# Foundation course - PHYSICS

## Lecture 7-1: Equilibrium and elasticity
## Gravitational force

- Equilibrium
- Elasticity and deformations
- Gravitational force

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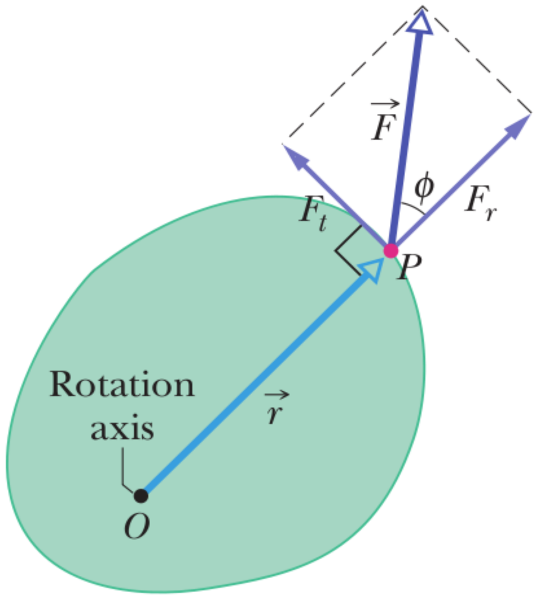
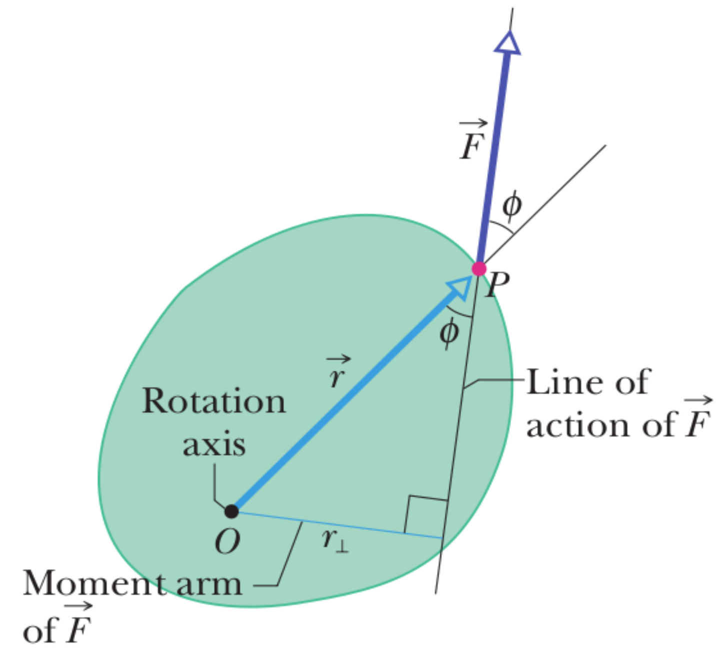
# Torque




$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$$

$$\tau = r\,F \sin\phi$$




But actually only the *tangential* component of the force causes the rotation.

You calculate the same torque by using this moment arm distance and the full force magnitude.

Two equivalent ways of computing the torque:

$$\tau = (r)(F \sin\phi) = r\,F_t$$

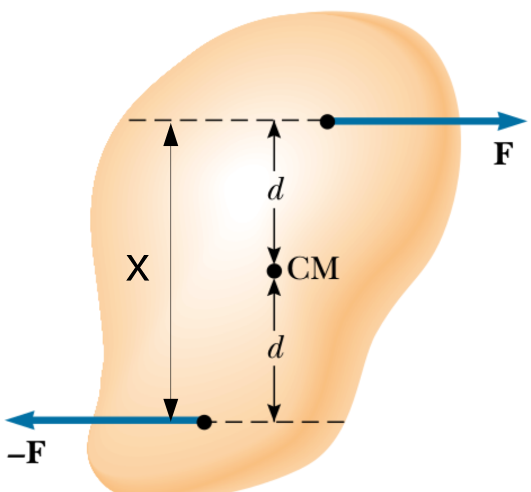
$$\tau = (r \sin\phi)(F) = r_\perp F$$

# Couple of forces

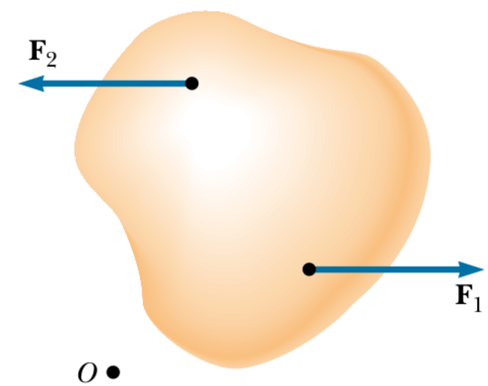
**Equivalent forces:**

- two forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ are **equivalent** if and only if $F_1 = F_2$ and if and only if the two produce the same torque about any axis

Object pivoted about an axis through its center of mass:




$$\tau = F\,x$$

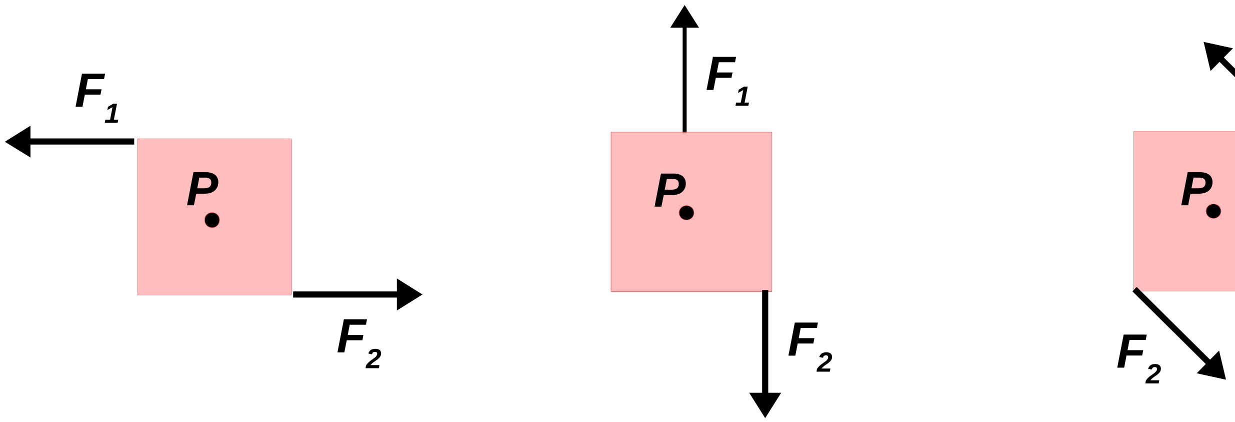



These forces are not equivalent

Two forces of equal magnitude form a **couple** if their lines of action are different parallel lines. Because each force produces the same torque *Fd*, the net torque has a magnitude of 2*Fd*.

**Example:**

The figure shows (in the overhead view) two forces of the same magnitude of 30 N acting on a square of 0.6 m side that can rotate about point *P*. What is the net torque about the pivot point?



# Momentum

**Linear momentum *p*:**

$$p = mv$$

**Angular momentum *L*:**

$$L = I\omega$$

**Newton's second law** expressed in terms of momentum:

$$F_{net} = \frac{dp}{dt}$$

$$\tau_{net} = \frac{dL}{dt}$$

$$F_{net} = ma = m\frac{dv}{dt}$$

$$\tau_{net} = I\alpha = I\frac{\Delta\omega}{\Delta t} = \frac{\Delta L}{\Delta t}$$

**Law of conservation of momentum:**

Momentum of any closed, isolated system does not change.

# Equilibrium

**Two requirements for the object's equilibrium:**

- the linear momentum ***P*** of the center of mass is constant
- the angular momentum ***L*** about the center of mass, or about any other point, is constant

$$\boldsymbol{P} = constant \qquad \boldsymbol{L} = constant$$

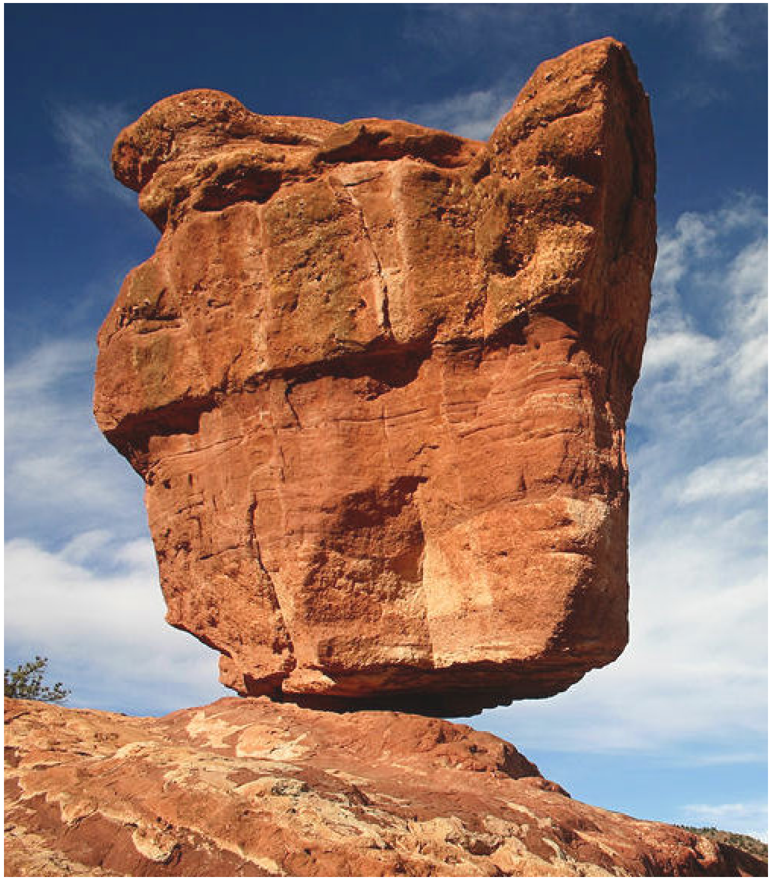
*Examples of objects in equilibrium:*

- a book resting on a table
- a hockey puck sliding with a constant velocity across a frictionless surface
- the rotating blades of a ceiling fan
- the wheel of a bicycle traveling along a straight path at constant speed

# Static equilibrium

**Static equilibrium**

- objects do not move in a given reference frame



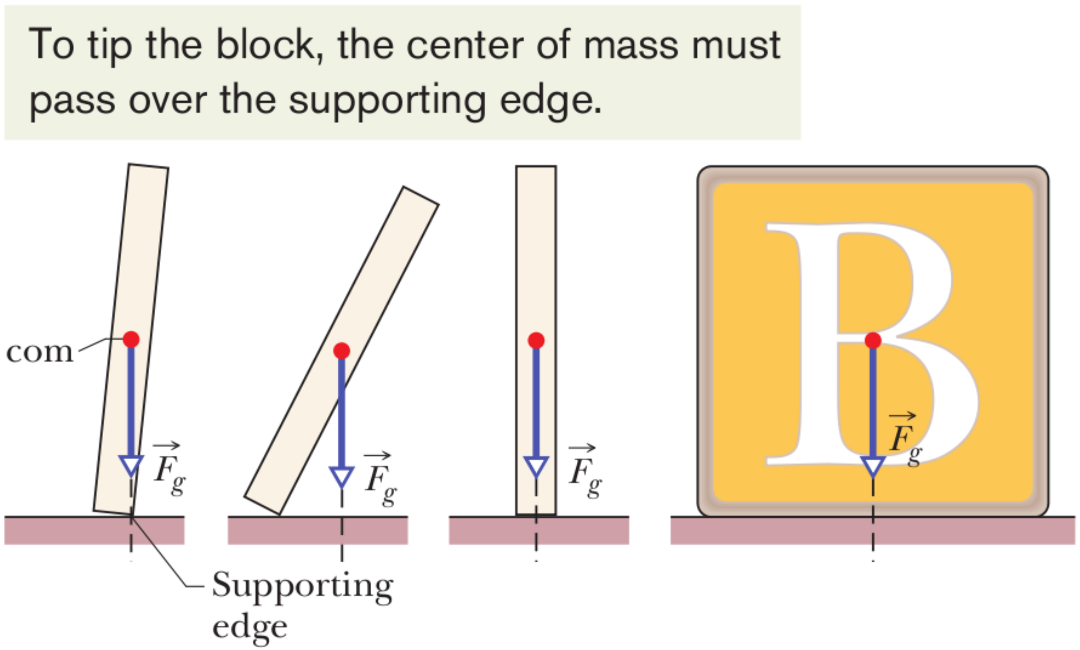
A balancing rock in static equilibrium

$$\boldsymbol{P} = 0 \qquad \boldsymbol{L} = 0$$

**An object displaced by a force from its equilibrium position**

- returns to the state of static equilibrium – **stable** static equilibrium
- moves to another state – **unstable** static equilibrium

# Static equilibrium




**Requirements of equilibrium:**

$$\boldsymbol{F}_{net} = \frac{\Delta \boldsymbol{P}}{\Delta t} \qquad \boldsymbol{F}_{net} = 0$$

$$\boldsymbol{\tau}_{net} = \frac{\Delta \boldsymbol{L}}{\Delta t} \qquad \boldsymbol{\tau}_{net} = 0$$

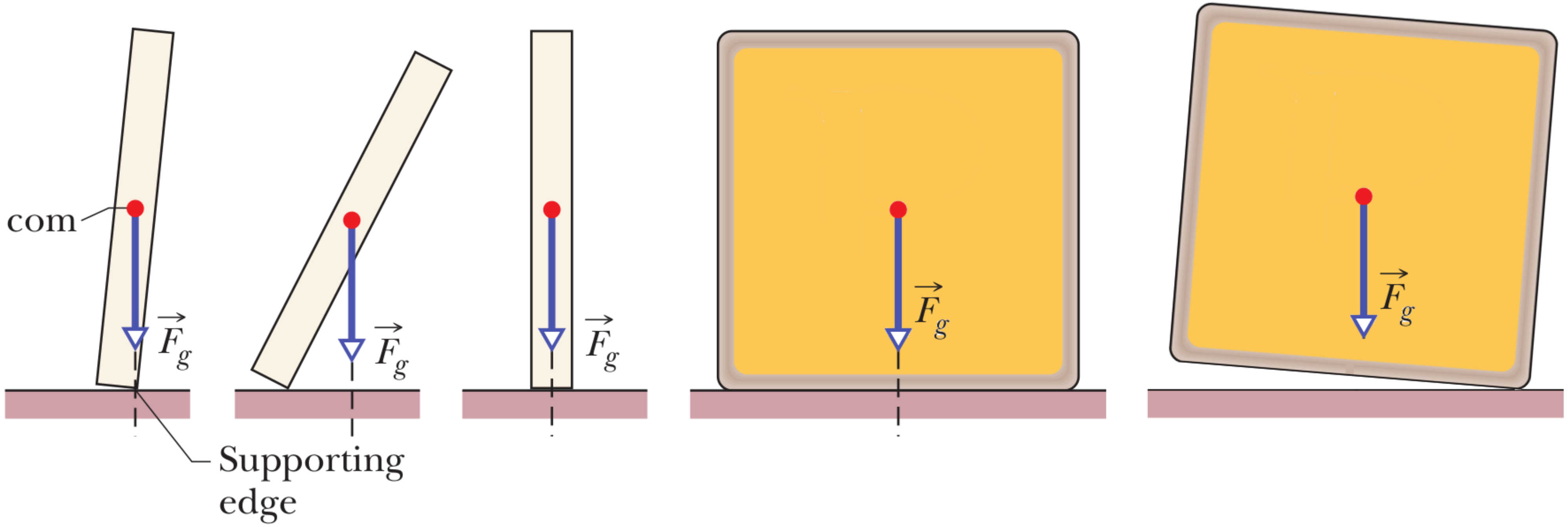
The vector sum of all the external forces that act on the body must be zero.

The vector sum of all external torques that act on the body, measured about *any* possible point, must also be zero.

**Forces in one *xy* plane:**

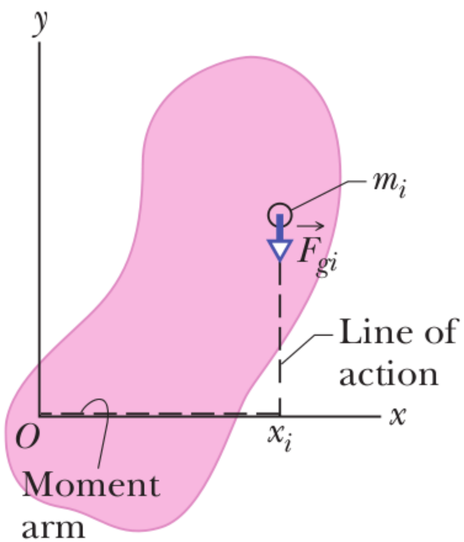
$$F_{net,x} = 0 \qquad F_{net,y} = 0 \qquad \tau_{net,z} = 0$$

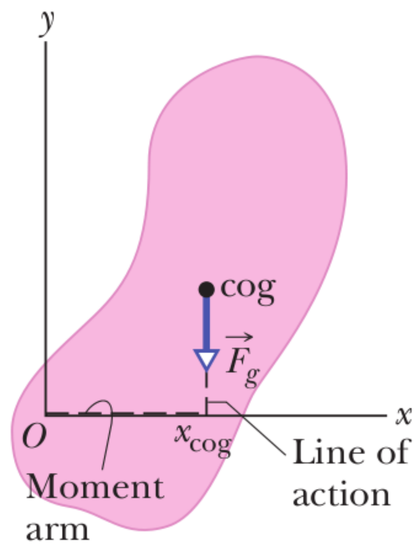
# Static equilibrium



# Center of gravity

The gravitational force $\boldsymbol{F_g}$ on a body effectively acts at a single point, called the **center of gravity (cog)** of the body.




**Homogeneous gravitational field:**

If $\boldsymbol{g}$ is the same for all elements of a body, then the body's **center of gravity** is coincident with the body's **center of mass.**

$$x_{cog} = x_{com}$$

$$\tau_i = x_i F_{gi} \qquad \tau = x_{cog} F_g$$

$$(m_1 g + m_2 g + \ldots + m_i g) x_{cog} = x_1 m_1 g + x_2 m_2 g + \ldots + x_i m_i g$$

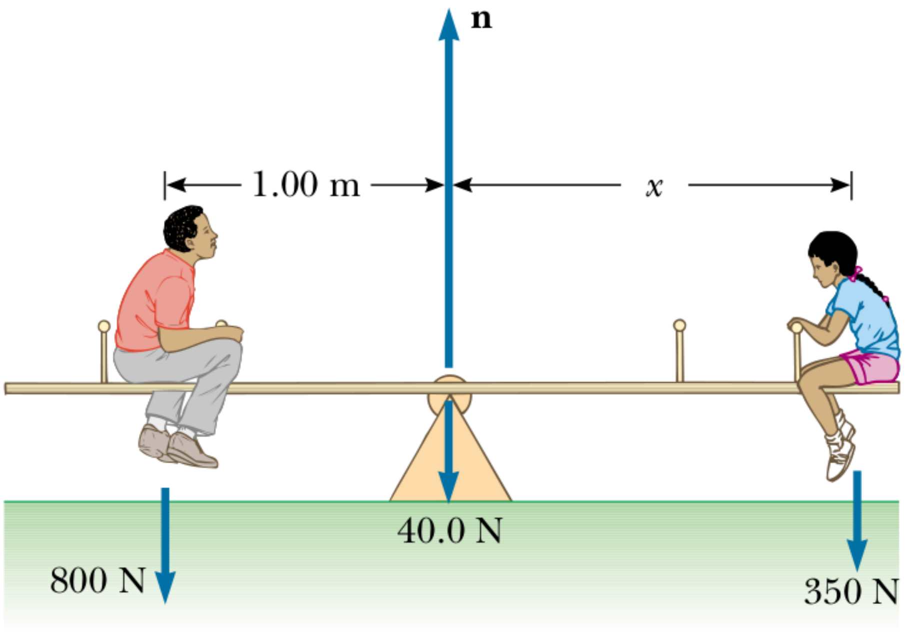
$$x_{cog} = \frac{x_1 m_1 + x_2 m_2 + \ldots + x_i m_i}{m_1 + m_2 + \ldots + m_i} = x_{com}$$

## Example of static equilibrium: The seesaw

A uniform 40.0 N board supports a father and daughter weighing 800 N and 350 N, respectively, as shown in the figure. If the support (called the fulcrum) is under the center of gravity of the board and if the father is 1.00 m from the center,

(a) determine the magnitude of the upward force n exerted on the board by the support.

(b) Determine where the child should sit to balance the system.



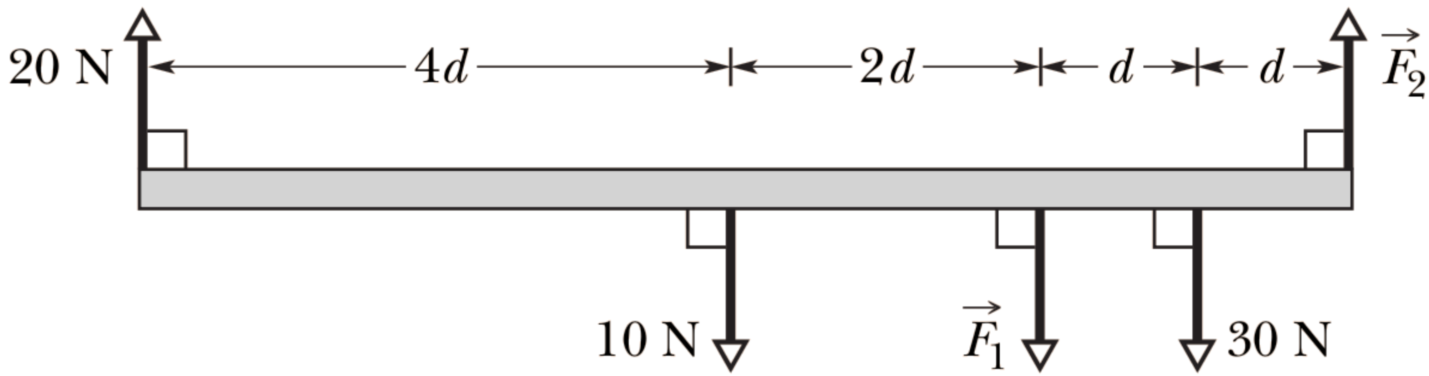
## Example of static equilibrium:

The figure gives an overhead view of a uniform rod in static equilibrium.

(a) Can you find the magnitudes of unknown forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ by balancing the forces?

(b) If you wish to find the magnitude of force $\boldsymbol{F_2}$ by using a balance of torques equation, where should you place a rotation axis to eliminate $\boldsymbol{F_1}$ from the equation?

(c) The magnitude of $\boldsymbol{F_2}$ turns out to be 65 N. What then is the magnitude of $\boldsymbol{F_1}$?

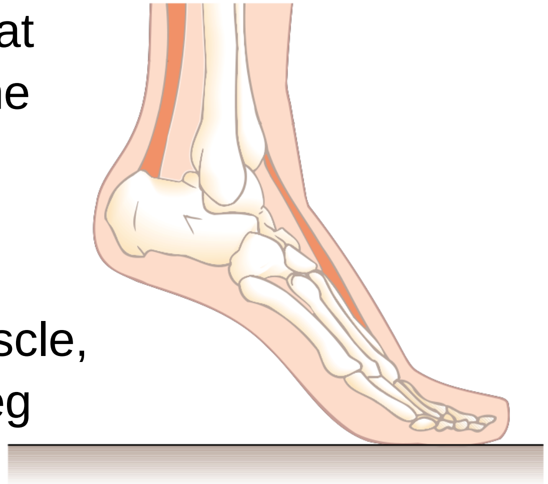


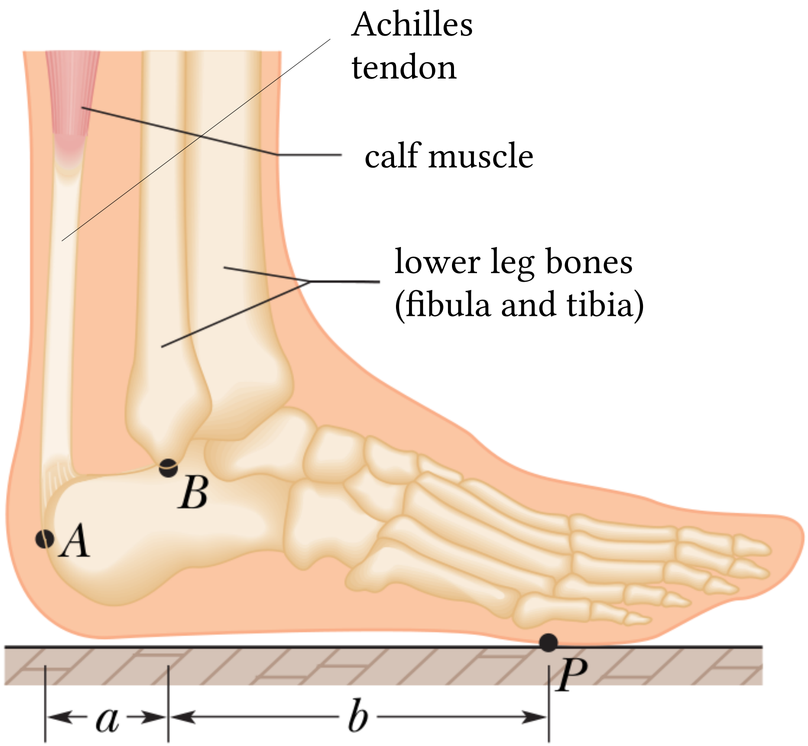
## Example of static equilibrium: Tiptoe standing

Figure shows the anatomical structures in the lower leg and foot that are involved in standing on tiptoe, with the heel raised slightly off the floor so that the foot effectively contacts the floor only at point *P*. Assume distance *a* = 5.0 cm, distance *b* = 15 cm, and the person's weight *W* = 900 N. Of the forces acting on the foot, what are the

(a) magnitude and direction of the force at point A from the calf muscle,

(b) magnitude and direction of the force at point B from the lower leg bones?

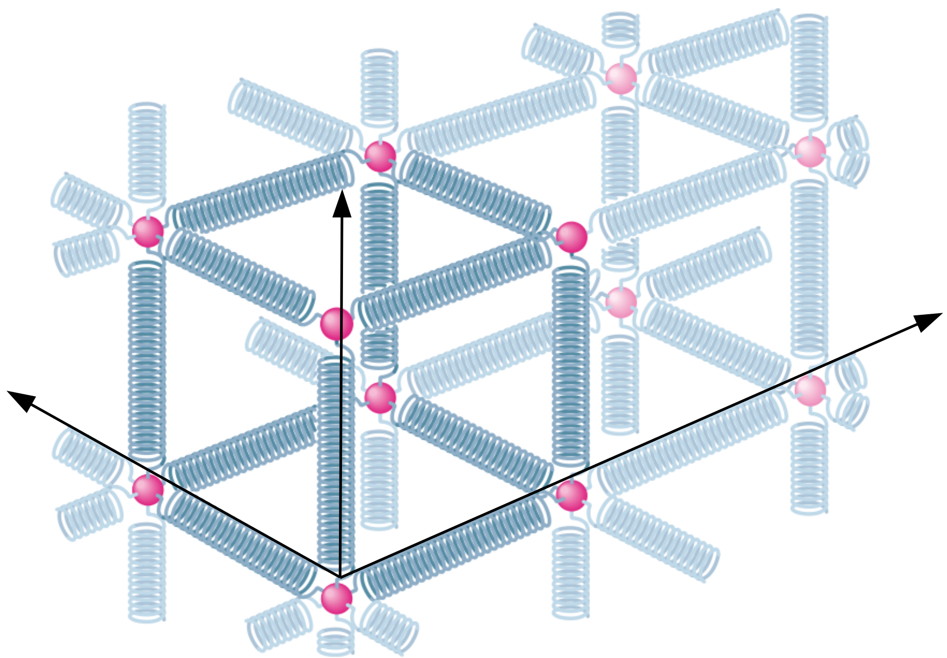




# Elasticity

**Solid matter**

- three-dimensional lattice – repetitive arrangement in which each atom is a well-defined equilibrium distance from its nearest neighbors
- atoms are held together by interatomic forces that are modeled as tiny springs
- rigid bodies are to some extent elastic (their dimensions can be slightly changed by pulling, pushing, twisting, or compressing them)



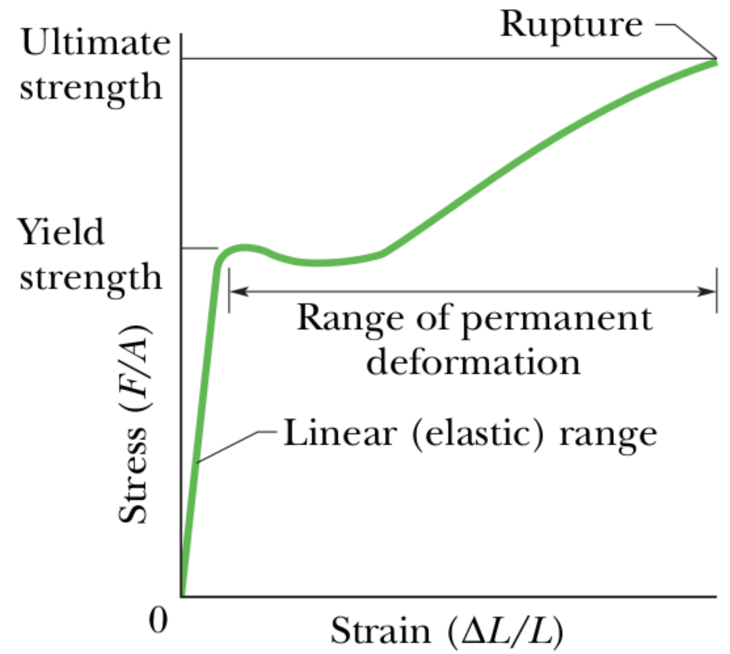
# Deformations of solids

## Stress

- quantity that is proportional to the force causing a deformation
- the external force acting on an object per unit cross-sectional area

## Strain

- measure of the degree of deformation




**Strain is proportional to stress**
(for sufficiently small stresses)

## Elastic modulus

- the constant of proportionality which depends on the material being deformed and on the nature of the deformation

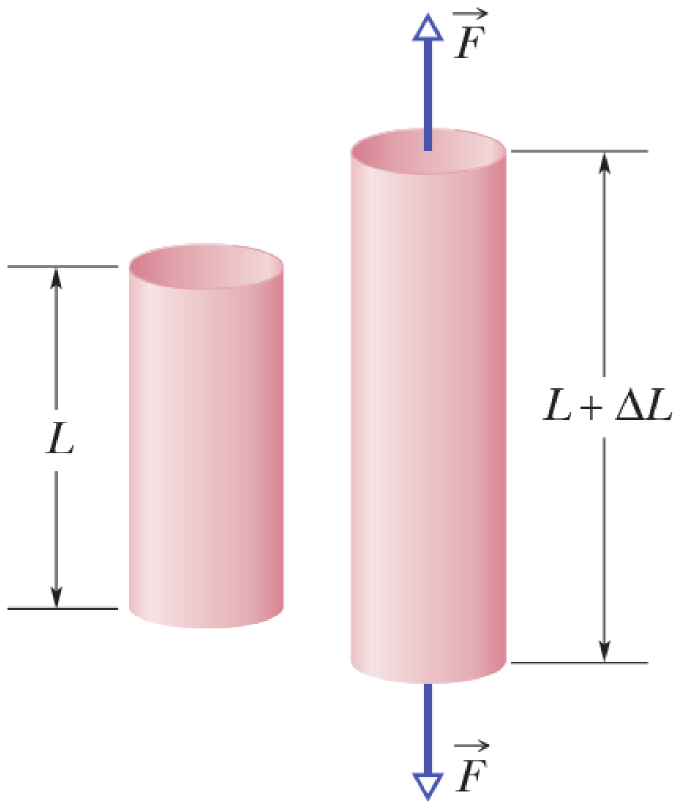
**stress = elastic modulus × strain**

**Three types of deformations:**

- elasticity in length
- elasticity of shape
- elasticity of volume

# Deformations of solids

## Tension and compression




σ  tensile stress
*E*  Young's modulus
ε  tensile strain (fractional change of length)

**Hooke's law:**

- linear dependence of a tensile stress on a relative elongation

$$\sigma = E\,\varepsilon$$

$$\sigma = \frac{F}{S}$$

$$\sigma = E\,\frac{\Delta L}{L}$$

**Force per unit area** = **pressure**

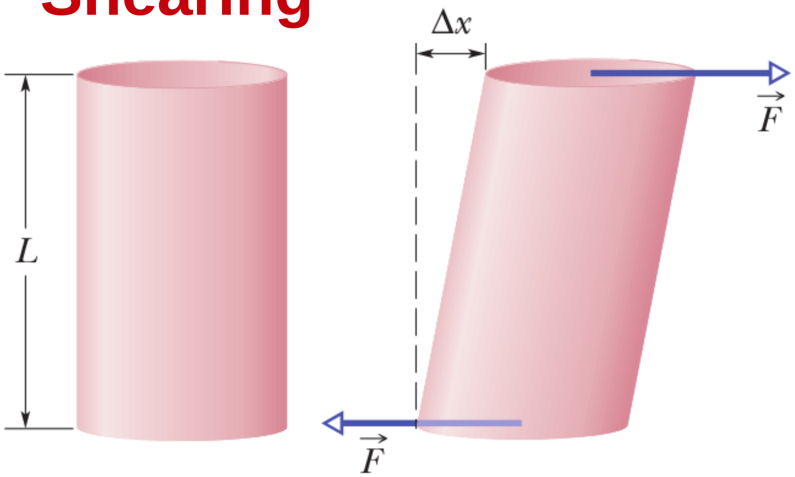
units: 1 pascal = 1 Pa = 1 $kg.m^{-1}.s^{-2}$

$\frac{\Delta L}{L}$ **relative elongation**

- dimensionless
- fractional (sometimes percentage) change in a length

# Deformations of solids

## Shearing

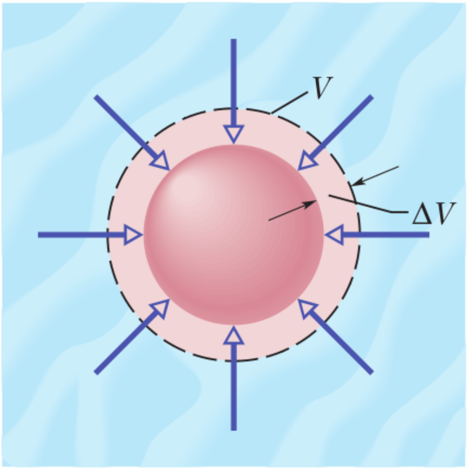



$$\sigma = G \frac{\Delta x}{L}$$

*σ* shear stress
*G* shear modulus

## Hydraulic stress

*p* pressure (hydraulic stress)
*B* bulk modulus




$$p = B \frac{\Delta V}{V}$$

The reciprocal of the bulk modulus is called the **compressibility**

Both solids and liquids have a bulk modulus.
No shear modulus and no Young's modulus are given for **liquids** because they do not sustain a shearing stress or a tensile stress (it flows instead).

**Example:**

What is the magnitude of the force acting on a steel guitar string of a length $L$ = 0.65 m and a cross-section area $S$ = 0.325 mm$^2$, if it elongates by 5 mm? Young's modulus of steel is 220 GPa.

# Newton's law of universal gravitation

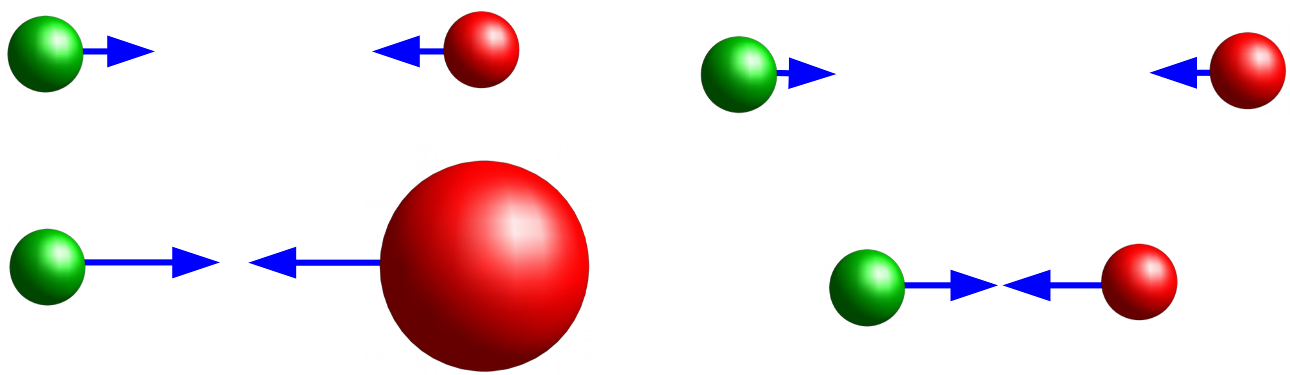
## Gravitational force

- the force of attraction between two objects which is proportional to the objects' masses
- this force acts between any two objects in the universe

## Law of universal gravitation

Objects attract other objects with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them.

$$F_g = G\frac{m_1 m_2}{r^2}$$



*G* is the **gravitational constant**: G = 6.67 × $10^{-11}$ N.m$^2$/kg$^2$

**Example:**

How would change the gravitational acceleration on the Earth's surface, if the Earth's radius is a half of its value and (a) the Earth's mass is the same, and (b) the Earth's density is the same?

# Gravitational potential energy

## Gravitational potential energy

- it characterizes a system of two particles
- we set $E_p = 0$ for $r = \infty$

$$E_p = -\frac{G m M}{r}$$

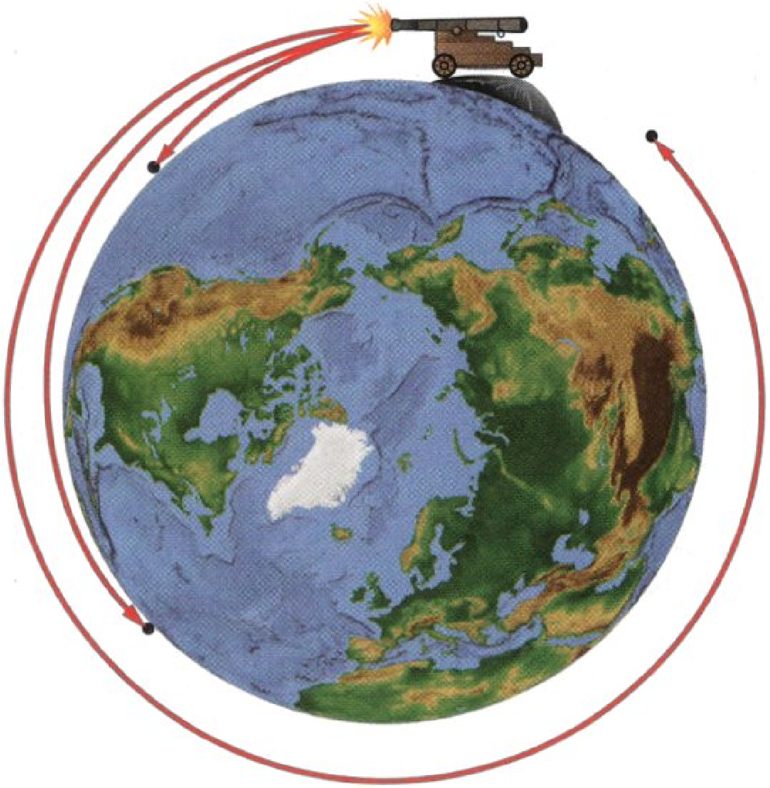
## Escape speed

- a minimal speed of the particle to escape Earth's gravitational field from its surface

$$E_k + E_p = \frac{1}{2} m v^2 - \frac{G m M}{r} = 0$$

$$v = \sqrt{\frac{2 G M}{r}}$$

# Orbits of planets and satellites



**Newton's thought experiment:**

- a cannon on a high mountain, firing a cannonball horizontally with a given horizontal speed
- in case of very high horizontal speed the cannonball would travel all the way around Earth
- to ignore air resistance, the distance from the surface must be more than 150 km

**Satellite in an orbit:** $F_{centripetal} = F_{gravitational}$

Satellite's speed:

$$\frac{m v^2}{r} = \frac{G M m}{r^2}$$

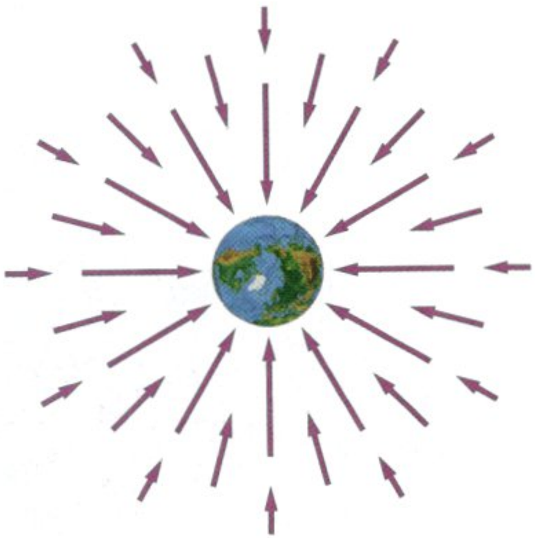
$$v = \sqrt{\frac{G M}{r}}$$

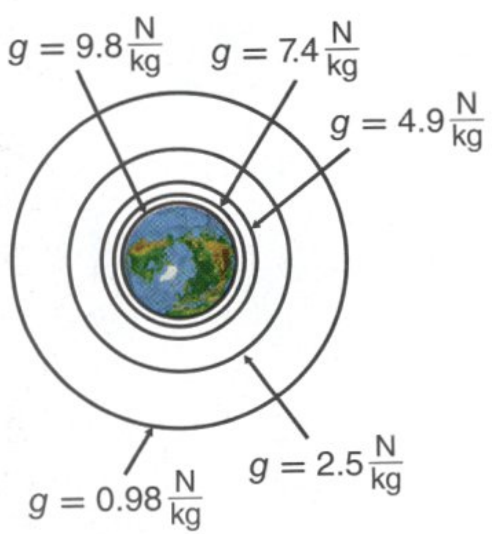
Satellite's orbital period:

$$\frac{m 4 \pi^2 r}{T^2} = \frac{G M m}{r^2}$$

$$T = 2\pi \sqrt{\frac{r^3}{G M}}$$

# The gravitational field






Gravitational acceleration:

$$F = \frac{GMm}{r^2} = m a_g \qquad a_g = \frac{GM}{r^2}$$

On Earth's surface $a_g = g$, $r = r_E$

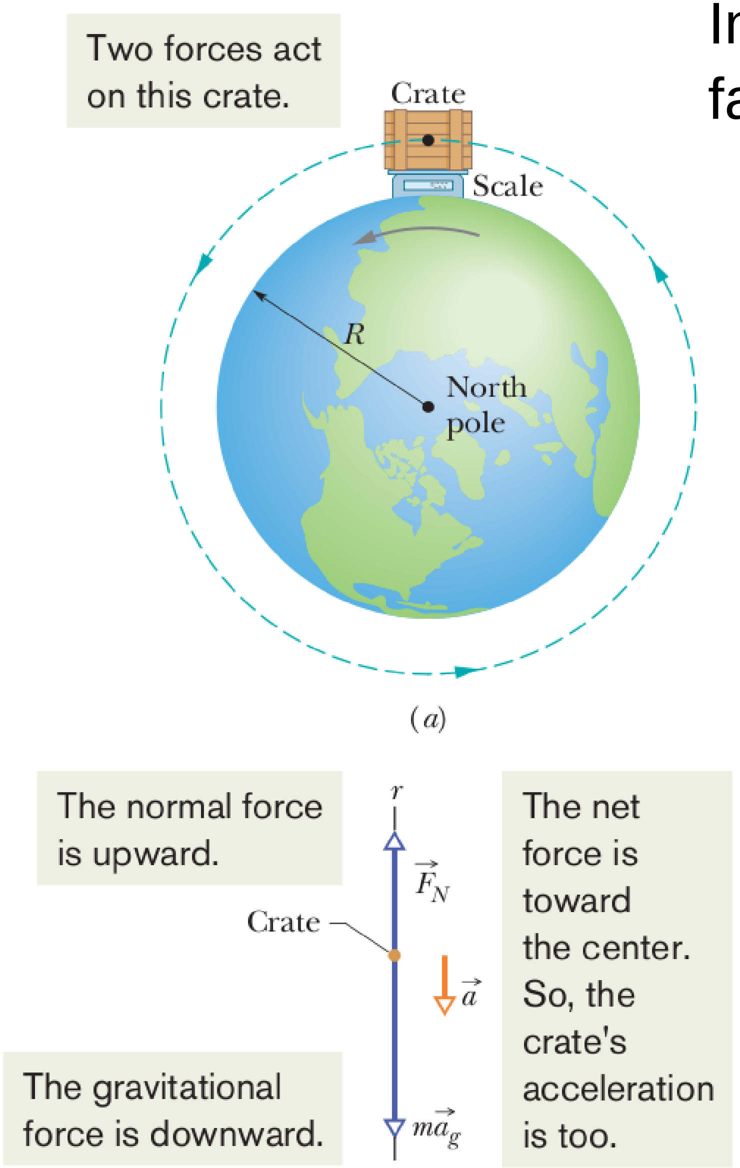
$$g = \frac{GM}{r_E^2} \qquad M = \frac{g r_E^2}{G}$$

$$\boxed{a_g = g\left(\frac{r_E}{r}\right)^2}$$

free-fall acceleration is distance-dependent

The strength of Earth's gravitational field varies **inversely** with the **square** of the **distance** from Earth's center.
Earth's gravitational field depends on **Earth's mass** but not on the mass of the object experiencing it.

# The gravitational field




Influence of the centripetal force to the free-fall acceleration:

$$F_N - m a_g = -m a_c$$

$$F_N - m a_g = -m \omega^2 R$$

$$m g = m a_g - m \omega^2 R$$

$$\boxed{g = a_g - \omega^2 R}$$

Difference in free-fall acceleration and gravitational acceleration is in centripetal acceleration (*g* is latitude-dependent).