Foundation course - PHYSICS

Lecture 6-1: Energy, work and power

Dynamics of solid bodies

- Kinetic energy
- Work
- Power
- Potential energy
- Conservation of mechanical energy

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Kinetic energy

Kinetic energy:

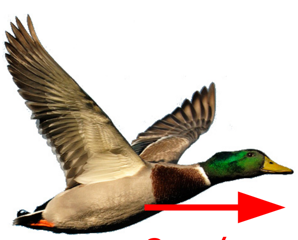
- energy associated with the **state of motion** of an object
- the object is moving faster, its kinetic energy increases
- kinetic energy of an stationary object is zero
- kinetic energy is always positive

$$\boxed{E_k = \frac{1}{2} m v^2}$$

Units of energy: 1 joule = 1 J = 1 kg.m^2.s^{-2}

Example:

m = 3 kg



E_K = 6 J

m = 1500 kg

E_K = 300 000 J

Work

Work:

- if an object accelerates to a greater speed by applying a force to this object, its kinetic energy increases (and vice versa)
- work *W* is the energy transferred to or from an object by means of a force acting on the object

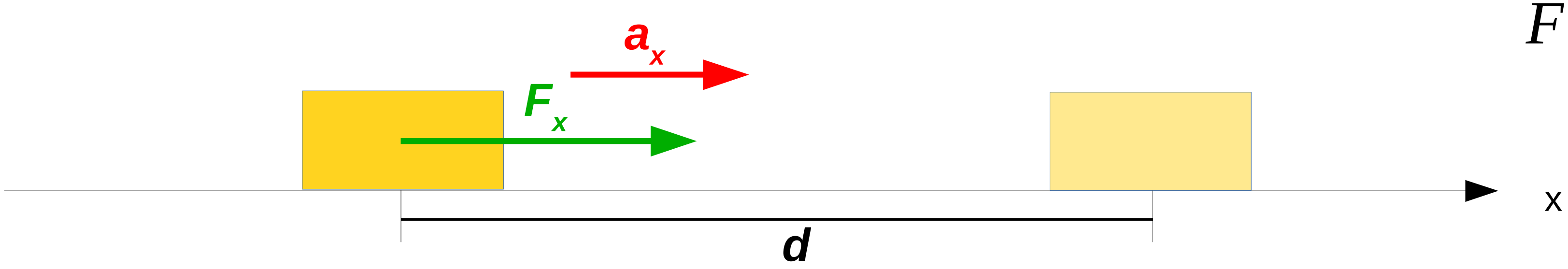
A constant force exerted on an object results to acceleration of the object:

$$F_x = m a_x$$

If acceleration is non-zero, velocity changes: $v = v_0 + a_x t$

Distance increases: $d = v_0 t + \frac{1}{2} a t^2$

Work



$$F_x = m a_x$$

Acceleration, velocity, and distance are related by the equation:

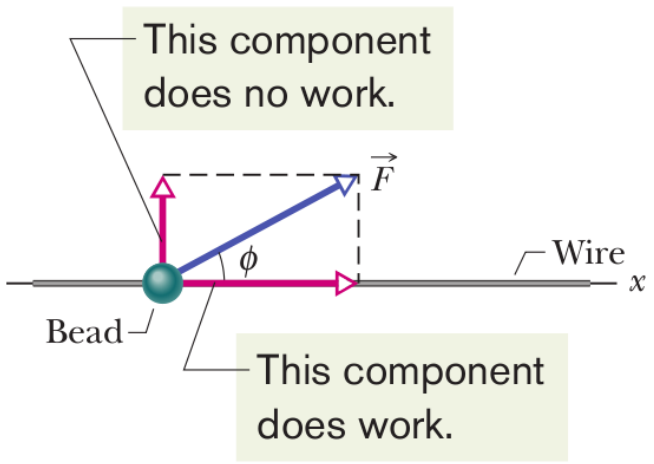
$$v^2 = v_0^2 + 2 a_x d$$

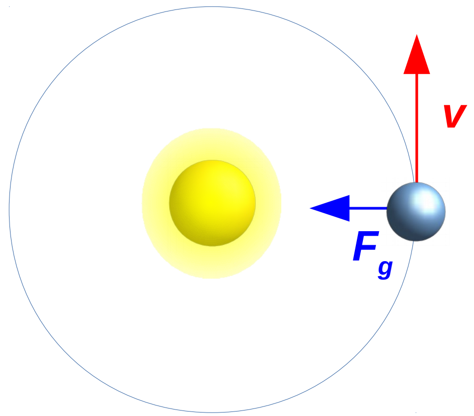
$$v^2 = v_0^2 + 2\left(\frac{F_x}{m}\right) d$$

work

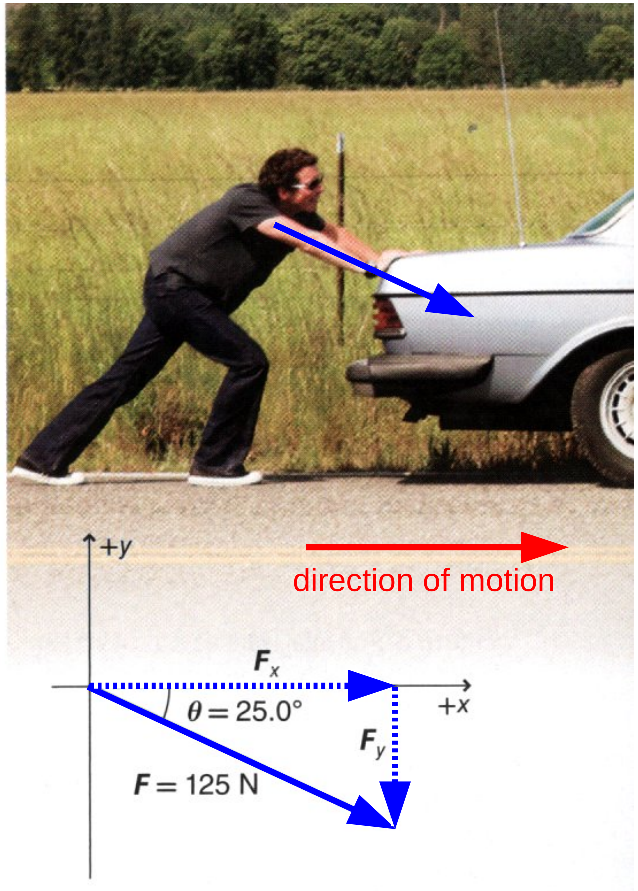
$$\frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 = F_x d$$

Work





F_g does no work
(θ = 90°)



Only the force component along the object's displacement does the work.

Work is the scalar (dot) product of ***F*** and ***d***

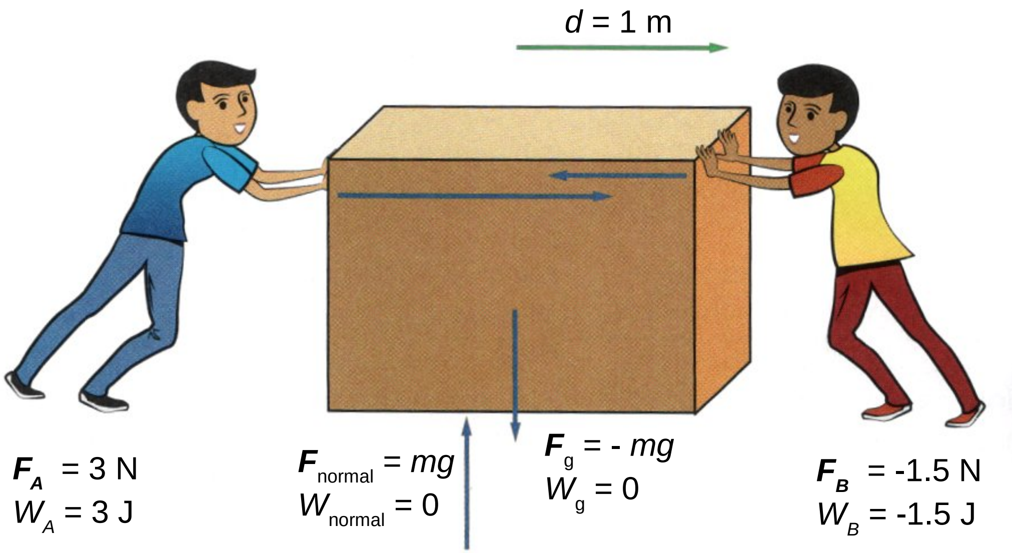
$$\boxed{W = \boldsymbol{F} \cdot \boldsymbol{d}}$$

$$\boxed{W = F\, d \cos\theta}$$

Work

Work done by many forces:

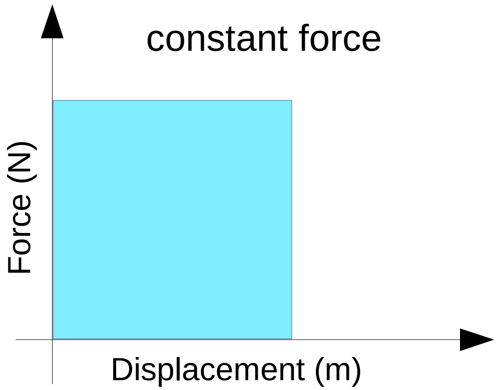
- forces <u>perpendicular</u> to the direction of motion do <u>no work</u>
  ($\theta = 90°$, $\cos\theta = 0$, $W = 0$)
- F_A is in the direction of motion:
  $\theta = 0°$, $\cos\theta = 1$, $W_A = F_A\, d$
- F_B is in the opposite direction of motion:
  $\theta = 180°$, $\cos\theta = -1$, $W_B = -F_B\, d$

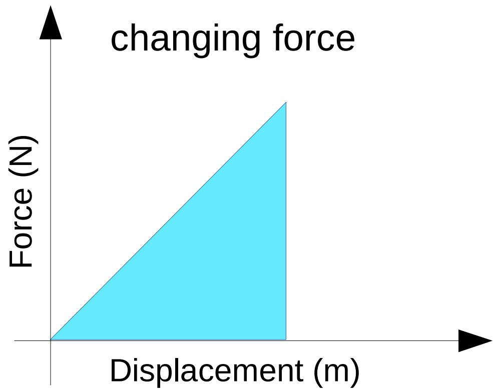


The total work done on the box is: $W = W_A + W_B = F_A\, d - F_B\, d$

Work done when forces change:

Work is equal to the area under the force-displacement graph





Kinetic energy

$$F\,d = \frac{1}{2}m\,v^2 - \frac{1}{2}m\,v_0^2$$

Work – kinetic energy theorem:

$$\boxed{W = \Delta E_k}$$

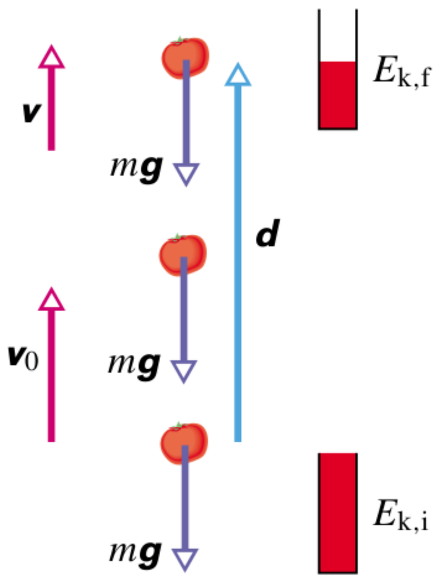
Work is the transfer of energy that occurs when a force is applied through a displacement.

Work on a system is equal to the change in the system's energy.

- the **external world does work** on a system – work is **positive** – energy of the system **increases**
- the **system does work** on the external world – work is **negative** – energy of the system **decreases**

Energy associated with motion = **kinetic energy**
Energy associated with changing position = **translational kinetic energy**

Work done by the gravitational force



Work W_g done on an object by the gravitational force:

$$\boxed{W_g = m\,g\,d\cos\theta}$$

Rising object: θ = 180°, cos θ = -1: $W_g = -m\,g\,d$

Falling object: θ = 0°, cos θ = +1: $W_g = +m\,g\,d$

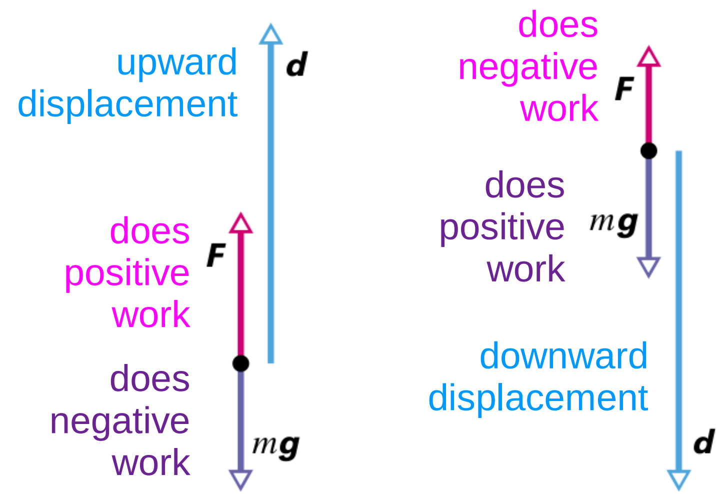
Work done in lifting and lowering an object:

$\Delta E_k = E_{k,final} - E_{k,initial} = W_{applied} + W_{gravitational}$

The object is stationary before and after the lift: $\Delta E_k = 0$

$W_a + W_g = 0$

$$\boxed{W_a = -W_g}$$

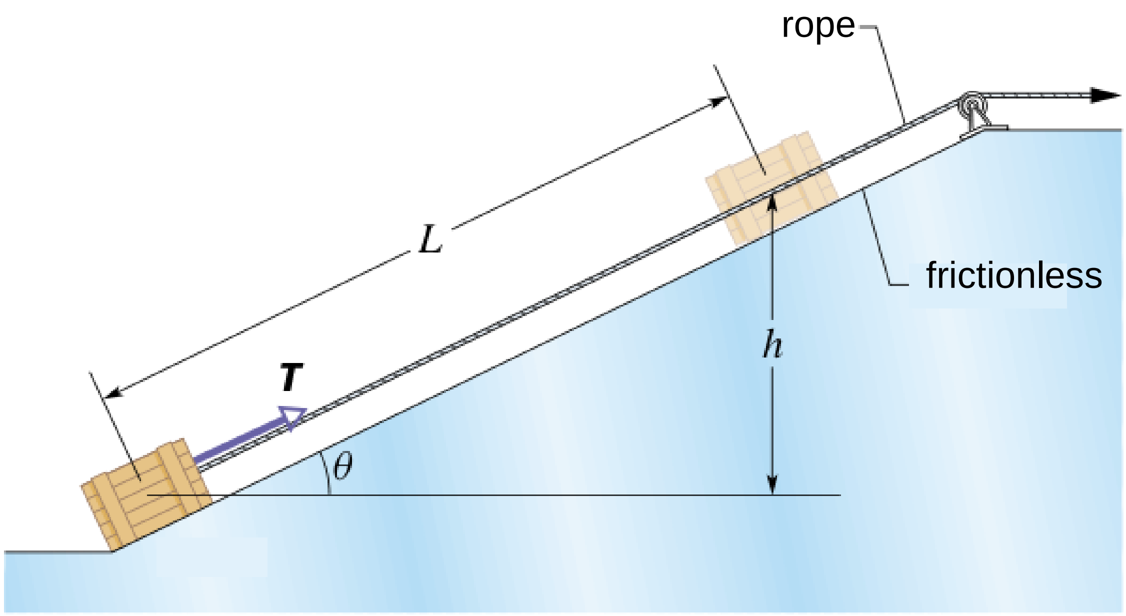


Example:

A box of mass m = 10 kg is pulled up an incline from rest and is stopped after traveling a distance L = 6 m to a height h = 2.5 m.

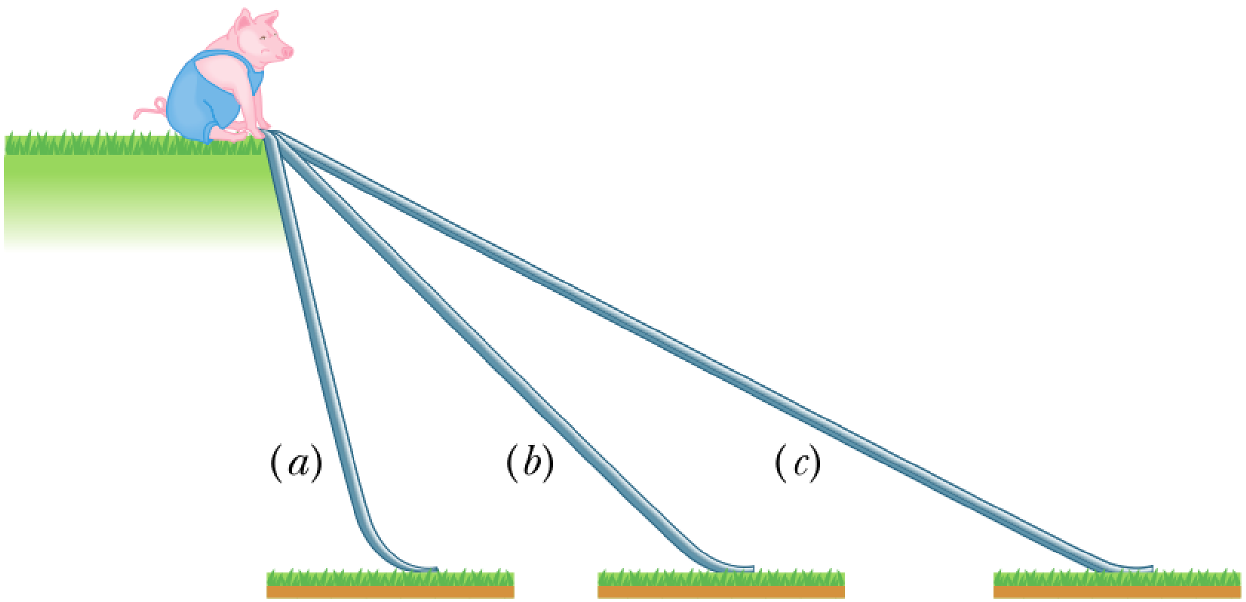
(a) What is the work done by the gravitational force $\boldsymbol{F_g}$?

(b) What is the work done by the tension force $\boldsymbol{T}$?



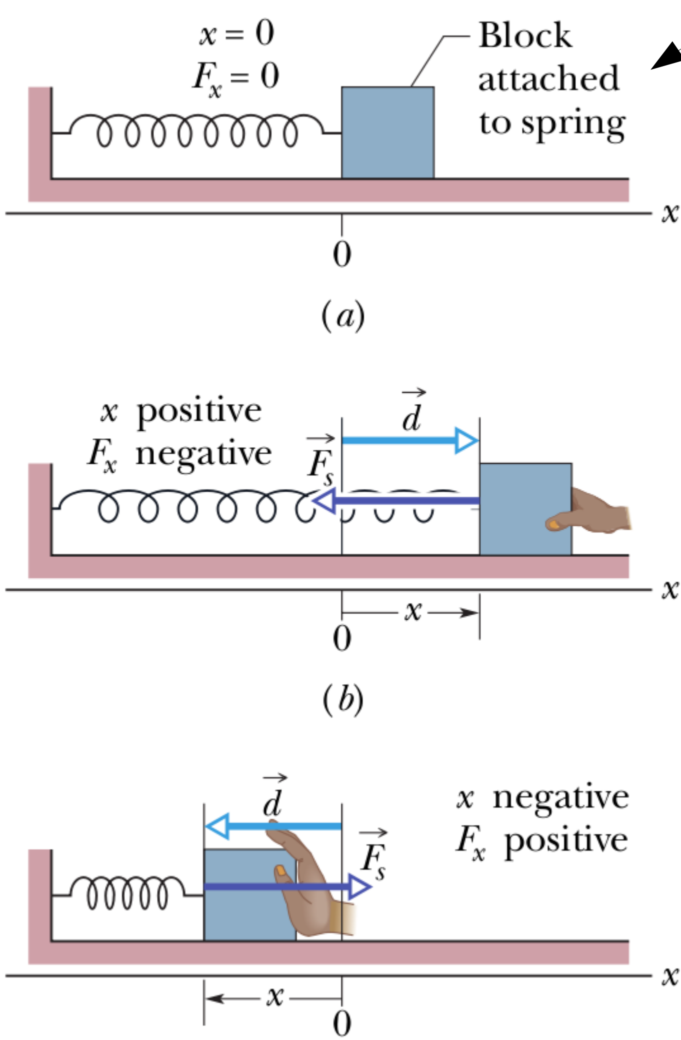
Checkpoint question:

A pig has a choice of three frictionless slides along which to slide to the ground. Rank the slides according to how much work the gravitational force does on the pig during the descent, greatest first.



Work done by a spring force

Spring force: variable force



relaxed state: $\boldsymbol{F}_s = 0$

Hooke's law for the spring force:

$$\boldsymbol{F}_s = -k\,\boldsymbol{d}$$

If the length of the spring is parallel with *x* axis and *x* = 0 represents the relaxed position, then:

$$F_x = -k\,x$$

The work done by the spring force:

$$\boxed{W_s = -\frac{1}{2}k\,x^2}$$

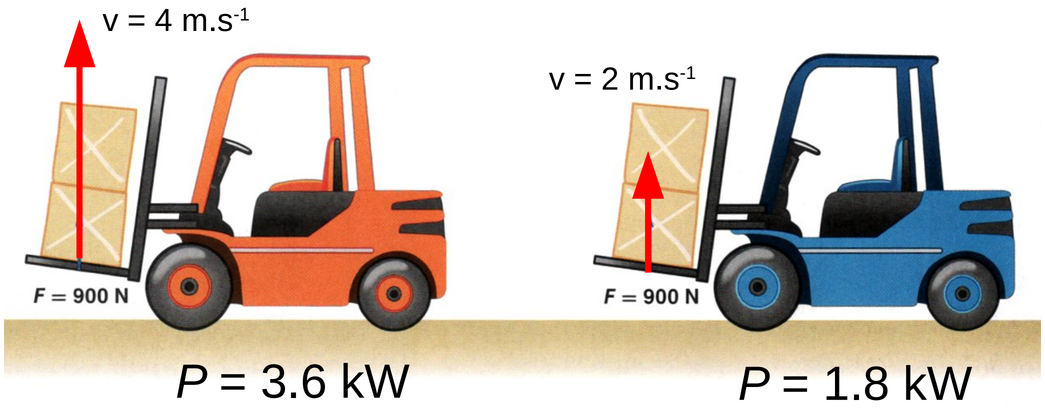
Power

Power is equal to the work done by a force divided by the time required for the change:

Average power P_{avg}: $$\boxed{P_{avg} = \frac{W}{\Delta t}}$$

Instantaneous power P: $$P = \frac{dW}{dt}$$

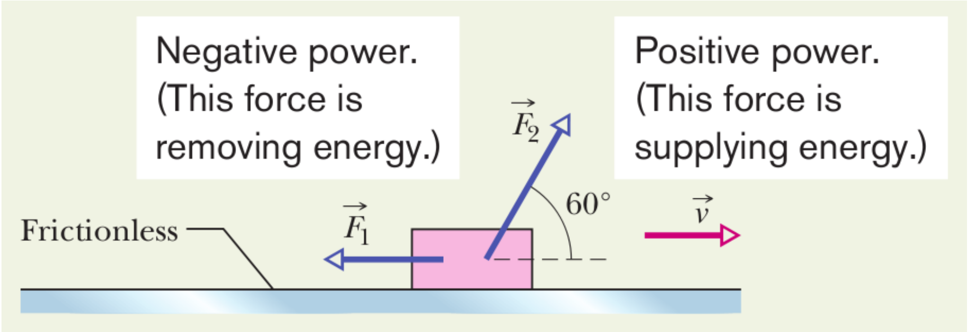
Units of power: 1 watt = 1 W = 1 J.s^{-1} = 1 kg.m^2.s^{-3}



Instantaneous power P:

If is a particle moving along a straight line and acted on by a constant force:

$$\boxed{P = \frac{\boldsymbol{F} \cdot \boldsymbol{d}}{t} = \boldsymbol{F} \cdot \boldsymbol{v} = F\, v \cos\theta}$$



Example:

A force of 5.0 N acts on a 15 kg body initially at rest. Compute the work done by the force in

(a) the first second

(b) the second second

(c) the third second

(d) the instantaneous power due to the force at the end of the third second.

Machines and their benefits

Machine:

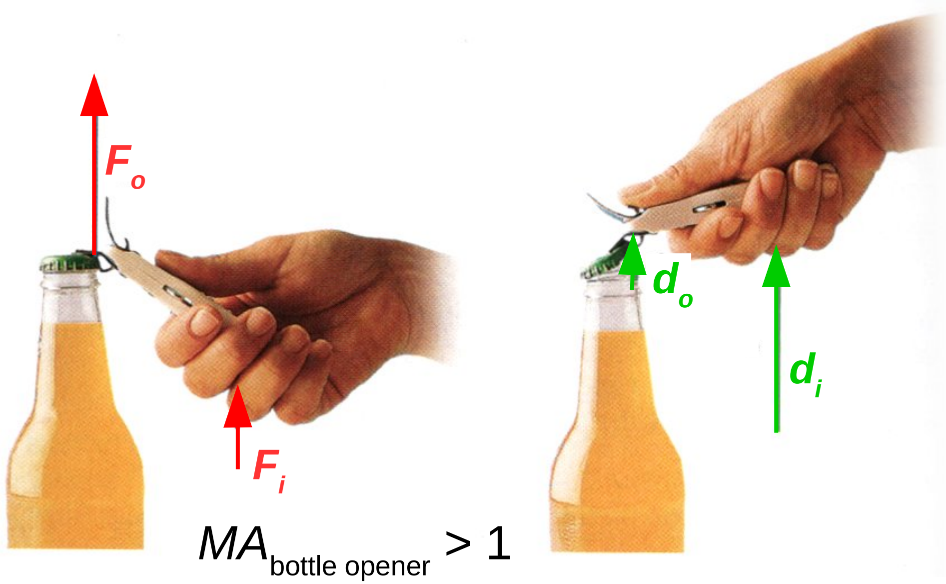
device that makes tasks easier by changing either the magnitude or the direction of the applied force

Mechanical advantage *MA*:

- **input force F_i**: the force exerted by a user on a machine
- **output force F_o**: the force exerted by the machine

$$MA = \frac{F_o}{F_i}$$

A machine can increase **force**, but it cannot increase **energy**.



$$W_o = W_i$$

$$F_o d_o = F_i d_i \quad \Rightarrow \quad \frac{F_o}{F_i} = \frac{d_i}{d_o}$$

Ideal mechanical advantage *IMA*:

$$IMA = \frac{d_i}{d_o}$$

Efficiency

In case of a real machine: $W_o \neq W_i$ The machine is less efficient.

The efficiency of a machine (in %) is equal to the output work, divided by the input work, multiplied by 100.

Efficiency:

An ideal machine has equal output and input work, and its efficiency is 100 percent.

$$e = \frac{W_o}{W_i} \times 100$$

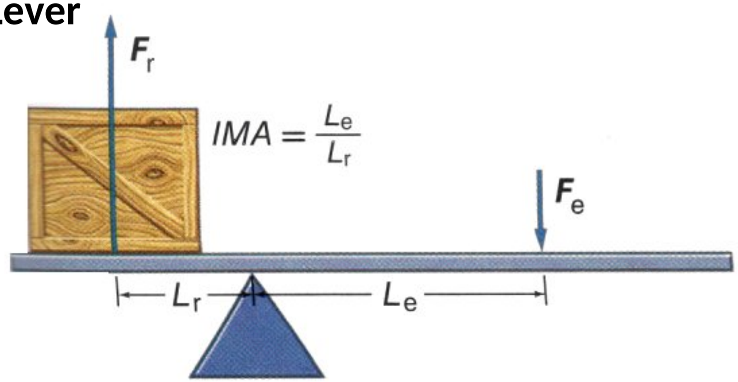
$$e = \frac{W_o}{W_i} = \frac{F_o d_o}{F_i d_i}$$

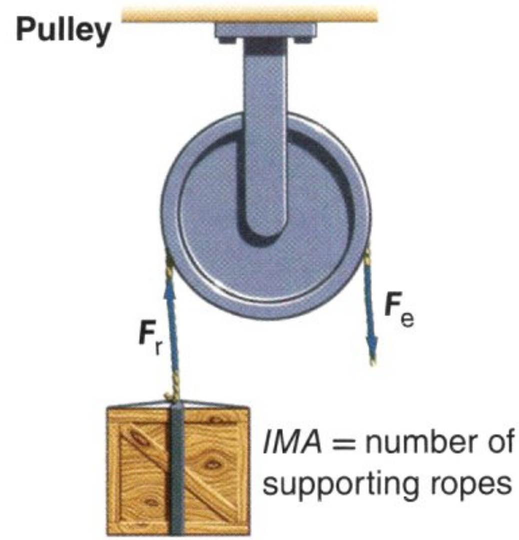
MA ***IMA***

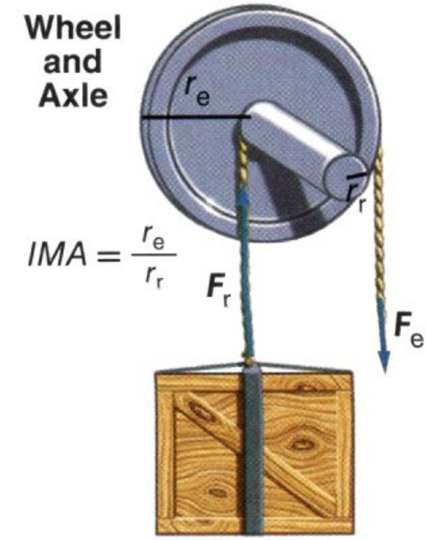
$$e = \frac{MA}{IMA} \times 100$$

Simple machines

Lever

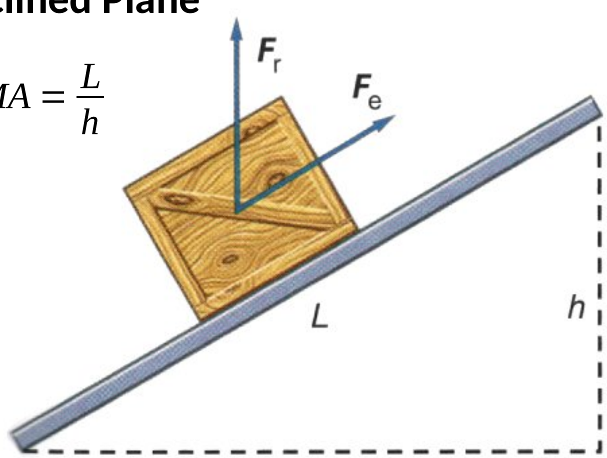


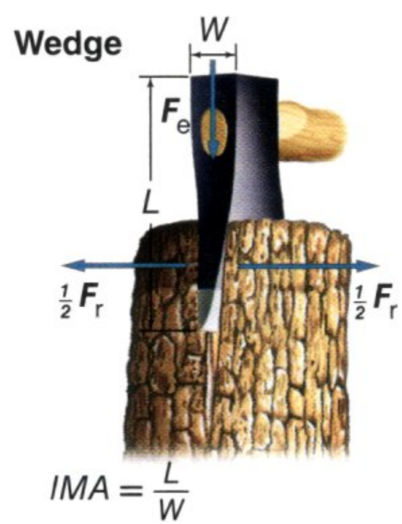


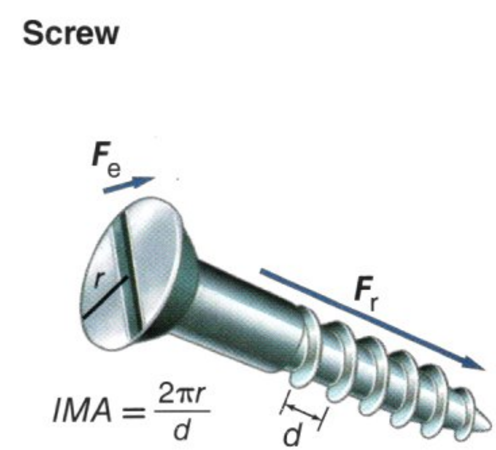


Inclined Plane

$$IMA = \frac{L}{h}$$





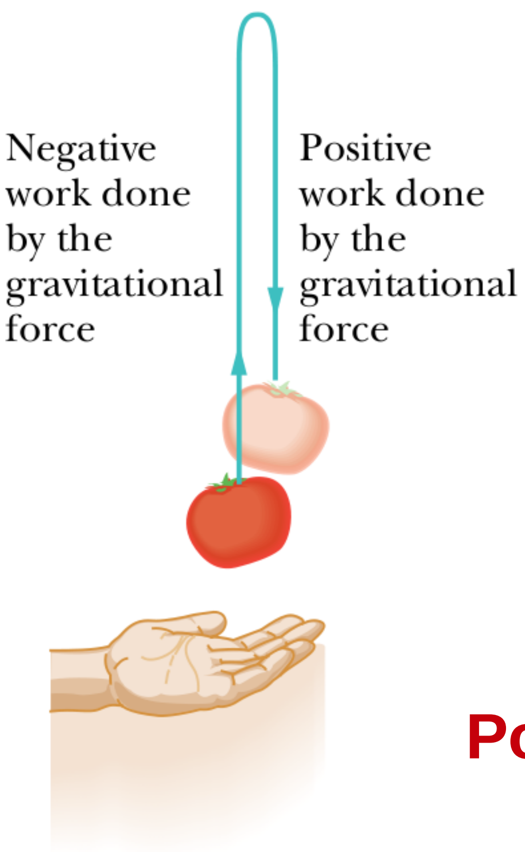


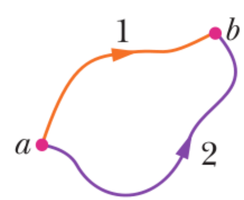
Potential energy

Potential energy is associated with the **configuration** (arrangement) of a system of objects that exert forces on one another

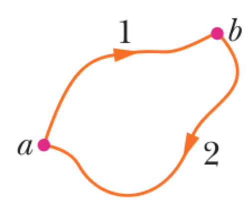
Conservative forces:

- gravitational and spring forces are conservative
- friction force is not conservative





The force is conservative. Any choice of path between the points gives the same amount of work.

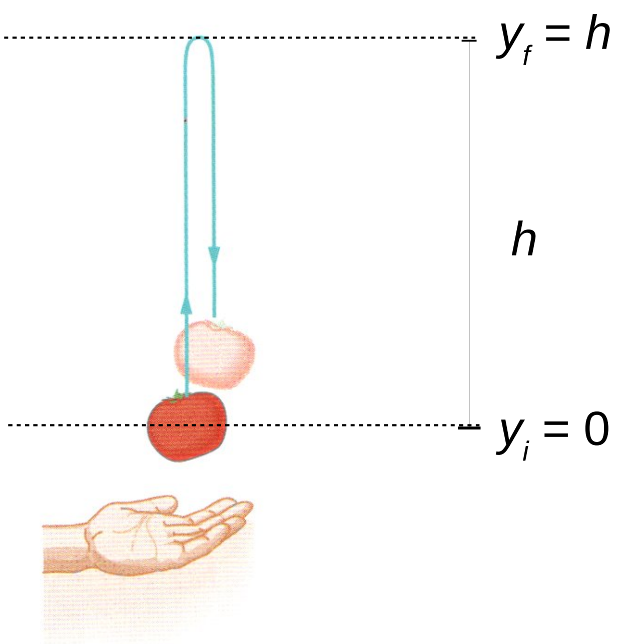


And a round trip gives a total work of zero.

Potential energy is defined as: $$\boxed{\Delta E_p = -W}$$

Potential energy

Gravitational potential energy (system body-Earth):



$$\boxed{\Delta E_p = -W_g}$$

$$\Delta E_p = -W_g = E_{pf} - E_{pi} = mg\left(y_f - y_i\right)$$

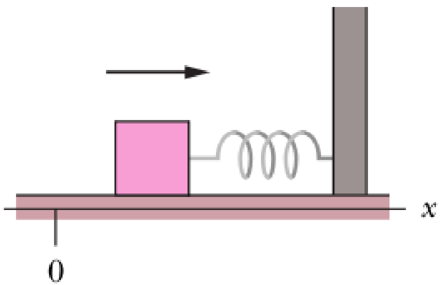
At a reference point $y_i = 0$ we set $E_{pi} = 0$:

$$E_p - E_{pi} = mg\left(y - y_i\right)$$

$$\boxed{E_P\left(y\right) = mgh}$$

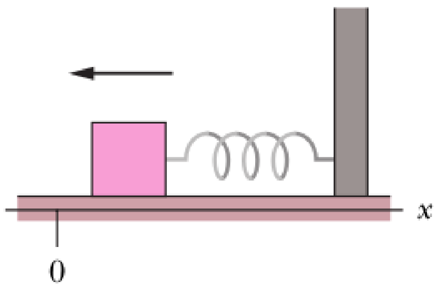
Potential energy

Elastic potential energy:



$$\boxed{\Delta E_p = -W_s}$$

$$E_{pf} - E_{pi} = \frac{1}{2} k\, x_f^2 - \frac{1}{2} k\, x_i^2$$

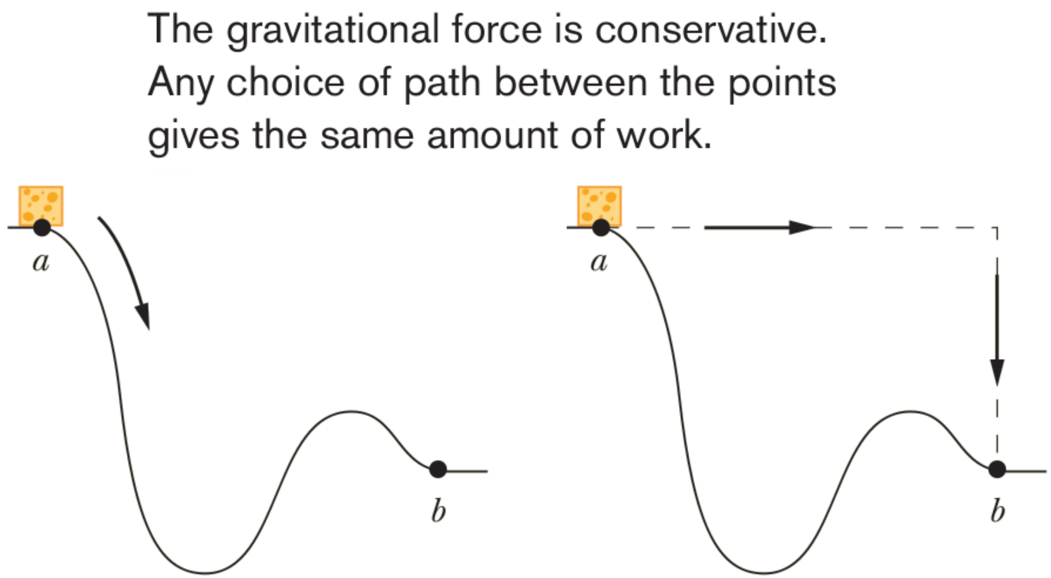


At a reference point $x_i = 0$ we set $E_{pi} = 0$:

$$\boxed{E_P(x) = \frac{1}{2} k\, x^2}$$

Example:

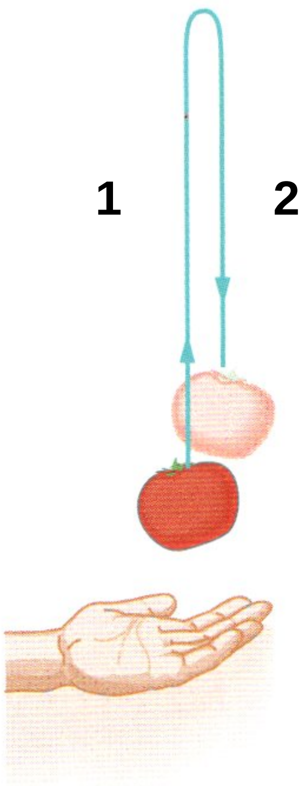
A 2.0 kg block of cheese slides along a frictionless track from point *a* to point *b*. The cheese travels through a total distance of 2.0 m along the track, and a net vertical distance of 0.80 m. How much work is done on the cheese by the gravitational force during the slide?

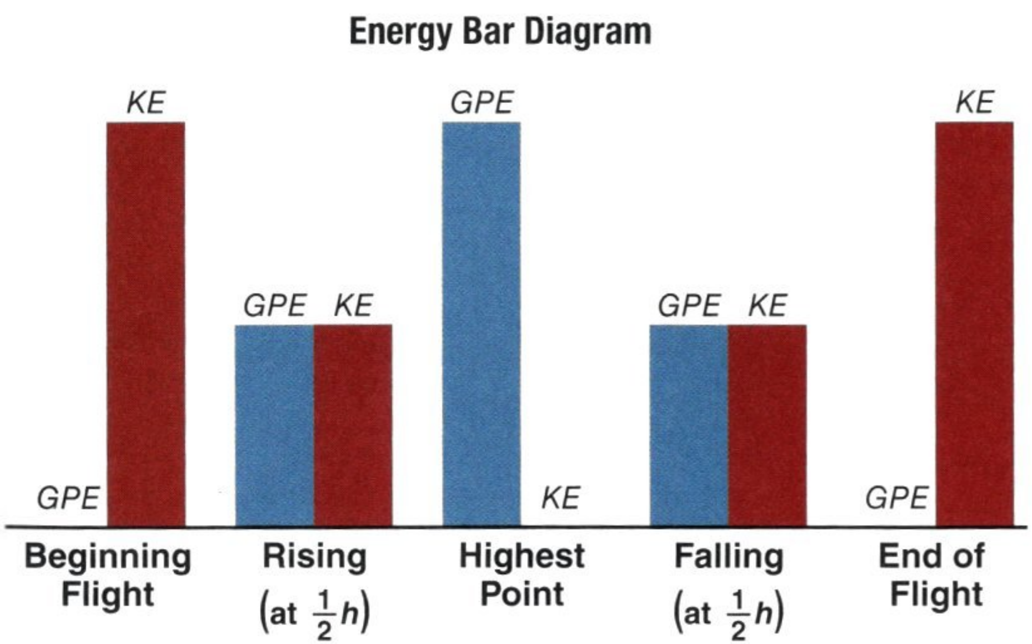


Energy: kinetic and potential

Translational kinetic energy: $$E_K = \frac{1}{2} m v^2$$

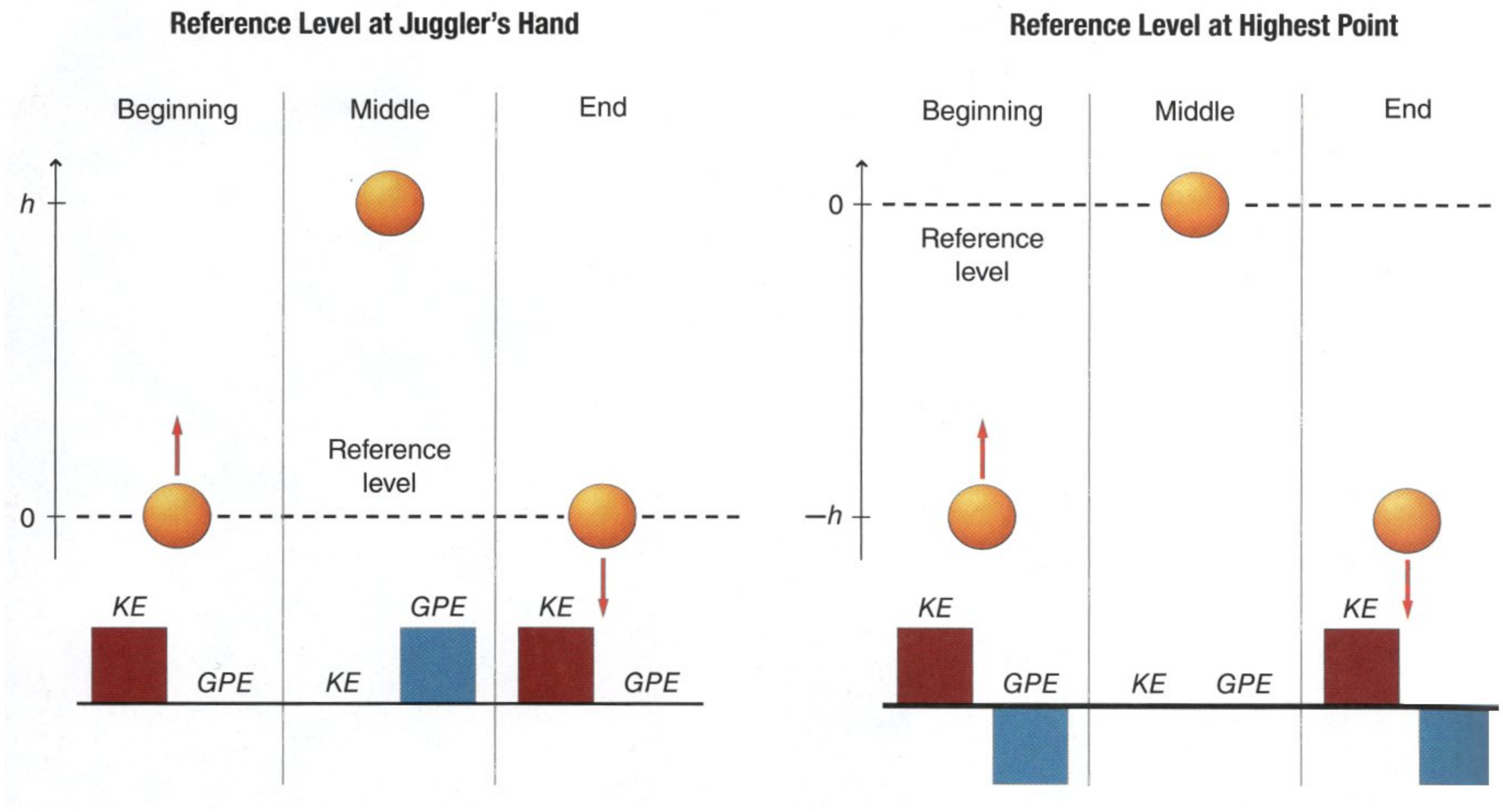
Gravitational potential energy: $$E_P = m g h$$





GPE… gravitational potential energy, *KE*… kinetic energy

Reference levels



The total mechanical energy of the system in each situation is **constant.**

Conservation of mechanical energy

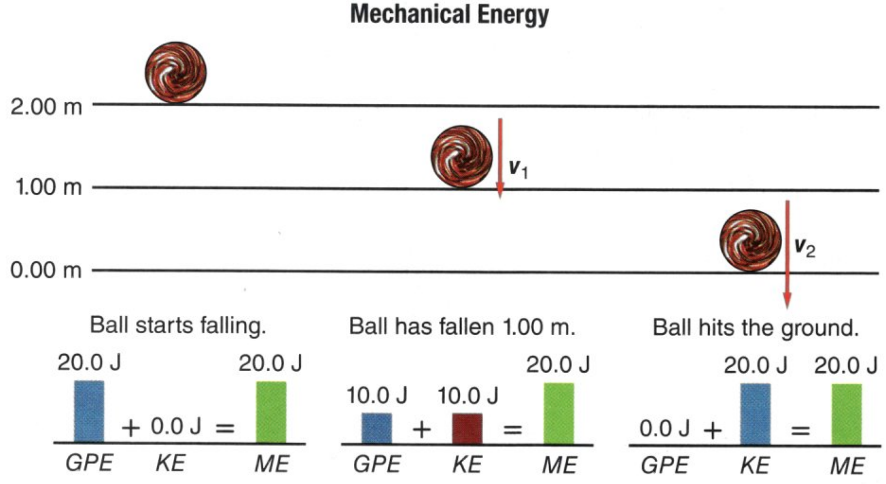
Mechanical energy of the system is equal to the **sum** of the **kinetic energy** and **potential energy** of the system's objects.

$$\boxed{E = E_k + E_p}$$

$$\Delta E_k = W \qquad \Delta E_p = -W$$

$$\Delta E_k = -\Delta E_p$$

$$E_{k2} - E_{k1} = -(E_{p2} - E_{p1})$$



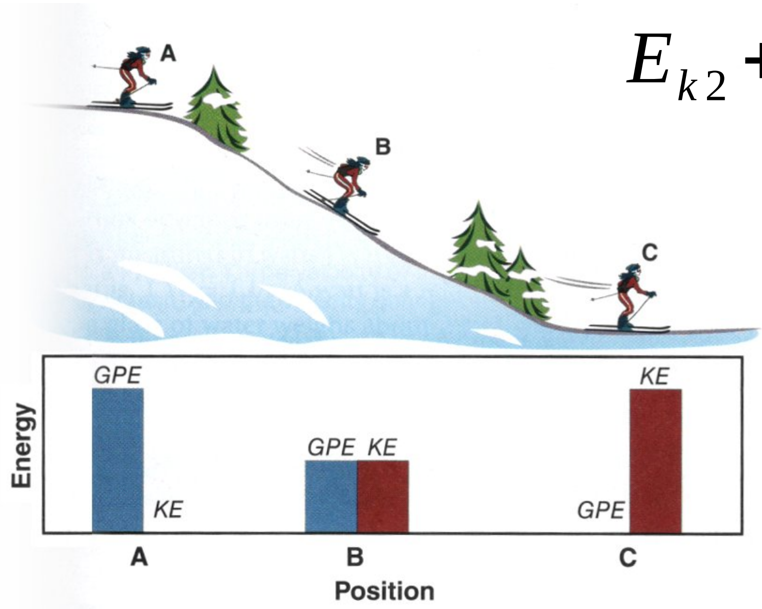
Conservation of mechanical energy:

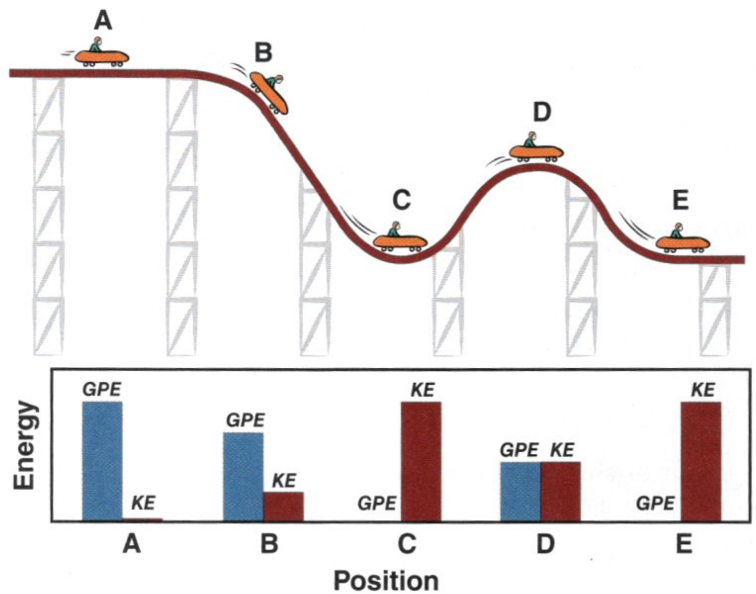
$$\boxed{E_{k2} + E_{p2} = E_{k1} + E_{p1}}$$

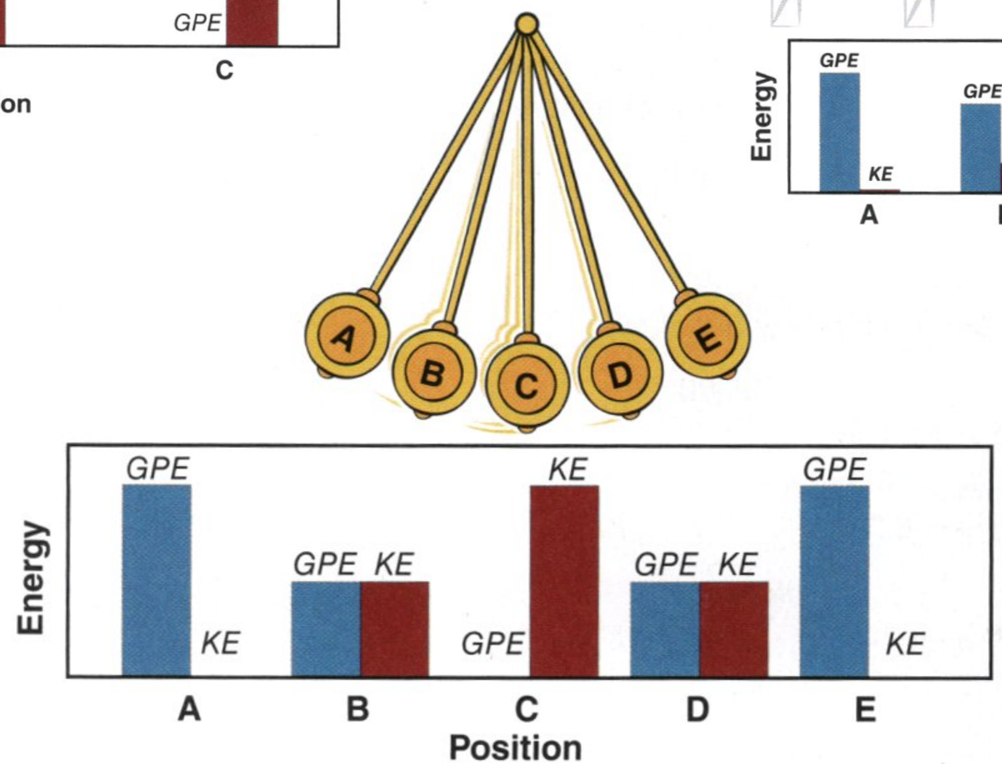
In an isolated system, where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy remains constant.

Conservation of mechanical energy

$$E_{k2} + E_{p2} = E_{k1} + E_{p1}$$





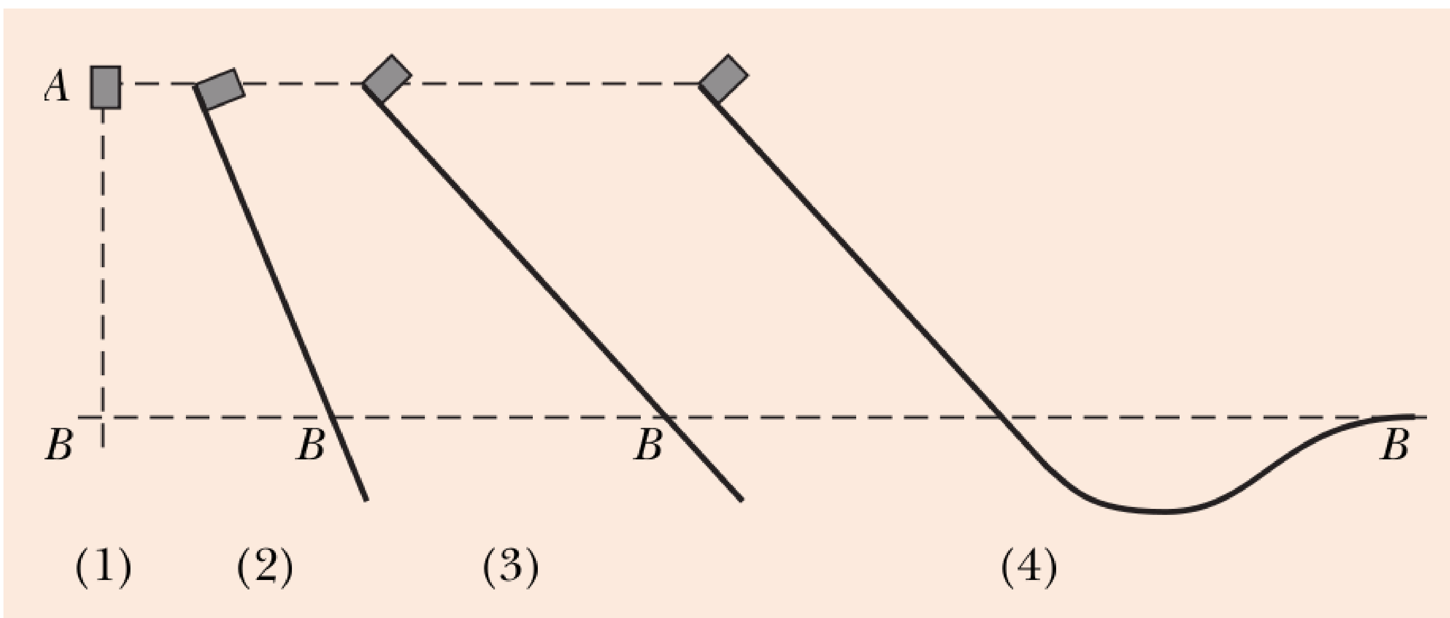


Checkpoint question:

The figure shows four situations – one in which an initially stationary block is dropped and three in which the block is allowed to slide down frictionless ramps.

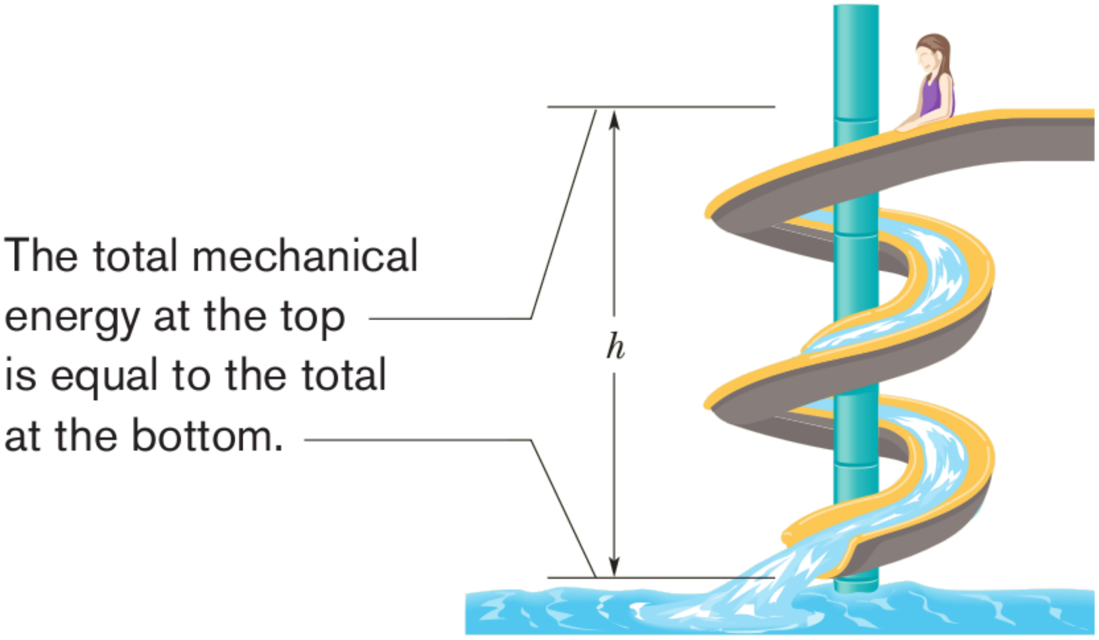
(a) Rank the situations according to the kinetic energy of the block at point *B*, greatest first.

(b) Rank them according to the speed of the block at point *B*, greatest first.



Example:

A child of mass m is released from rest at the top of a water slide, at height h = 8.5 m above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.



Summary

Work:

$$\boxed{W = \boldsymbol{F} \cdot \boldsymbol{d}}$$

$$\boxed{W = F\,d \cos\theta}$$

Kinetic energy:

$$\boxed{W = \Delta E_k}$$

$$\boxed{E_k = \frac{1}{2} m v^2}$$

Potential energy:

$$\boxed{\Delta E_p = -W}$$

Gravitational potential energy:

$$\boxed{\Delta E_p = -W_g}$$

$$\boxed{E_P(y) = m g h}$$

Elastic potential energy:

$$\boxed{\Delta E_p = -W_s}$$

$$\boxed{E_P(x) = \frac{1}{2} k x^2}$$

Conservation of mechanical energy:

$$\boxed{E_{k2} + E_{p2} = E_{k1} + E_{p1}}$$