Foundation course - PHYSICS

Lecture 5-1: Kinematics of a particle

- motion in two dimensions
- angled launches
- uniform and nonuniform circular motion
- relative motion

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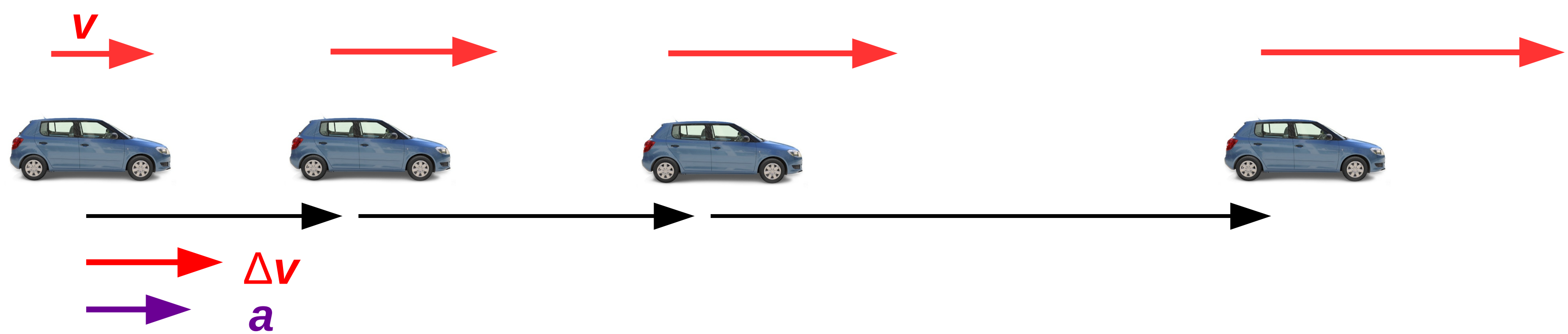
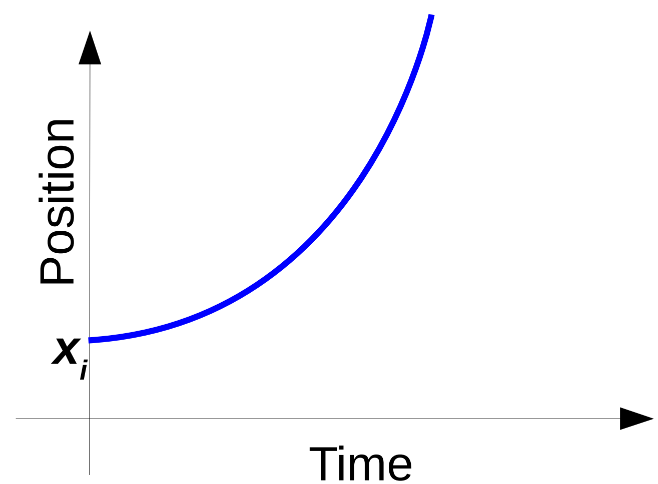
Summary

Motion with a **constant** acceleration



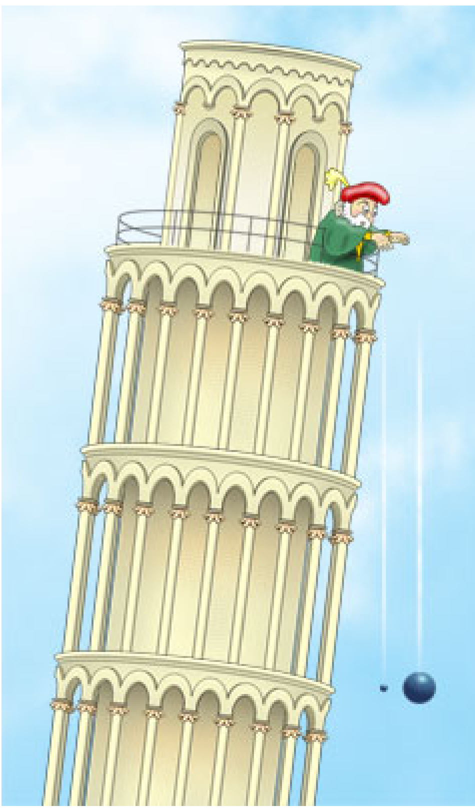
$$\boldsymbol{v} = \boldsymbol{v}_0 + \boldsymbol{a}\,t$$

$$\boldsymbol{x} = \boldsymbol{x}_0 + \boldsymbol{v}_0\,t + \frac{1}{2}\boldsymbol{a}\,t^2$$

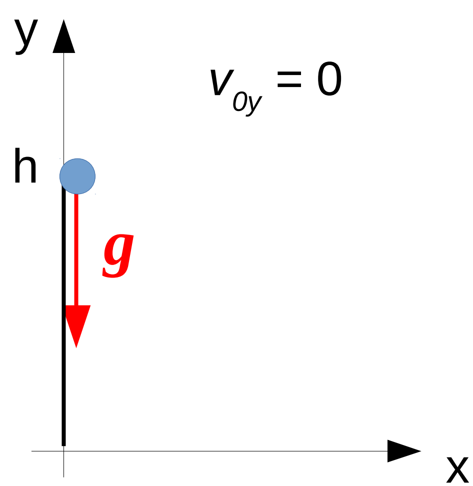


Free fall

Free fall is the motion of an object when **gravity** is the only significant force acting on it.



Galileo's experiments with free fall motion



$$a_y = -g$$

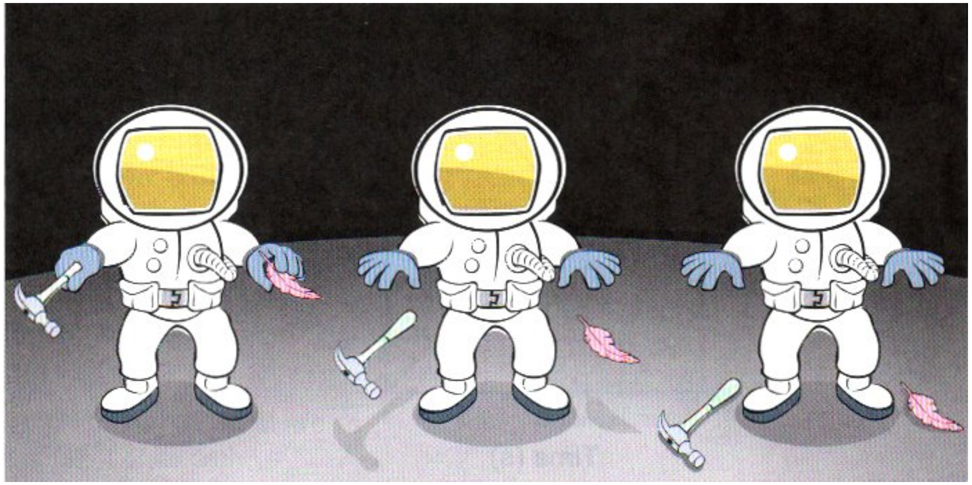
$$v_y = -gt$$

$$y = h - \frac{1}{2}gt^2$$

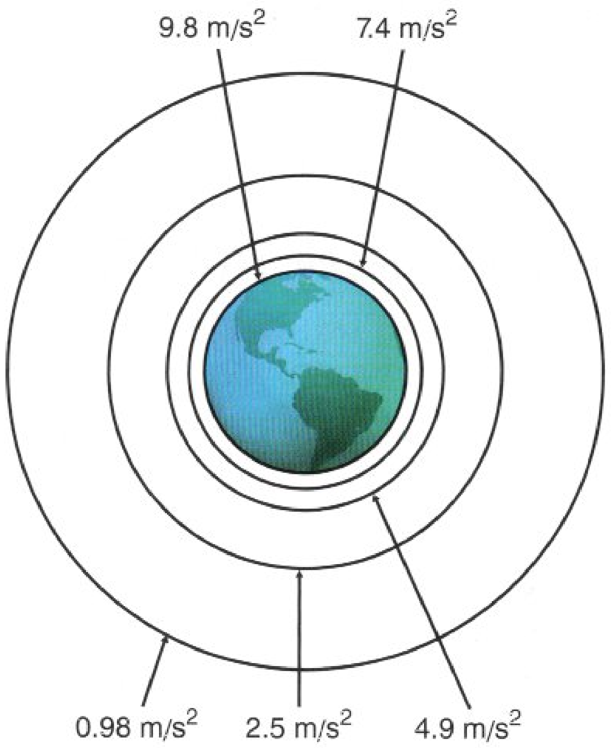
Free fall

Free-fall acceleration:

- near Earth's surface is about 9.8 m/s^2 downward (each second velocity increases by 9.8 m/s)
- it is not dependent on mass, density and shape of the falling object
- its magnitude depends on distance from the Earth



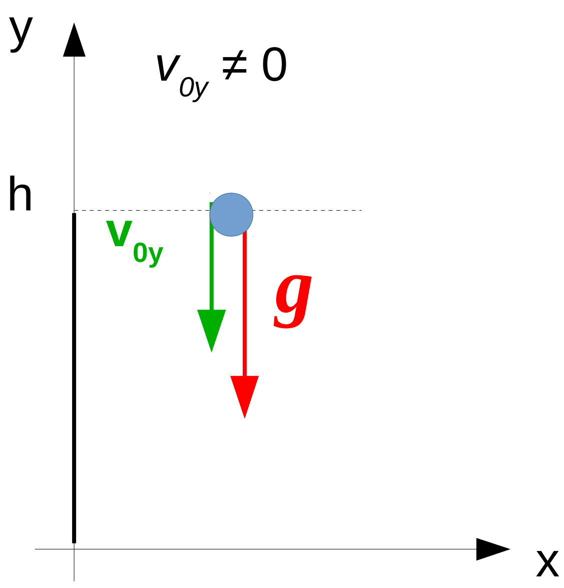
Without air friction (in vacuum) all objects are falling equally.



Downward throw

Downward throw

- initial velocity is not zero and is in the same direction as free fall acceleration



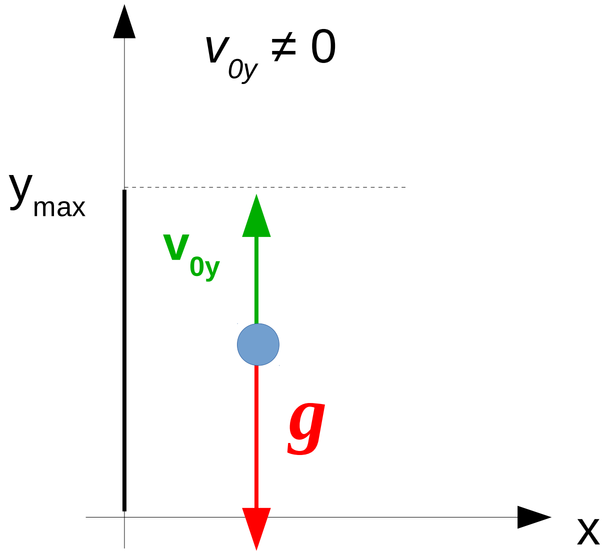
$$v_y = -v_{0y} - gt$$

$$y = h - v_{0y}t - \frac{1}{2}gt^2$$

Upward throw

Upward throw

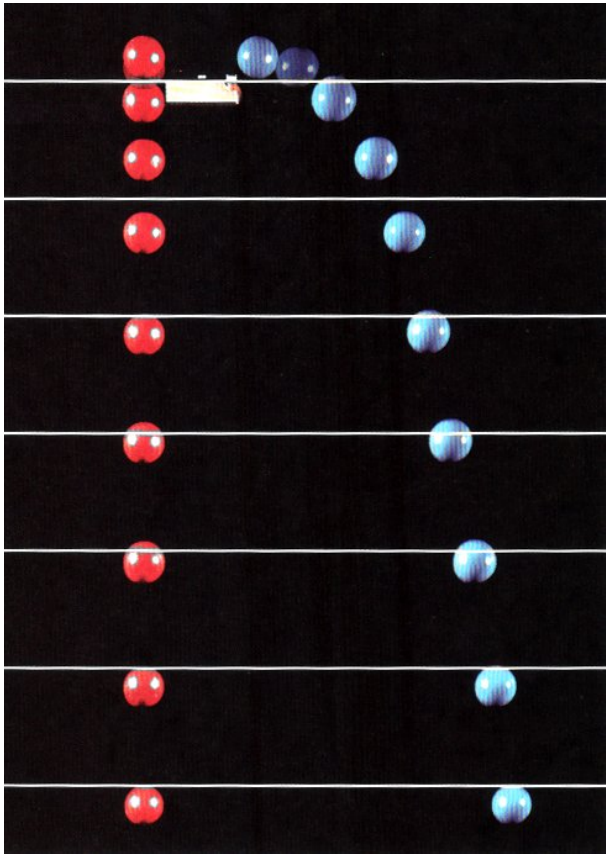
- initial velocity is not zero and is in the opposite direction as free fall acceleration



$$v_y = v_{0y} - g\,t$$

$$y = y_0 + v_{0y}t - \frac{1}{2}g\,t^2$$

Motion in two dimensions



Independence of motion in two dimensions

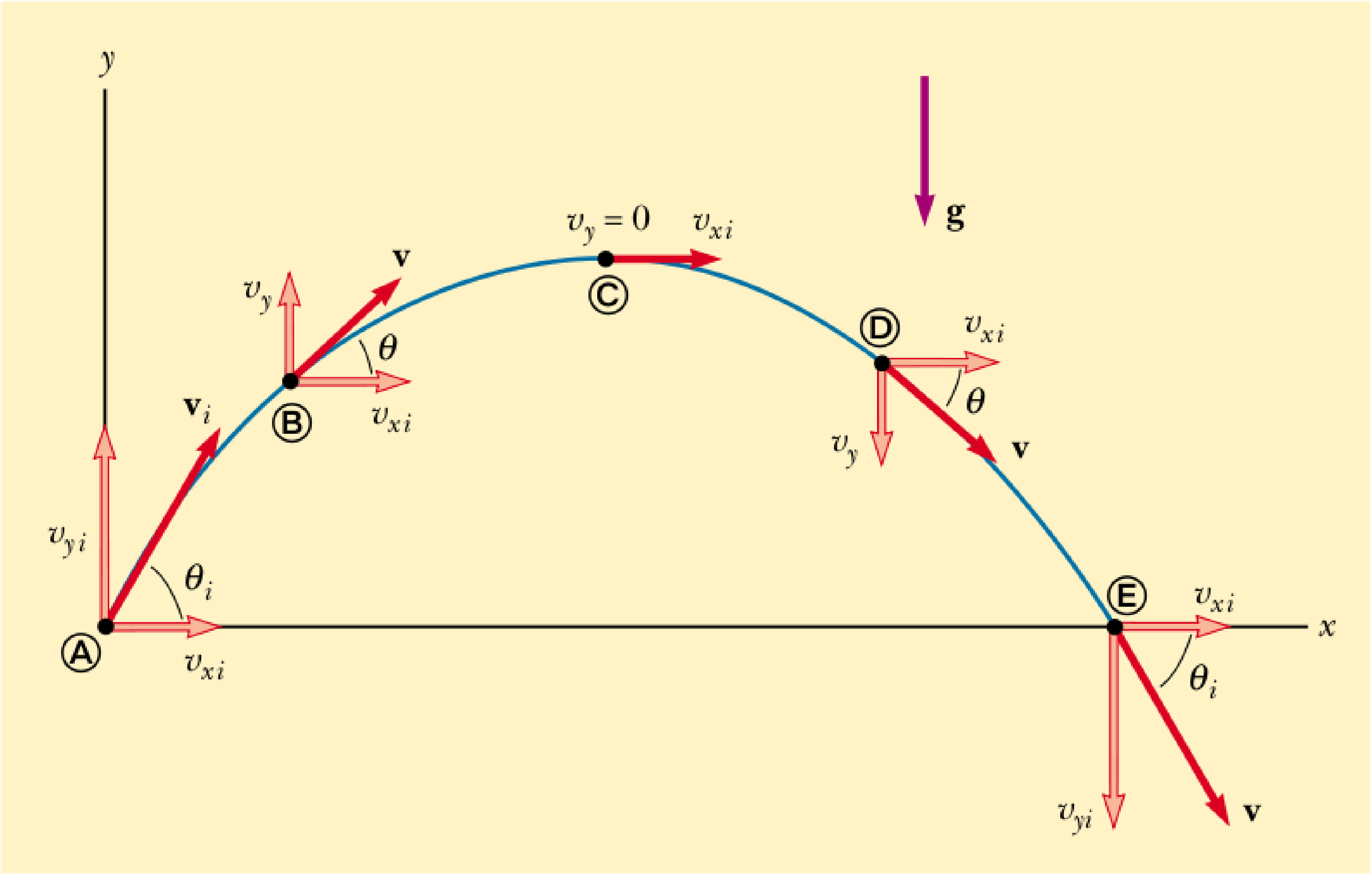
- motion of a projectile (blue ball) is a combination of two motions – horizontal and vertical

Horizontal motion does not affect vertical motion and vice versa

Downward velocity increases regularly due to free-fall acceleration, horizontal velocity (blue ball) is constant

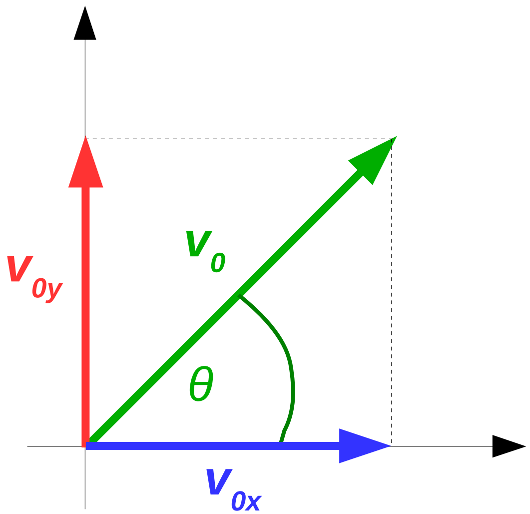
red ball = no initial velocity
blue ball = initial horizontal velocity
balls have the same vertical motion as they fall

Angled launches



When a projectile is launched at an angle, the initial velocity has a **vertical** component as well as **horizontal** component.

Angled launches



Separation of vertical and horizontal motions:

- horizontal motion → horizontal velocity component v_{0x}
- vertical motion → vertical velocity component v_{0y}

We know: We want to know:

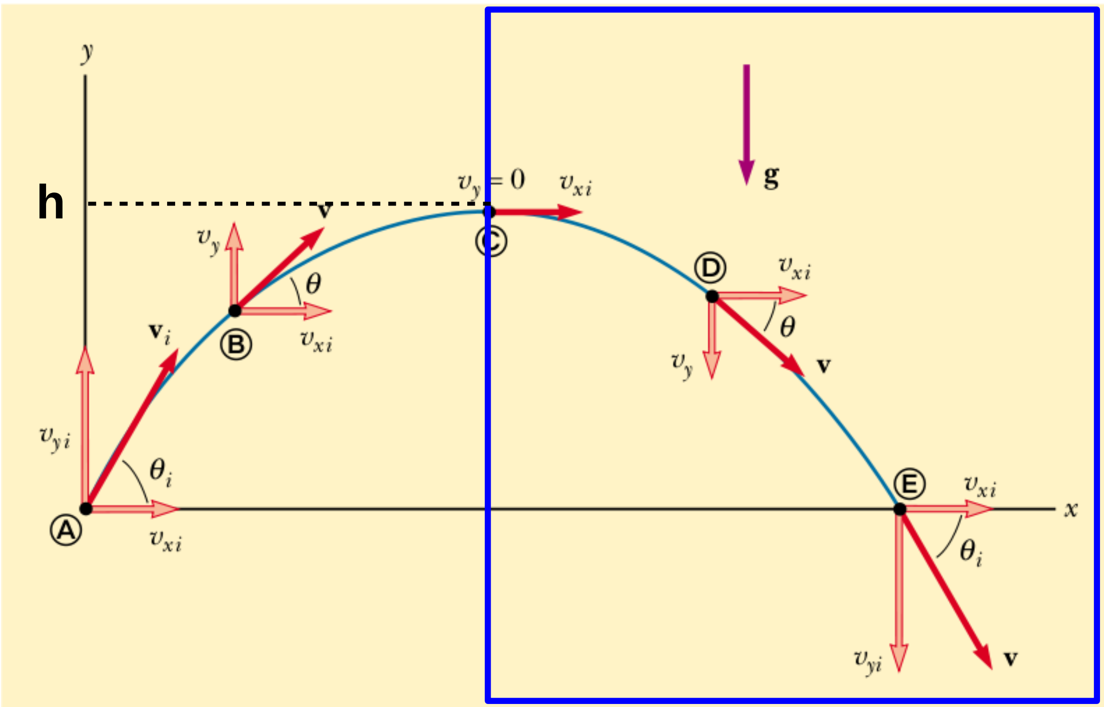
magnitude v_0 and direction θ ⟶ **v_{0x} and v_{0y} components**

$$v_{0x} = v_0 \cos\theta \qquad v_{0y} = v_0 \sin\theta$$

v_{0x} and v_{0y} components ⟶ **magnitude v_0 and direction θ**

$$v_0 = \sqrt{{v_{0x}}^2 + {v_{0y}}^2} \qquad \tan\theta = \frac{v_{0y}}{v_{0x}}$$

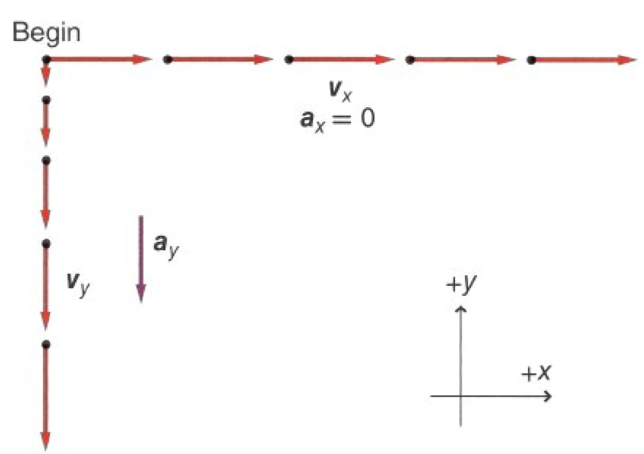
Horizontally launched projectile

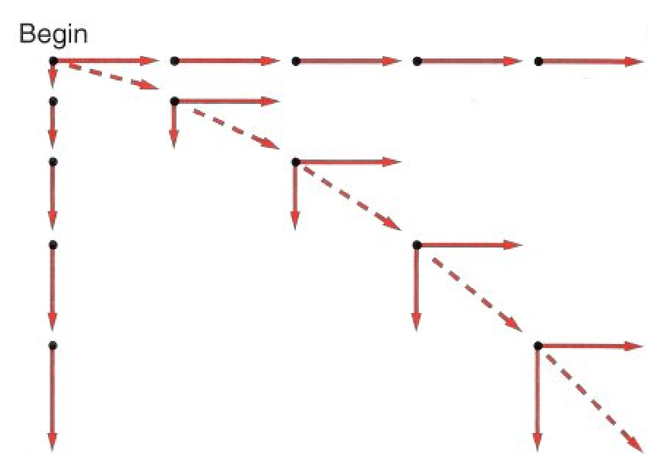


horizontal motion: constant velocity
vertical motion: constant acceleration

The horizontal and vertical velocity components at each moment are added to form the velocity vector at that moment.

The trajectory has a parabolic shape.





$$x = v_0 t + x_0$$

$$y = h - \frac{1}{2} g t^2$$

Example: horizontally launched projectile

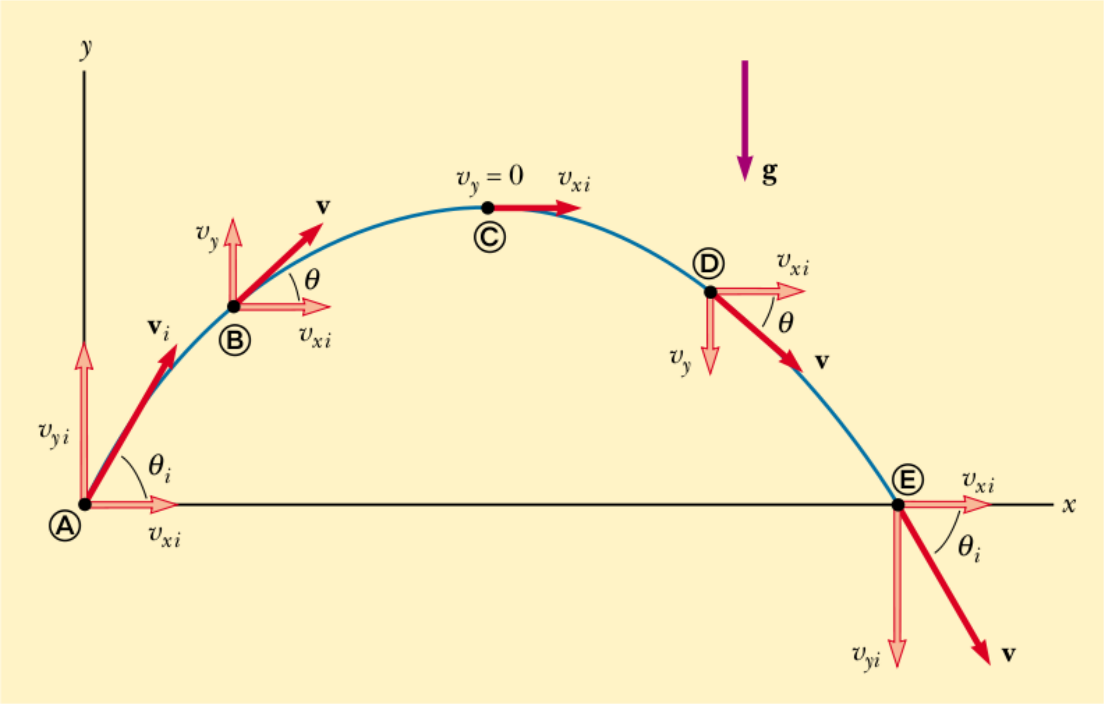
A projectile is fired horizontally from a gun that is 45.0 m above flat ground, emerging from the gun with a speed of 250 m/s.

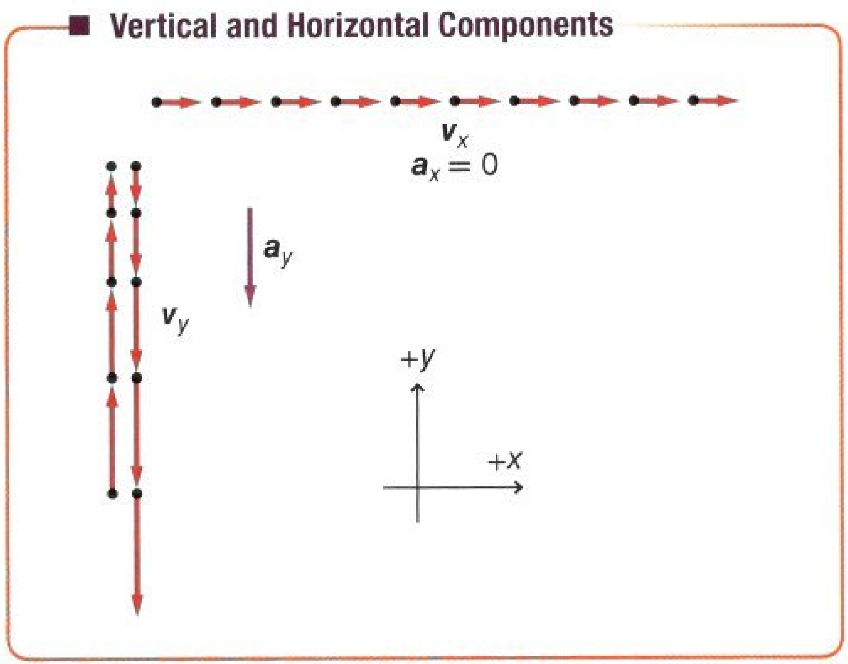
(a) How long does the projectile remain in the air?

(b) At what horizontal distance from the firing point does it strike the ground?

(c) What is the magnitude of the vertical component of its velocity as it strikes the ground?

Angled launches





When a projectile is launched at an angle, the initial velocity has a **vertical** component as well as **horizontal** component.

horizontal motion: constant velocity
vertical motion: constant acceleration

Vertical upward and downward motion:

- at each point the velocity of the object has the same magnitude
- the directions of the velocities are opposite

Angled launches

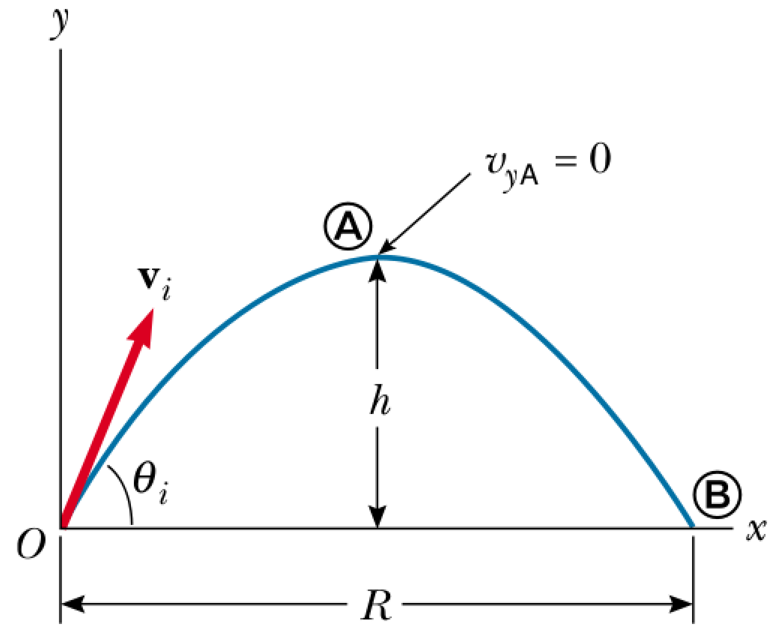
$$x - x_0 = v_{0x} t$$

$$y - y_0 = v_{0y} t - \frac{1}{2} g t^2$$

$$v_y = v_{0y} - g t$$

$$v_{0x} = v_0 \cos\theta$$

$$v_{0y} = v_0 \sin\theta$$



Ⓐ **Maximum height:**

- vertical velocity ***v_y* is zero**

$$h = \frac{v_0^2 \sin^2\theta}{2g}$$

Angled launches

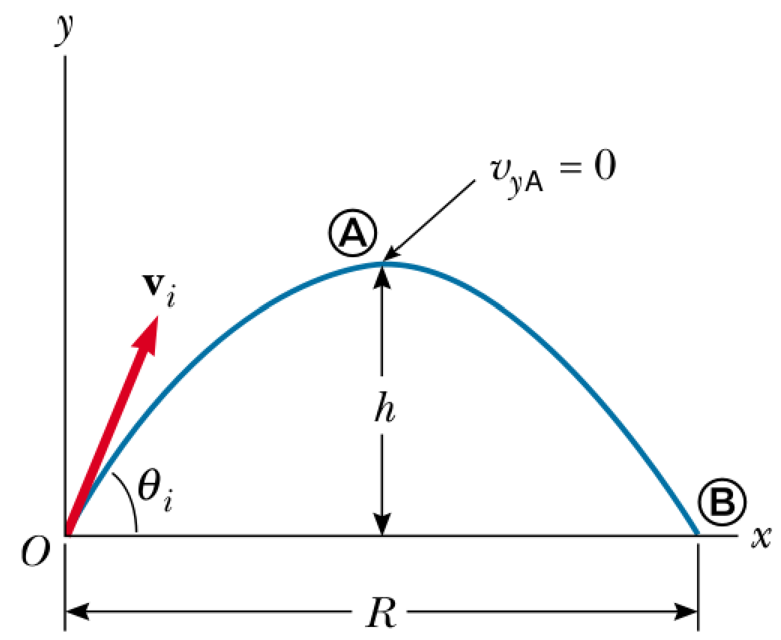
$$x - x_0 = v_{0x} t$$

$$y - y_0 = v_{0y} t - \frac{1}{2} g t^2$$

$$v_y = v_{0y} - g t$$

$$v_{0x} = v_0 \cos\theta$$

$$v_{0y} = v_0 \sin\theta$$



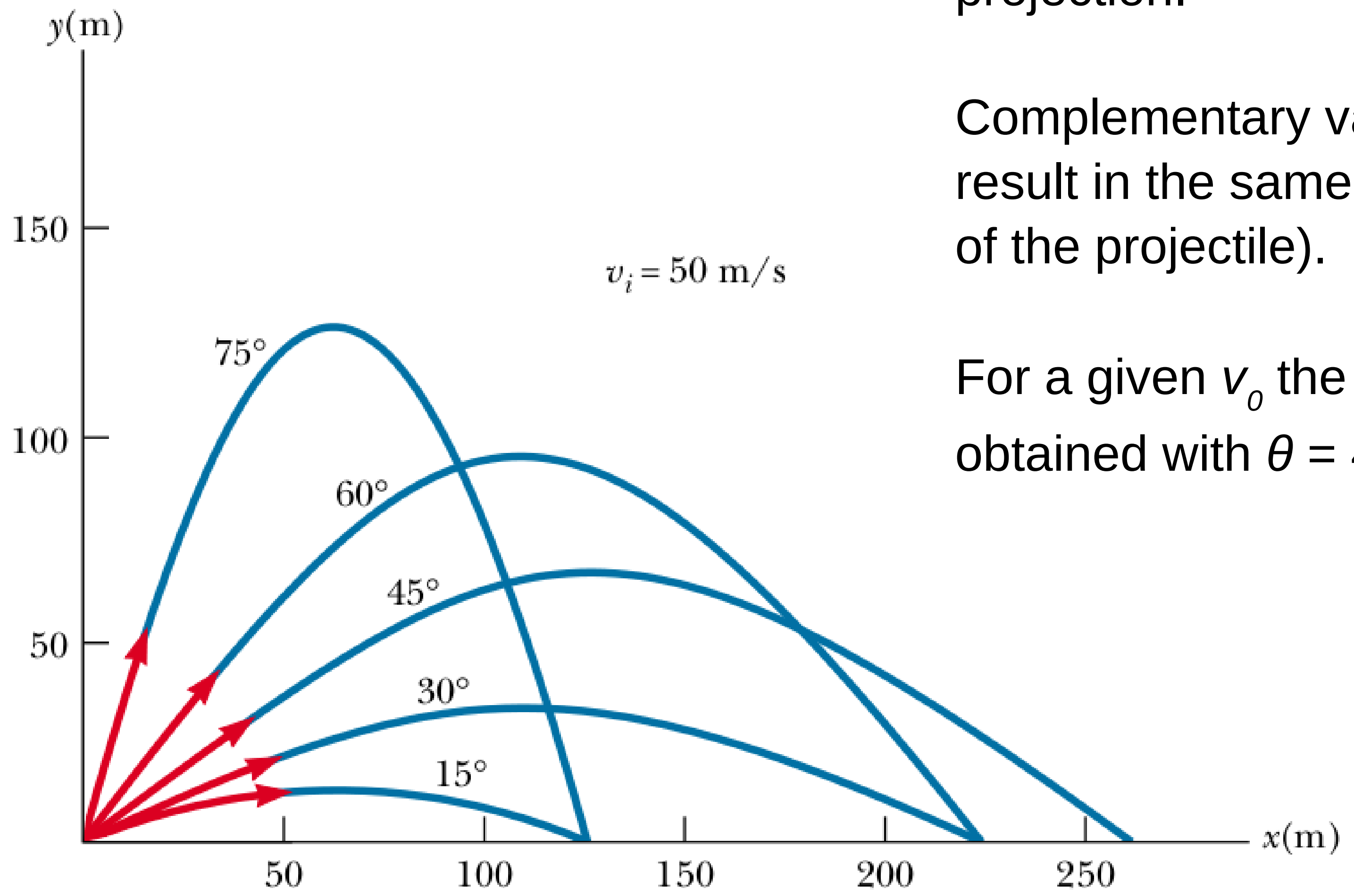
Ⓑ **Range:**

- horizontal distance when initial and final heights are the same (***$y - y_0$* is zero**)

$$R = \frac{v_0^2}{g} \sin 2\theta$$

Angled launches

Range-angle dependency:



A projectile fired from the origin with a given speed at various angles of projection.

Complementary values of the angle result in the same value of x (range of the projectile).

For a given v_0 the maximal range is obtained with θ = 45°.

Checkpoint question:

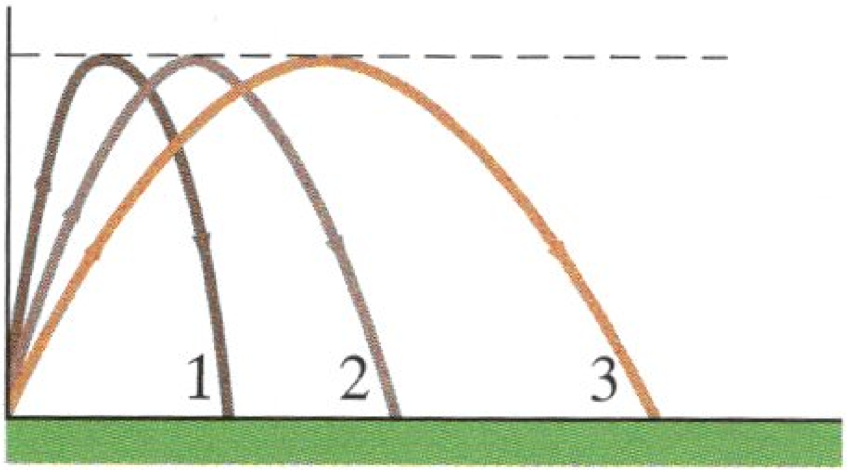


Figure shows three paths for a football kicked from ground level. Ignoring the effects of air, rank the paths according to:

(a) time of flight

(b) initial vertical velocity component

(c) initial horizontal velocity component

(d) initial speed

Example: Cannonball to pirate ship

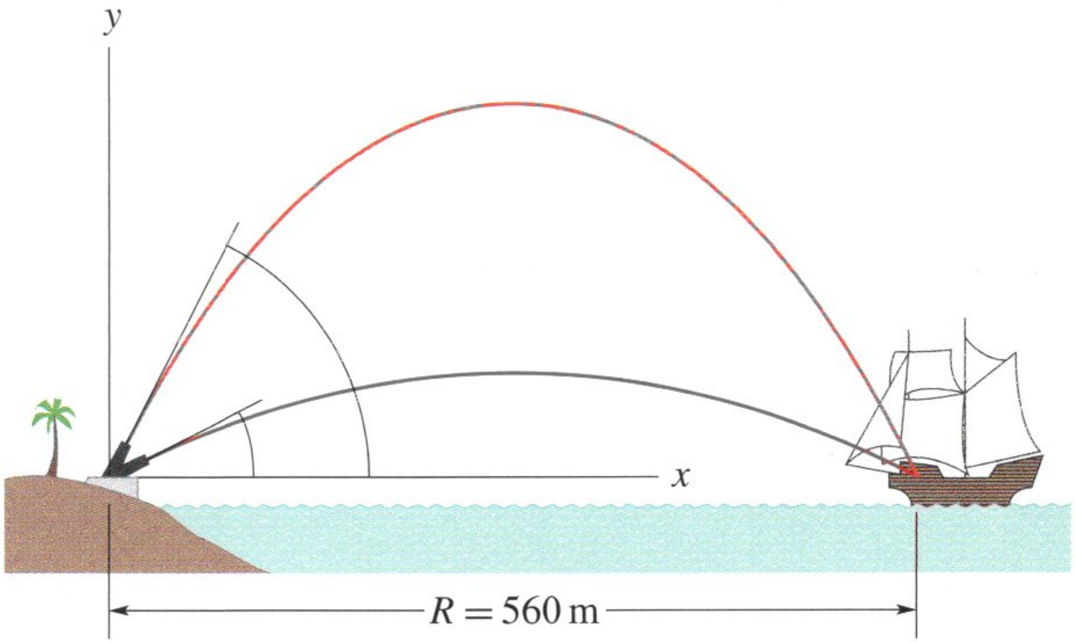


Figure shows a pirate ship 560 m from a fort defending a harbor entrance. A defense cannon, located at sea level, fires balls at initial speed v_0 = 82 m/s. At what angle θ_0 from the horizontal must a ball be fired to hit the ship?

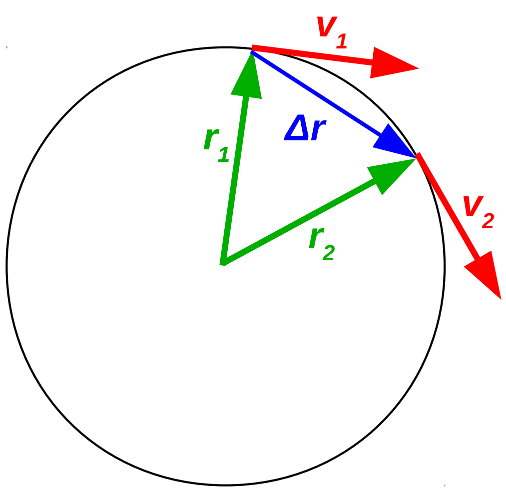
Uniform circular motion

It is a motion with **acceleration** related to the **change in velocity direction**.

Uniform circular motion

- movement of an object at a constant speed around a circle with a fixed radius
- position of the object is given by the position vector ***r***

Position vectors, velocity vectors, displacement vector

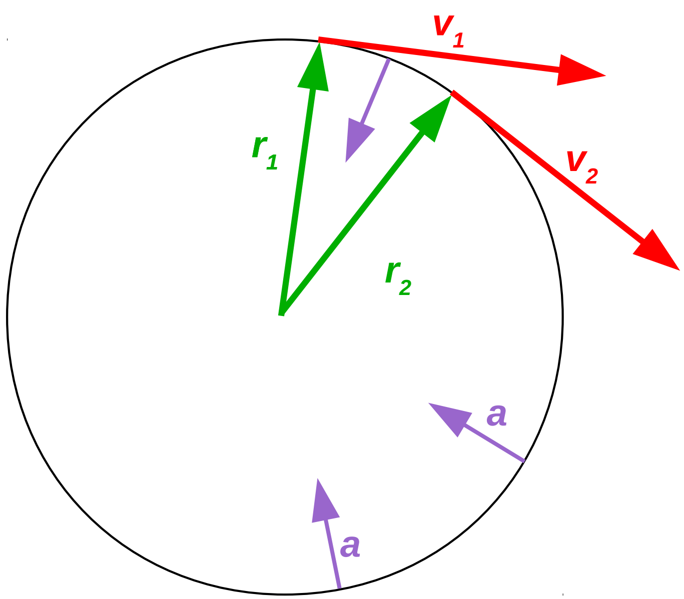


Object's velocity is defined as: $\bar{\boldsymbol{v}} = \frac{\Delta \boldsymbol{x}}{\Delta t}$

In case of circular motion: $\bar{\boldsymbol{v}} = \frac{\Delta \boldsymbol{r}}{\Delta t}$

Velocity vector is tangent to the circular path.

Uniform circular motion

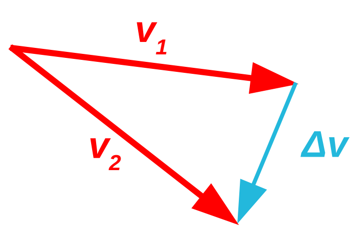


The average acceleration: $\bar{\boldsymbol{a}} = \frac{\Delta \boldsymbol{v}}{\Delta t}$

The average acceleration has the same direction as Δ***v***.

Centripetal acceleration *a*:

For a very small time interval, ***a*** points toward the center of the circle.



$$\frac{\Delta r}{r} = \frac{\Delta v}{v} \longrightarrow \frac{\Delta r}{r \Delta t} = \frac{\Delta v}{v \Delta t}$$

$$\frac{v}{r} = \frac{a}{v} \longrightarrow \boxed{a = \frac{v^2}{r}}$$

Uniform circular motion

Period of revolution T:

- time needed for the object to make one complete revolution
- during this time the object travels a distance equal to the circumference of the circle (*2πr*)

$$T = \frac{2\pi r}{v}$$

Analogical to the equation for the uniform motion with constant velocity $s = v\,t$

Frequency f:

$$f = \frac{1}{T}$$

Frequency units: 1 Hz (Hertz) = 1 s^{-1}

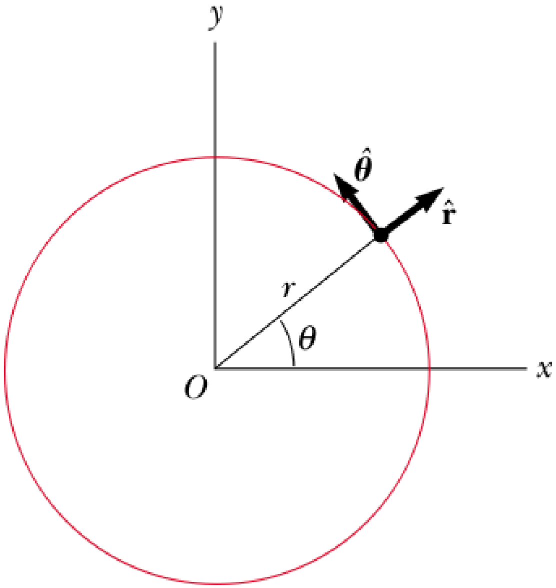
$$v = \frac{2\pi r}{T} \quad \rightarrow \quad \frac{2\pi}{T} = \omega \quad \rightarrow \quad v = \omega r$$

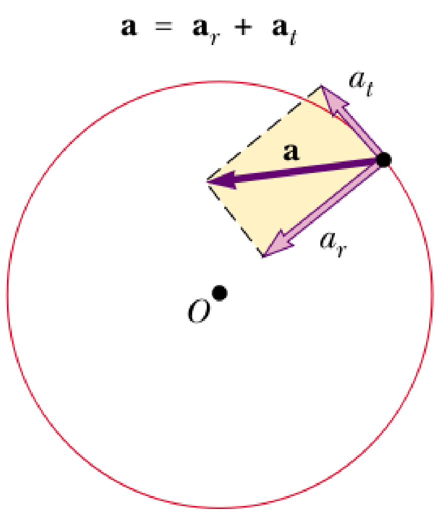
Angular velocity ω

Angular velocity is measured in radians (dimensionless unit). Radian is related to the ratio of the circumference of a circle to its radius.

Nonuniform circular motion

Velocity vector changes not only its **direction**, but also its **magnitude**.





unit vectors $\boldsymbol{\theta}$ and $\boldsymbol{r}$ determine radial and tangential direction

$$\boldsymbol{a} = \boldsymbol{a}_r + \boldsymbol{a}_t$$

Acceleration is the sum of its radial and tangential components:

- **radial component $\boldsymbol{a}_r$** arises from the change in direction of the velocity
- vector and is directed toward the center of curvature
- **tangential component $\boldsymbol{a}_t$** causes the change in magnitude of the velocity vector (a_t becomes zero, if the particle follows uniform circular motion)

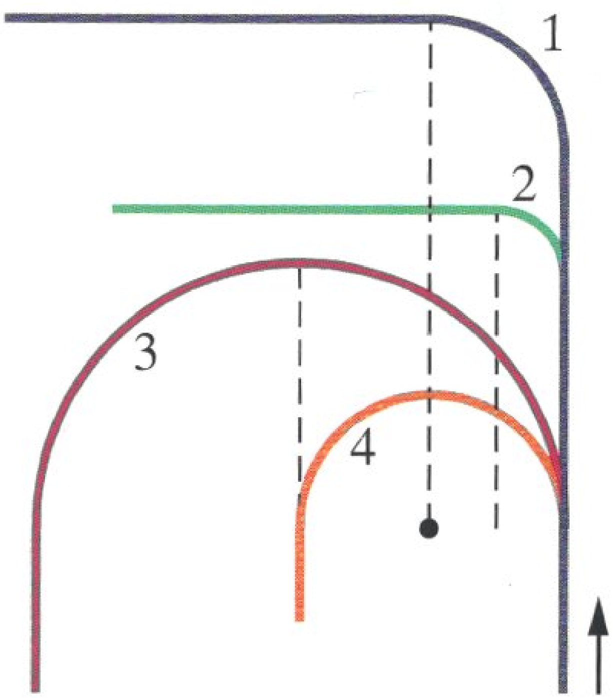
Checkpoint questions:

(a) Is it possible to be accelerating while traveling at constant speed?

(b) Is it possible to round a curve with zero acceleration?

(c) Is it possible to round a curve with a constant magnitude of acceleration?

Figure shows four tracks (either half- or quarter-circles) that can be taken by a train, which moves at a constant speed.
Rank the tracks according to the magnitude of a train's acceleration on the curved portion, greatest first.



Example: uniform circular motion

An Earth satellite moves in a circular orbit 640 km above Earth’s surface with a period of 98.0 min. What are the

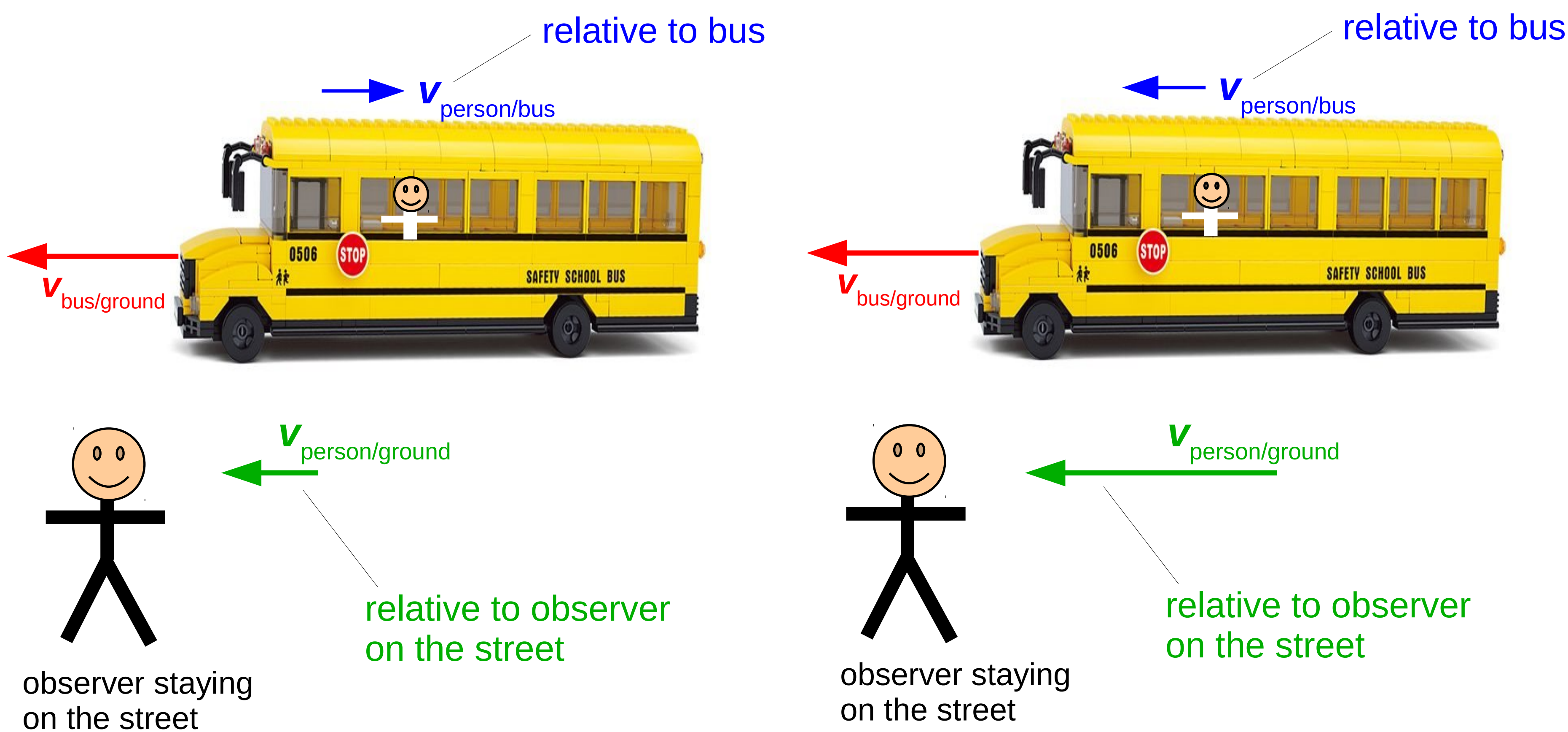
(a) speed,

(b) magnitude of the centripetal acceleration of the satellite?

Earth’s radius is 6371 km.

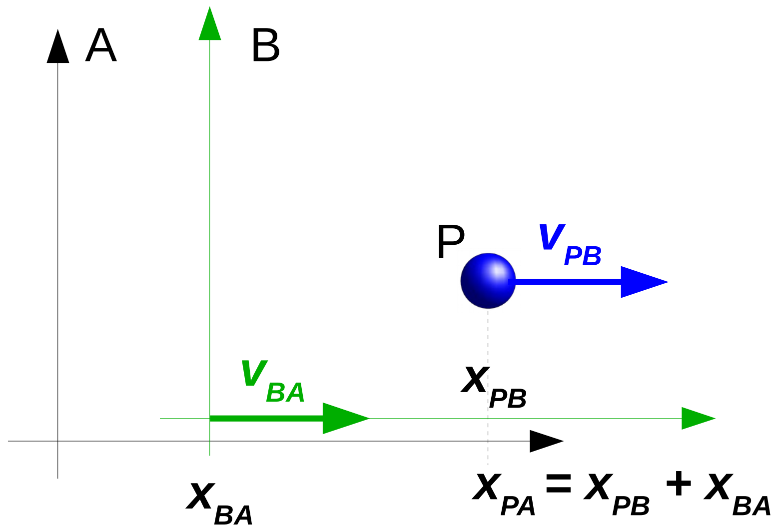
Relative motion in one dimension

Velocity of an object depends on the reference frame where it is observed or measured.



Relative motion in one dimension

Reference frames A and B move at constant velocity relative to each other.



Position (displacement) of the particle:

$$x_{PA} = x_{PB} + x_{BA}$$

Velocity of the particle:

$$v_{PA} = v_{PB} + v_{BA}$$

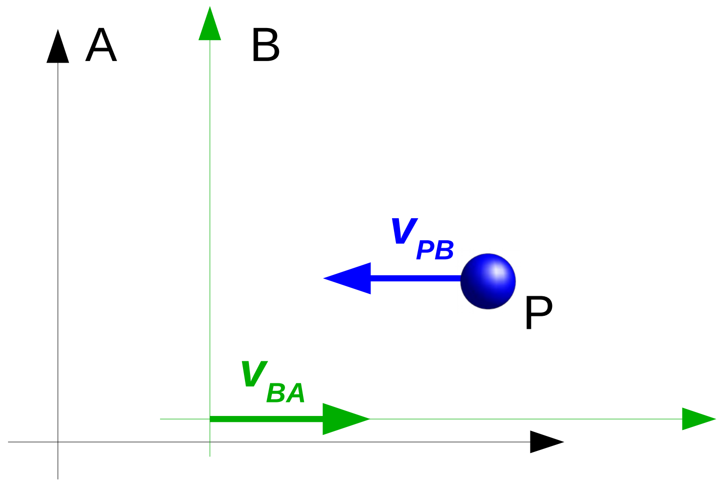
Acceleration of the particle:

v_{BA} is constant → a_{BA} is zero

$$a_{PA} = a_{PB}$$

Observers on different frames of reference that move at **constant velocity** relative to each other will measure **the same acceleration** for a moving particle.

Example:



A… observer Alex
B… observer Barbara
P… car

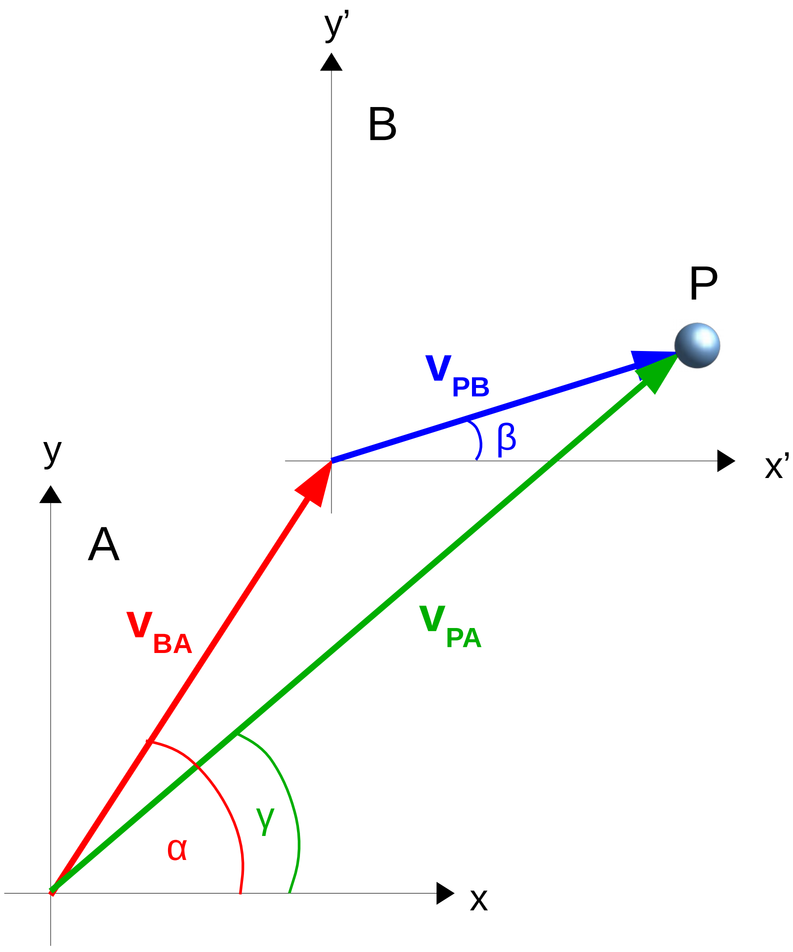
In figure above suppose that Barbara's velocity relative to Alex is constant v_{BA} = 52 km/h and car *P* is moving in the negative direction of the *x* axis.

(a) If Alex measures a constant v_{PA} = *-78* km/h for car *P*, what velocity v_{PB} will Barbara measure?

(b) If car *P* brakes to a stop relative to Alex in time *t* = 10 s at constant acceleration, what is its acceleration a_{PA} relative to Alex?

(c) What is the acceleration a_{PB} of car *P* relative to Barbara during the braking?

Relative motion in two dimensions



$$\boldsymbol{v}_{PB} + \boldsymbol{v}_{BA} = \boldsymbol{v}_{PA}$$

Combining velocities:

- resolve the vectors into x- and y-components

$\mathbf{v}_{BA}$:

x-component: $v_{BAx} = v_{BA} \cos \alpha$

y-component: $v_{BAy} = v_{BA} \sin \alpha$

$\mathbf{v}_{PB}$:

x-component: $v_{PBx} = v_{PB} \cos \beta$

y-component: $v_{PBy} = v_{PB} \sin \beta$

$\mathbf{v}_{PA}$:

x-component: $v_{PAx} = v_{PB} \cos \beta + v_{BA} \cos \alpha$

y-component: $v_{PAy} = v_{PB} \sin \beta + v_{BA} \sin \alpha$

$$v_{PA}^2 = v_{PAx}^2 + v_{PAy}^2$$

Example: relative motion in two dimensions

A train travels due south at 30 m/s (relative to the ground) in a rain that is blown toward the south by the wind. The path of each raindrop makes an angle of 70° with the vertical, as measured by an observer stationary on the ground. An observer on the train, however, sees the drops fall perfectly vertically. Determine the speed of the raindrops relative to the ground.