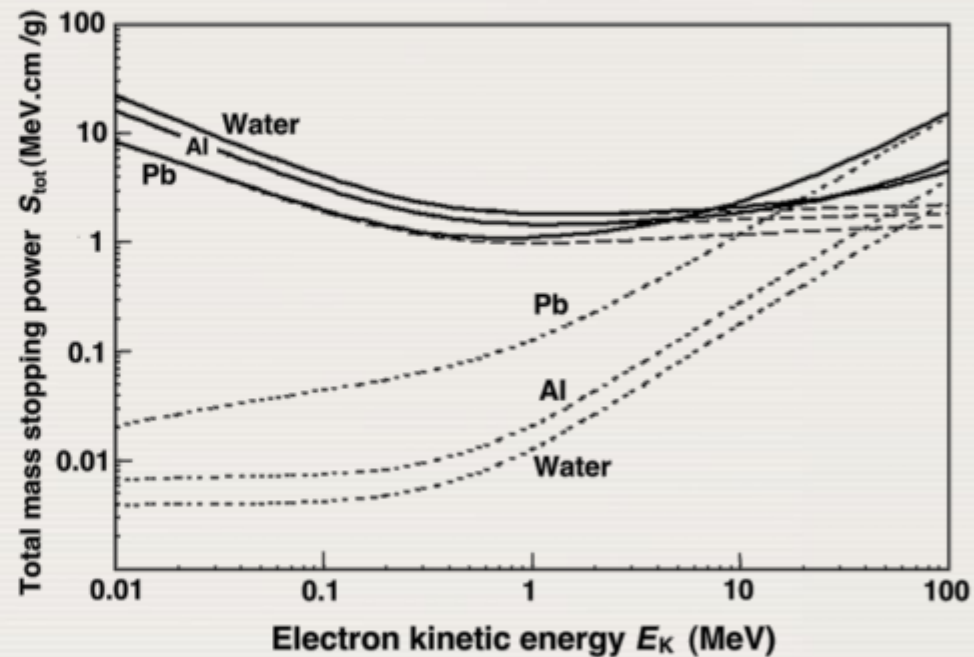
total mass stopping power

Dashed lines:

mass collision stopping power

Dotted lines:

mass radiation stopping power





## 1.3 ELECTRON INTERACTIONS

### 1.3.3 Stopping power

- $(S/\rho)_{\text{tot}}$  is used in the calculation of **particle range  $R$**

$$R = \int_0^{E_K} \left( \frac{S}{\rho} (E_K) \right)_{\text{tot}}^{-1} dE_K$$

- Both  $(S/\rho)_{\text{tot}}$  and  $(S/\rho)_{\text{rad}}$  are used in the determination of **radiation yield  $Y(E_K)$**

$$Y = \frac{1}{E_K} \int_0^{E_K} \frac{(S/\rho)_{\text{rad}}}{(S/\rho)_{\text{tot}}} dE_K$$

## 1.3 ELECTRON INTERACTIONS

### 1.3.4 Mass angular scattering power

- Angular and spatial spread of a pencil electron beam traversing an absorbing medium can be approximated with a Gaussian distribution.
- Multiple Coulomb scattering of electrons traversing a path length  $\ell$  is commonly described by the mean square scattering angle  $\theta^2$  proportional to the mass thickness  $\rho\ell$ .
- Mass angular scattering power  $T/\rho$  is defined as

$$\frac{T}{\rho} = \frac{1}{\rho} \frac{d\overline{\theta^2}}{d\ell} = \frac{\overline{\theta^2}}{\rho\ell}$$

## 1.4 PHOTON INTERACTIONS

### 1.4.1 Types of indirectly ionizing photon irradiations

Ionizing photon radiation is classified into four categories:

- ☐ **Characteristic x ray**

Results from electronic transitions between atomic shells

- ☐ **Bremsstrahlung**

Results mainly from electron-nucleus Coulomb interactions

- ☐ **Gamma ray**

Results from nuclear transitions

- ☐ **Annihilation quantum (annihilation radiation)**

Results from positron-electron annihilation

## 1.4 PHOTON INTERACTIONS

### 1.4.1 Types of indirectly ionizing photon irradiations

- In penetrating an absorbing medium, photons may experience various interactions with the atoms of the medium, involving:
  - Absorbing **atom** as a whole
  - **Nuclei** of the absorbing medium
  - **Orbital electrons** of the absorbing medium.

## 1.4 PHOTON INTERACTIONS

### 1.4.1 Types of indirectly ionizing photon irradiations

- Interactions of photons with nuclei may be:
  - Direct photon-nucleus interactions (photodisintegration)  
or
  - Interactions between the photon and the electrostatic field of the nucleus (pair production).
  
- Photon-orbital electron interactions are characterized as interactions between the photon and either
  - Loosely bound electron (Compton effect, triplet production)  
or
  - Tightly bound electron (photoelectric effect).

## 1.4 PHOTON INTERACTIONS

### 1.4.1 Types of indirectly ionizing photon irradiations

- **Loosely bound electron** is an electron whose binding energy  $E_B$  to the nucleus is small compared to the photon energy  $h\nu$ .  
$$E_B \ll h\nu$$
- Interaction between a photon and a loosely bound electron is considered to be an interaction between a photon and a free (unbound) electron.

## 1.4 PHOTON INTERACTIONS

### 1.4.1 Types of indirectly ionizing photon irradiations

- ❑ **Tightly bound electron** is an electron whose binding energy  $E_B$  is comparable to, larger than, or slightly smaller than the photon energy  $h\nu$ .
- ❑ For a photon interaction to occur with a tightly bound electron, the binding energy  $E_B$  of the electron must be of the order of, but slightly smaller, than the photon energy.

$$E_B \leq h\nu$$

- ❑ Interaction between a photon and a tightly bound electron is considered an interaction between a photon and the atom as a whole.

## 1.4 PHOTON INTERACTIONS

### 1.4.1 Types of indirectly ionizing photon irradiations

- As far as the **photon fate** after the interaction with an atom is concerned there are two possible outcomes:
  - **Photon disappears** (i.e., is absorbed completely) and a portion of its energy is transferred to light charged particles (electrons and positrons in the absorbing medium).
  - **Photon is scattered** and two outcomes are possible:
    - The resulting photon has the same energy as the incident photon and no light charged particles are released in the interaction.
    - The resulting scattered photon has a lower energy than the incident photon and the energy excess is transferred to a light charged particle (electron).



# 1.4 PHOTON INTERACTIONS

## 1.4.1 Types of indirectly ionizing photon irradiations

☐ Light charged particles produced in the absorbing medium through photon interactions will:

- Either deposit their energy to the medium through Coulomb interactions with orbital electrons of the absorbing medium (collision loss also referred to as ionization loss).
- Or radiate their kinetic energy away through Coulomb interactions with the nuclei of the absorbing medium (radiation loss).

# 1.4 PHOTON INTERACTIONS

## 1.4.2 Photon beam attenuation

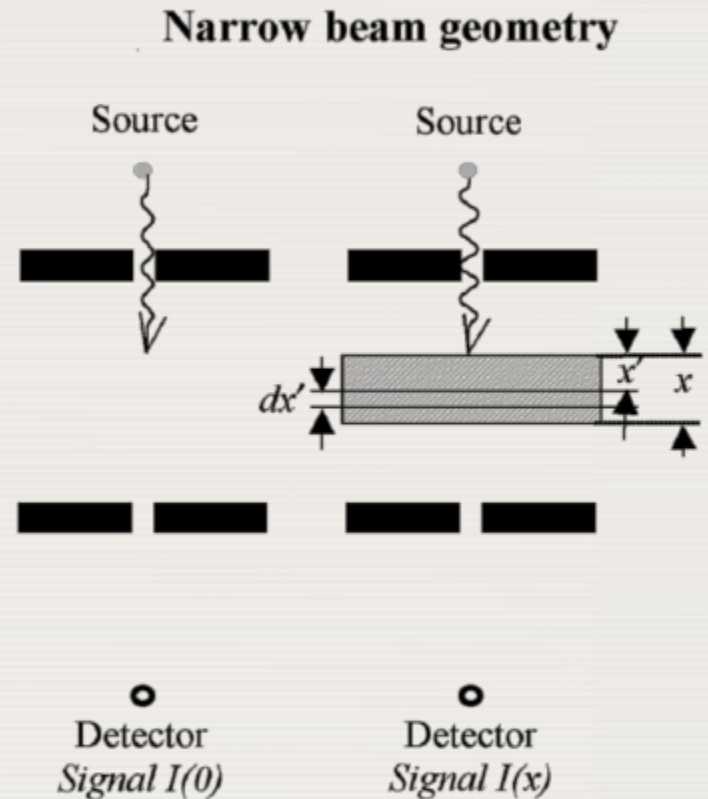
- ❑ The most important parameter used for characterization of x-ray or gamma ray penetration into absorbing media is the **linear attenuation coefficient  $\mu$** .
- ❑ Linear attenuation coefficient  $\mu$  depends upon:
  - Energy  $h\nu$  of the photon beam
  - Atomic number  $Z$  of the absorber
- ❑ Linear attenuation coefficient may be described as the **probability per unit path length** that a photon will have an interaction with the absorber.

# 1.4 PHOTON INTERACTIONS

## 1.4.2 Photon beam attenuation

□ Attenuation coefficient  $\mu$  is determined experimentally using the so-called **narrow beam geometry technique** that implies a narrowly collimated source of mono-energetic photons and a narrowly collimated detector.

- $x$  represents total thickness of the absorber
- $x'$  represents the thickness variable.

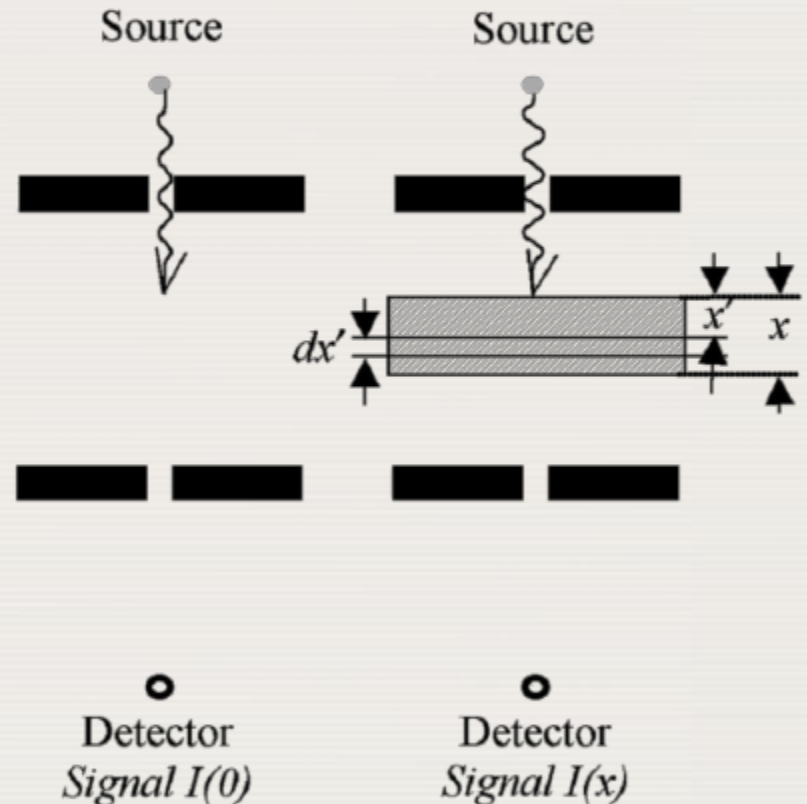


# 1.4 PHOTON INTERACTIONS

## 1.4.2 Photon beam attenuation

- A slab of absorber material of thickness  $x$  decreases the detector signal intensity from  $I(0)$  to  $I(x)$ .
- A layer of thickness  $dx'$  reduces the beam intensity by  $dI$  and the fractional reduction in intensity,  $-dI/I$  is proportional to
  - Attenuation coefficient  $\mu$ .
  - Layer thickness  $dx'$ .

### Narrow beam geometry



# 1.4 PHOTON INTERACTIONS

## 1.4.2 Photon beam attenuation

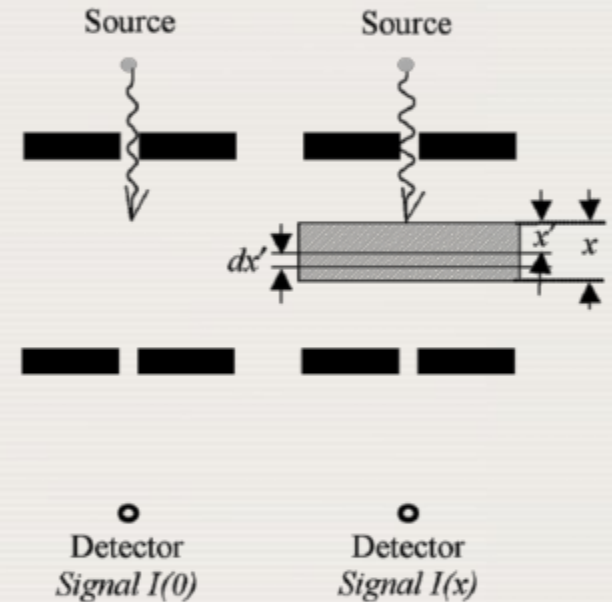
- Fractional reduction in intensity is given as:

$$-\frac{dI}{I} = \mu dx$$

- After integration from 0 to  $x$  we obtain

$$\int_{I(0)}^{I(x)} \frac{dI}{I} = -\int_0^x \mu dx' \quad \text{or} \quad I(x) = I(0)e^{-\int_0^x \mu dx'}$$

### Narrow beam geometry



# 1.4 PHOTON INTERACTIONS

## 1.4.2 Photon beam attenuation

- For a homogeneous medium  $\mu = \text{const}$  and one gets the standard exponential relationship valid for monoenergetic photon beams:

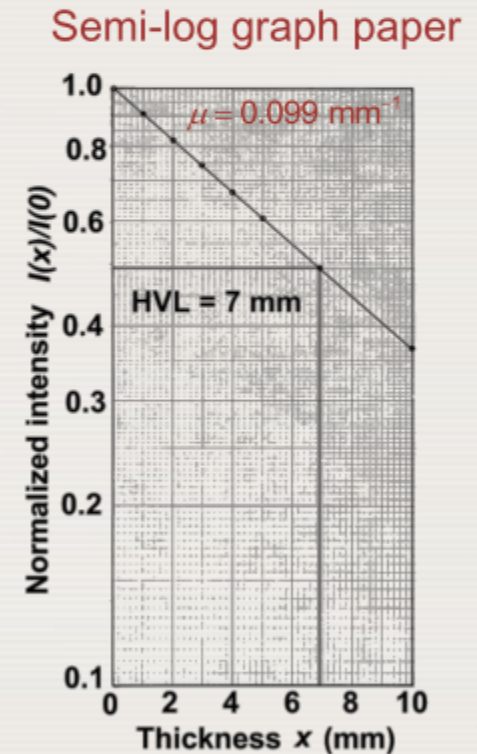
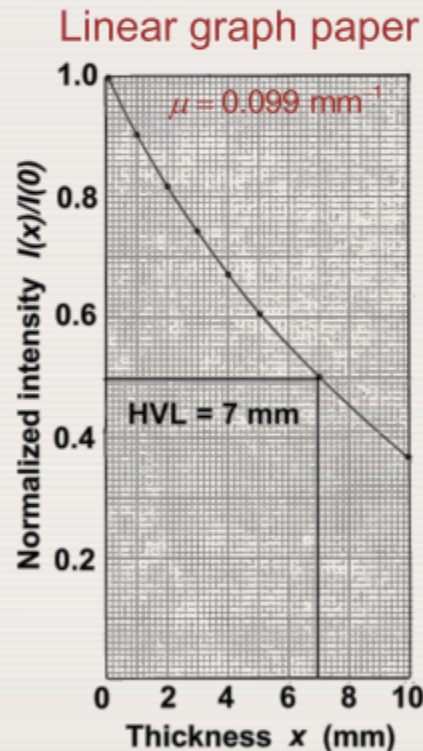
$$I(x) = I(0)e^{-\mu x}$$

or

$$I(x) / I(0) = e^{-\mu x}$$

For  $x = \text{HVL}$

$$\frac{I(x)}{I(0)} = 0.5$$



## 1.4 PHOTON INTERACTIONS

### 1.4.2 Photon beam attenuation

- Several thicknesses of special interest are defined as parameters for mono-energetic photon beam characterization in narrow beam geometry:
  - **Half-value layer (HVL<sub>1</sub> or  $x_{1/2}$ )**  
Absorber thickness that attenuates the original intensity to 50 %.
  - **Mean free path (MFP or  $\bar{x}$ )**  
Absorber thickness which attenuates the beam intensity to  $1/e = 36.8$  %.
  - **Tenth-value layer (TVL or  $x_{1/10}$ )**  
Absorber thickness which attenuates the beam intensity to 10 %.

## 1.4 PHOTON INTERACTIONS

### 1.4.2 Photon beam attenuation

- The relationship for  $x_{1/2}$ ,  $\bar{x}$ , and  $x_{1/10}$  is:

$$\mu = \frac{\ln 2}{x_{1/2}} = \frac{1}{\bar{x}} = \frac{\ln 10}{x_{1/10}}$$

or

$$x_{1/2} = (\ln 2)\bar{x} = \frac{\ln 2}{\ln 10} x_{1/10} \approx 0.3x_{1/10}$$



# 1.4 PHOTON INTERACTIONS

## 1.4.2 Photon beam attenuation

- In addition to the linear attenuation coefficient  $\mu$  other related attenuation coefficients and cross sections are in use for describing photon beam attenuation:
- Mass attenuation coefficient  $\mu_m$
  - Atomic cross section  ${}_a\mu$
  - Electronic cross section  ${}_e\mu$

## 1.4 PHOTON INTERACTIONS

### 1.4.2 Photon beam attenuation

□ Basic relationships:

$$\mu = \rho \mu_m = n_a \mu = n_e Z_e \mu$$

$$n_a = \frac{N_a}{V} = \rho \frac{N_a}{m} = \rho \frac{N_A}{A}$$

where  $n_a$  is the number of atoms per volume of absorber with density  $\rho$  and atomic mass  $A$ .

# 1.4 PHOTON INTERACTIONS

## 1.4.2 Photon beam attenuation

	<i>Symbol</i>	<i>Relationship to <math>\mu</math></i>	<i>Units</i>
<i>Linear attenuation coefficient</i>	$\mu$	$\mu$	$\text{cm}^{-1}$
<i>Mass attenuation coefficient</i>	$\mu_m$	$\frac{\mu}{\rho}$	$\text{cm}^2 / \text{g}$
<i>Atomic cross section</i>	$\sigma_a \mu$	$\frac{\mu}{n^a}$	$\text{cm}^2 / \text{atom}$
<i>Electronic cross section</i>	$\sigma_e \mu$	$\frac{\mu}{Zn^e}$	$\text{cm}^2 / \text{electron}$

## 1.4 PHOTON INTERACTIONS

### 1.4.2 Photon beam attenuation

□ Energy transfer coefficient  $\mu_{tr} = \mu \frac{\bar{E}_{tr}}{h\nu}$ ,

with  $\bar{E}_{tr}$  the average energy transferred from the primary photon with energy  $h\nu$  to kinetic energy of charged particles ( $e^-$  and  $e^+$ ).

□ Energy absorption coefficient  $\mu_{ab} = \mu \frac{\bar{E}_{ab}}{h\nu}$ ,

with  $\bar{E}_{ab}$  the average energy absorbed in the volume of interest in the absorbing medium.

In the literature,  $\mu_{en}$  is usually used instead of  $\mu_{ab}$ , however, the use of subscript “ab” for energy absorbed compared to the subscript “tr” for energy transferred seems more logical.

## 1.4 PHOTON INTERACTIONS

### 1.4.2 Photon beam attenuation

- Average (mean) energy absorbed in the volume of interest

$$\bar{E}_{ab} = \bar{E}_{tr} - \bar{E}_{rad}$$

with  $\bar{E}_{rad}$  the average energy component of  $\bar{E}_{tr}$  which the charged particles lose in the form of radiation collisions (bremsstrahlung) and is not absorbed in the volume of interest.

# 1.4 PHOTON INTERACTIONS

## 1.4.2 Photon beam attenuation

□ Linear energy absorption coefficient is

$$\mu_{ab} = \mu \frac{\bar{E}_{ab}}{h\nu} = \mu \frac{\bar{E}_{tr} - \bar{E}_{rad}}{h\nu} = \mu_{tr} - \mu_{tr} \frac{\bar{E}_{rad}}{\bar{E}_{tr}} = \mu_{tr} (1 - \bar{g})$$

where  $\bar{g}$  is the so-called **radiation fraction** (the average fraction of the energy lost in radiation interactions by the secondary charged particles, as they travel through the absorber).

## 1.4 PHOTON INTERACTIONS

### 1.4.2 Photon beam attenuation

- **Mass attenuation coefficient** of a compound or a mixture is approximated by a summation of a weighted average of its constituents:

$$\frac{\mu}{\rho} = \sum_i w_i \frac{\mu_i}{\rho}$$

- $w_i$  is the proportion by weight of the i-th constituent.
- $\mu_i/\rho$  is the mass attenuation coefficient of the i-th constituent.

# 1.4 PHOTON INTERACTIONS

## 1.4.2 Photon beam attenuation

- ☐ **Attenuation coefficient**  $\mu$  has a specific value for a given photon energy  $h\nu$  and absorber atomic number  $Z$ .
- ☐ The value for the attenuation coefficient  $\mu(h\nu, Z)$  for a given photon energy  $h\nu$  and absorber atomic number  $Z$  represents a sum of values for all individual interactions that a photon may have with an atom:

$$\mu = \sum_i \mu_i$$



# 1.4 PHOTON INTERACTIONS

## 1.4.3 Types of photon interactions with absorber

- According to the **type of target** there are two possibilities for photon interaction with an atom:
  - Photon - orbital electron interaction
  - Photon - nucleus interaction
  
- According to the **type of event** there are two possibilities for photon interaction with an atom:
  - Complete absorption of the photon
  - Scattering of the photon

# 1.4 PHOTON INTERACTIONS

## 1.4.3 Types of photon interactions with absorber

- In medical physics photon interactions fall into four groups:
  - Interactions of major importance:
    - Photoelectric effect.
    - Compton scattering by free electron.
    - Pair production (including triplet production).
  - Interactions of moderate importance:
    - Rayleigh scattering.
    - Thomson scattering by free electron.
  - Interactions of minor importance
    - Photonuclear reactions (nuclear photoelectric effect)
  - Negligible interactions:
    - Thomson and Compton scattering by the nucleus.
    - Meson production.
    - Delbrück scattering.

# 1.4 PHOTON INTERACTIONS

## 1.4.3 Types of photon interactions with absorber

Interaction	Symbol for electronic cross section	Symbol for atomic cross section	Symbol for linear attenuation coefficient
Thomson scattering	${}_e\sigma_{Th}$	${}_a\sigma_{Th}$	$\sigma_{Th}$
Rayleigh scattering	-	${}_a\sigma_R$	$\sigma_R$
Compton scattering	${}_e\sigma_c$	${}_a\sigma_c$	$\sigma_C$
Photoelectric effect	-	${}_a\tau$	$\tau$
Pair production	-	${}_aK_{pp}$	$K_p$
Triplet production	${}_eK_{tp}$	${}_aK_{tp}$	$K_t$
Photodisintegration	-	${}_a\sigma_{pn}$	$\sigma_{pn}$

# 1.4 PHOTON INTERACTIONS

## 1.4.3 Types of photon interactions with absorber

### ☐ TYPES OF TARGETS IN PHOTON INTERACTIONS

#### Photon-orbital electron interaction

- with bound electron
  - Photoelectric effect
  - Rayleigh scattering
- with “free” electrons
  - Thomson scattering
  - Compton scattering
- with Coulomb field of electron
  - Triplet production

#### Photon-nucleus interaction

- with nucleus directly
  - Photodisintegration
- with Coulomb field of nucleus
  - Pair production

## 1.4 PHOTON INTERACTIONS

### 1.4.3 Types of photon interactions with absorber

#### ☐ Types of photon-atom interactions

##### **Complete absorption of photon**

Photoelectric effect  
Pair production  
Triplet production  
Photodisintegration

##### **Photon scattering**

Thomson scattering  
Rayleigh scattering  
Compton scattering

# 1.4 PHOTON INTERACTIONS

## 1.4.4 Photoelectric effect

### Photoelectric effect:

- ❑ Photon of energy  $h\nu$  interacts with a **tightly bound electron**, i.e., with whole atom.
- ❑ Photon disappears.
- ❑ Conservation of energy and momentum considerations show that photoelectric effect can occur only on a tightly bound electron rather than on a loosely bound (“free”) electron.

# 1.4 PHOTON INTERACTIONS

## 1.4.4 Photoelectric effect

- Orbital electron is ejected from the atom with kinetic energy

$$E_K = h\nu - E_B,$$

where  $E_B$  is the binding energy of the orbital electron.

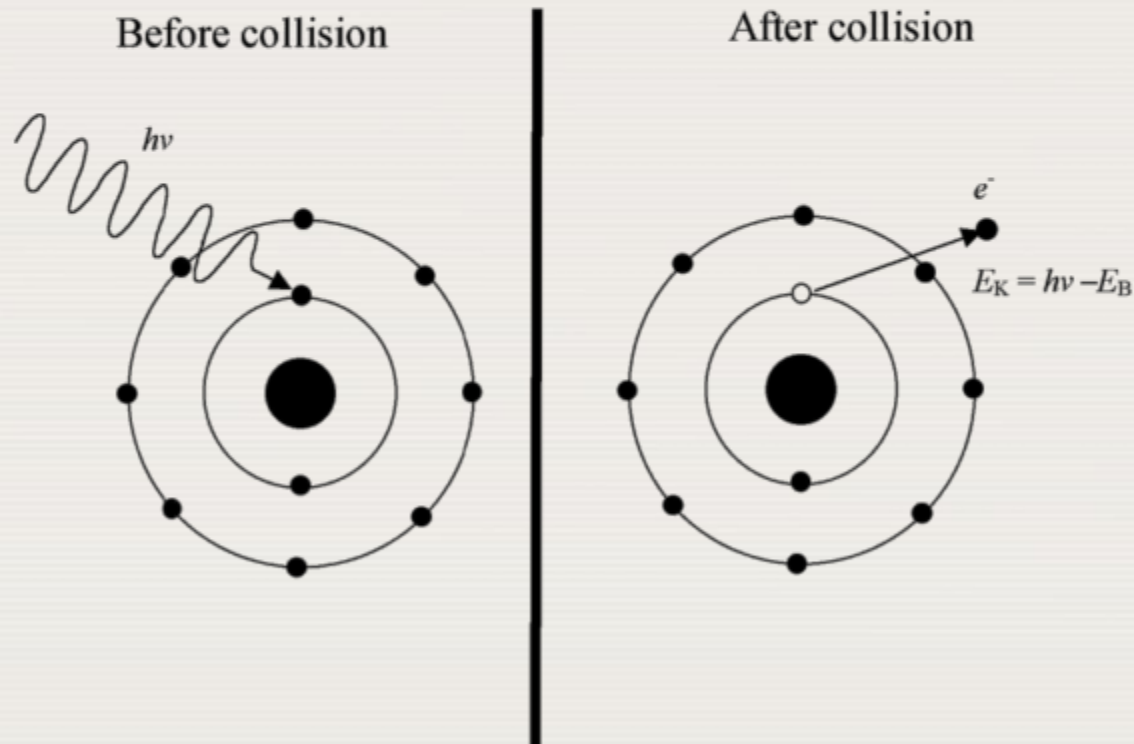
- Ejected orbital electron is called a **photoelectron**.
- When the photon energy  $h\nu$  exceeds the K-shell binding energy  $E_B(K)$  of the absorber atom, the photoelectric effect is most likely to occur with a K-shell electron in comparison with higher shell electrons.

# 1.4 PHOTON INTERACTIONS

## 1.4.4 Photoelectric effect

### □ Schematic diagram of the photoelectric effect

- A photon with energy  $h\nu$  interacts with a K-shell orbital electron.
- Orbital electron is emitted from the atom as a photoelectron.

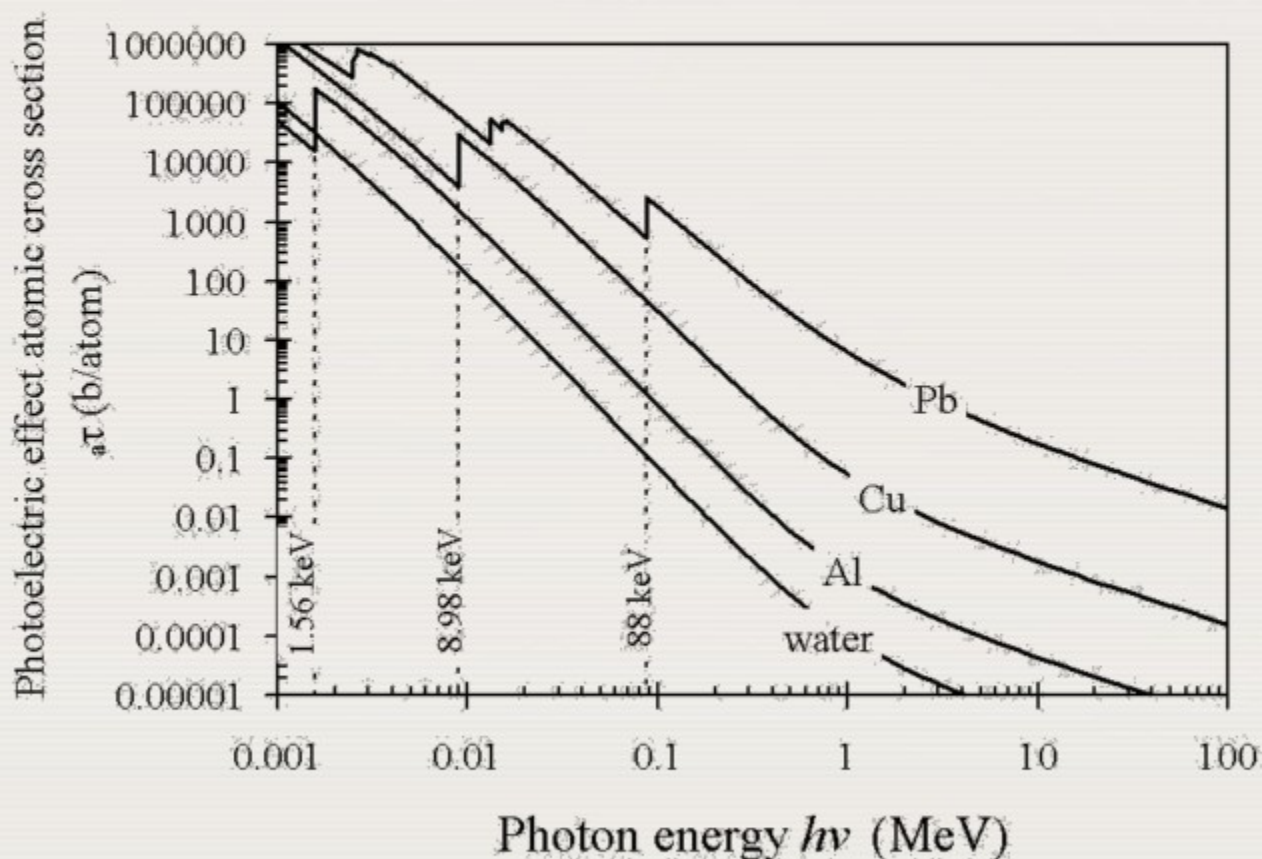




# 1.4 PHOTON INTERACTIONS

## 1.4.4 Photoelectric effect

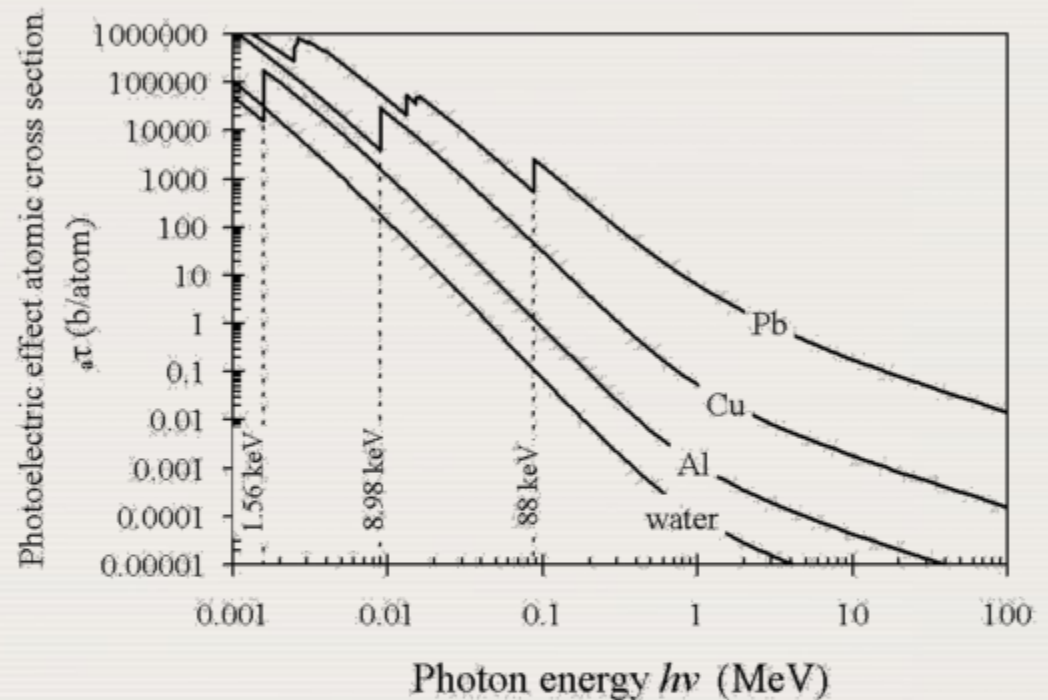
- ☐ Photoelectric atomic cross sections for water, aluminum, copper and lead against photon energy.



# 1.4 PHOTON INTERACTIONS

## 1.4.4 Photoelectric effect

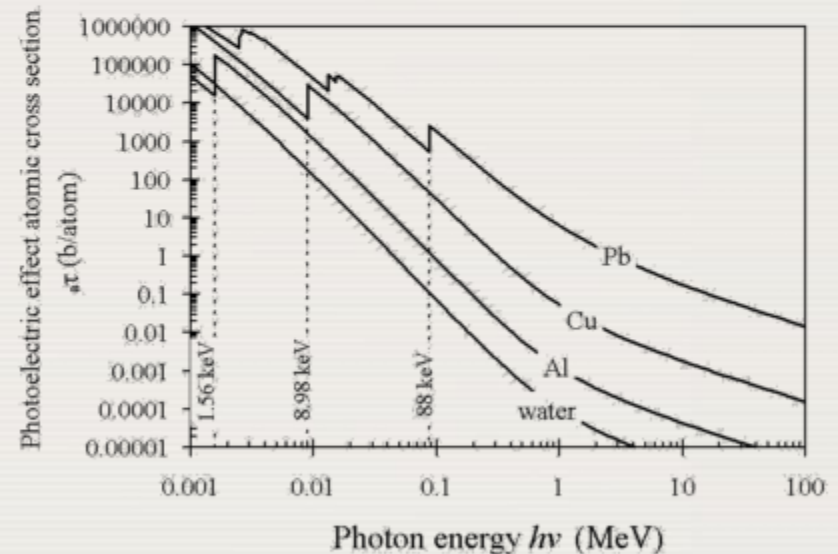
- Atomic attenuation coefficient  $\tau_a$  for photoelectric effect is proportional to  $Z^4/(h\nu)^3$ .
- Mass attenuation coefficient  $\tau_m$  for photoelectric effect is proportional to  $Z^3/(h\nu)^3$ .



# 1.4 PHOTON INTERACTIONS

## 1.4.4 Photoelectric effect

- A plot of  $\tau_m$  against  $h\nu$  shows, in addition to a steady decrease in  $\tau_m$  with increasing photon energy, sharp discontinuities when  $h\nu$  equals the binding energy  $E_B$  for a particular electronic shell of the absorber.
- These discontinuities, called **absorption edges**, reflect the fact that for  $h\nu < E_B$  photons cannot undergo photoelectric effect with electrons in the given shell, while for  $h\nu \geq E_B$  they can.



# 1.4 PHOTON INTERACTIONS

## 1.4.4 Photoelectric effect

- Average (mean) energy transferred from a photon with energy  $h\nu > E_B(K)$  to electrons,  $(\bar{E}_K)_{tr}^{PE}$ , is given as:

$$(\bar{E}_K)_{tr}^{PE} = h\nu - P_K \omega_K E_B(K)$$

with

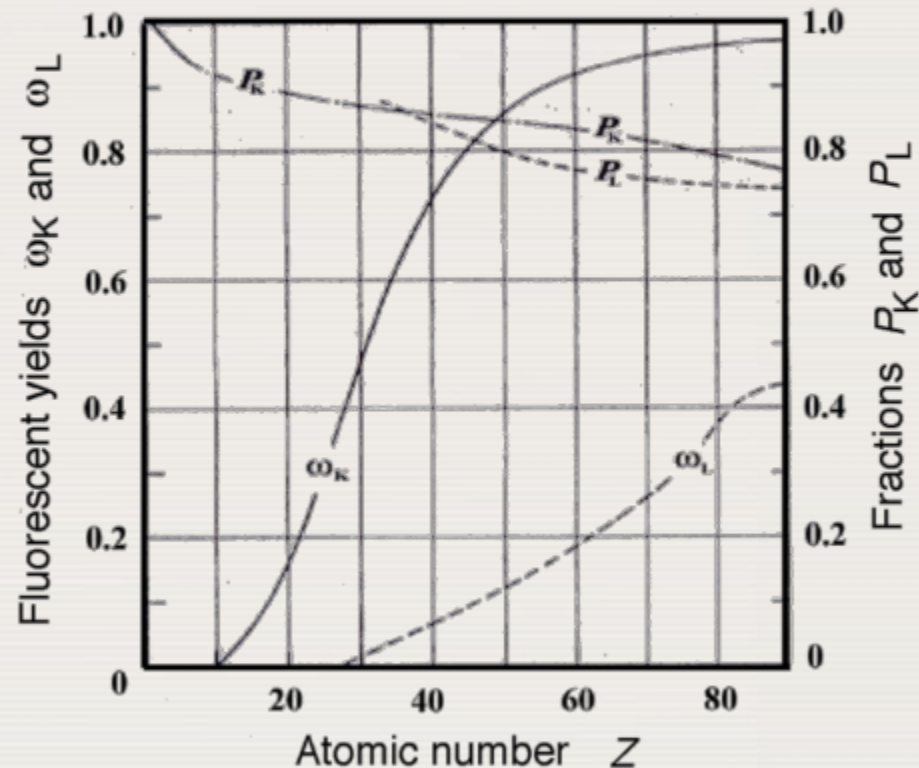
- $E_B(K)$  binding energy of the K-shell electron (photoelectron).
- $P_K$  fraction of all photoelectric interactions in the K shell.
- $\omega_K$  fluorescence yield for the K shell.

# 1.4 PHOTON INTERACTIONS

## 1.4.4 Photoelectric effect

### Fluorescence yield $\omega_x$ and function $P_x$

- Fluorescence yield  $\omega_x$  is defined as the number of photons emitted per vacancy in a given atomic shell X.
- Function  $P_x$  for a given shell gives the proportion of photoelectric events in given shell compared to the total number of photoelectric events in the whole atom.



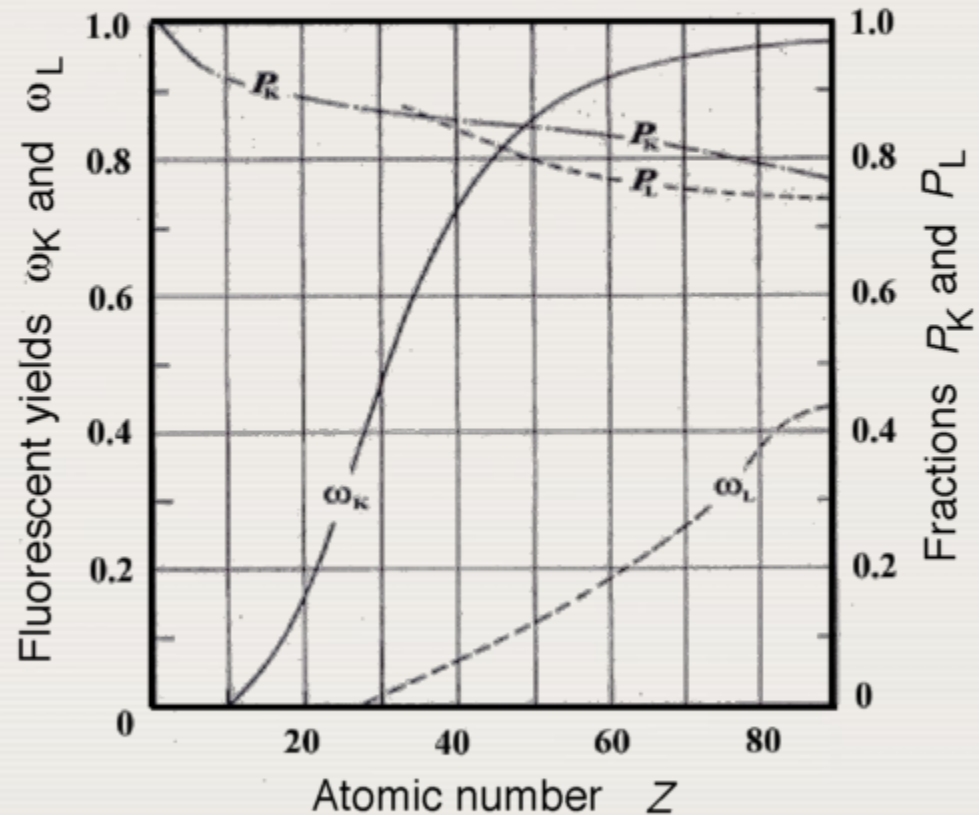
# 1.4 PHOTON INTERACTIONS

## 1.4.4 Photoelectric effect

### □ Fluorescence yields $\omega_K$ and $\omega_L$ and functions $P_K$ and $P_L$

The range of  $P_K$  is from 1.0 at low atomic numbers  $Z$  to 0.8 at high atomic numbers  $Z$  of the absorber.

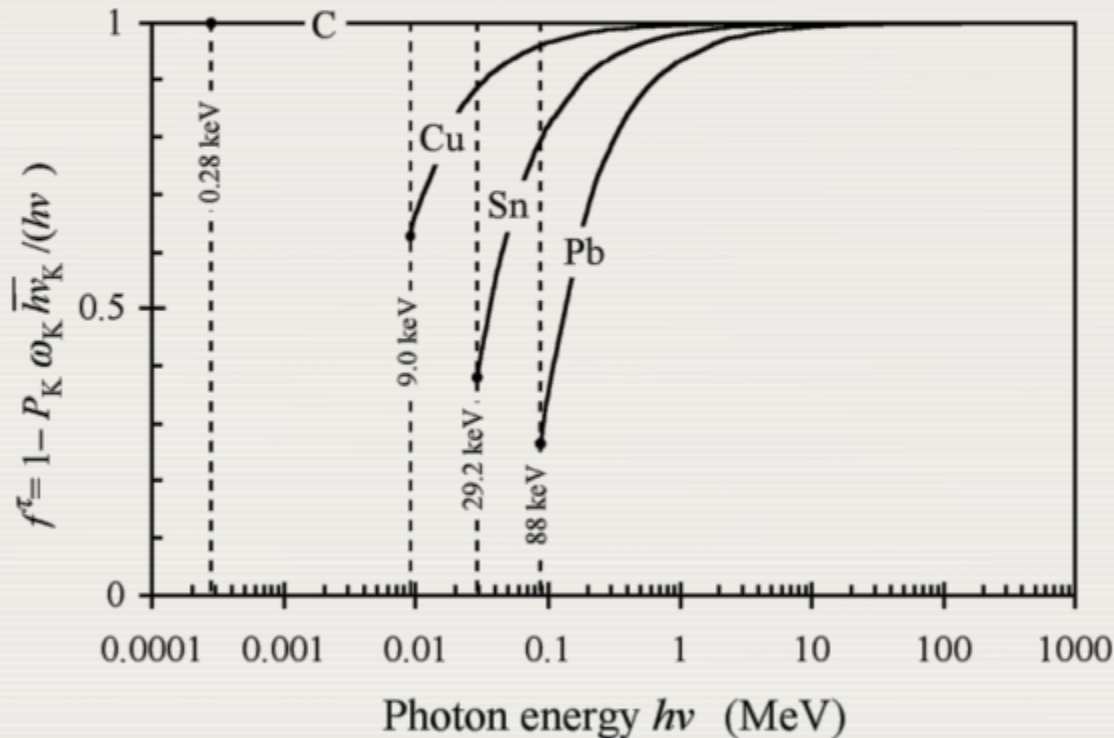
The range in  $\omega_K$  is from 0 at low atomic numbers  $Z$  through 0.5 at  $Z = 30$  to 0.96 at high  $Z$ .



# 1.4 PHOTON INTERACTIONS

## 1.4.4 Photoelectric effect

- Mean energy transfer fraction for photoelectric effect  $\bar{f}_{PE}$  is:



$$\bar{f}_{PE} = \frac{hv - (\bar{E}_K)_{tr}^{PE}}{hv}$$
$$= 1 - \frac{P_K \omega_K E_B(K)}{hv}$$

# 1.4 PHOTON INTERACTIONS

## 1.4.5 Coherent (Rayleigh) scattering

### Coherent (Rayleigh) scattering:

- ❑ In coherent (Rayleigh) scattering the photon interacts with a bound orbital electron, i.e., with the combined action of the whole atom.
- ❑ The event is elastic as the photon loses essentially none of its energy and is scattered through only a small angle.
- ❑ No energy transfer occurs from the photon to charged particles in the absorber; thus Rayleigh scattering plays no role in the energy transfer coefficient but it contributes to the attenuation coefficient.

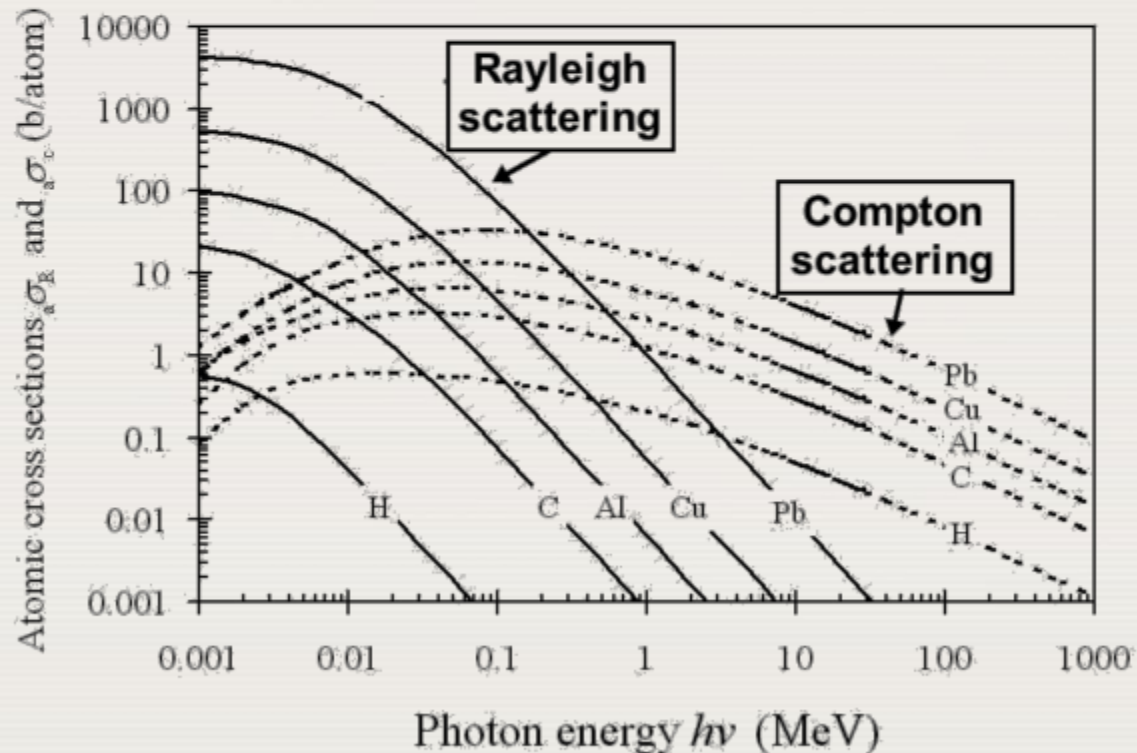


# 1.4 PHOTON INTERACTIONS

## 1.4.5 Coherent (Rayleigh) scattering

### □ Coefficients for coherent (Rayleigh) scattering

- Atomic cross section is proportional to  $(Z/h\nu)^2$ .
- Mass attenuation coefficient is proportional to  $(Z/h\nu)^2$ .



# 1.4 PHOTON INTERACTIONS

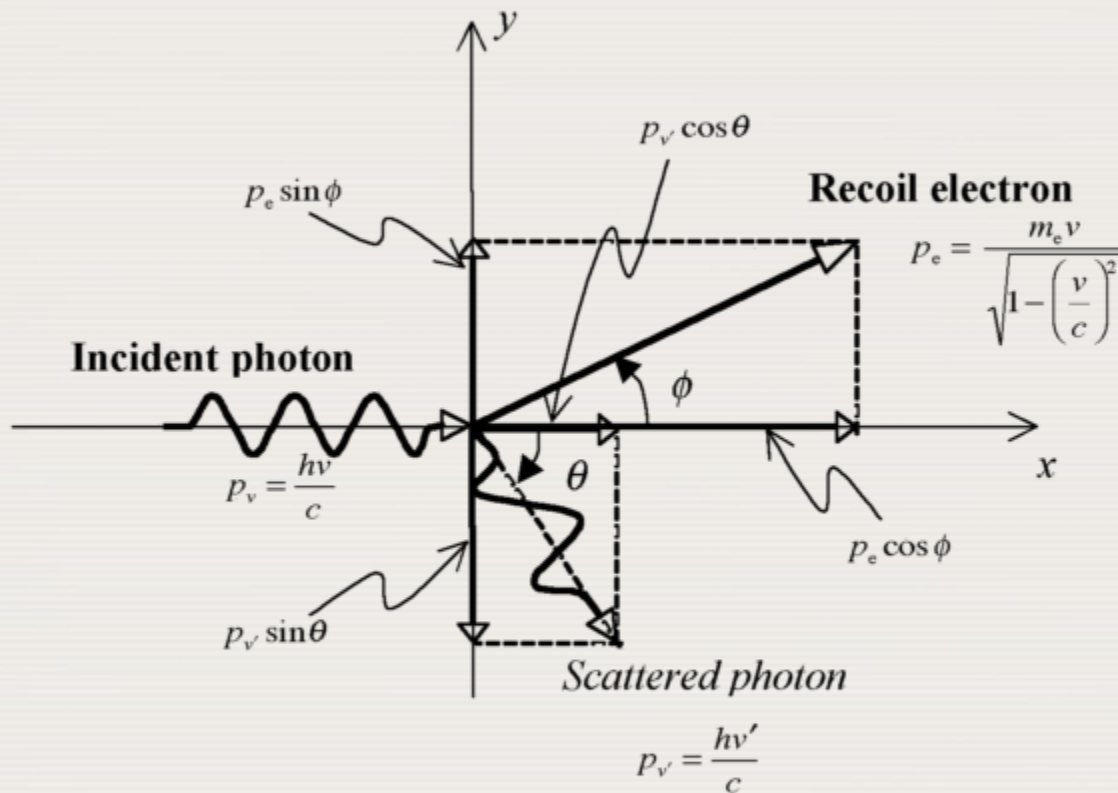
## 1.4.6 Compton (Incoherent) scattering

### Compton (incoherent) scattering

- ❑ In Compton effect (incoherent scattering) a photon with energy  $h\nu$  interacts with a loosely bound (“free”) electron.
- ❑ Part of the incident photon energy is transferred to the “free” orbital electron which is emitted from the atom as the Compton (recoil) electron.
- ❑ Photon is scattered through a scattering angle  $\theta$  and its energy  $h\nu'$  is lower than the incident photon energy  $h\nu$ .
- ❑ Angle  $\phi$  represents the angle between the incident photon direction and the direction of the recoil electron.

# 1.4 PHOTON INTERACTIONS

## 1.4.6 Compton scattering



Conservation of energy

$$h\nu + m_e c^2 = h\nu' + m_e c^2 + E_K$$

Conservation of momentum (x axis)

$$p_v = p_{v'} \cos \theta + p_e \cos \phi$$

Conservation of momentum (y axis)

$$0 = -p_{v'} \sin \theta + p_e \sin \phi$$

Compton expression:

$$\Delta\lambda = \lambda_C (1 - \cos \theta)$$

$$\lambda_C = \frac{h}{m_e c} = 0.024 \text{ \AA}$$

# 1.4 PHOTON INTERACTIONS

## 1.4.6 Compton scattering

- Scattering angle  $\theta$  and recoil angle  $\phi$  are related as follows

$$\cot \phi = (1 + \varepsilon) \tan \frac{\theta}{2} \qquad \varepsilon = \frac{h\nu}{m_e c^2}$$

- Relationship between the scattered photon energy  $h\nu'$  and the incident photon energy  $h\nu$  is:

$$h\nu' = h\nu \frac{1}{1 + \varepsilon(1 - \cos \theta)} \qquad \varepsilon = \frac{h\nu}{m_e c^2}$$

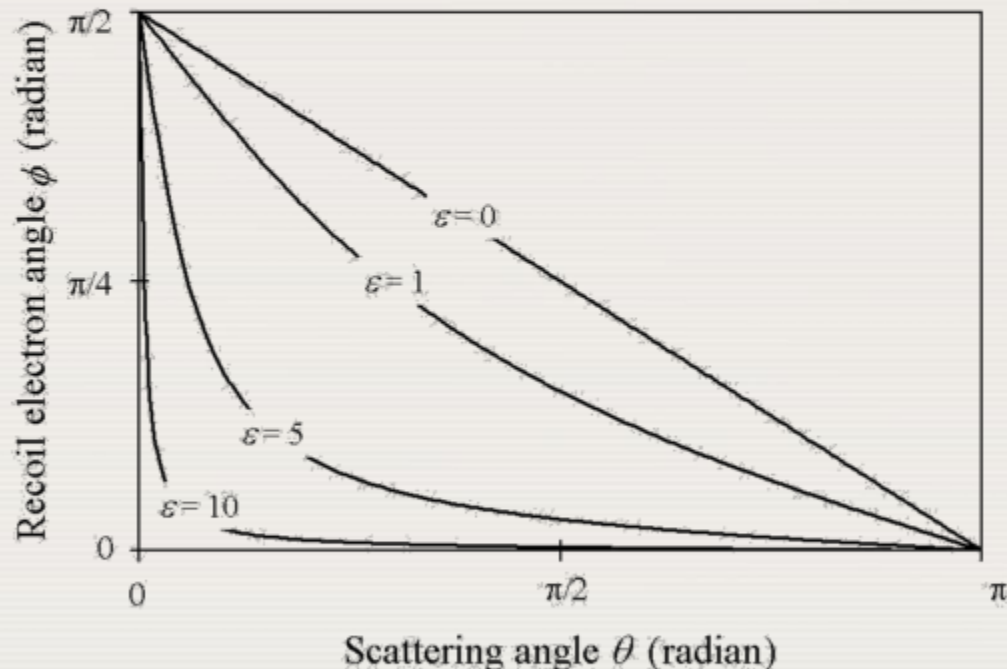
- Relationship between the kinetic energy of the recoil electron  $E_K$  and the energy of the incident photon  $h\nu$  is:

$$E_K = h\nu \frac{\varepsilon(1 - \cos \theta)}{1 + \varepsilon(1 - \cos \theta)} \qquad \varepsilon = \frac{h\nu}{m_e c^2}$$

## 1.4 PHOTON INTERACTIONS

### 1.4.6 Compton scattering

- Relationship between the photon scattering angle  $\theta$  and the recoil angle  $\phi$  of the Compton electron:



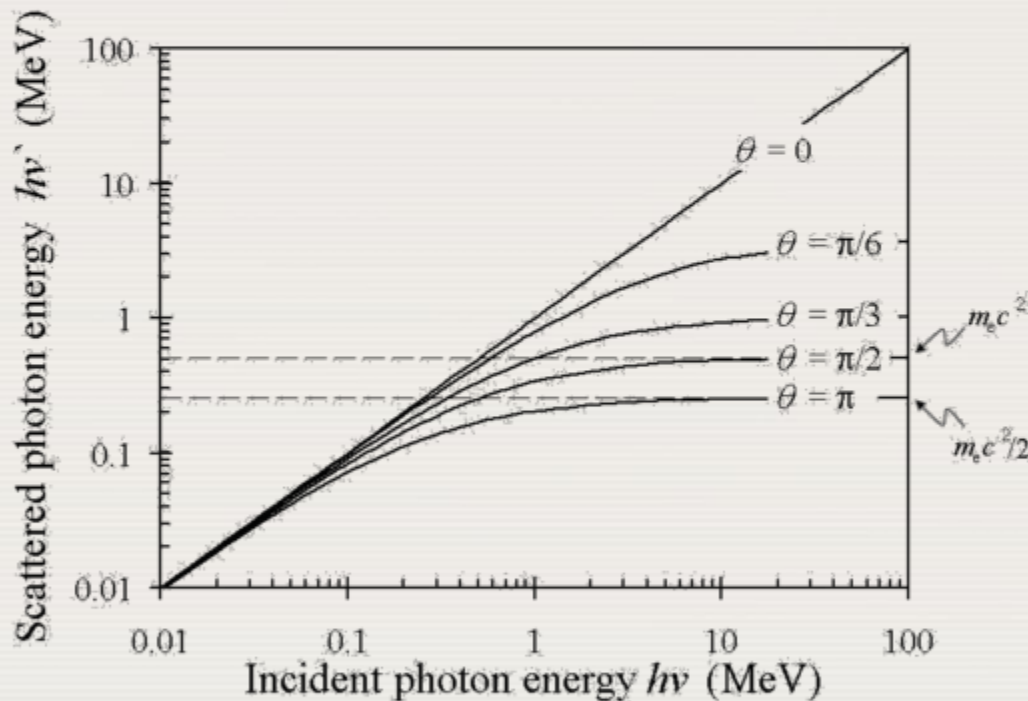
$$\cot \phi = (1 + \epsilon) \tan \frac{\theta}{2}$$

$$\epsilon = \frac{h\nu}{m_e c^2}$$

# 1.4 PHOTON INTERACTIONS

## 1.4.6 Compton scattering

- Relationship between the scattered photon energy  $h\nu'$  and the incident photon energy  $h\nu$ :



$$h\nu' = h\nu \frac{1}{1 + \varepsilon(1 - \cos \theta)}$$

$$\varepsilon = \frac{h\nu}{m_e c^2}$$

# 1.4 PHOTON INTERACTIONS

## 1.4.6 Compton scattering

- Energy of Compton scattered photons  $h\nu'$  is expressed as:

$$h\nu' = h\nu \frac{1}{1 + \varepsilon(1 - \cos \theta)}$$

- Energy of photons scattered at  $\theta = 90^\circ$

$$h\nu'(\theta = \pi/2) = \frac{h\nu}{1 + \varepsilon} \quad h\nu'_{\max}(\theta = \pi/2) = \lim_{h\nu \rightarrow \infty} \frac{h\nu}{1 + \varepsilon} = m_e c^2 = 0.511 \text{ MeV}$$

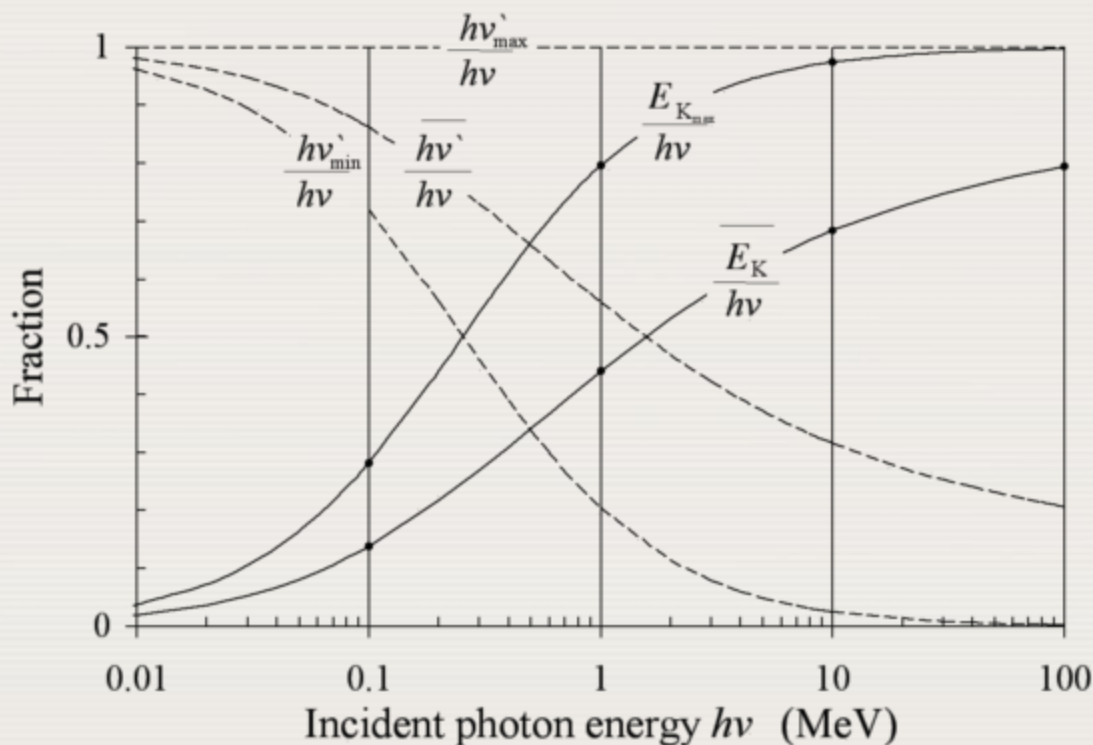
- Energy of photons scattered at  $\theta = \pi$

$$h\nu'(\theta = \pi) = \frac{h\nu}{1 + 2\varepsilon} \quad h\nu'_{\max}(\theta = \pi) = \lim_{h\nu \rightarrow \infty} \frac{h\nu}{1 + 2\varepsilon} = \frac{m_e c^2}{2} = 0.255 \text{ MeV}$$

# 1.4 PHOTON INTERACTIONS

## 1.4.6 Compton scattering

- Maximum and mean fractions of incident photon energy  $h\nu$  given to the scattered photon  $h\nu'$  and to Compton (recoil) electron  $E_K$ .



$$\frac{h\nu'}{h\nu} = \frac{1}{1 + \epsilon(1 - \cos \theta)}$$

$$\frac{h\nu'_{\max}}{h\nu} = \frac{h\nu'}{h\nu} (\theta = 0) = 1$$

$$\frac{E_K}{h\nu} = \frac{\epsilon(1 - \cos \theta)}{1 + \epsilon(1 - \cos \theta)}$$

$$\frac{(E_K)_{\max}}{h\nu} = \frac{E_K}{h\nu} (\theta = \pi) = \frac{2\epsilon}{1 + 2\epsilon}$$

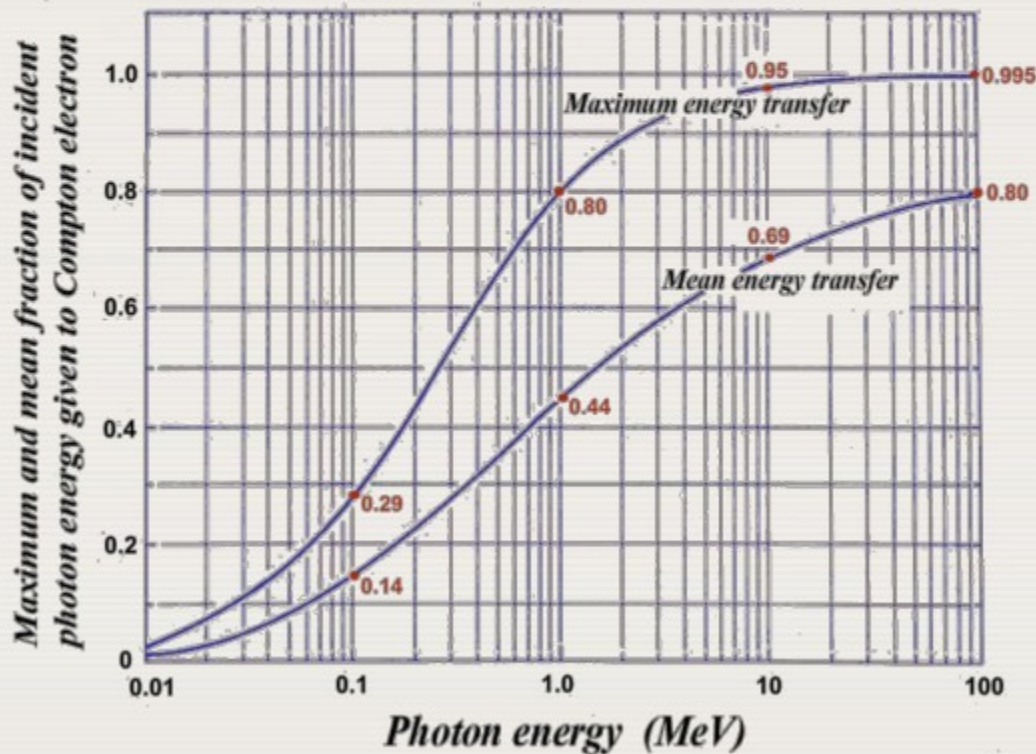
$$\frac{h\nu'_{\min}}{h\nu} = \frac{h\nu'}{h\nu} (\theta = \pi) = \frac{1}{1 + 2\epsilon}$$



# 1.4 PHOTON INTERACTIONS

## 1.4.6 Compton scattering

- Maximum and mean energy transfer from the photon with energy  $h\nu$  to Compton (recoil) electron (“Compton Graph #1”).
- Mean energy transfer fraction for Compton effect  $\bar{f}_C$ .



$$\frac{E_K}{h\nu} = \frac{\epsilon(1 - \cos\theta)}{1 + \epsilon(1 - \cos\theta)}$$

$$\epsilon = \frac{h\nu}{m_e c^2}$$

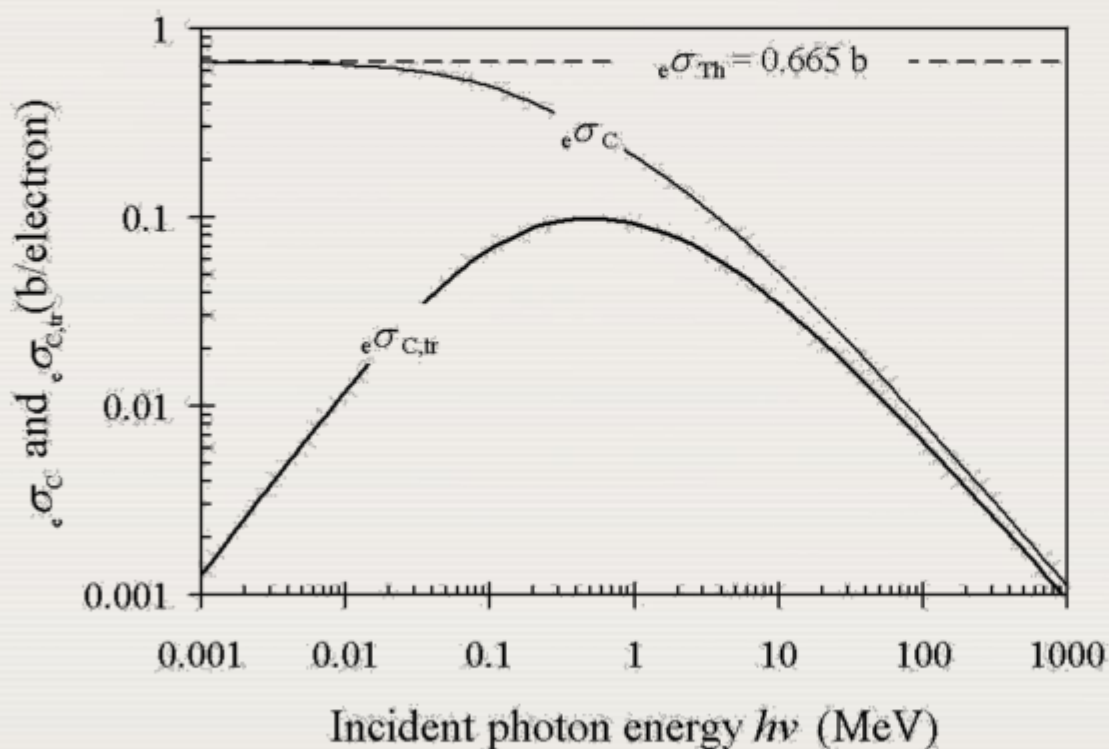
$$\frac{(E_K)_{\max}}{h\nu} = \frac{2\epsilon}{1 + 2\epsilon}$$

$$\bar{f}_C = \frac{(\bar{E}_K)_{\text{tr}}^C}{h\nu}$$

# 1.4 PHOTON INTERACTIONS

## 1.4.6 Compton scattering

- Electronic Compton attenuation coefficient  ${}_e\sigma_C$  steadily decreases with increasing photon energy  $h\nu$ .



$$({}_e\sigma_C)_{tr} = {}_e\sigma_C \times f_C$$

$$\bar{f}_C = \frac{(\bar{E}_K)_{tr}^C}{h\nu}$$

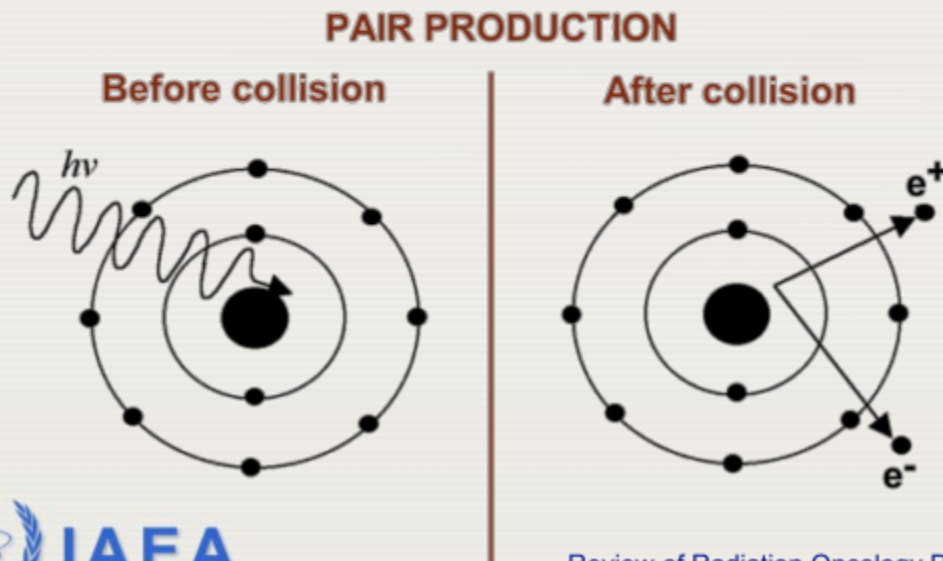
# 1.4 PHOTON INTERACTIONS

## 1.4.7 Pair production

### Pair production (in field of the nucleus or orbital electron)

#### □ In nuclear pair production

- Photon disappears.
- An electron-positron pair with a combined kinetic energy equal to  $h\nu - 2m_e c^2$  is produced in the nuclear Coulomb field.
- Threshold energy  $h\nu_{\text{thr}}$  for nuclear pair production is:



$$h\nu_{\text{thr}} = 2m_e c^2 \left\{ 1 + \frac{m_e c^2}{M_A c^2} \right\} \approx 2m_e c^2$$

$m_e$  electron mass

$M_A$  mass of nucleus

$$m_e c^2 = 0.511 \text{ MeV}$$

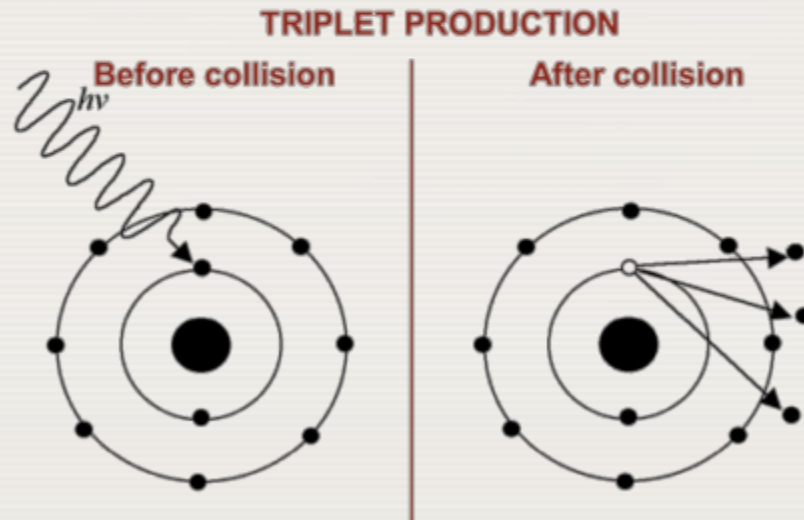
# 1.4 PHOTON INTERACTIONS

## 1.4.7 Pair production

□ In triplet production (also called electronic pair production):

- Photon disappears.
- An electron-positron pair is produced in the Coulomb field of an orbital electron, and a triplet (two electrons and one positron) leave the site of interaction.
- Threshold energy for triplet production is:  $h\nu_{\text{thr}} = 4m_e c^2 = 2.04 \text{ MeV}$

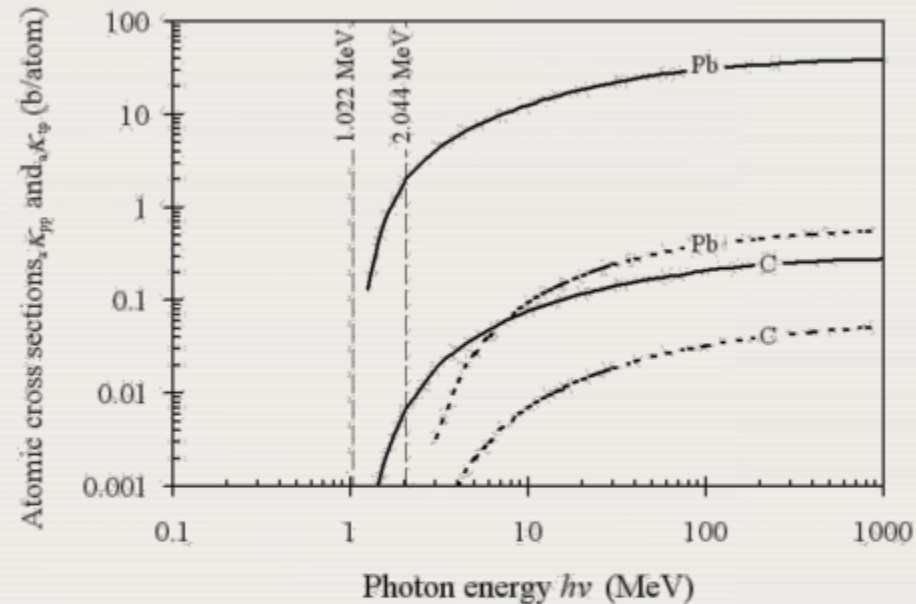
$$m_e c^2 = 0.511 \text{ MeV}$$



# 1.4 PHOTON INTERACTIONS

## 1.4.7 Pair production

- **Atomic cross sections** for pair production and triplet production equal zero for photon energies below the threshold energy.
- **Atomic cross section** for pair production and triplet production increase rapidly with photon energy above the threshold energy.



Atomic cross sections for pair production: **solid curves**

Atomic cross sections for triplet production: **dashed curves**

## 1.4 PHOTON INTERACTIONS

### 1.4.7 Pair production

- Atomic cross section for nuclear pair production  ${}_a\kappa_{NPP}$  varies approximately as the square of the atomic number  $Z$  of the absorber.
- Atomic cross section for triplet production  ${}_a\kappa_{TP}$  varies approximately linearly with  $Z$ , the atomic number of the absorber.

## 1.4 PHOTON INTERACTIONS

### 1.4.7 Pair production

- Mass attenuation coefficient for **nuclear pair production**  $(\kappa/\rho)_{\text{NPP}}$  varies approximately linearly with  $Z$ , the atomic number of the absorber.
- Mass attenuation coefficient for **triplet production**  $(\kappa/\rho)_{\text{TP}}$  is essentially independent of the atomic number  $Z$  of the absorber.

## 1.4 PHOTON INTERACTIONS

### 1.4.7 Pair production

- Attenuation coefficient for nuclear pair production exceeds significantly the attenuation coefficient for triplet production at same photon energy and atomic number of absorber.
- ${}_a K_{TP}$  is at most about 30 % of  ${}_a K_{NPP}$  for  $Z = 1$  and less than 1 % for high  $Z$  absorbers.
- Usually, the tabulated values for pair production  $K$  include contribution of both the pair production in the field of the nucleus and the pair production in the field of electron, i.e.,

$${}_a K = {}_a K_{NPP} + {}_a K_{TP}$$



## 1.4 PHOTON INTERACTIONS

### 1.4.7 Pair production

- Total kinetic energy transferred from the photon to charged particles (electron and positron) in pair production is

$$h\nu - 2m_e c^2$$

- Mass attenuation coefficient  $\kappa/\rho$  is calculated from the atomic cross section  ${}_a\kappa$  as follows

$$\frac{\kappa}{\rho} = {}_a\kappa \frac{N_A}{A}$$

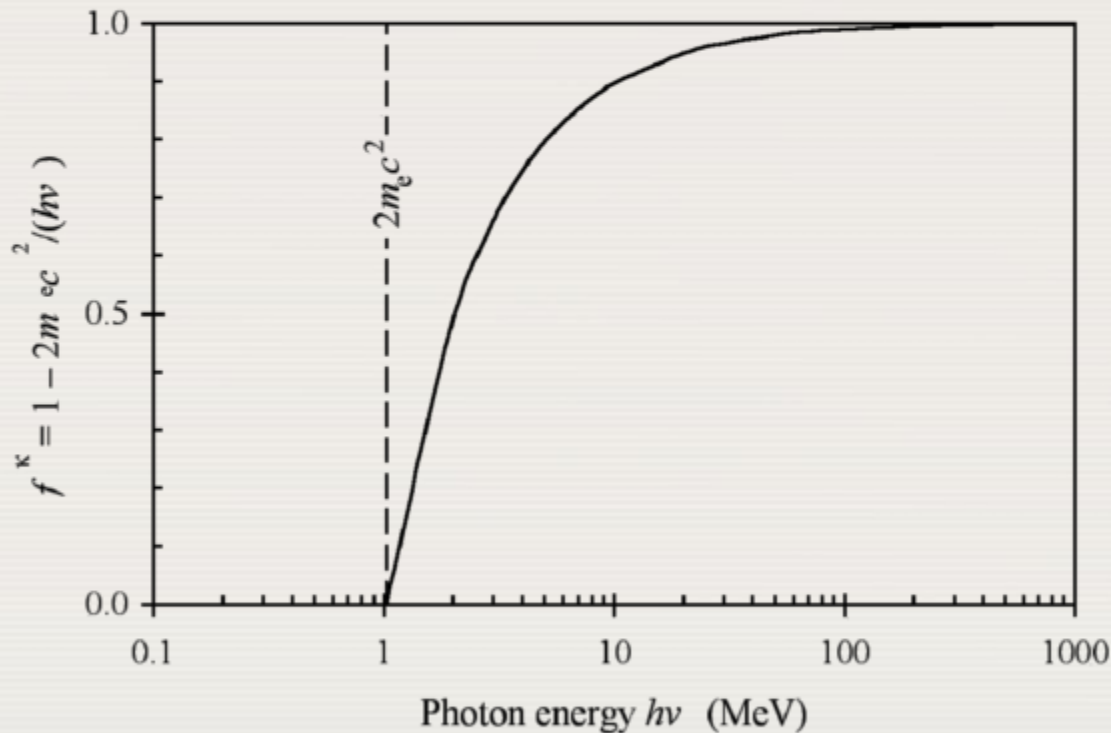
- Mass energy transfer coefficient  $(\kappa/\rho)_{tr}$  is:

$$\left(\frac{\kappa}{\rho}\right)_{tr} = f_{PP} \frac{\kappa}{\rho} = \frac{\kappa}{\rho} \left(1 - \frac{2m_e c^2}{h\nu}\right)$$

# 1.4 PHOTON INTERACTIONS

## 1.4.7 Pair production

- Mean energy transfer fraction for pair production  $\bar{f}_{PP}$

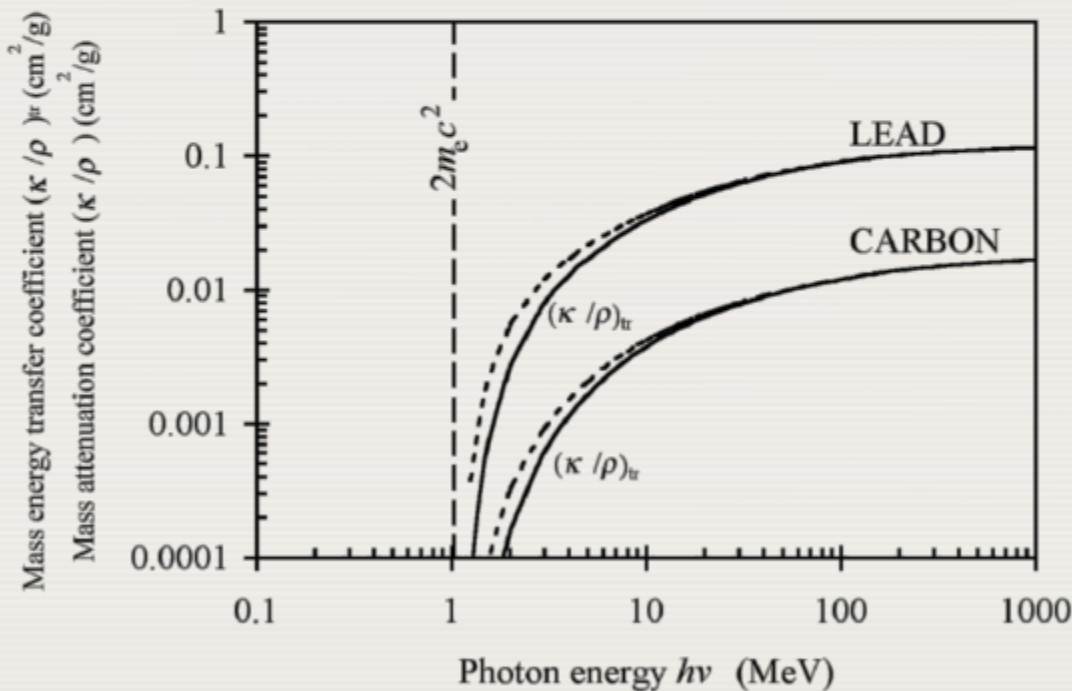


$$\begin{aligned}\bar{f}_{PP} &= \frac{h\nu - \bar{E}_{tr}^{PP}}{h\nu} \\ &= 1 - \frac{2m_e c^2}{h\nu}\end{aligned}$$

# 1.4 PHOTON INTERACTIONS

## 1.4.7 Pair production

- Mass attenuation coefficient  $\kappa/\rho$  and mass energy transfer coefficient  $(\kappa/\rho)_{tr}$  for pair production against photon energy  $h\nu$ .



Mass attenuation coefficient:  
**dashed curves**

Mass energy transfer  
coefficient: **solid curves**

## 1.4 PHOTON INTERACTIONS

### 1.4.8 Photonuclear reactions

- ❑ Photonuclear reactions (nuclear photoelectric effect):
  - High energy photon is absorbed by the nucleus of the absorber.
  - A neutron or a proton is emitted.
  - Absorber atom is transformed into a radioactive reaction product.
- ❑ **Threshold** is of the order of  $\sim 10$  MeV or higher, with the exception of the deuteron and beryllium-9 ( $\sim 2$  MeV).
- ❑ Probability for photonuclear reactions is much smaller than that for other photon atomic interactions; photonuclear reactions are thus usually neglected in medical physics.

# 1.4 PHOTON INTERACTIONS

## 1.4.9 Contribution to attenuation coefficients

□ For a given  $h\nu$  and  $Z$ :

- Linear attenuation coefficient  $\mu$
- Linear energy transfer coefficient  $\mu_{\text{tr}}$
- Linear energy absorption coefficient  $\mu_{\text{ab}}$  (often designated  $\mu_{\text{en}}$ )

are given as a **sum of coefficients** for individual photon interactions

$$\mu = \tau + \sigma_{\text{R}} + \sigma_{\text{C}} + \kappa$$

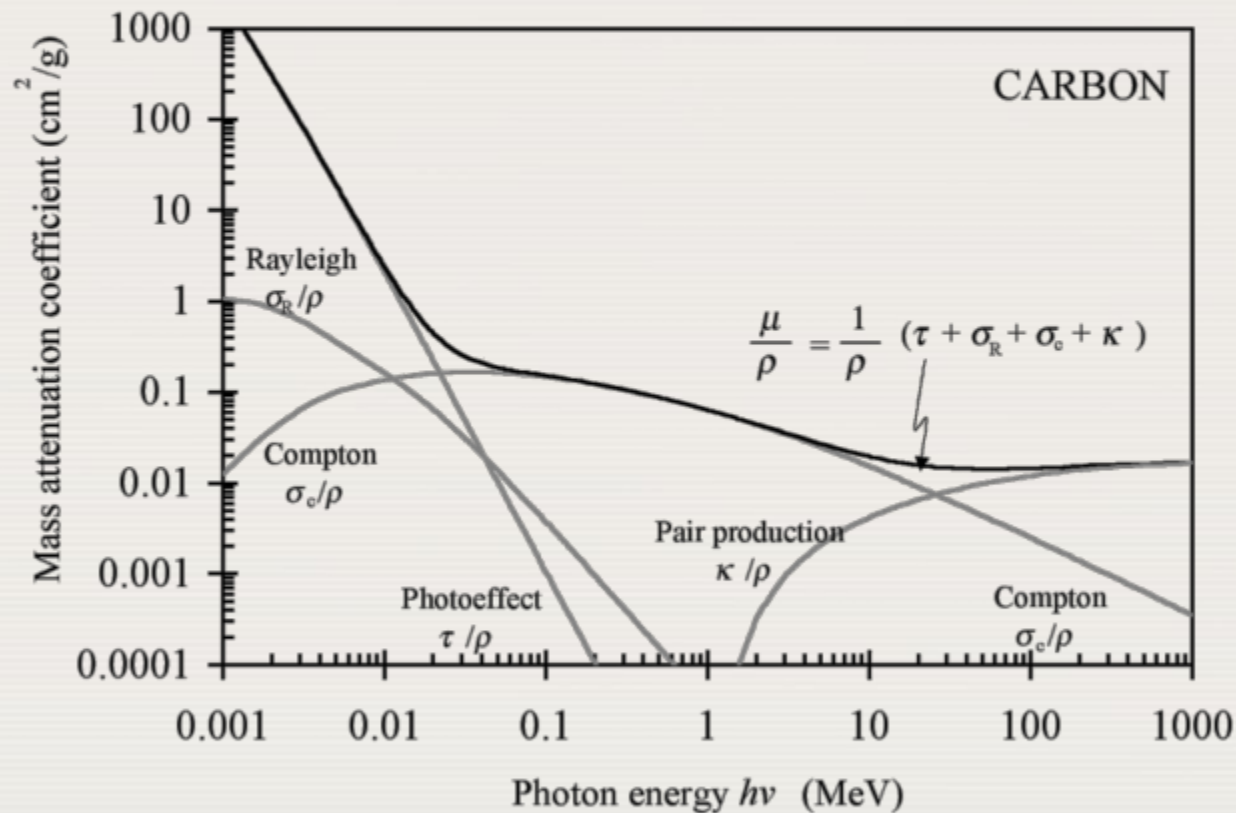
$$\mu_{\text{tr}} = \tau_{\text{tr}} + (\sigma_{\text{R}})_{\text{tr}} + (\sigma_{\text{C}})_{\text{tr}} + \kappa_{\text{tr}} = f_{\text{PE}} \tau + f_{\text{C}} \sigma_{\text{C}} + f_{\text{PP}} \kappa$$

$$\mu_{\text{ab}} \equiv \mu_{\text{en}} = \mu_{\text{tr}} (1 - \bar{g})$$

# 1.4 PHOTON INTERACTIONS

## 1.4.9 Contribution to attenuation coefficients

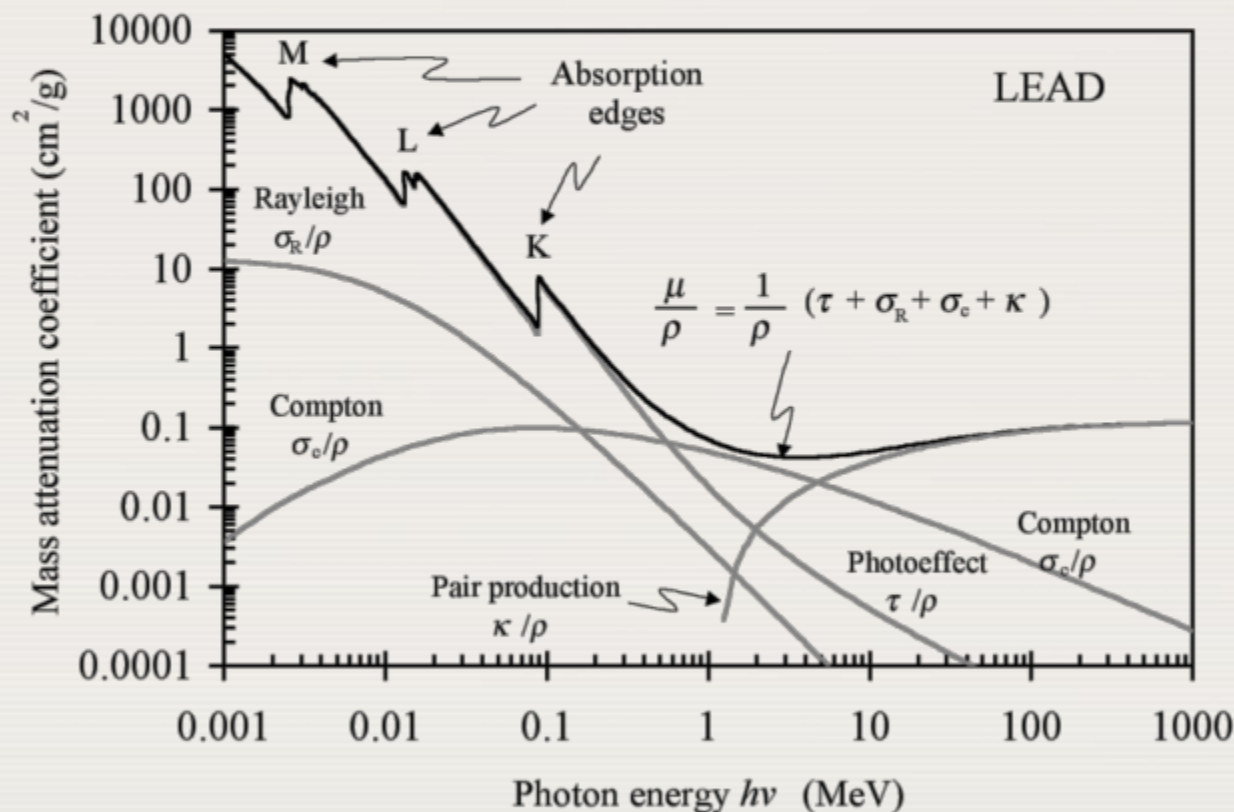
Mass attenuation coefficient against photon energy for carbon



# 1.4 PHOTON INTERACTIONS

## 1.4.9 Contribution to attenuation coefficients

Mass attenuation coefficient against photon energy for lead



## 1.4 PHOTON INTERACTIONS

### 1.4.10 Relative predominance of individual effects

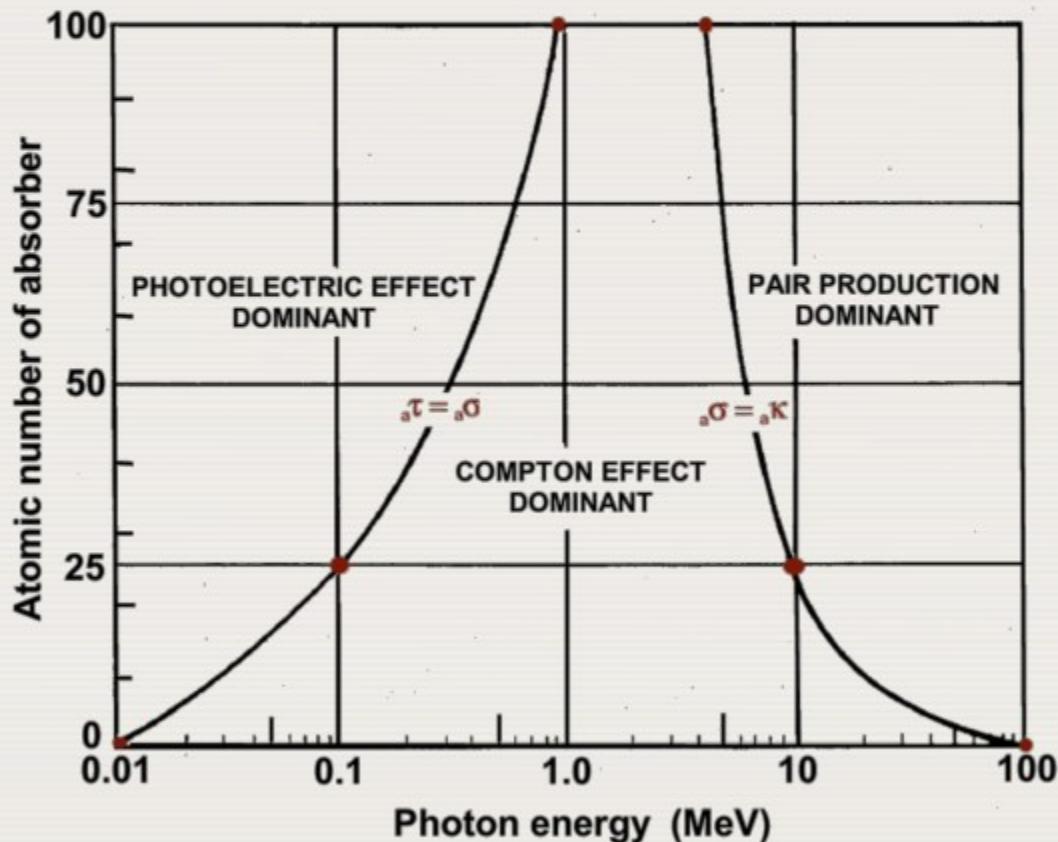
- ❑ **Probability** for a photon to undergo any one of the various interaction phenomena with an atom of the absorber depends:
  - On the energy  $h\nu$  of the photon.
  - On the atomic number  $Z$  of the absorber.
  
- ❑ In general,
  - Photoelectric effect predominates at low photon energies.
  - Compton effect predominates at intermediate photon energies.
  - Pair production predominates at high photon energies.



# 1.4 PHOTON INTERACTIONS

## 1.4.10 Relative predominance of individual effects

- Regions of relative predominance of the three main forms of photon interaction with absorber.



## 1.4 PHOTON INTERACTIONS

### 1.4.11 Effects following photon interactions

- ❑ In photoelectric effect, Compton scattering and triplet production **vacancies** are produced in atomic shells through ejection of an orbital electron.
- ❑ Vacancies are filled with orbital electrons making **transitions** from higher to lower level atomic shells.
- ❑ Electronic transitions are followed by emission of **characteristic x rays** or **Auger electrons**; the proportion governed by the fluorescence yield.

## 1.4 PHOTON INTERACTIONS

### 1.4.11 Effects following photon interactions

- ❑ Pair production and triplet production are followed by the **annihilation of the positron**, which lost almost all its kinetic energy through Coulomb interactions with absorber atoms, with a “free” electron producing two **annihilation quanta**.
- ❑ The two annihilation quanta have most commonly energy of 0.511 MeV each, and are emitted at approximately  $180^\circ$  to each other to satisfy the conservation of momentum and energy.
- ❑ Annihilation may also occur of an energetic positron with an electron and this rare event is referred to as **annihilation-in-flight**.

# 1.4 PHOTON INTERACTIONS

## 1.4.12 Summary of photon interactions

	<i>Photoelectric effect</i>	<i>Rayleigh scattering</i>	<i>Compton effect</i>	<i>Pair production</i>
<b><i>Photon interaction</i></b>	With whole atom (bound electron)	With bound electrons	With free electron	With nuclear Coulomb field
<b><i>Mode of photon interaction</i></b>	Photon disappears	Photon scattered	Photon scattered	Photon disappears
<b><i>Energy dependence</i></b>	$\frac{1}{(h\nu)^3}$	$\frac{1}{(h\nu)^2}$	Decreases with energy	Increases with energy
<b><i>Threshold</i></b>	Shell binding energy	No	Shell binding energy	$\sim 2m_e c^2$
<b><i>Particles released in absorber</i></b>	Photoelectron	None	Compton (recoil) electron	Electron-positron pair

# 1.4 PHOTON INTERACTIONS

## 1.4.12 Summary of photon interactions

	<i>Photoelectric effect</i>	<i>Rayleigh scattering</i>	<i>Compton effect</i>	<i>Pair production</i>
<i>Linear attenuation coefficient</i>	$\tau$	$\sigma_R$	$\sigma_C$	$\kappa$
<i>Atomic coefficient dependence upon Z</i>	${}_a\tau \propto Z^4$	${}_a\sigma_R \propto Z^2$	${}_a\sigma_C \propto Z$	${}_a\kappa \propto Z^2$
<i>Mass coefficient dependence upon Z</i>	$\frac{\tau}{\rho} \propto Z^3$	$\frac{\sigma_R}{\rho} \propto Z$	Independent of Z	$\frac{\kappa}{\rho} \propto Z$

# 1.4 PHOTON INTERACTIONS

## 1.4.12 Summary of photon interactions

	<i>Photoelectric effect</i>	<i>Rayleigh scattering</i>	<i>Compton effect</i>	<i>Pair production</i>
<i>Mean energy transferred to charged part's</i>	$h\nu - P_K \omega_K h\bar{\nu}_K$	0	$\bar{E}_{tr}^C$ (Compton graph)	$h\nu - 2m_e c^2$
<i>Fraction of energy <math>h\nu</math> transferred</i>	$1 - \frac{P_K \omega_K h\bar{\nu}_K}{h\nu}$	0	$\frac{\bar{E}_{tr}^C}{h\nu}$ (Compton graph)	$1 - \frac{2m_e c^2}{h\nu}$
<i>Subsequent effect</i>	Characteristic x ray, Auger effect	None	Characteristic x ray, Auger effect	Positron annihilation radiation
<i>Predominant energy region for water</i>	< 20 keV	–	20 keV – 20 MeV	> 20 MeV
<i>Predominant energy region for lead</i>	< 500 MeV	–	500 keV – 5 MeV	> 5 MeV

# 1.4 PHOTON INTERACTIONS

## 1.4.13 Example of photon attenuation

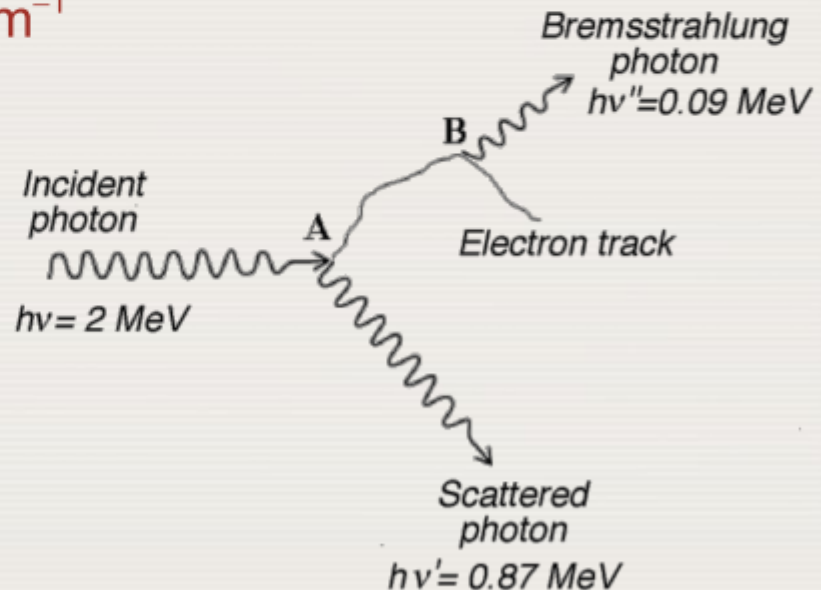
□ For 2 MeV photons in lead ( $Z = 82$ ;  $A = 207.2$ ;  $\rho = 11.36 \text{ g/cm}^3$ ) the linear attenuation coefficients are as follows:

- Photoelectric effect:  $\tau = 0.055 \text{ cm}^{-1}$
- Coherent (Rayleigh) scattering:  $\sigma_R = 0.008 \text{ cm}^{-1}$
- Compton scattering:  $\sigma_C = 0.395 \text{ cm}^{-1}$
- Pair production:  $\kappa = 0.056 \text{ cm}^{-1}$

□ Mean energy transferred to charged particles:

$$(\bar{E}_K)_{tr} = 1.13 \text{ MeV}$$

□ Mean energy absorbed in lead:  $(\bar{E}_K)_{ab} = 1.04 \text{ MeV}$



## 1.4 PHOTON INTERACTIONS

### 1.4.13 Example of photon attenuation

$$\tau = 0.055 \text{ m}^{-1} \quad \sigma_R = 0.008 \text{ cm}^{-1} \quad \sigma_C = 0.395 \text{ cm}^{-1} \quad \kappa = 0.056 \text{ cm}^{-1}$$

- Linear attenuation coefficient:

$$\mu = \tau + \sigma_R + \sigma_C + \kappa = (0.055 + 0.008 + 0.395 + 0.056) \text{ cm}^{-1} = 0.514 \text{ cm}^{-1}$$

- Mass attenuation coefficient:

$$\mu_m = \frac{\mu}{\rho} = \frac{0.514 \text{ cm}^{-1}}{11.36 \text{ g/cm}^3} = 0.0453 \text{ cm}^2/\text{g}$$

- Atomic attenuation coefficient:

$$\begin{aligned} {}_a\mu &= \left( \frac{\rho N_A}{A} \right)^{-1} \mu = \frac{207.2 \text{ (g/mol)} \times 0.514 \text{ cm}^{-1}}{11.36 \text{ (g/cm}^3) \times 6.022 \times 10^{23} \text{ (atom/mol)}} \\ &= 1.56 \times 10^{-23} \text{ cm}^2/\text{atom} \end{aligned}$$



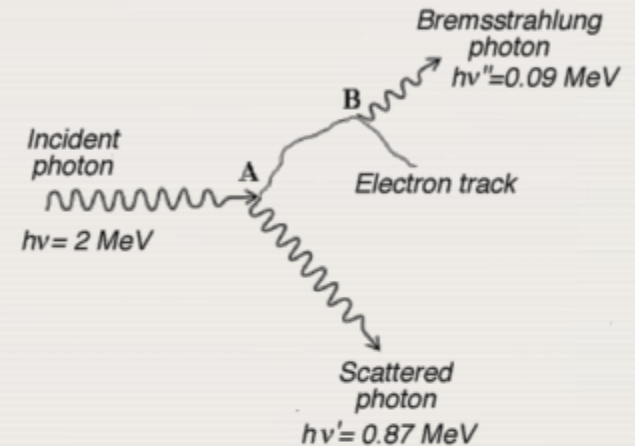
# 1.4 PHOTON INTERACTIONS

## 1.4.13 Example of photon attenuation

$$(\bar{E}_K)_{tr} = 1.13 \text{ MeV}$$

$$\mu_m = \frac{\mu}{\rho} = 0.0453 \text{ cm}^2/\text{g}$$

$$(\bar{E}_K)_{ab} = 1.04 \text{ MeV}$$



□ Mass energy transfer coefficient:

$$\frac{\mu_{tr}}{\rho} = \frac{(\bar{E}_K)_{tr}}{h\nu} \frac{\mu}{\rho} = \frac{1.13 \text{ MeV} \times 0.0453 \text{ cm}^2/\text{g}}{2 \text{ MeV}} = 0.0256 \text{ cm}^2/\text{g}$$

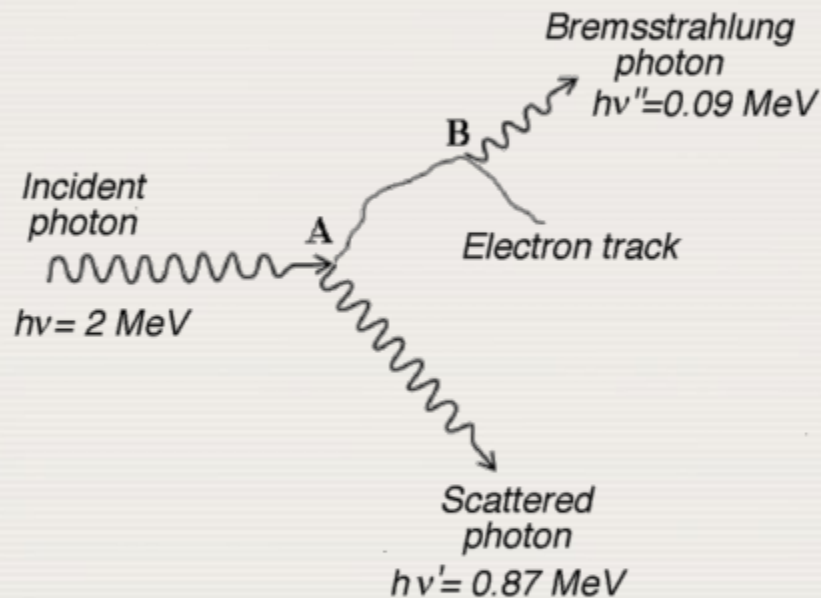
□ Mass energy absorption coefficient:

$$\frac{\mu_{ab}}{\rho} = \frac{(\bar{E}_K)_{ab}}{h\nu} \frac{\mu}{\rho} = \frac{1.04 \text{ MeV} \times 0.0453 \text{ cm}^2/\text{g}}{2 \text{ MeV}} = 0.0236 \text{ cm}^2/\text{g}$$

# 1.4 PHOTON INTERACTIONS

## 1.4.13 Example of photon attenuation

### Conclusion:



□ For a 2 MeV photon in lead on the average:

- 1.13 MeV will be transferred to charged particles (electrons and positrons).
- 0.87 MeV will be scattered through Rayleigh and Compton scattering.
- Of the 1.13 MeV transferred to charged particles:
  - 1.04 MeV will be absorbed in lead.
  - 0.09 MeV will be re-emitted in the form of bremsstrahlung photons.
- Radiation fraction  $\bar{g}$  for 2 MeV photons in lead is 0.08.

## 1.4 PHOTON INTERACTIONS

### 1.4.14 Production of vacancies in atomic shells

- There are 8 main means for producing vacancies in atomic shells and transforming the atom from a neutral state into an excited positive ion:
  - Coulomb interaction (1) of energetic charged particle with orbital electron.
  - Photon interactions
    - Photoelectric effect (2)
    - Compton effect (3)
    - Triplet production (4)
  - Nuclear decay
    - Electron capture (5)
    - Internal conversion (6)
  - Positron annihilation (7)
  - Auger effect (8)

## 1.4 PHOTON INTERACTIONS

### 1.4.14 Production of vacancies in atomic shells

- ❑ **Note:** Pair production does not produce shell vacancies, because the electron-positron pair is produced in the field of the nucleus.
  
- ❑ **Vacancies in inner atomic shells are not stable;** they are followed:
  - Either by emission of characteristic photons
  - Or by emission of Auger electronsand cascade to the outer shell of the ionized atom (ion).
  
- ❑ **Ion eventually attracts a free electron** from its surroundings and reverts to a neutral atom (ionic recombination).