Foundation course - PHYSICS

Lecture 4-2: Kinematics of a particle

- position and displacement
- average velocity, average speed, instantaneous velocity
- average acceleration, instantaneous acceleration
- one-dimensional motion
- free fall, upward and downward throws

Naďa Špačková spackova@physics.muni.cz

Kinematics

Essential questions:

- How fast and how far an object moves
- In which direction the object is moving
- Whether the object is speeding up or slowing down
- Whether the object is standing still or moving at a constant speed

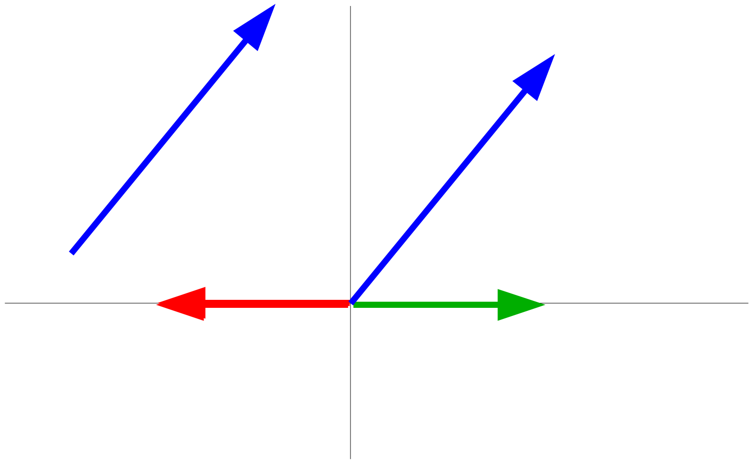
Motion:

- Objects move in many different ways
- The least complicated motion is a movement along a straight line
- Description of motion is a description of place and time
- Other characteristics of motion are velocity (speed) and acceleration

Coordinate systems

- gives the location of the zero point of the variable you are studying and the direction in which the values of the variable increase
- the **origin** = the point with zero value of all variables
- the **position** is described by the **distance** and **direction**
- **vectors** = quantities characterized by both **magnitude** (size) and **direction**

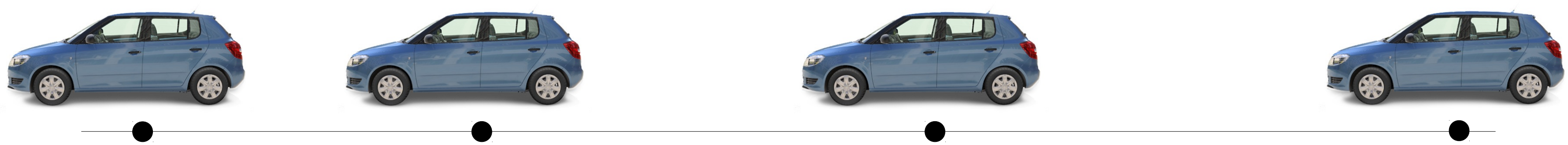
The red vector has the same magnitude as the green vector, but opposite direction. Blue arrows represent the same vector.



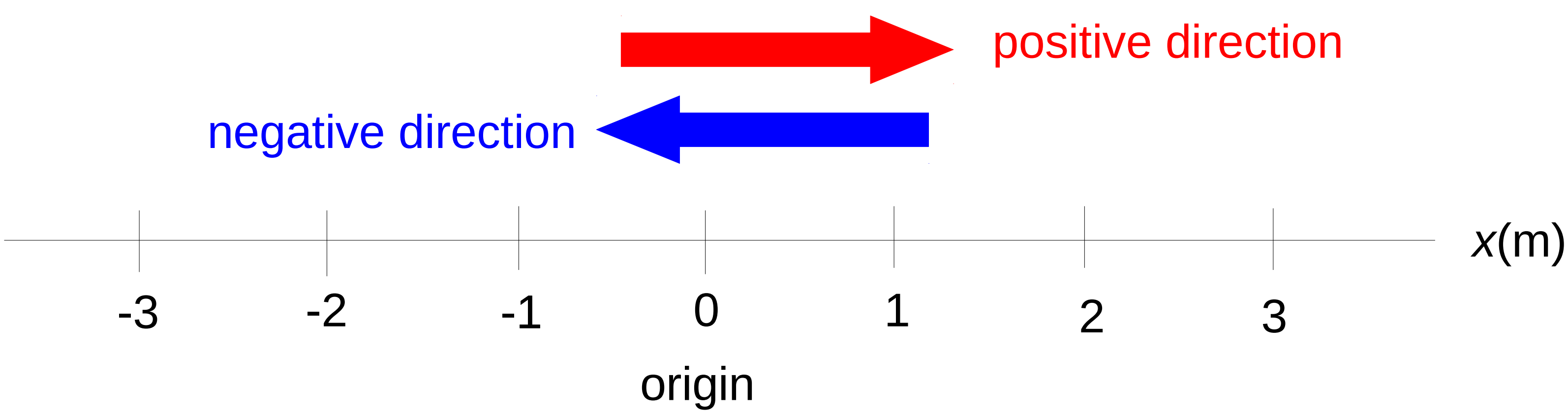
- **scalars** = quantities characterized only by its **size**

Particle Models

- the object of interest is replaced with a single point
- the object's size must be much less than the distance it moves
- the object's internal motions are ignored



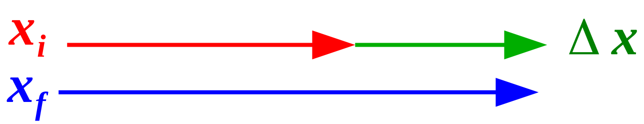
Position and displacement



Vectors and scalars

- time intervals = scalars
- time interval $\Delta t = t_{final} - t_{initial}$
- positions and displacements = vectors
- displacement $\Delta \boldsymbol{x} = \boldsymbol{x}_{final} - \boldsymbol{x}_{initial}$
- vector addition and subtraction:

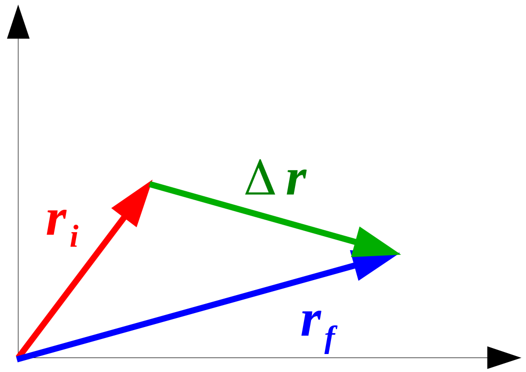
One-dimensional situation:



$$\Delta \boldsymbol{x} = \boldsymbol{x}_f - \boldsymbol{x}_i \qquad \boldsymbol{x}_f = \Delta \boldsymbol{x} + \boldsymbol{x}_i$$

R = A – B = A + (-B)

Two-dimensional situation:



$$\Delta \boldsymbol{r} = \boldsymbol{r}_f - \boldsymbol{r}_i$$

Position-time graph

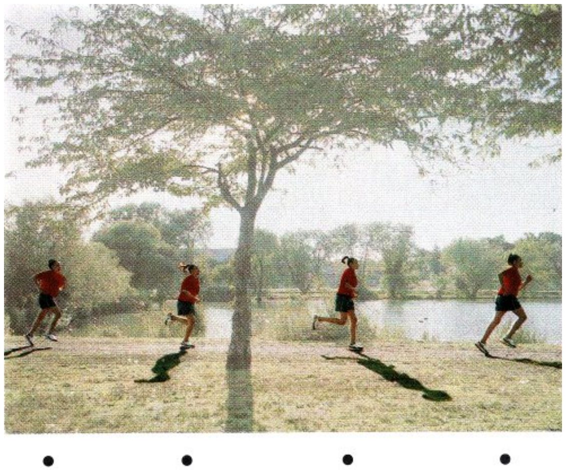
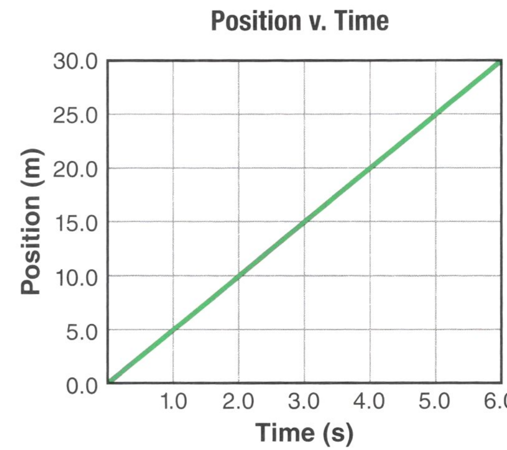
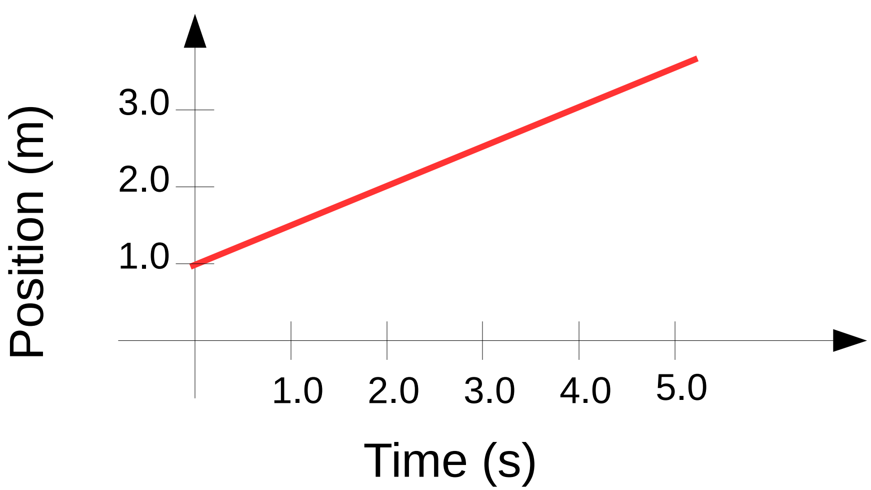


Table 1 Position v. Time

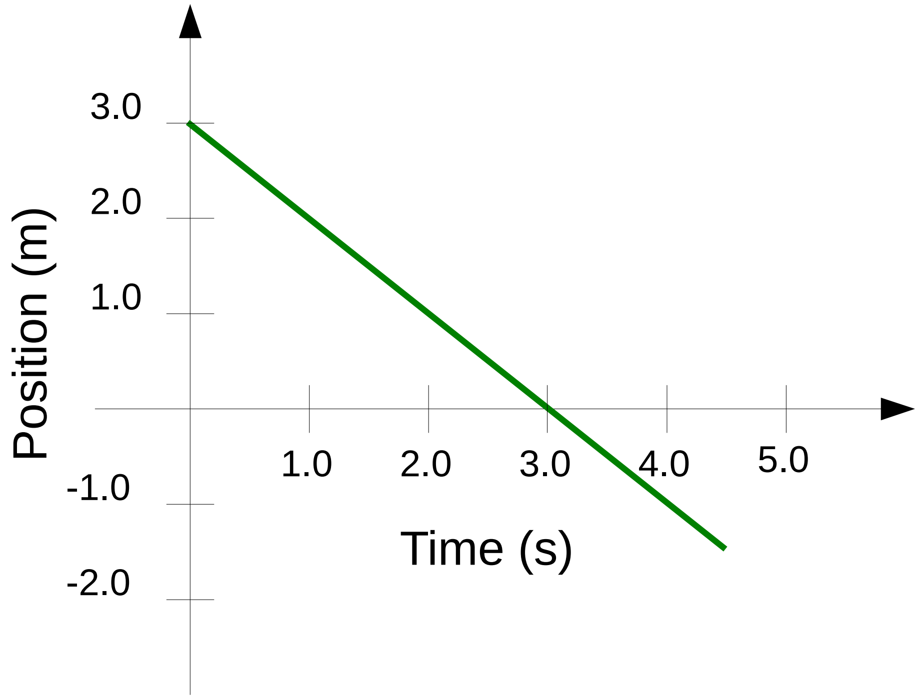
Time (s)	Position (m)
0.0	0.0
1.0	5.0
2.0	10.0
3.0	15.0
4.0	20.0
5.0	25.0



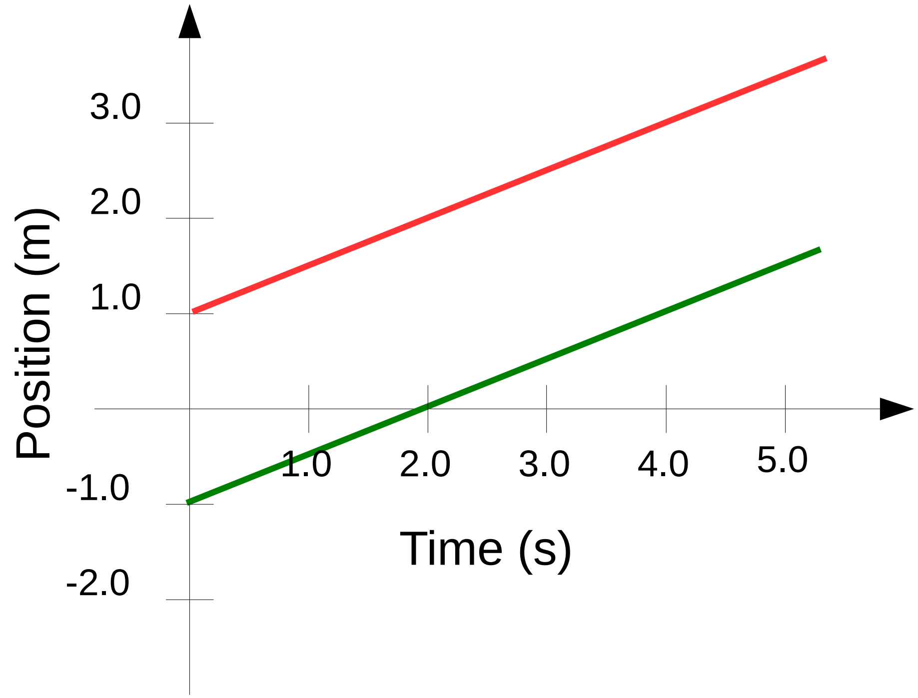


Position-time graph

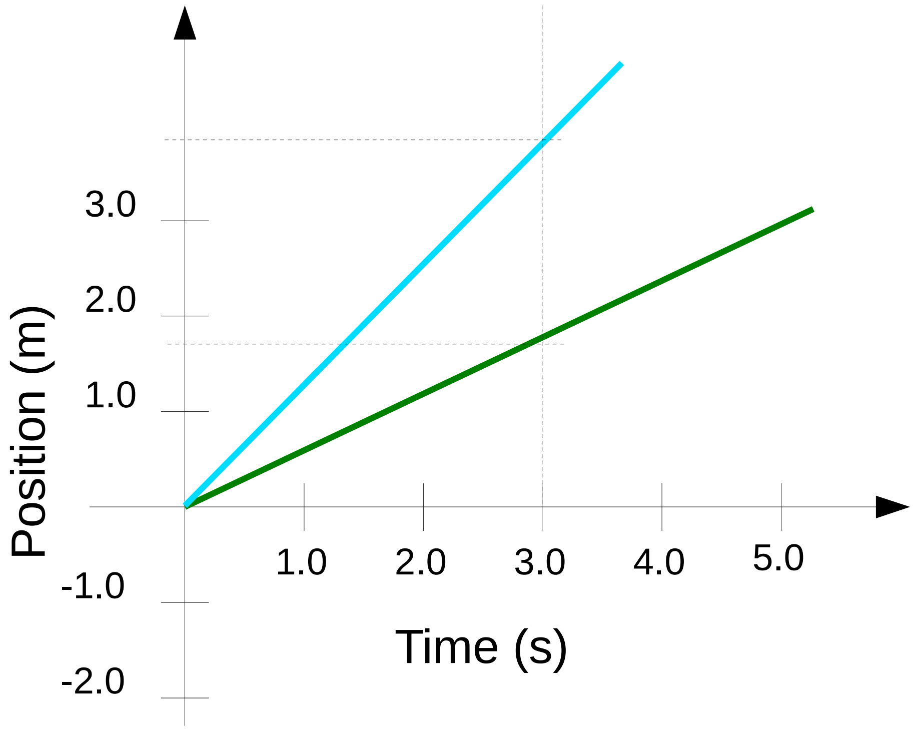
Example 1: motion in negative direction



Example 2: two motions in positive direction, the same speed, different initial position



Average velocity and average speed



For a fixed time interval the magnitude of the displacement is greater for the cyan object: **this object is moving faster**

Average velocity

$$\bar{\boldsymbol{v}} \equiv \frac{\Delta \boldsymbol{x}}{\Delta t} = \frac{\boldsymbol{x}_{final} - \boldsymbol{x}_{initial}}{t_{final} - t_{initial}}$$

Average speed

$$v_{avg} = \frac{total\ distance}{\Delta t}$$

Velocity (speed) units: 1 $m.s^{-1}$

Caution:

- average velocity = vector
- average speed = scalar

Examples: average velocity and average speed

An automobile travels on a straight road for 40 km at 30 km/h. It then continues in the same direction for another 40 km at 60 km/h. (a) What is the average velocity of the car during the full 80 km trip? (Assume that it moves in the positive *x* direction.) (b) What is the average speed?

An automobile travels on a straight road for 40 km at 30 km/h. It then continues in the ***opposite*** direction for another 40 km at 60 km/h. (a) What is the average velocity of the car during the full 80 km trip? (b) What is the average speed?

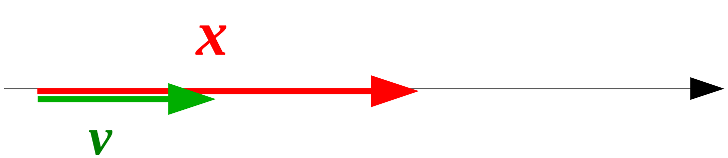
Instantaneous velocity and speed

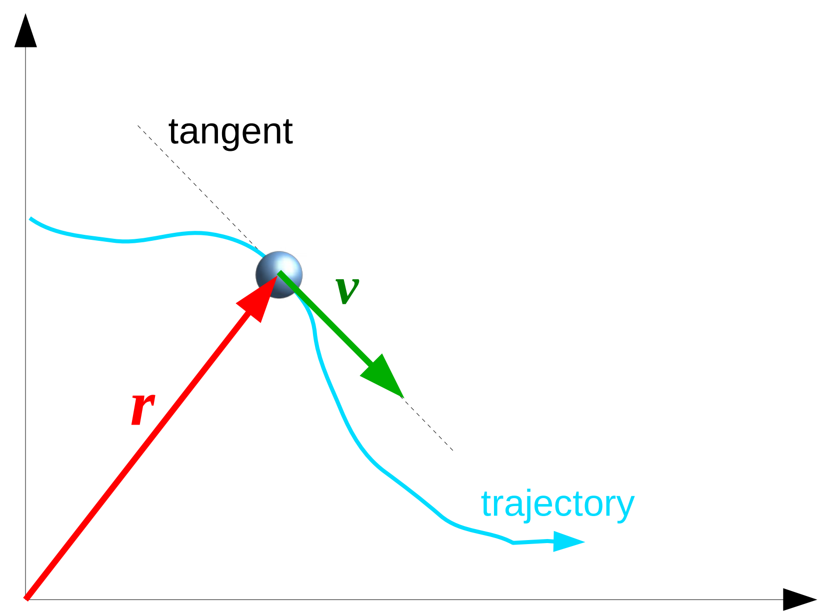
Instantaneous velocity

$$\boldsymbol{v} = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{x}}{\Delta t} = \frac{d\,\boldsymbol{x}}{d\,t}$$

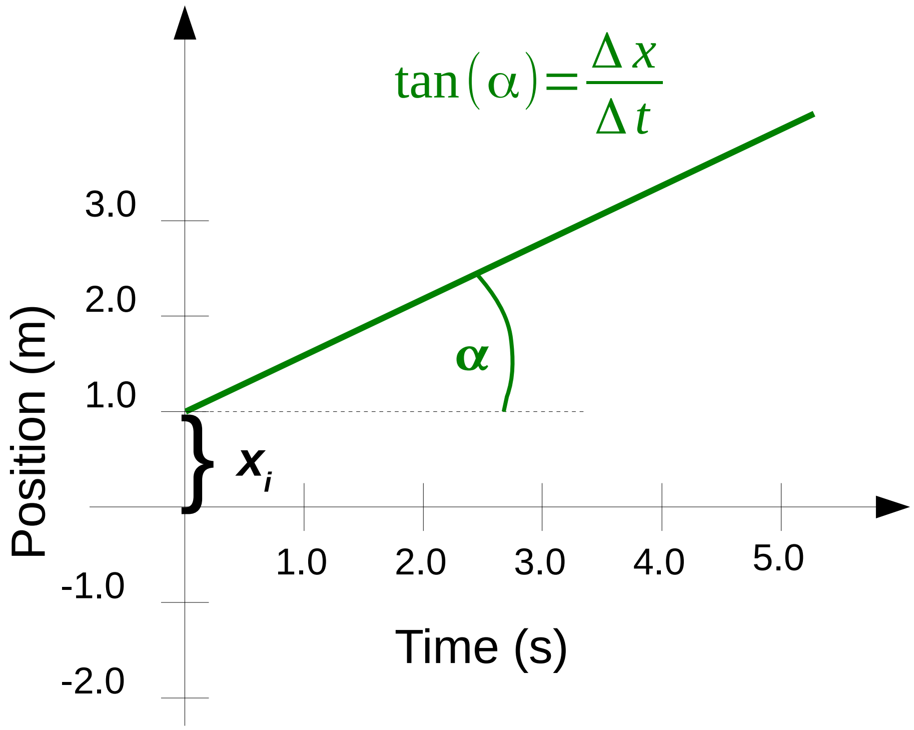
Two-dimensional situation:
Velocity vector is always tangent to the particle’s path at the particle’s position.

Motion along the straight line:
instantaneous velocity is of the same direction as displacement





Equation of motion



General representation of the linear function:

$$y = mx + b$$

y...quantity on the vertical axis
m...line's slope
x… quantity on the horizontal axis
b...line's y-intercept

Equation of motion for a position v. time graph: $\boldsymbol{x} = \bar{\boldsymbol{v}}\, t + \boldsymbol{x}_i$

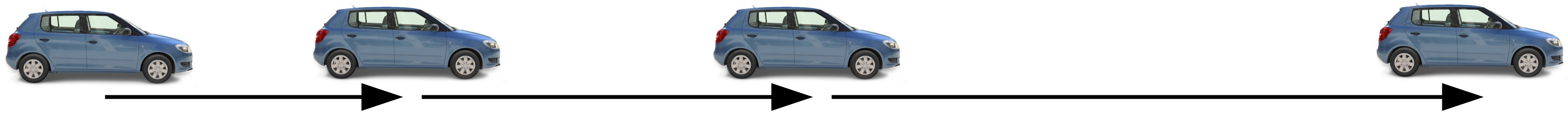
Uniform and nonuniform motion

Uniform motion

- moving along the a straight line with an unchanging velocity

Nonuniform motion

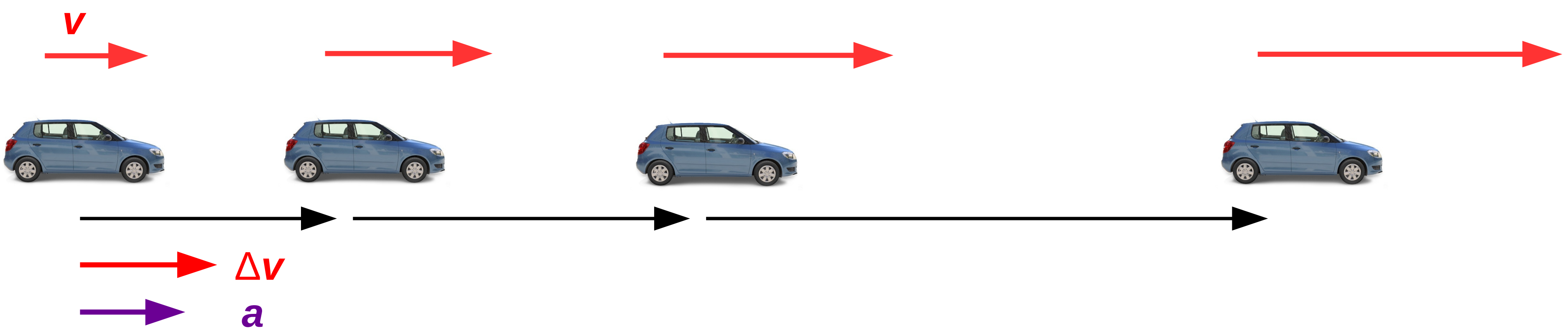
- velocity is changing

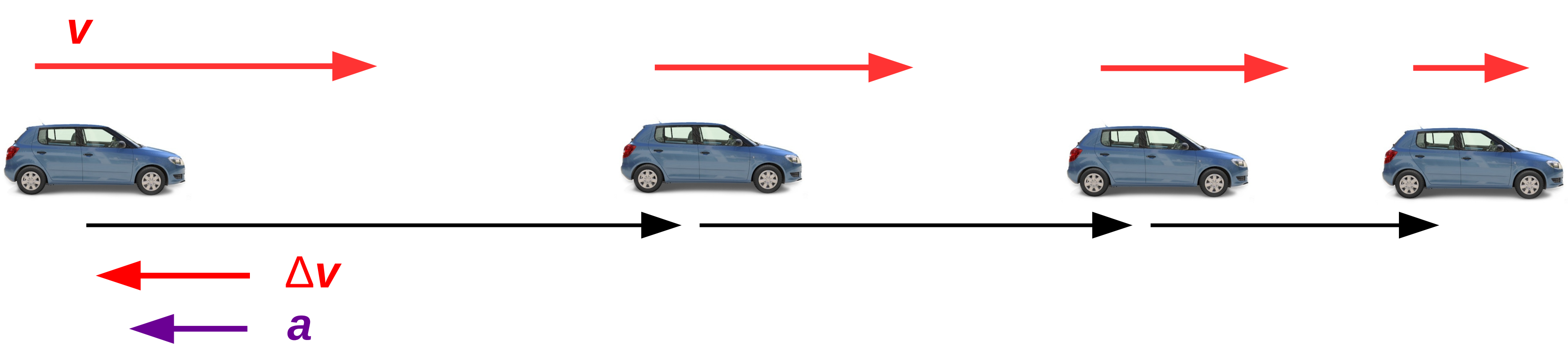


Acceleration

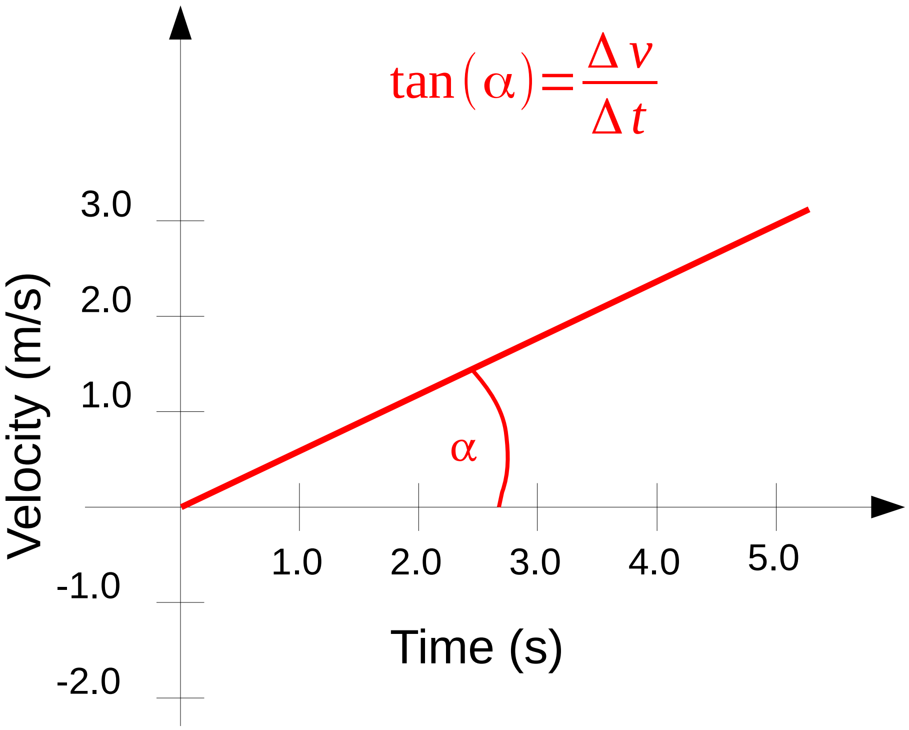
is the rate at which the object's velocity changes

Acceleration is a vector (magnitude, direction)



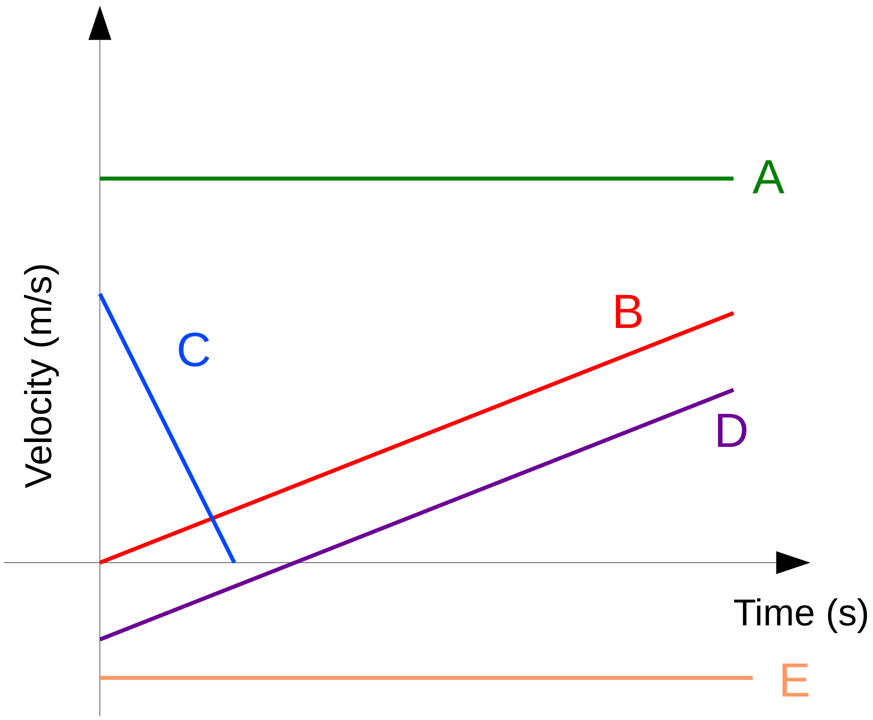


Velocity-time graphs



The slope of *v(t)* represents acceleration

Examples of various accelerations:
a = zero (A, E)
a = positive (B, D)
a = negative (C)



Acceleration

Average acceleration

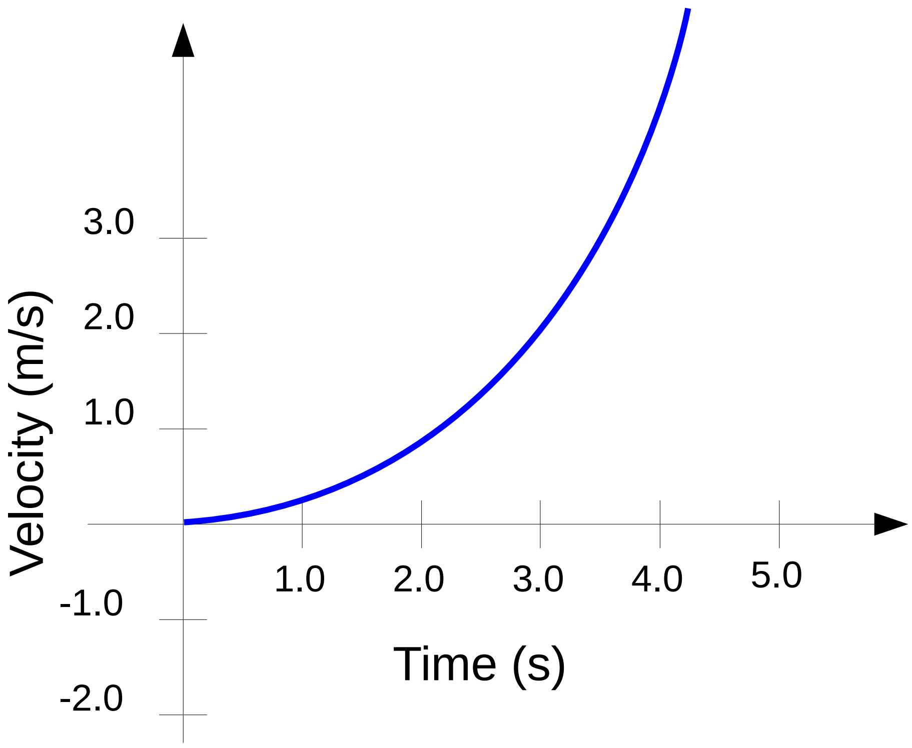
$$\bar{\boldsymbol{a}} \equiv \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{\boldsymbol{v}_{final} - \boldsymbol{v}_{initial}}{t_{final} - t_{initial}}$$

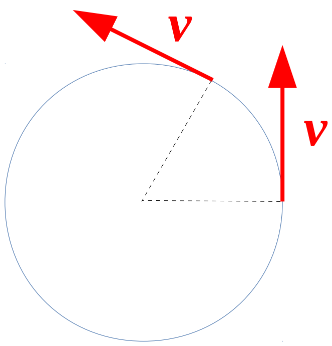
Instantaneous acceleration

$$\boldsymbol{a} = \lim_{\Delta t \to 0} \frac{\Delta \boldsymbol{v}}{\Delta t} = \frac{d\boldsymbol{v}}{dt}$$

Acceleration units: 1 m.s^{-2}

Example: Non-constant acceleration

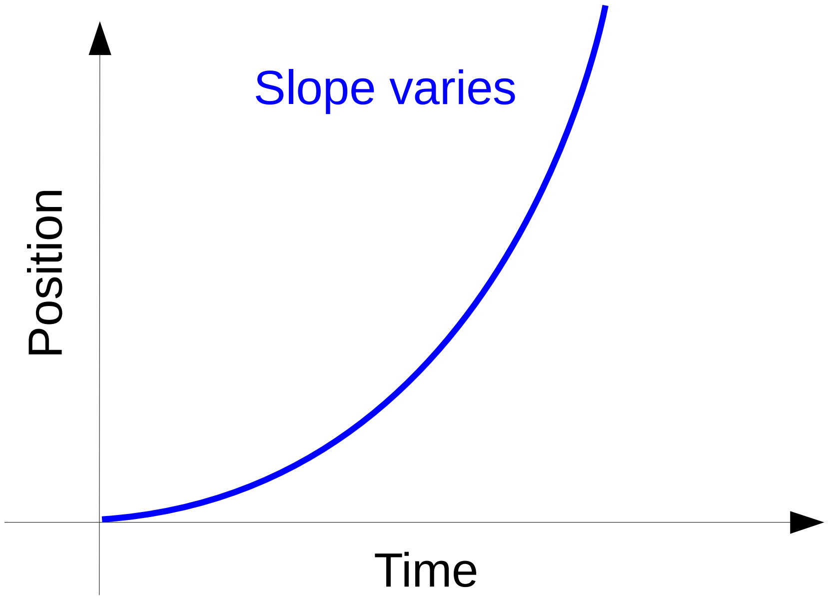


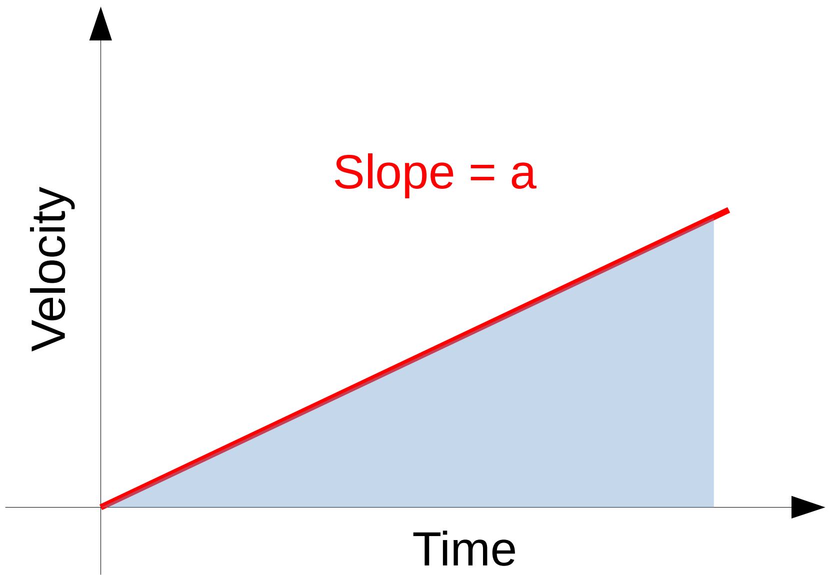


Acceleration with constant speed

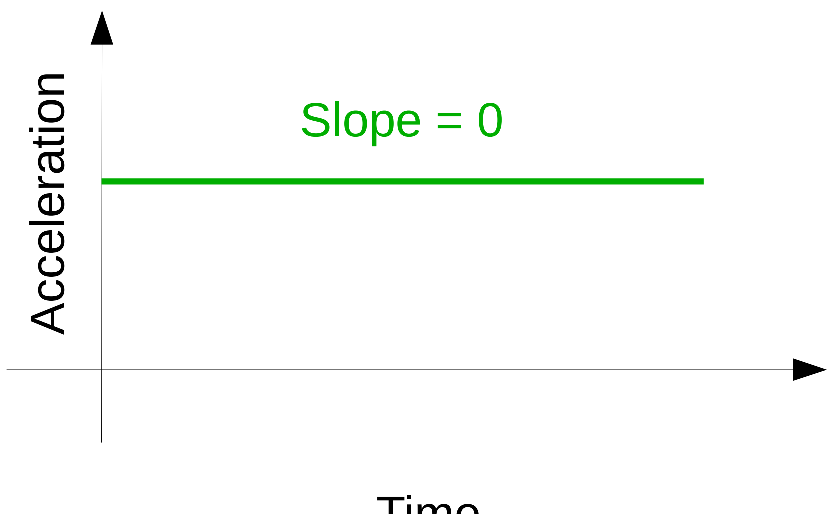
acceleration is associated with a change in the direction of motion

Motion with constant acceleration



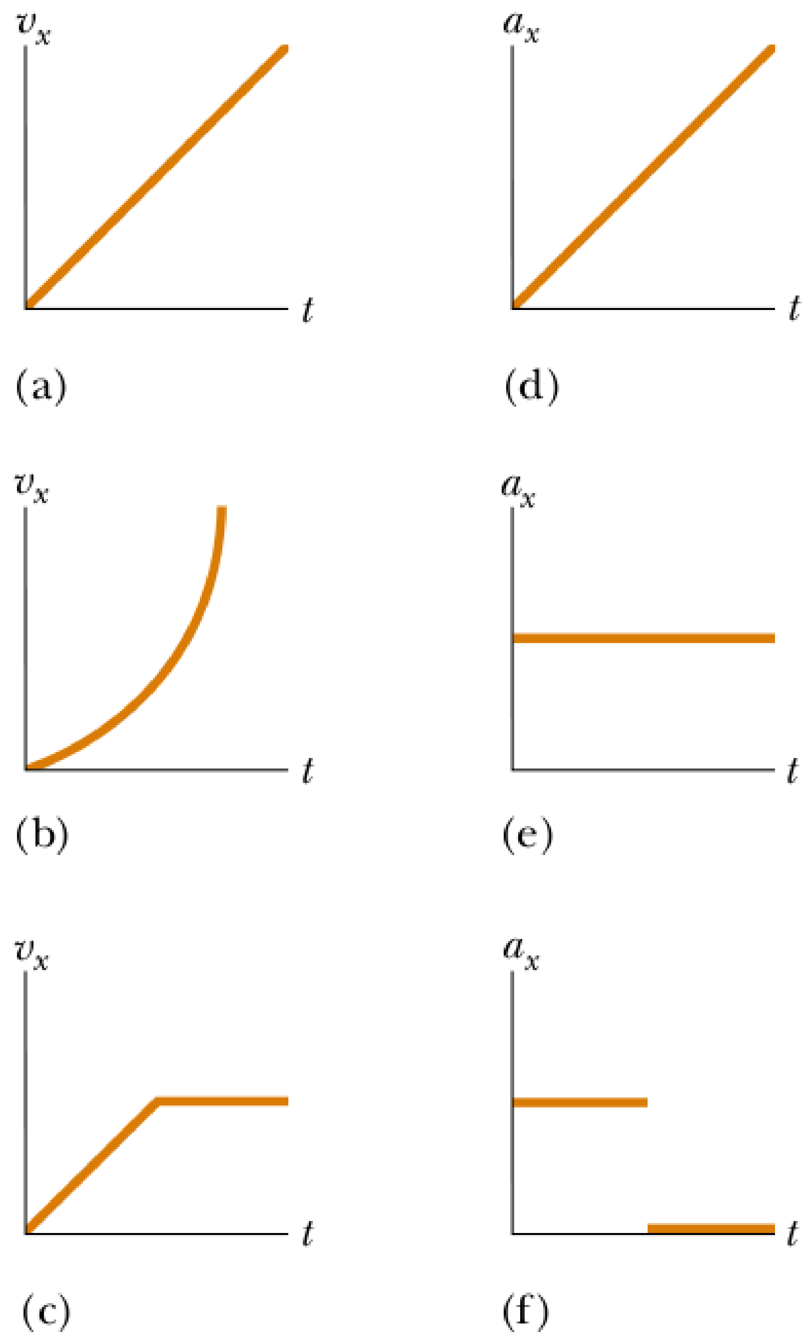


Area under the graph represents displacement of the particle



Checkpoint question:

Match each v_x-t graph with the a_x-t graph that best describes the motion.



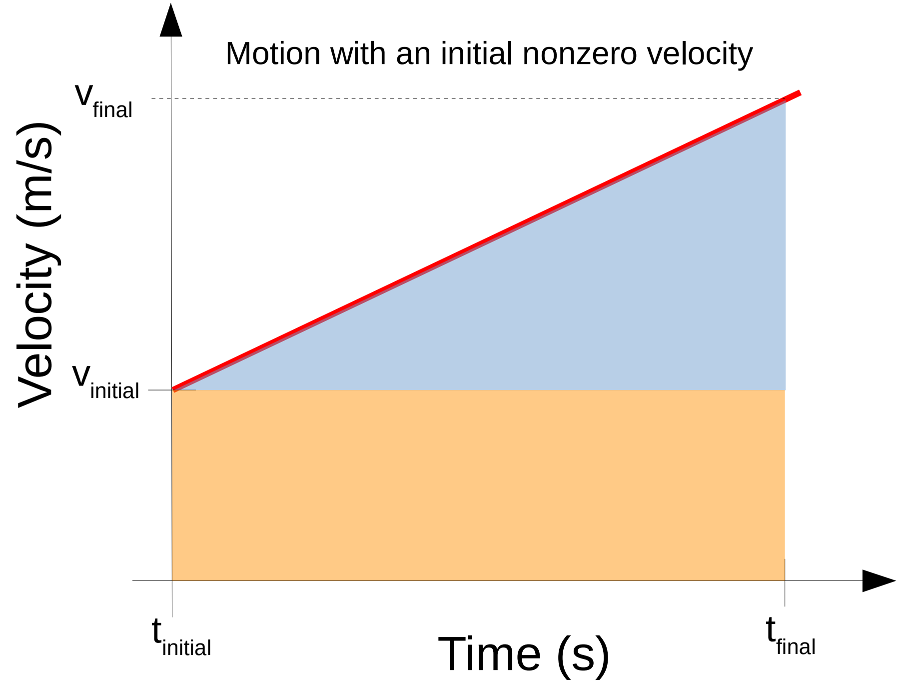
Velocity with average acceleration

$$\bar{\boldsymbol{a}} \equiv \frac{\Delta \boldsymbol{v}}{\Delta t}$$

$$\Delta \boldsymbol{v} = \bar{\boldsymbol{a}}\, \Delta t$$

$$\boldsymbol{v}_f - \boldsymbol{v}_i = \bar{\boldsymbol{a}}\, \Delta t$$

$$\boldsymbol{v}_f = \boldsymbol{v}_i + \bar{\boldsymbol{a}}\, \Delta t$$



When acceleration is constant, the average acceleration is the same as the instantaneous acceleration.

$$\Delta \boldsymbol{x} = \Delta \boldsymbol{x}_{rectangle} + \Delta \boldsymbol{x}_{triangle} = \boldsymbol{v}_i \Delta t + \frac{1}{2} \Delta \boldsymbol{v} \Delta t = \boldsymbol{v}_i \Delta t + \frac{1}{2} \boldsymbol{a} (\Delta t)^2$$

$$\boldsymbol{x}_f - \boldsymbol{x}_i = \boldsymbol{v}_i \Delta t + \frac{1}{2} \boldsymbol{a} (\Delta t)^2$$

if $t_i = 0$ →

$$\boldsymbol{x}_f = \boldsymbol{x}_i + \boldsymbol{v}_i t_f + \frac{1}{2} \boldsymbol{a} t_f^2$$

$$\boldsymbol{v}_f = \boldsymbol{v}_i + \boldsymbol{a}\, t_f$$

$$v = v_0 + at$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2$$

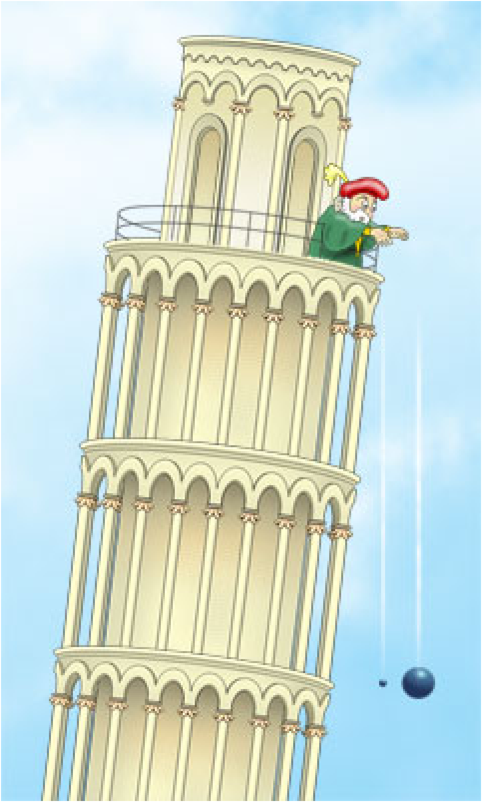
$$x - x_{0,} v_{0,} v, a, t$$

Examples: equations of motion

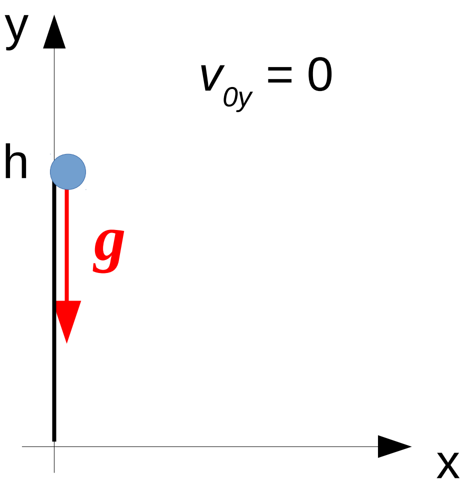
A car accelerates from rest with a uniform acceleration. After traveling a distance of 160 m its velocity is 26 m/s. What is its acceleration?

Free fall

Free fall is the motion of an object when **gravity** is the only significant force acting on it.



Galileo's experiments with free fall motion



$$a_y = -g$$

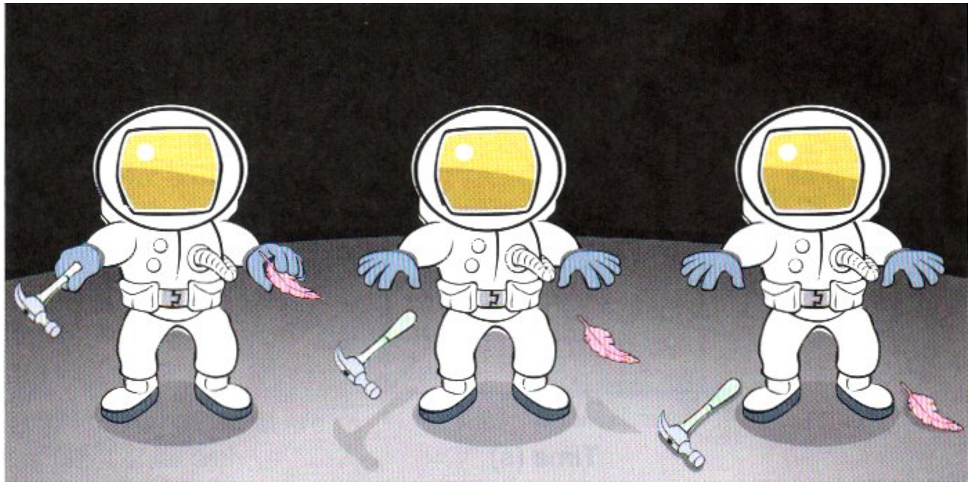
$$v_y = -gt$$

$$y = h - \frac{1}{2} g t^2$$

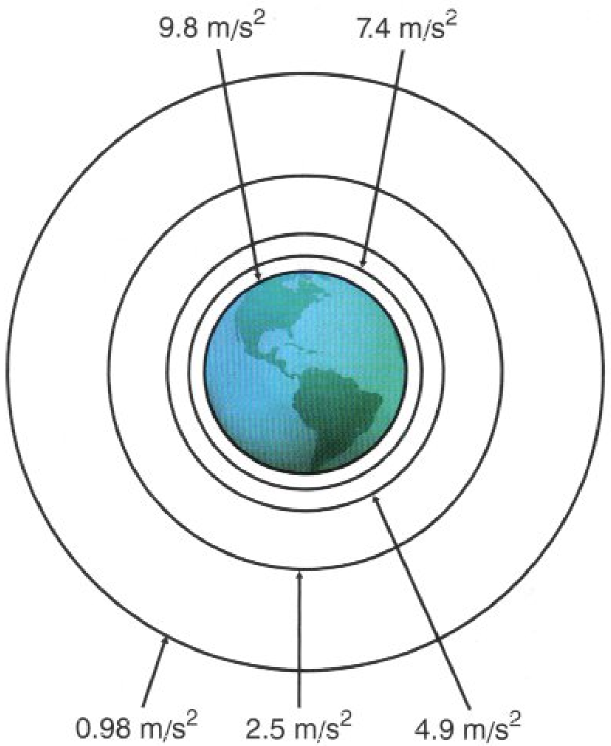
Free fall

Free-fall acceleration:

- near Earth's surface is about 9.8 m/s^2 downward (each second velocity increases by 9.8 m/s)
- it is not dependent on mass, density and shape of the falling object
- its magnitude depends on distance from the Earth



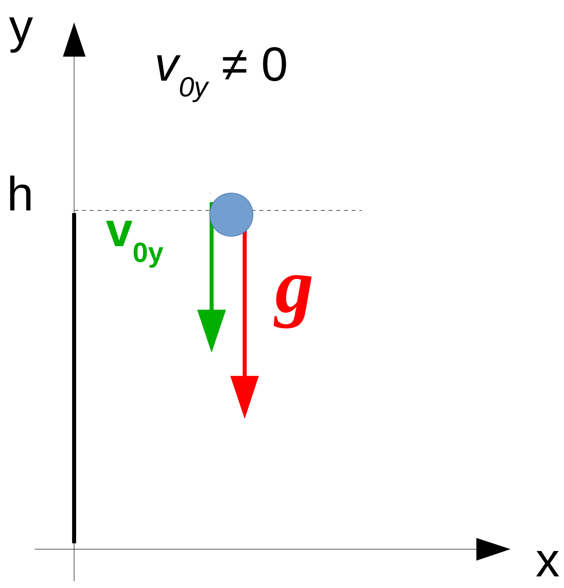
Without air friction (in vacuum) all objects are falling equally.



Downward throw

Downward throw

- initial velocity is not zero and is in the same direction as free fall acceleration



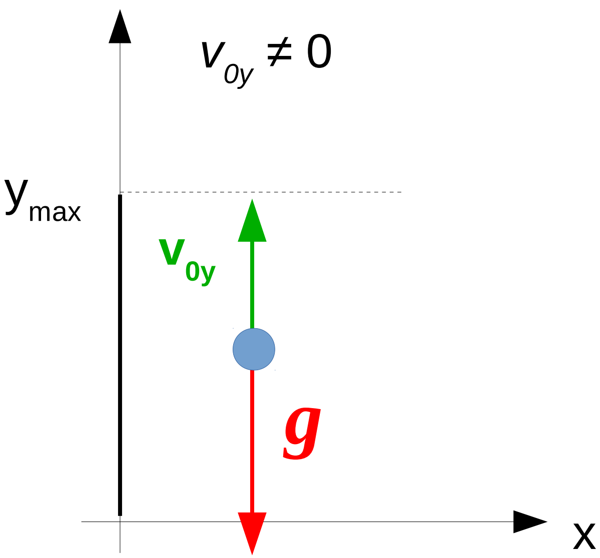
$$v_y = -v_{0y} - g t$$

$$y = h - v_{0y} t - \frac{1}{2} g t^2$$

Upward throw

Upward throw

- initial velocity is not zero and is in the opposite direction as free fall acceleration



$$v_y = v_{0y} - g t$$

$$y = y_0 + v_{0y} t - \frac{1}{2} g t^2$$

Summary

Motion with **constant** acceleration

$$\boldsymbol{v}_f = \boldsymbol{v}_i + \boldsymbol{a}\, t_f$$

$$\boldsymbol{x}_f = \boldsymbol{x}_i + \boldsymbol{v}_i t_f + \frac{1}{2} \boldsymbol{a}\, t_f^2$$